Multifactor Model of the Digital Cryptocurrency Market as a Computational Core of the Information System

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Abstract
A multifactor model for the process of trading operations with Digital Cryptocurrencies (DCC) is introduced. This model is intended to be integrated into the computational core of an intelligent information system for investors in the DCC market. The model demonstrates that the controllability of the process of buying and selling a set of DCCs can be depicted using a game-theoretical approach. Adopting this approach will enable investors to craft strategies that bolster currency stability in the DCC market. What sets the introduced model apart is its ability to accurately depict the buying and selling processes in the multi-currency DCC market. This insight has facilitated the use of a constructive methodology to deduce the buy-sell strategies of market participants by solving bilinear multistep quality games with multiple terminal surfaces. The results from a computational experiment are also presented. In this experiment, various parameter relations describing the DCC trading operations were analyzed. These findings offer all stakeholders, especially the market participants of DCCs, tools to ensure stability not only in the traditional currency market but also the DCC market.

Keywords
Digital cryptocurrency, game model, buy-sell, multistep game, strategy.

1. Introduction

Digital Cryptocurrencies (hereinafter referred to as DCCs) are swiftly gaining traction globally. As the interest in DCCs surges, funds are increasingly being directed into the DCC market, which remains unregulated in most countries. Additionally, in many nations, central banks have yet to exert any influence over these activities. To retain their ability to regulate the financial system, address economic crises, manage inflation rates, and influence the prices of goods and services, central banks have started exploring various means of adapting to this evolving landscape.

One such approach has been the issuance of their DCCs. As per estimates from [1], currently, nearly 200 million people globally own DCCs. Predictions from [1] and [2] suggest this number will only increase. Many analysts anticipate that, in the coming years, the number of DCC owners will approach 300 million.

A vast body of scientific literature is dedicated to the study of CCE markets and the analysis and assessment of the efficiency of investments in DCC. However, the challenges of managing the process of buying and selling DCCs in an ambiguous setting—especially when leveraging both game theory and fuzzy
mathematics—have not been sufficiently addressed. As will be demonstrated subsequently, utilizing game-theoretic modeling in the management of DCC purchase and sale procedures (while considering uncertainty, conflict, and the resultant economic risk for investors) provides a means to evaluate the reliability of potential transactions in the DCC market. This, in turn, can help mitigate economic risks for DCC investors. This backdrop has underscored the relevance and interest in the subject of this study.

2. Literature Review and Analysis

With the swiftly growing interest of investors in the DCC market, researchers from a diverse range of scientific fields, from mathematics to applied psychology, have started to closely examine the phenomenon of DCC [3–4].

Successful economic development [5–6] necessitates a stable currency. This became particularly evident when the leaders of the global economy, especially the G7 member states, initiated the digital transformation of their economies. To ensure the stability of national currencies, the world’s premier institutions have devised various models for their preservation. Notably, several of these models incorporate new factors stemming from the shift to DCC in the age of pervasive informatization, in addition to traditional ones.

The review of scientific publications indicates that the attractiveness of investments in DCC remains underexplored. While a majority of studies center on DCC volatility [7–11] and forecasting of DCC rates, their authors often view DCC as an alternative to traditional currency. Yet, many researchers also highlight that DCC represents a considerably risky investment [1, 22–26].

It should be noted that the greatest value is represented by models that allow to application of direct methods of maintaining currency stability. Such models are presented in [12–14]. And it is such a model that is proposed in our work.

From a review of recent publications, it’s evident that there currently isn’t a comprehensive methodological framework capable of offering a thorough description of DCC markets. More critically, such a framework is needed to provide a clear forecast of the market’s prospects.

All of the above determined the relevance and objectives of our research.

3. The Purpose of the Work and the Objectives of the Study

Development of a multifactor model of trading operations with DCCs based on the use of the apparatus of bilinear multistep quality games with multiple terminal surfaces.

4. Methods and Models

Multifactor model of trading operations with digital cryptocurrencies.

We assume that there are two players involved in trading operations on the DCC market. Players participate in the process of buying and selling several DCCs, such as ADA, BTC, DOT, EOS, ETC, ETH, LINK, LTC, XRP, and others [1–23].

The difference between the considered multifactor model of the DCC market and those previously considered in [15] is that players conduct their market transactions with multiple DCCs instead of with pairs of DCCs.

Let us describe the procedure of buying and selling DCCs, which is the basis for the creation of the multifactor model of DCCs.

So, Player I, having set DCC1 equal to \( x(0) = (x_1(0), \ldots, x_K(0)) \), buys (or sells) set DCC2 equal to \( y(0) = (y_1(0), \ldots, y_M(0)) \), from Player II. Player II, having DCC2, sells (or buys) DCC1. Before the beginning of the trading session, the spot rates of DCC1 against DCC2 and vice versa are set. Let’s \( \Psi \) denote the matrix of spot rates of the currencies of the second group to the currencies of the first group. An element \( \psi_{ij} \) of the matrix \( \Psi \) means the spot rate of the currency of \( j \) is the second group against \( i \) is the currency of the first group \( (i = 1, \ldots, K; j = 1, \ldots, M) \). Through \( \Phi \) denote the matrix of spot rates of currencies of the first group to currencies of the second group. An element \( \phi_{ij} \) of the matrix \( \Phi \) means the spot rate of \( i \) is currency of the first group against \( j \) is currency of the second group \( (i = 1, \ldots, M; j = 1, \ldots, K) \).

At the moment of \( t = 0 \) (start of trading) Player I have a set \( x(0) \) (DCC1) to buy a set of
DCC2. Player II has \( y(0) \) a set of DCC2 to buy DCC1.

Here is a description of the model of trade operations with the selected DCCs. At the moment \( t = 0 \), Players I and II replenish their sets of DCCs \( x(0) \) (DCC1) and \( y(0) \) (DCC2) and have the following volumes of DCCs \( A \cdot x(0) \) and \( B \cdot y(0) \). Here, respectively, \( A \) and \( B \) are the transformation matrices of the DCC1 and DCC2 sets (analogous to the growth rates in the univariate case), respectively. \( \Lambda \) is an order \( K \) matrix with positive elements that are greater than (or equal to) the elements (elements) of the unit order \( K \) matrix \( E \); \( B \) is an order \( M \) matrix with positive elements that are greater than (or equal to) the elements (elements) of the unit order \( M \) matrix \( E \).

Then players allocate, respectively, \( U(0) \cdot A \cdot x(0) \) DCC1 and \( V(0) \cdot B \cdot y(0) \) DCC2 to purchase DCC2 and DCC1. Here \( U(0) \cdot A \cdot x(0) \) — a vector, the components of which characterize the expenditures of this particular DCC of Player II in the process of purchasing DCC of Player II; \( U(0) \) a diagonal matrix of order \( K \), consisting of the elements 
\[
 u_i(0) : u_i(0) \geq 0, \sum_{i=1}^{K} u_i(0) = 1. \]
Besides \( V(0) \cdot B \cdot y(0) \) is a vector, the components of which characterize the expenditures of this particular DCC of Player II in the process of purchasing DCC of Player I; \( V(0) \) a diagonal matrix of order \( M \), consisting of the elements
\[
 v_j(0) : v_j(0) \geq 0, \sum_{j=1}^{M} v_j(0) = 1. \]

It is assumed that at the moment of the trading session the players know the matrices \( G_{\text{pok}}, D_{\text{prod}}, D_{\text{prod}}^* \), where:

\( G_{\text{pok}} \) is the matrix of purchases of each DCC of the second group of DCCs of the first group. The purchase matrix \( G_{\text{pok}} \) consists of the elements \( g_{ij}^{\text{pok}} \). This matrix of dimension \( K \times M \).

\( D_{\text{prod}} \) is the matrix of sales of each DCC of the second group to the DCCs of the first group. The sales matrix consists of the elements \( q_{ij}^{\text{prod}} \). This matrix of dimension \( M \times K \).

\( D_{\text{prod}}^* \) is a matrix with elements \( 1/q_{ij}^{\text{prod}} \). This matrix is also of dimension \( M \times K \).

Then, the volumes of the sets of DCC1 and DCC2 of Players I and II at the moment \( t = 1 \) will be \( x(1) \) and \( y(1) \) respectively, where \( x(1) \) and \( y(1) \) are determined from the relations:

\[
x(1) = A \cdot x(0) + [-E + \Gamma \cdot \Lambda \cdot D_{\text{prod}}^*] \cdot U(0) \cdot A \cdot x(0) + [\Theta \cdot \Psi - \Omega \cdot G_{\text{pok}}] \cdot V(0) \cdot B \cdot y(0) \tag{1}
\]

\[
y(1) = B \cdot y(0) + [-E + \Gamma^* \cdot \Phi \cdot R \cdot G_{\text{pok}}] \cdot V(0) \cdot B \cdot y(0) + [L \cdot \Phi - \Omega^* \cdot D_{\text{prod}}^*] \cdot U(0) \cdot A \cdot x(0) \tag{2}
\]

Let us explain relations (1) and (2) step by step.

In the beginning, we will describe the actions of Player I, then of Player II.

**Step 1.** Player I increased the volume of the set of his DCCs1 from \( x(0) \) to \( A \cdot x(0) \).

**Step 2.** Player I allocated part of the set of his DCCs1 to buy a set of DCCs2 in the form of a set of currencies \( U(0) \cdot A \cdot x(0) \).

**Step 3.** Player I has determined (based on statistics of previous trading sessions) the structure (share) of his investments (expenses) of the sets of his DCC1 in each currency of the second group. This structure is given by diagonal elements \( \lambda_j \) (\( j = 1, \ldots, M \)):

\[
\lambda_j \geq 0, \sum_{j=1}^{M} \lambda_j = 1. \]

Let us denote by \( \Lambda \) is the matrix of diagonal elements \( \lambda_j \).

**Explanation 1.**

If there is a set of currencies \( U(0) \cdot A \cdot x(0) \) of Player I, then if we operate: \( D_{\text{prod}}^* \cdot U(0) \cdot A \cdot x(0) \), then we get a \( M \)-dimensional vector which "as if" means the volume of the set of currencies of Player II. However, this product allows us to define only one component of this \( M \)-dimensional vector, since the entire vector \( U(0) \cdot A \cdot x(0) \) will be spent to buy only this one component. There is no more of Player I’s currency available for the other components (other currency) of Player I. It has already been used up. Therefore, it is necessary to partition the set of currencies into \( M \) parts so that the entire range of Player II’s currencies can be purchased. This is done by introducing the set: \( \lambda_j \) (\( j = 1, \ldots, M \)):
\( \lambda_j \geq 0, \sum_{j=1}^{M} \lambda_j = 1 \). Note that the selection of these coefficients can be done in other ways, not necessarily as in Step 3.

**Step 4.** To determine each currency of Player II (each component) that Player I bought, it is enough to multiply the matrix \( D_{\text{prod}}^* \) by the vector \( \lambda_j \cdot U(0) \cdot A \cdot x(0) \), and then the \( j^{th} \) currency (component) of the second group will be determined. Thus, the product \( \Lambda \cdot D_{\text{prod}}^* \cdot U(0) \cdot A \cdot x(0) \) will determine the volume of Player I’s currency if the volume of Player I’s currency set is distributed using the coefficients \( \lambda_j \) (\( j = 1, \ldots, M \)).

**Step 5.** Conversion of the set of currencies of Player II into the set of currencies of Player I takes place. For this process a matrix \( \Psi \) is applied—the matrix of spot rates of the currencies of Player I to the currencies of Player II. This corresponds to the fact that we have the expression: \( \Psi \cdot \Lambda \cdot D_{\text{prod}}^* \cdot U(0) \cdot A \cdot x(0) \), which defines the volume of Player I’s currency set in the case when all of Player II’s currencies are converted into one particular currency (one component).

*Note.* The currency conversion situation is similar to the currency purchase situation (see Explanation, Step 3). This is taken into account in Step 6.

**Step 6.** In Step 6, the structure of the currencies of the first group is determined by specifying a diagonal matrix \( \Gamma \) of order \( K \), with diagonal elements \( \gamma_i \), which characterizes the volume of Player II’s \( i \) is currency from converting \( \gamma_i \) a fraction of the volume of the set of currencies \( \Psi \cdot \Lambda \cdot D_{\text{prod}}^* \cdot U(0) \cdot A \cdot x(0) \) of Player I. This matrix characterizes the structure of the exchange rate of the currency set of Player II’s currencies to the currency set of Player I:

\[
\Gamma \cdot \Psi \cdot \Lambda \cdot D_{\text{prod}}^* \cdot U(0) \cdot A \cdot x(0),
\]

**Step 7.** Player II, as well as Player I, allocates to the purchase of currencies of Player I the value of \( V(0) \cdot B \cdot y(0) \).

**Step 8.** In this step, we will convert the set of currencies \( \Psi \cdot V(0) \cdot B \cdot y(0) \).

Considering remark 1, to determine the structure of the set of currencies of Player I, we define a matrix (see Step 9).

**Step 9.** Let us denote by \( \Theta \) is a diagonal matrix of order \( K \) with elements \( \theta_i, \theta_i \geq 0, \sum_{i=1}^{K} \theta_i = 1 \). Then we obtain:

\[
\Theta \cdot \Psi \cdot V(0) \cdot B \cdot y(0),
\]

the volume of Player I’s currency set after Player II has allocated a set \( V(0) \cdot B \cdot y(0) \) to buy Player I’s currency set.

**Step 10.** Let’s describe the payment by Player I to purchase Player II’s set of currencies.

To do this, we will do a multiplication \( G_{\text{prod}} \) by the vector \( V(0) \cdot B \cdot y(0) \). We obtain:

\[
G_{\text{prod}} \cdot V(0) \cdot B \cdot y(0).
\]

And, taking into account “Explanation 1” in Step 3, we perform Step 11.

**Step 11.** Let’s denote by \( \Omega \) the diagonal matrix of order \( K \) with elements \( \omega_i, \omega_i \geq 0, \sum_{i=1}^{K} \omega_i = 1 \). We obtain: \( \Omega \cdot G_{\text{prod}} \cdot V(0) \cdot B \cdot y(0) \), “structured” (i.e., distributed over the components of Player I’s currency set) payment for the volume of Player II’s currency set.

Thus it is possible to determine the values of Player I’s currencies at the time \( t = 1 \):

\[
x(1) = A \cdot x(0) + \left[ -E + \Gamma \cdot \Psi \cdot \Lambda \cdot D_{\text{prod}}^* \right] \cdot U(0) \cdot A \cdot x(0) + \left[ \Theta \cdot \Psi \cdot \Omega \cdot G_{\text{prod}} \right] \cdot V(0) \cdot B \cdot y(0);
\]

Let’s explain the set of actions on the part of Player I.

**Step 12.** The second player has increased the volume of the DCC set \( y(0) \) to \( B \cdot y(0) \).

**Step 13.** Player II allocated a part of the volume of his set of DCC to buy some volume of Player I’s set of DCC in the form of a set of currencies \( V(0) \cdot B \cdot y(0) \).

**Step 14.** The second group has determined (based on the statistics of previous trading sessions) the structure (share) of its investments (expenditures) of its currency in each currency of the first group. This structure is given by the diagonal elements \( r_i \) (\( i = 1, \ldots, K \)):

\[
r_i \geq 0, \sum_{i=1}^{K} r_i = 1.
\]

These elements form a diagonal matrix \( R \).

*Explanation 2.* If there is a set of currencies \( V(0) \cdot B \cdot y(0) \) of Player I, then if we operate: \( G_{\text{prod}} \cdot V(0) \cdot B \cdot y(0) \), then we get a \( K \)-dimensional vector which “as if” means the volume of the set of currencies of
Player I. However, this product allows us to define only one component of this $K$-dimensional vector, since the entire vector $V(0) \cdot B \cdot y(0)$ will be spent to buy only this one component of Player I. There is no more second-group currency for the other components (other currency) of Player I. It has already been used up. Therefore, it is necessary to partition the set $K$ of currencies into parts so that the entire range of Group I currencies can be purchased. This is done by introducing the set: $r_i$ (i = 1, ..., $K$), $r_i \geq 0, \sum_{i=1}^{K} r_i = 1$. The selection of these ratios can be done in different ways (see "Explanation 1").

Step 15. To determine each currency of Player I (each component) it is enough to multiply the matrix $G_{pok}$ by a vector $r_i \cdot V(0) \cdot B \cdot y(0)$ and then the $i^{th}$ currency (component) of Player I will be determined. Thus, denoting by $R$ is a diagonal matrix of order, with diagonal elements $r_i$, we obtain that the product $R \cdot G_{pok} \cdot V(0) \cdot B \cdot y(0)$ will determine the volume of the set of currencies of Player I if the volume of the set of currencies of Player II was distributed using the coefficients $r_i$ (i = 1, ..., $K$).

Step 16. Step 16 converts the set of currencies of Player I to the set of currencies of Player II. This corresponds to the fact that we have the expression: $\Phi \cdot R \cdot G_{pok} \cdot V(0) \cdot B \cdot y(0)$.

Considering "Observation 1", we proceed to Step 17.

Step 17. In Step 17, the structure of Player II’s currency set is determined by specifying a diagonal matrix $\Gamma$ of order $M$, with diagonal elements $\gamma_i^i$, which characterizes the volume of the second group’s currency from the conversion $\gamma_i$ of a fraction of the volume of Player II’s $i$-currency set $\Phi \cdot R \cdot G_{pok} \cdot V(0) \cdot B \cdot y(0)$. This matrix characterizes the exchange rate structure of Player I’s set of currencies to Player I’s set of currencies: $\Gamma \cdot R \cdot G_{pok} \cdot V(0) \cdot B \cdot y(0)$.

Step 18. At this step, we will convert a set of currencies $\Phi \cdot U(0) \cdot A \cdot x(0)$.

Taking into account "Remark 2" to determine the structure of Player II’s currency set, we define a matrix $L$ (see Step 19).

Step 19. Let’s denote by $L$ is a diagonal matrix of order $M$ with elements $l_i \cdot l_i \geq 0, \sum_{i=1}^{M} l_i = 1$.

Then we obtain: $L \cdot \Phi \cdot U(0) \cdot A \cdot x(0)$.

Step 20. In Step 20, we will describe Player II’s payment for the purchase of Player I’s set of currencies.

To do this, we will do a multiplication $D_{prod}^*$ by the vector $U(0) \cdot A \cdot x(0)$. We get: $D_{prod}^* \cdot U(0) \cdot A \cdot x(0)$.

And, as before, let’s take into account Remark 3 (see below).

Remark 3.

The situation with the purchase, in this case, is similar to the situation of currency purchase by Player I (see "Explanation", Step 3).

Step 21. Let us denote by $\Omega^*$ is the diagonal matrix of order $M$ with elements $\omega_i^* \cdot \omega_i^* \geq 0, \sum_{i=1}^{M} \omega_i^* = 1$. We obtain:

$$\Omega^* \cdot D_{prod}^* \cdot U(0) \cdot A \cdot x(0)$$

This set denotes the structured payment by Player II of the set of currencies of Player I.

Thus it is possible to determine the values of the currencies of Player II at the time $t = 1$:

$$y(1) = B \cdot y(0) + [-E + \Gamma^* \cdot \Phi \cdot R \cdot G_{pok} \cdot V(0) \cdot B \cdot y(0) + [L \cdot \Phi - \Omega^* \cdot D_{prod}^* \cdot U(0) \cdot A \cdot x(0)]$$

Consequently, we have that the values of the players’ currency set at the moment $t = 1$ are written using relations (1) and (2).

The conditions for the end of the trading session at the moment $t = 1$ will be the fulfillment of conditions (3), (4), or (5):

The following options are possible at the moment $t = 1$:

$$(x(1), y(1)) \in S_0,$$  \hspace{1cm} (3)
$$(x(1), y(1)) \in F_0,$$  \hspace{1cm} (4)
$$(x(1), y(1)) \in D_0,$$  \hspace{1cm} (5)
$$(x(1), y(1)) \in H_0,$$  \hspace{1cm} (6)

where $S_0, F_0, D_0$ and $H_0$ as:

$$S_0 = \bigcup_{i=1}^{M} \{(x, y) : (x, y) \in R_{K+M}, x \geq 0, y_i < 0\},$$
$$F_0 = \bigcup_{i=1}^{K} \{(x, y) : (x, y) \in R_{K+M}, x_i < 0, y \geq 0\},$$
\[ D_0 = \bigcup_{i=1}^{M} \{(x, y) : (x, y) \in R^{K+M}, y_i < 0\} \\bigcup_{i=1}^{K} \{(x, y) : (x, y) \in R^{K+M}, x_i < 0\} \]

\[ H_0 = R^{K+M}_{+} \]

Case (3) is desirable for Player I, and case (4) is desirable for Player II. In case (5), the players also stop interacting since they cannot continue. In case (6), they continue the interaction for moments \( i > 1 \).

Due to symmetry, we restrict ourselves to considering the problem from the point of view of Player I [16]. The second problem is solved similarly.

The definition of the pure strategy and the preference set of Player I was given in [15–16]. Recall that the preference set of Player I is the set of such initial states of the players that have the property that if the game starts from them, Player I can choose his strategy \( U_s(\ldots, \ldots) \) to ensure the fulfillment of condition (3) at one of the time moments. At the same time, this strategy chosen by Player I contributes to preventing Player II from fulfilling conditions (4) and (5) at previous moments.

The set of such states will be called the set of preferences of Player I. Accordingly, the strategies \( U_s(\ldots, \ldots) \) of Player I possessing the above properties are his optimal strategies.

The solution to Problem 1 consists in finding the sets of “preferences” of Player I allies \( W_i \) and their optimal strategies. Similarly, the problem is posed from the point of view of Player II.

**Solution of Problem 1**

The solution to the problem depends on the ratio of parameters defining the procedure of confrontation between the allied player and Player II opponent.

All cases of the ratio of parameters defining the multifactor model of the DCC market will be presented in the form of two cases:

1. \[ L \cdot \Phi - \Omega^* \cdot D^*_{\text{prod}} \leq 0, \quad \llbracket -E + G^* \cdot \Phi \cdot R \cdot G_{\text{pot}} \rrbracket \leq 0. \]
2. \[ \Theta \cdot \Psi - \Omega \cdot G_{\text{pot}} \geq 0, \quad \llbracket -E + G^* \cdot \Phi \cdot R \cdot G_{\text{pot}} \rrbracket \leq 0. \]

Let us consider Case 1 and introduce the notations:

\[ H(l) = B, \quad F(l) = [-L \cdot \Phi + \Omega^* \cdot D^*_{\text{prod}}] \cdot A; \]

\[ H(k + 1) = H(k) \cdot B, \quad F(k + 1) = \llbracket F(k) \cdot [\Gamma \cdot \Psi \cdot \Lambda \cdot D^*_{\text{prod}}] \rrbracket \cdot A - \llbracket -H(k) \cdot [L \cdot \Phi - \Omega^* \cdot D^*_{\text{prod}}] \rrbracket \cdot A; \]

\( E \) is a singular matrix of order \( K; \ k = 1, \ldots \).

Let us denote by \( W^k_i \) is the set of such initial states of players, which have the property that if the game starts from them, then Player I can choose his strategy \( U_{s(\ldots, \ldots)} \) to ensure the fulfillment of condition (3) at the moment \( k, \ (k = 1, \ldots) \). At the same time, this strategy chosen by Player I contributes to preventing Player II from fulfilling conditions (4) and (5) at previous moments. Through \( (W_i^k) \), is denotes the set of such initial states of players, which have the property that if the game starts from them, then Player I can choose his strategy \( U_{s(\ldots, \ldots)} \) to ensure at the moment \( k = 1, \ldots \) the fulfillment of condition (3) on the \( i-th \) component of Player II. At the same time, this strategy chosen by Player I contributes to preventing Player II from fulfilling conditions (4) and (5) at previous moments.

It is not difficult to see that \( W_i = \bigcup_{j=1}^{M} (W_j^k) \), and

\[ W_i^k = \bigcup_{j=1}^{M} (W_j^k) \]

If Condition 1 is satisfied, the sets \( (W_i^k) \) are written as follows:

\[ (W_i^k) = \{(x(0), y(0)) : (x(0), y(0)) \in R^{K+M}, \llbracket H(k) \cdot y(0) \rrbracket \leq \llbracket F(k) \cdot x(0) \rrbracket \}. \]

At such ratios of parameters the optimal strategy of Player I

\[ U^*(\cdot) \cdot U_{\text{opt}}^*(x, y) = \begin{cases} E, & (x, y) \in W_i; \\ \text{not determined}, & (x, y) \not\in W_i; \end{cases} \]

Realization of the strategy of countereaction of Player II to Player I is such: \( V_{\text{op}}^*(x, y) = 0. \)

Note that if the conditions of Case 1 are met, the preference sets are finite in number, and for each component of Player I. Hence, we obtain that the procedure of constructing the preference sets of Player I is finite. Let us add that if the conditions of Case 1 are satisfied, the components of the vector of Player I are positive for any moment and any strategies of players.
Finding the preference sets and optimal strategies of allied Player I in Case 2 is done similarly. The construction of preference sets and optimal strategies of ally Player II is done similarly.

Note. Since the model uses diagonal matrices that specify the structures of the DCC vectors, it is possible to manipulate them by changing the diagonal elements to obtain the desired result.

5. Computational Experiment

The computational experiment concerning the problem of identifying the area of investor preference in DCC2 was conducted using the Matlab environment. The results are depicted in Fig. 1. On the abscissa, investments in the set of DCCs by Player I are represented. Conversely, on the ordinate axis, investments in the set of DCCs by Player II are shown. The red straight line (1) illustrates the balance beam.

The blue dashed line shows the trajectory of the player’s movement in the area of preference of Player II, which is above the equilibrium ray. At present, the results of theoretical deductions and experimental studies have formed the basis of the developed intelligent system for DCC trading.

![Figure 1: Results of a computational experiment. The trajectory of the players’ movement](image)

6. Discussion of the Results of the Computational Experiment

Fig. 1 depicts a scenario where Player II (the buyer of DCC1), leveraging the suboptimal behavior of the DCC1 buyer at the onset, manages to shift the system’s state to his terminal area. If the trajectory aligns with the equilibrium ray—which demarcates the boundary of Player I’s preference set—it signifies a situation where the DCC rate is at equilibrium. In this somewhat rare case, both players navigate along this ray, employing their optimal strategies, resulting in a state that simultaneously satisfies both parties. Conversely, if the trajectory falls below the equilibrium ray, it demonstrates a scenario where Player I (the buyer of DCC2) holds an advantageous position in the parameter ratio, meaning the situation is within Player I’s preference set. Under these circumstances, Player I, by applying his optimal strategy, will realize his objective—essentially steering the system’s state to his terminal zone.

7. Conclusions

A multifactor game model of the DCC market has been explored. The research demonstrates that the controllability of processes within a trading session can be articulated through a game approach. This approach is grounded in solving a system of discrete bilinear equations with multivariate variables. The model’s distinctiveness lies in its deviation from existing methodologies, specifically by addressing a bilinear multistep quality game with multiple terminal surfaces. For the first time, a solution to this new bilinear multistep quality game, which considers dependent motions, has been identified. We also showcase the outcomes of a computational experiment, wherein diverse parameter relationships that outline the DCC buying and selling process are considered.

The findings detailed in this paper can be instrumental in preventing instances of exchange rate instability in the DCC investment market, which are commonly observed in practice. Consequently, this model might also prove beneficial for predicting situations on trading floors that deal with DCCs. Additionally, the results offer insights into selecting control measures to sustain exchange rate stability in the DCC investment market, especially for major banking entities.
References


