An Image-Schematic Analysis of Hasse and Euler Diagrams

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Abstract

We extend Stapleton & al.'s theory of *observational advantage* for analysing the effectiveness of diagrams by taking into account the cognitive complexity of the act of observation. We do so by modelling the sensemaking of diagrams as conceptual blends of geometric configurations with image schemas. We analyse an example of reasoning with a Hasse and an Euler diagram, and we posit that, while their observational advantage is theoretically equivalent, the Hasse diagram requires a much more complex network of conceptual blends to model certain acts of observation. We believe our approach adds to the theoretical discussion on what factors influence the effectiveness of a diagram and provides new avenues for the exploration of how our embodied experiences contribute to the way we do diagrammatic reasoning.

This is an extended abstract of the paper published at the DIAGRAMS 2022 conference [1].

Keywords

conceptual blending, image schemas, sensemaking, diagram effectiveness, observational advantage

1. Introduction

To account for the effectiveness of a representation, Stapleton & al. put forward a formal theory of *observation* and *observational advantage* that distinguishes between the information that is observable in a given representation and the one that has to be inferred from it; and they formally prove the *observational advantage* of Euler diagrams over set-theoretic sentences when it comes to conveying information about claims concerning set equality and inclusion [2]. In order to achieve that, the authors resort to an abstract notation for Euler diagrams that is detached from the cognitive aspects of the act of observing and making sense of diagrams [3]. To account for such cognitive factors, we have proposed to model the act of making sense of a diagram as a network of conceptual blends of image schemas with the geometric configuration of the diagram [4, 5, 6, 7].

In this paper, we show that, while we can draw a Hasse diagram that has an equivalent *observational advantage* as a given Euler diagram (according to Stapleton & al.'s theory), such

The Seventh Image Schema Day (ISD7), September 2, 2023, Rhodes, Greece

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CEUR Workshop Proceedings (CEUR-WS.org)

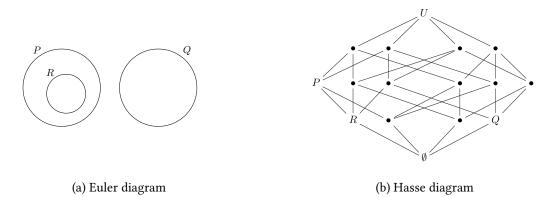


Figure 1: Euler and Hasse diagrams of equivalent observational advantage (according to Stapleton et al.'s theory) that are semantically equivalent to the set of set-theoretical sentences $S = \{P \cap Q = \emptyset, R \subseteq P\}$.

a diagram would require a more complex network of conceptual blends to model the way we make sense of it. Our hypothesis is that, among several different diagrams conveying the same information, the more cognitively effective ones would be those requiring simpler networks of conceptual blends.

Our work is based on various theories of cognitive science. First, the notion of sense-making refers to how agents actively create meaning by perceiving and acting within their environment [8, 9]. Image schemas are mental structures acquired through infancy, as humans interact with their environment, and reflect the basic structure of sensorimotor contingencies experienced repeatedly, such as CONTAINER, LINK, and PATH [10, 11]. Conceptual blending is a theory that posits that novel meaning emerges as we integrate existing concepts with each other [12]. Integrating all these theories, and applying them to the domain of diagrammatic reasoning, our proposal is the following: The geometry of a diagram is not meaningful on its own. We make sense of it, and reason with it, by integrating with it certain image schemas that are suitable to actively draw conclusions about its semantics [4, 5, 6].

2. Image Schemas In the Act of Observing Diagrams

We introduce an Euler and a Hasse diagram that have an equivalent *observational advantage* because any entailment about sets that can be *observed* in one diagram can also be *observed* in the other. However, we claim that the act of observing a particular set-theoretic claim is cognitively more complicated in the Hasse diagram. We will show this by describing the act of observation in these diagrams as integration networks that make explicit the conceptual blends [12] of the image schemas [13, 14] with the geometrical elements of the diagram, and we show that the integration network that corresponds to the Hasse diagram is more complex than the one corresponding to the Euler diagram.

2.1. Working Example

Take, for example, the set of set-theoretic sentences $\mathcal{S} = \{P \cap Q = \emptyset, R \subseteq P\}$ over a set of labels $\mathcal{L} = \{P, Q, R\}$ (two additional symbols, \emptyset and U, are also part of the syntax, to denote the empty set and the universal set, respectively). An observationally complete Euler diagram that is semantically equivalent to S is shown in Fig. 1(a). All set-theoretic sentences that are entailed by \mathcal{S} can be observed from this Euler diagram. We can also draw a semantically equivalent Hasse diagram for S, such as the one shown in Fig. 1(b). This Hasse diagram represents the lattice of all regions of the Euler diagram, generated as the lattice of sets closed under finite union and intersections, such that $A \lor B = A \cup B$ and $A \land B = A \cap B$. Formally, all labels in \mathscr{L} are attached to some of the lattice elements (i.e., there exists a labelling function $\lambda : \mathcal{L} \to \mathfrak{L}$, where \mathfrak{L} denotes this lattice of regions), the maximum is labelled with the additional symbol U, and the minimum is labelled with the additional symbol Ø. In general, given an Euler diagram whose curves are labelled with labels \mathscr{L} , the corresponding Hasse diagram will represent a lattice with 2^n elements, where $0 \le n \le 2^{|\mathscr{L}|}$. As with the Euler diagram of in Fig. 1(a), all set-theoretic sentences that are entailed by S can be observed from the Hasse diagram of Fig. 1(b). In what follows, we will describe these observations using integration networks of image schemas with the geometry, and compare the complexity of the integration networks corresponding to the two diagrams.

2.2. The Act of Observation In Hasse Diagrams

To observe if a certain set-theoretic claim $S \subseteq T$ or S = T holds in a given Hasse diagram (where *S* and *T* are labels or complex set-theoretic expressions formed using the operators \cap , \cup , \setminus , and \neg), we must first identify the nodes of the Hasse diagram representing set-expressions *S* and *T*, and then check if there is an upward path between these nodes (for set inclusion) or if they are the same (for set equality). The existence of an upward path can be immediately ruled out if the nodes representing *S* and *T* are distinct nodes at the same level of the Hasse diagram. Let us denote this identification task with a function **node** that assigns to each set-theoretic expression *S* over a set of labels \mathcal{L} a node **node**(*S*) in the Hasse diagram:

- if $S \in \mathcal{L}$, then **node**(*S*) = $\lambda(S)$, the node labeled with *S*
- if $S = S_1 \cup S_2$, then
 - if there is a downward path from $node(S_1)$ to $node(S_2)$, then $node(S) = node(S_1)$
 - if there is a upward path from $node(S_1)$ to $node(S_2)$, then $node(S) = node(S_2)$
 - if there is neither an upward nor a downward path between $\mathbf{node}(S_1)$ and $\mathbf{node}(S_2)$, then $\mathbf{node}(S)$ is the lowest of all those nodes that are on a meeting point between an upward path from $\mathbf{node}(S_1)$ to $\mathbf{node}(U)$, and a upward path from $\mathbf{node}(S_2)$ to $\mathbf{node}(U)$
- if $S = S_1 \cap S_2$, then
 - if there is a downward path from $node(S_1)$ to $node(S_2)$, then $node(S) = node(S_2)$
 - if there is a upward path from $node(S_1)$ to $node(S_2)$, then $node(S) = node(S_1)$

- if there is neither an upward nor a downward path between **node**(S_1) and **node**(S_2), then **node**(S) is the highest of all those nodes that are on a meeting point between a downward path from **node**(S_1) to **node**(\emptyset), and a downward path from **node**(S_2) to **node**(\emptyset)
- if $S = S_1 \setminus S_2$, then
 - if there is a downward path from $node(S_1)$ to $node(S_2)$, then
 - * if $node(S_2) = node(\emptyset)$, then $node(S) = node(S_1)$
 - * if $\mathbf{node}(S_2) \neq \mathbf{node}(\emptyset)$, then $\mathbf{node}(S)$ is the highest among all those nodes (excluding $\mathbf{node}(S_1)$) that are on all downward paths from $\mathbf{node}(S_1)$ to $\mathbf{node}(\emptyset)$ that do not go through $\mathbf{node}(S_2)$
 - if there is a upward path from $node(S_1)$ to $node(S_2)$, then $node(S) = node(\emptyset)$;
 - if there is neither an upward nor a downward path between $\mathbf{node}(S_1)$ and $\mathbf{node}(S_2)$, then
 - * if $\mathbf{node}(S_1 \cap S_2) \neq \mathbf{node}(\emptyset)$, then $\mathbf{node}(S)$ is the highest among all those nodes (excluding $\mathbf{node}(S_1)$) that are on all downward paths from $\mathbf{node}(S_1)$ to $\mathbf{node}(\emptyset)$ that do not go through $\mathbf{node}(S_1 \cap S_2)$
 - * if $\mathbf{node}(S_1 \cap S_2) = \mathbf{node}(\emptyset)$, then $\mathbf{node}(S) = \mathbf{node}(S_1)$
- if $S = \overline{S_1}$, then
 - if $\mathbf{node}(S_1) = \mathbf{node}(\emptyset)$, then $\mathbf{node}(S) = \mathbf{node}(U)$,
 - if $\mathbf{node}(S_1) \neq \mathbf{node}(\emptyset)$, then $\mathbf{node}(S)$ is the highest among all those nodes (excluding $\mathbf{node}(U)$) that are on all downward paths from $\mathbf{node}(U)$ to $\mathbf{node}(\emptyset)$ that do not go through $\mathbf{node}(S_1)$

In the above description of the way we observe set-theoretic claims in a given Hasse diagram, we can identify several image schemas such as LINK, PATH, VERTICALITY, and SCALE, which hint at how we make sense of the diagram in an embodied way. We thus describe the cognitive process of observation as constructing a network of blends involving some instances of the aforementioned image schemas, and parts of the geometric configuration of the Hasse diagram.

Concretely, to observe, for instance, whether $Q \subseteq P \setminus R$, we need to check if we can reach a target location **node**($P \setminus R$) starting from a source location **node**(Q) by traversing a path of contiguous node locations going upwards. Since Q is already denoted in the diagram, there is no need to locate it by way of our enactive cognition. We would, however, need to identify the target location **node**($P \setminus R$) in the Hasse diagram. To do so, we would need to check first if we can reach **node**(R) on a downward path from **node**(P), blending the base of the VERTICALITY schema to the lowest node, i.e., **node**(\emptyset), and a LINK schema and a PATH schema on the edge from **node**(P) to **node**(R). Since this is possible, we next need to find all downward paths from **node**(P) to **node**(\emptyset) that do not go through **node**(R). This blends a VERTICALITY schema, two LINK schemas and a PATH schema on the Hasse diagram, in order to traverse the two steps on the path from **node**(P) to **node**(\emptyset) via the node location that is not labelled with R. The highest location on our path down (excluding **node**(P)) is the node we were looking for. Subsequently, we return to our original question, whether $Q \subseteq P \setminus R$. Now, we have to check whether there is

an upward path from $\mathbf{node}(Q)$ to the node we have identified as $\mathbf{node}(P \setminus R)$. Here, the SCALE schema comes into play. The way this particular Hasse diagram is drawn, a user can easily put in correspondence the base of the VERTICALITY schema with the geometrically lowest shape of the Hasse diagram, i.e., the node representing \emptyset , and one level of a SCALE to each group of points that are on the same horizontal plane. This way, the user can observe that $\mathbf{node}(Q)$ and the node we identified as $\mathbf{node}(P \setminus R)$ are on the same level. Our embodied experience with paths, scales and the vertical dimension equips us with the knowledge that if two objects are on the same level of a vertical scale, it is impossible to traverse an upward path from one towards the other. Thus, it is immediately clear to us that there is no upward path from $\mathbf{node}(Q)$ to $\mathbf{node}(P \setminus R)$ and therefore $Q \subseteq P \setminus R$ does not hold.

2.3. The Act of Observation In Euler Diagrams

To observe if a certain set-theoretic claim $S \subseteq T$ or S = T holds in a given Euler diagram, as the one in Fig. 1(a), we must first identify the regions of the Euler diagram representing set-expressions *S* and *T*, and then check if the first region is inside the second (for set inclusion), or if they are the same region (for set identity). Let us denote this identification task with a function **region** that assigns to each set-theoretic expression *S* over a set of labels \mathcal{L} a region region(*S*) in the Euler diagram:

- if $S \in \mathcal{L}$, then **region**(*S*) is the region inside the closed curve labeled with *S*
- if S = S₁ ∪ S₂, then region(S) is the region made up of the combination of the insides of region(S₁) and region(S₂)
- if $S = S_1 \cap S_2$, then **region**(S) is the region that is both inside **region**(S_1) and inside **region**(S_2)
- if $S = S_1 \setminus S_2$, then **region**(S) is the part of **region**(S₁) outside of **region**(S₂)
- if $S = \overline{S_1}$, then **region**(S) is the region outside **region**(S_1)

In the above description of the way we observe set-theoretic claims in a given Euler diagram, we can identify several times the image schema CONTAINER underlying the manner we make sense of the diagram in an embodied way. We model this cognitive process as a network of conceptual blends involving some instances of the CONTAINER schema and parts of the geometric configuration of the Euler diagram.

For instance, to observe $Q \subseteq P \setminus R$, we need to check if **region**(*Q*) is contained in **region**($P \setminus R$). This points to two instances of the CONTAINER schema blended upon the geometric configuration of the Euler diagram, capturing our sense-making of the inside, boundary, and outside of **region**($P \setminus R$), and of **region**(*Q*), together with the containment relationship between the two CONTAINER schemas. Concretely, the integration network involved is as follows: first, to identify $P \setminus R$, we put in correspondence the boundary of one CONTAINER schema with the curves labelled P and R, the inside with the area between curves P and R, and the outside with the area outside curve P and the area inside curve R. With this blend, we model the way we observe **region**($P \setminus R$) in the diagram as a container. Subsequently, to check if $Q \subseteq P \setminus R$, we construct another blend between a second CONTAINER schema and the same geometrical configuration. This time the boundary, inside and outside of the CONTAINER will correspond to the curve labelled Q, its interior, and its exterior. Checking whether $Q \subseteq P \setminus R$ amounts to

observing that the boundary of the CONTAINER schema we put in correspondence with the former is located on the outside of the CONTAINER schema we put in correspondence with the latter. This observation again comes from our experience with containers, leading to the realisation that if Q is on the outside of $P \setminus R$ then it cannot be on its inside, and thus $Q \subseteq P \setminus R$ does not hold.

Comparing the complexity of the integration networks required to model the observations of $Q \subseteq P \setminus R$ from the Hasse and Euler diagrams, we can note that the integration network for the Euler diagram contains fewer different image schemas, fewer instances of image schemas, the diagram geometry itself contains much fewer elements, and the correspondences are also fewer. Concerning the blended space, blending the boundaries of CONTAINER schemas with the closed curves in a diagram imbues the latter with a sense of enclosure and separation. This sense emerges in the conceptual blends, where geometrical and image-schematic elements are integrated with each other, into elements that are simultaneously geometric and image-schematic. As we have seen, what constitutes the interior, boundary and exterior of a configuration of closed curves representing a set-theoretic expression, such as $P \setminus R$, arises in the way a CONTAINER schema is blended with the said configuration; not from the geometry itself.

3. Discussion

According to our framework, the effectiveness of Euler diagrams for representing set inclusion and disjointness (demonstrated in behavioral experiments [15, 16]) can be explained as follows: The geometry of an Euler diagram can be put in correspondence with instances of the CONTAINER schema. Through the process of constructing these correspondences, and thus integration networks, facts like $R \cap Q = \emptyset$ in Fig. 1(a) become immediately apparent. This integration network models how a user cognitively structures set *P* as a container, surrounding and enveloping curve *R*, thus keeping it from getting into contact with set *Q*—in agreement with [17].

In contrast, when reasoning with a Hasse diagram, we think about paths, links, vertical orientation, and levels of scales. Some indication that image schemas are implicitly used to cognitively structure diagrams is provided by the informal language researchers use when describing how Hasse diagrams should be used for reasoning [18, 19, 20, 17, 21]. Additional support comes from behavioural experiments showing that being upright, as opposed to slanted, explicitly showing levels (i.e., having points placed on horizontal parallels), and having non-crossed lines makes Hasse diagrams faster to interpret [22, 23]. These findings are consistent with our claims that observation in Hasse diagrams can be modelled as blends of VERTICALITY, SCALE, LINK and PATH.

An additional contribution of our work is defining in more detail what Stapleton et al. call 'meaning-carrying relationships' [2]. The definition of observation that Stapleton et al. use includes this term, forcing them to address concrete geometric and cognitive properties of the diagram; a meaning-carrying relationship is defined as a visuospatial relationship between syntactic elements of a visual representation, that expresses a certain meaning. One of our contributions here is that what counts as a meaning-carrying relationship can be explained in

terms of blends with image schemas.

4. Conclusions

We have explored the notion of *observational advantage* of Stapleton et al. [2] in a more cognitively-inspired way. In most approaches to diagrammatic reasoning, the specific meaning-carrying relations involved are taken as a given, and treated abstractly. In contrast, we believe our framework explores how they can emerge through the interplay of image schemas —which crystallize our early embodied experiences— with the diagram geometry. Our model simply accounts for the differences in the image schemas at play, keeping all else equal. We do not model all processes and factors that could affect the cognitive cost, e.g., the user's experience with the diagrammatic formalism, domain knowledge and cognitive strategies.

Acknowledgments

The present research was supported by CORPORIS (PID2019-109677RB-I00) funded by Spain's *Agencia Estatal de Investigación*; by DIAGRAFIS (202050E243) funded by CSIC; by WENET (H2020, 823783) funded by the European Commission; and by the *Ajuts per a grups de recerca de Catalunya* (2021 SGR 0075) funded by *AGAUR*, *Generalitat de Catalunya*.

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