

Spatio-temporal Feature Extraction of Magnetic Patterns for Indoor Positioning

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Abstract

Geomagnetic-based indoor positioning has been widely studied as an alternative to WiFi-based indoor positioning systems. However, the geomagnetic-based approach is challenging in practice due to its limited feature diversity and susceptibility to noise. Also, geomagnetic ground-truth locations are difficult to be obtained. This study aims to overcome the inherent limitation of magnetic data and discrepancies between testing conditions and real-world scenarios, where the ground-truths are unknown.

By employing the dynamic time warping with global invariances(DTW-GI), we figure out that the time-series magnetic sensor readings can become a representative feature set, which can be mapped as spatio-temporal features according to a moving trajectory regardless of the ground-truth information. Because we find out the magnetic data is spatial and temporal, a deep multimodal neural network with autoencoders is designed for positioning estimation model, and its objective function is made to map them on target trajectories.

From the landmark experiments in which only we know corner locations, the proposed positioning model is evaluated effectively to encode representations of the magnetic data and decode them to the trajectories between the landmarks with 96.90 percent accuracy.

Keywords

Geomagnetism, Smartphone sensors, Spatio-temporal features, Dynamic time warping with global invariances, Autoencoders

1. Introduction

Indoor positioning has received significant attention over the years due to the growth of the location-based services market. Recent technological advances and demand from various industries further accelerate research and development of indoor positioning solutions. Particularly, modern smartphones with various types of sensors provide accessibility to ubiquitous positioning systems[1, 2, 3].

While technologies such as Ultra-Wideband (UWB)[4], Bluetooth Low Energy (BLE)[5], and WiFi[6, 7] are the reliable and typical implementations for indoor positioning, their dependency on physical anchors limits usage where device installation is undesirable. To address the issue, geomagnetic-based approach has arisen as an alternative [8, 9]. With magnetic

Proceedings of the Work-in-Progress Papers at the 13th International Conference on Indoor Positioning and Indoor Navigation (IPIN-WiP 2023), September 25 - 28, 2023, Nuremberg, Germany

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 CEUR Workshop Proceedings (CEUR-WS.org)

patterns generated by ferromagnetic objects like columns, elevators, and walls within buildings, geomagnetic-based positioning distinguishes different indoor locations[10, 11].

The primary challenge encountered in geomagnetic-based indoor positioning is its inherent physical limitations. Unlike WiFi-based methods that leverage the number of access points as rich feature dimensions, geomagnetic-based approach relies solely on three-axis measurements[12]. This limited feature diversity challenges the accurate positioning process based on magnetic patterns[13]. Moreover, sensitivity of magnetometers necessitates adequate calibration. Magnetometers are prone to biases induced by common materials with soft- and hard-iron properties.[14] In other words, daily exposure to magnetic objects can distort the magnetic field of devices, resulting in sensor bias and adversely affecting the accuracy of the data acquisition.

Thus, calibration is conventionally necessary to mitigate biases and compensate for sensor distortions caused by external elements. However, calibrating for every usage in practical applications using smartphone sensors is not feasible, leading to discrepancies between real-world environments and controlled testing conditions, resulting in inaccurate indoor positioning[15].

This study aims to reduce the discrepancies generated from uncalibrated sensors and identify the representations of space-oriented features. Accordingly, we (1) extract spatio-temporal features from both magnetic patterns and trajectories with time-series similarity measures of latent space and (2) develop a position inference model that points to the target space with consideration of deep-embedded features.

2. Background

Because magnetic patterns can be captured by a movement along the space, different walking speed and pattern generates different pattern. Also, noise and fluctuation of sensors generate different anomalies in the same area. Therefore, researches on geomagnetic indoor positioning systems involve feature extraction and pattern recognition to identify robust representation.[16, 13, 17].

Qu Wang et al.[16] proposed an indoor positioning system that combines the detection of magnetic loop closure with the calibration of pedestrian dead reckoning (PDR) trajectories using formerly collected magnetic patterns stored in a database. By detecting magnetic loop closure, when a person passes through a physical location previously visited, the system can identify known locations and utilize them to calibrate the PDR trajectory. The magnetic patterns stored in the database are reference points for accurate positioning. AMID(Accurate Magnetic Indoor Localization Using Deep Learning) system[13] is one of the earliest studies of geomagnetic-based indoor positioning using a deep neural network by converting magnetic patterns into images, which are then classified to enable precise indoor positioning.

Both studies focus on magnetic patterns for indoor positioning. They acknowledge magnetic patterns as reliable indicators of specific locations within indoor environments. Additionally, both studies employ a sliding window technique to extract magnetic patterns instead of relying solely on the absolute intensity of a reference point. By identifying magnetic pattern changes by implementing a sliding window approach, these studies aim to improve the performance and reliability of indoor positioning systems in real-time applications. However, when considering the practical application of magnetic patterns for indoor positioning, several challenges exist:

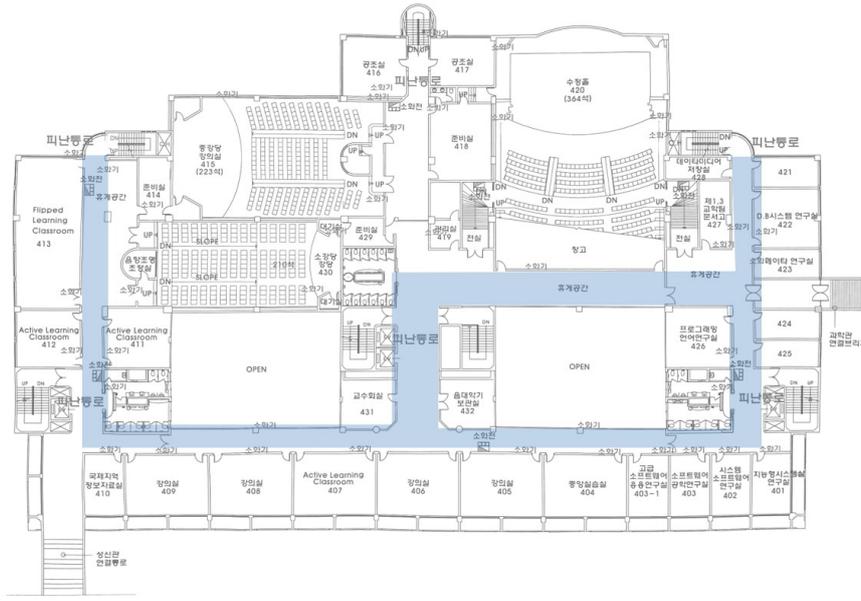


Figure 1: The floor plan of the target building with the accessible track highlighted in color.

- Indoor magnetic patterns change dynamically[18].
- Sensor calibration is frequently required to ensure accurate results[19].
- Data collection conditions are not constant, further complicating the positioning process.

As a result, it becomes essential to consider both temporal and spatial information, highlighting the need for more robust feature extraction methods. Asadi et al.[20] developed a model with deep-embedded learning using autoencoders to consider both spatial and temporal features of trajectory data. The model aims to minimize spatial, reconstruction, and clustering loss simultaneously, enabling the extraction of meaningful representations that capture the underlying patterns and relationships within the trajectory data. Vayer et al.[21] introduced DTW-GI (Dynamic Time Warping with Global Invariances) as an enhanced version of the traditional dynamic time warping technique[22]. DTW-GI incorporates additional considerations of axis discrepancies and rotation, making it suitable for comparing sequences that exhibit both temporal and spatial distortions. DTW-GI metrics, therefore, have found applications in trajectory analysis, human pose estimation, and other fields.

In this paper, we propose a DNN model of different parallel autoencoder structures with a fully-connected network combined. Autoencoders are designed to build latent feature representations from magnetic and trajectory data. This feature extracting part reduces discrepancies between devices resulting from lack of calibration by adapting DTW-GI metrics. Then, the rest part of the model infers the current location from the latent features. The DTW-GI metric is also adapted for window size determination of input data features before the model development steps.

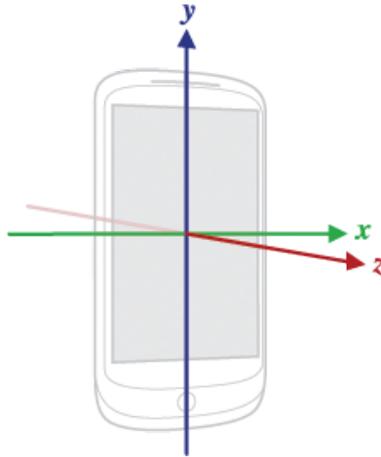


Figure 2: The relative coordinate system of an Android smartphone device, via Android Developers. (<https://developer.android.com/guide/>)

3. Smartphone sensor data analysis

Experimental data was collected using magnetometers and inertial measurement units(IMU) on two different human-held smartphones, for two hours along the same tracks. Figure 1 shows the accessible corridors within the target building. The data acquisition process was carried out assuming typical daily indoor positioning conditions without sensor calibration. The magnetometer sample rate was set at 400Hz, while the IMU had a sample rate of 100Hz which is the default sampling rate of Android sensors. A lower sampling rate of 4 Hz was chosen to synchronize and integrate the magnetometer and IMU data. Data preprocessing step includes converting from the local reference frame to the global reference frame using game rotation vectors provided by the Android sensor system.

3.1. DTW-GI analysis on magnetic sequence data

A magnetometer measures the 3-axis magnetic intensities at a specific location. However, point-referenced magnetic intensities are insufficient for discerning distinct coordinates accurately. Sliding window technique is popularly used for enhancing time series data features. Sliding windows identify space-oriented indoor magnetic patterns making more feature dimensions. For instance, existing ferromagnetic objects within a particular spatial unit, such as the characteristic pattern associated with the presence of an elevator, can be treated as indicators or landmarks of specific locations. By converting sequential magnetic data into time series data representative of the target space, spatial features can be extracted from the magnetic field and effectively augment temporal information for indoor positioning purposes.

Magnetic sequence data presents identical patterns at the same position, but uncalibrated sensors lead to discrepancies between the axis by different devices, as shown in Figure 3. Also, the fluctuations or variations in magnetic patterns are so minor and subtle, making magnetic-based indoor positioning challenging.

Therefore, we apply the DTW-GI (Dynamic Time Warping with Global Invariances) technique

to measure temporal similarities or differences between sequential data. Unlike conventional DTW, DTW-GI incorporates the consideration of both axis discrepancies and rotation, making it suitable for comparing sequences with temporal and spatial distortions, disregarding variations due to different devices, speeds, and noise levels. Furthermore, by warping the time axis, DTW-GI enables optimal alignment between two sequences, allowing for non-linear and variable-speed alignments, thereby enhancing the accuracy and precision of similarity measurements, particularly for space and time-dependent sequences.

The key concept behind Dynamic Time Warping (DTW) is to find the best alignment by minimizing the cumulative distance between corresponding points in the sequences. It dynamically constructs a warping path connecting corresponding points in the sequences, allowing for local deformations in the time axis. The path is determined by considering the distances between points and neighbors to minimize the overall alignment cost. Given two time series $\mathbf{x} \in \mathbb{R}^{T \times p}$ and $\mathbf{y} \in \mathbb{R}^{T' \times p}$ with the feature dimension p , the vanilla DTW is calculated as follows:

$$\text{DTW}(\mathbf{x}, \mathbf{y}) = \min_{\pi \in \mathcal{A}(\mathbf{x}, \mathbf{y})} \sum_{(i,j) \in \pi} d(\mathbf{x}_i, \mathbf{y}_j) \quad (1)$$

where $\mathcal{A}(\mathbf{x}, \mathbf{y})$ is the set of all alignments, d is a ground metric (mostly Euclidean distance), and π is pairs of time series [22].

DTW_γ extends the vanilla DTW by a soft-min operator to let DTW differentiable calculation. In case of $\gamma = 0$, DTW_γ is equivalent to the DTW calculation [23].

$$\text{DTW}_\gamma(\mathbf{x}, \mathbf{y}) = \min_{\pi \in \mathcal{A}(\mathbf{x}, \mathbf{y})} \gamma \sum_{(i,j) \in \pi} d(\mathbf{x}_i, \mathbf{y}_j) = -\gamma \log \left(\sum_{\pi \in \mathcal{A}(\mathbf{x}, \mathbf{y})} e^{-\sum_{(i,j) \in \pi} d(\mathbf{x}_i, \mathbf{y}_j) / \gamma} \right) \quad (2)$$

By considering axial rotation through manifold alignment, the DTW-GI algorithm becomes a powerful tool for capturing and analyzing spatio-temporal patterns, providing more accurate and robust results for various applications, including indoor positioning and trajectory mapping. The DTW-GI calculation is expressed as the cross-similarity matrix C between samples from \mathbf{x} and $f(\mathbf{y})$, where \mathcal{F} stands for a family of functions, f for a short-cut optimization function, and W_π denotes the Frobenius inner product [21]:

$$\begin{aligned} \text{DTW}_\gamma\text{-GI}(\mathbf{x}, \mathbf{y}) &= \min_{f \in \mathcal{F}} \min_{\pi \in \mathcal{A}(\mathbf{x}, \mathbf{y})} \gamma \langle W_\pi, C(\mathbf{x}, f(\mathbf{y})) \rangle \\ &= \min_{f \in \mathcal{F}} -\gamma \log \sum_{\pi \in \mathcal{A}(\mathbf{x}, \mathbf{y})} e^{\langle W_\pi, C(\mathbf{x}, f(\mathbf{y})) \rangle / \gamma} \end{aligned} \quad (3)$$

To specify a trajectory within the global context, similarity scores are calculated for each of the three axes, measuring the extent of resemblance between corresponding locations in the sequences. Once the DTW-GI scores for each axis are obtained, their summation yields a comprehensive measure of total sequential similarity. This summation consolidates information from all three axes, providing a holistic assessment of the representative patterns in the data. Scatter plots facilitate the visualization of temporal correspondence, as depicted in Figure 3. Lighter points in the scatter plots indicate a higher likelihood of representing the same trajectory. The DTW-GI-based analysis revealed that a 10-second window was sufficient for identifying spatial representatives from the magnetic data.

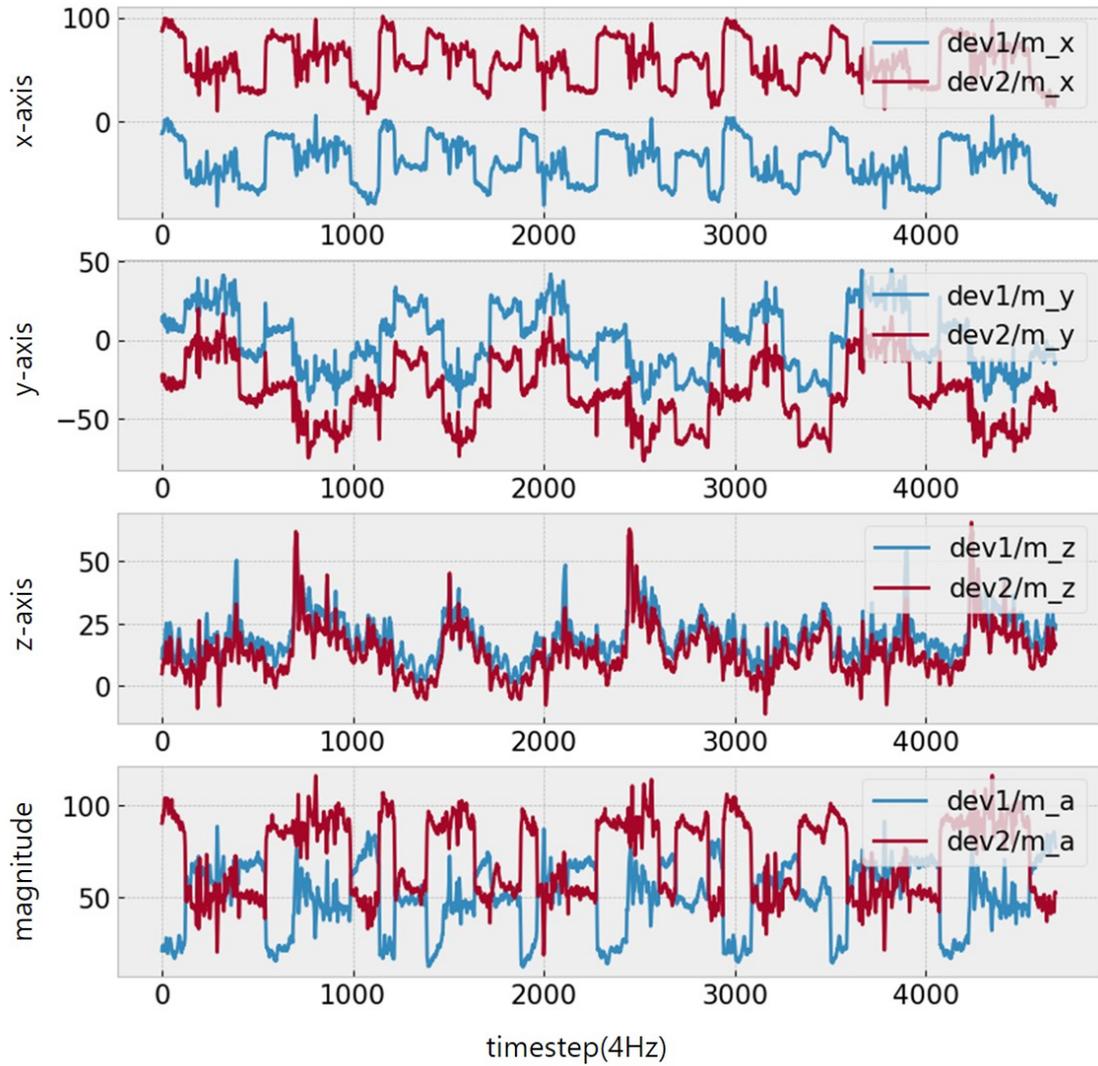


Figure 3: Magnetic field variations in each axis captured by smartphone sensors and magnitude.

3.2. AI-IMU for trajectory calculation

The Inertial Measurement Units (IMU) consist of accelerometers and gyroscopes, informing the relative displacement and motion. By integrating data from these sensors, IMU can track changes in position, velocity, and orientation, thereby facilitating the mapping of complete trajectory or path within the indoor environment. The AI-IMU model[2] was utilized to calculate a moving trajectory in this paper.

As mentioned, the sliding window technique has been applied to enhance the magnetic pattern within the target space. Consequently, the resulting features inherently pertain to the trajectory rather than being point-referenced. Thus, aligning them along the corresponding

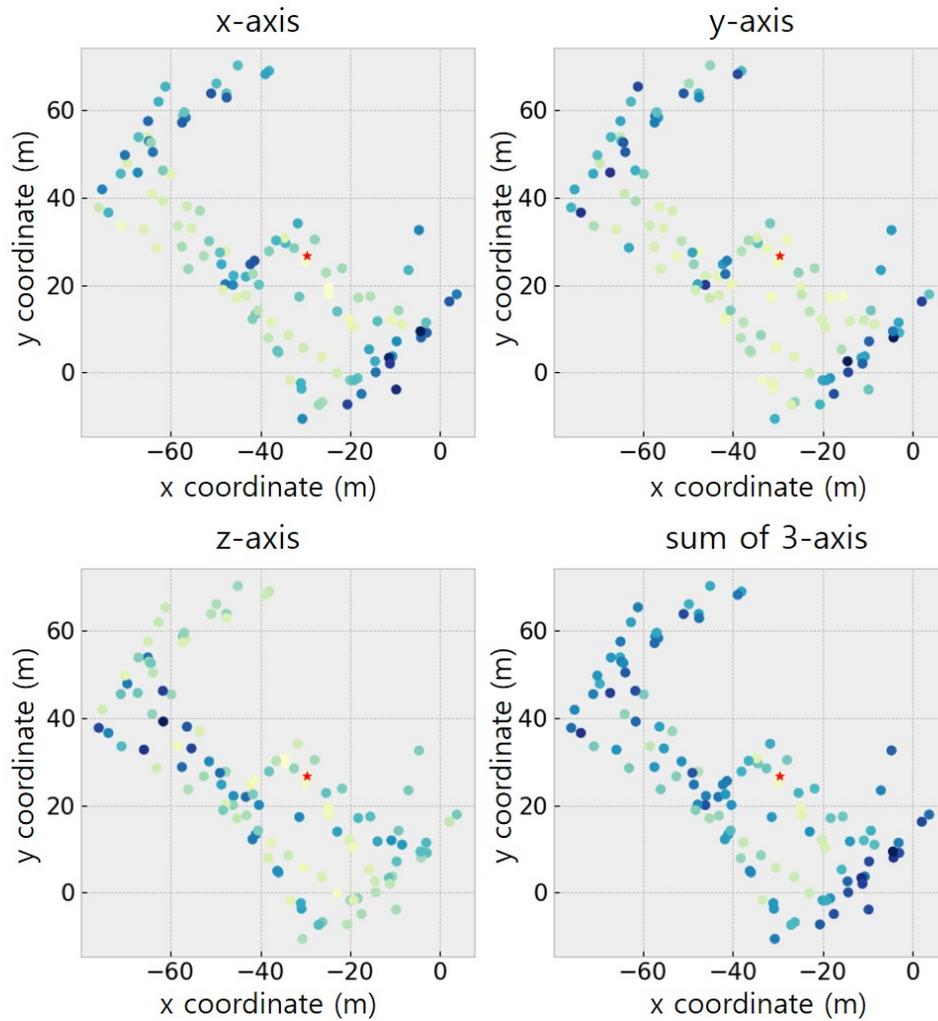


Figure 4: Scatter plot of DTW-GI analysis results (red representing the reference points for similarity calculation).

trajectory is essential to represent and analyze the space-oriented magnetic features accurately. Furthermore, this alignment facilitates a comprehensive connection of the spatial feature of the magnetic patterns. By integrating data from accelerometers and gyroscopes, the IMU captures changes in position and computes the relative trajectory.

However, it is essential to acknowledge that the presence of integral error in the gyroscope readings can introduce drift and inaccuracies during the trajectory mapping process. Despite

the identical nature of the tracks in real-world scenarios, the cumulative effects of integral error in the gyroscope can lead to deviations from the ground-truth paths in the estimated trajectories. Various techniques, such as pattern matching, feature extraction, or similarity measures, can achieve this track identification and alignment. In the next chapter, these spatio-temporal features are combined to develop a position inference model, and the trajectories with features will be aligned with the deep learning technique.

4. Spatio-temporal features extraction with autoencoders

Autoencoders are unsupervised learning models that reconstruct input data by compressing it into a lower-dimensional representation, known as the latent space or bottleneck. The autoencoder architecture consists of an encoder, which maps the input data to the latent space, and a decoder, which reconstructs the data from the latent representation. By training the autoencoders to minimize the reconstruction error, it learns to extract the most representative features from the input data.

In this research, autoencoders were built to capture the spatio-temporal similarities present in both the magnetic sequences and trajectories. Consequently, two separate autoencoders were constructed to extract features from the magnetic patterns and trajectories, respectively, as shown in figure 5. For both magnetic sequence autoencoder and trajectory autoencoder, each model reconstructs its respective data with a fixed window size of 10 seconds and a 0.25-second interval. Each autoencoder in the proposed method consists of five layers, with the third layer as the bottleneck layer.

The sequential data undergo normalization within the sliding window in the trajectory encoder before feeding an autoencoder as input data. This approach focuses solely on the local trajectory and ensures blindness to the global context. Normalizing the trajectory into sliding windows narrows the analysis and feature extraction process to the local segment within each window. This localized approach allows for a detailed examination of the temporal dynamics within the time horizon, enabling accurate analysis and capturing of relevant spatio-temporal patterns. Furthermore, by being blind to the global context, the trajectory encoder can extract features and characteristics specific to the local trajectory by understanding the data in the context of the given time window.

Two autoencoders generate deep embeddings at the bottleneck layer. These deep embeddings, which represent the encoded features, are subsequently combined. The resulting concatenated features are then mapped onto the trajectory using a bias term to facilitate the inference of the current indoor position. This multimodal structure, therefore, consists of two autoencoders and one deep neural network minimizing the reconstruction losses (RL_1, RL_2) and the prediction loss (L). Therefore the loss function for feature reconstruction is as follows where m for magnetic, t stands for trajectory:

$$\sum_{m=1}^m \|y_m - f_1(x_m)\|^2 + \sum_{t=1}^t \|y_t - f_2(x_t)\|^2 \quad (4)$$

This approach enables the incorporation of spatio-temporal similarities into the positioning process. The dataset was divided into a 2:1 ratio, with two-thirds of the data used for training

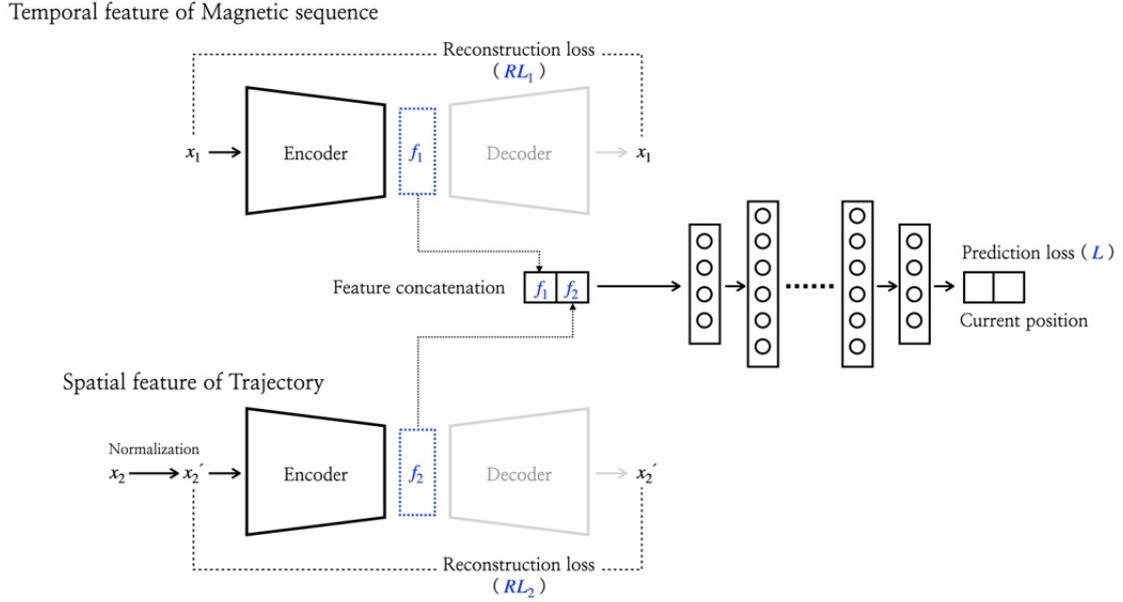


Figure 5: The structure of two autoencoders for spatio-temporal feature extraction and the positioning model of neural networks with the deep embedding concatenation.

and the remaining one-third for testing. The positioning model of the neural network structure was trained to identify and indicate the location along the relative trajectory.

5. Experimental results

We designed the model to pair inputs from two different datasets of magnetic sequences in the training phase to align the devices. The structure of the model is one single multimodal block combining autoencoders and fully connected layers. Two magnetic sequences and one trajectory are fed to train the representation for the same position, sharing the weights of the magnetic feature extractor. Soft DTW-GI scores of magnetic sequence in latent space are calculated as an additional metric to synchronize coinciding data.

The model is designed to predict the position from the magnetic patterns along the trajectory by minimizing sum of reconstruction losses and soft DTW-GI loss with the window size of 10 seconds, updating with 0.25-second intervals. The magnetic sequence autoencoder and the trajectory autoencoder demonstrated a reconstruction error of less than 3 percent, as shown in Table 1, indicating their effectiveness in capturing the underlying patterns and features. Additionally, the mapping accuracy reached 96 percent, highlighting the performance of the

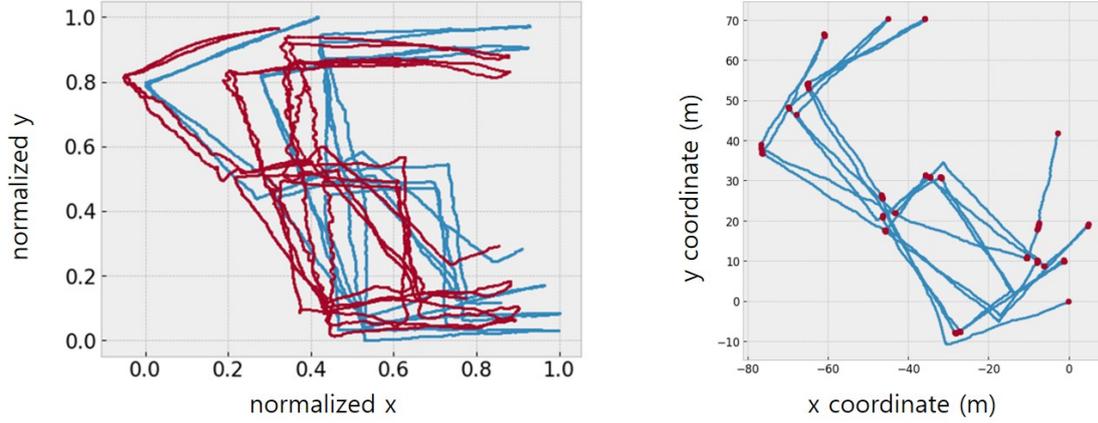


Figure 6: (left) The prediction results of positioning model. (right) An example of corner detection with change point analysis.

Table 1

Testing accuracy of prediction outputs by metrics.

	MSE (Mean squared error)	MAE (Mean absolute error)	MAPE (Mean absolute percentage error)
Mag	0.00	0.03	2.37
Traj	0.01	0.07	4.20
Mapping	0.01	0.04	3.10

models to map the trajectories accurately.

A change point analysis extracted landmarks in the corner points to identify the specific location. The analysis method involves the computation of trajectory data derivatives within each window, enabling the identification of notable variations in the data. The corner points are selected when the sampled data exceeds a predefined threshold. Post-processing is necessary once corners are defined by the change point algorithms due to possibilities for misidentification caused by fluctuations in the trajectory. After the corner definition step, the proposed indoor positioning model maps one unified trajectory and achieves precise localization performance and accurate positioning by mapping each node at the corners in the target area.

To summarize, integrating multimodal learning for feature extraction and results from change point analysis make it possible to compute absolute indoor positioning coordinates, resulting in a comprehensive and accurate representation of the indoor space. This approach ensures reliable and precise indoor positioning.

6. Conclusion and future study

This study presents a practical implementation with analysis of magnetic sequence data and IMU-oriented trajectories collected from smartphone sensors. The analysis leverages the DTW-GI technique in conjunction with a sliding window approach to extract spatio-temporal features

and identify space-oriented magnetic patterns. By measuring temporal similarities using DTW-GI, similarity scores are computed for each axis and aggregated to provide an overall measure of sequential similarity. The findings demonstrate that a 10-second window is sufficient for identifying spatial representatives from the data.

To capture the spatio-temporal similarities in magnetic sequences and trajectories, autoencoders, one type of unsupervised learning, are employed. These models compress the data into a lower-dimensional latent space and reconstruct it, focusing on minimizing reconstruction errors and extracting informative features. The trajectory encoder specifically operates within the sliding window, enabling a detailed analysis of temporal dynamics within each window while disregarding the global context. This localized approach facilitates the capture of intricate spatio-temporal patterns. Moreover, the trajectory encoder extracts features specific to the local trajectory, enhancing understanding and utilization within the given time window.

Supervised learning techniques using deep neural networks are then applied, using relative trajectories as target labels. The models are trained on labeled data to predict relative trajectories based on landmark nodes. This approach significantly improves the system's performance to estimate positions and movements within the target building. In future studies, the integration of self-supervised learning methods with the DTW-GI technique is planned to map relative trajectories to precise positioning. This research contributes to developing effective and robust indoor positioning systems in real-world practical scenarios without sensor calibration, focusing on the representative pattern.

Acknowledgments

This work was supported by the Tech Incubator Program for Start-up(S3321488) funded by the Ministry of SMEs and Startups(MSS, Korea)

References

- [1] C. Chen, X. Lu, A. Markham, N. Trigoni, Ionet: Learning to cure the curse of drift in inertial odometry, in: *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32, 2018.
- [2] S. Herath, H. Yan, Y. Furukawa, Ronin: Robust neural inertial navigation in the wild: Benchmark, evaluations, & new methods, in: *2020 IEEE International Conference on Robotics and Automation (ICRA)*, IEEE, 2020, pp. 3146–3152.
- [3] S. Subedi, D.-H. Kim, B.-H. Kim, J.-Y. Pyun, Improved smartphone-based indoor localization system using lightweight fingerprinting and inertial sensors, *IEEE Access* 9 (2021) 53343–53357.
- [4] A. Alarifi, A. Al-Salman, M. Alsaleh, A. Alnafessah, S. Al-Hadhrami, M. A. Al-Ammar, H. S. Al-Khalifa, Ultra wideband indoor positioning technologies: Analysis and recent advances, *Sensors* 16 (2016) 707.
- [5] R. Faragher, R. Harle, An analysis of the accuracy of bluetooth low energy for indoor positioning applications, in: *Proceedings of the 27th International Technical Meeting of The Satellite Division of the Institute of Navigation (ION GNSS+ 2014)*, 2014, pp. 201–210.

- [6] L. Antsfeld, B. Chidlovskii, E. Sansano-Sansano, Deep smartphone sensors-wifi fusion for indoor positioning and tracking, arXiv preprint arXiv:2011.10799 (2020).
- [7] A. Sahar, D. Han, An lstm-based indoor positioning method using wi-fi signals, in: Proceedings of the 2nd International Conference on Vision, Image and Signal Processing, 2018, pp. 1–5.
- [8] S. He, K. G. Shin, Geomagnetism for smartphone-based indoor localization: Challenges, advances, and comparisons, ACM Computing Surveys (CSUR) 50 (2017) 1–37.
- [9] Z. Jin, R. Kang, H. Su, Multi-scale fusion localization based on magnetic trajectory sequence, Sensors 23 (2023) 449.
- [10] J. Chung, M. Donahoe, C. Schmandt, I.-J. Kim, P. Razavai, M. Wiseman, Indoor location sensing using geo-magnetism, in: Proceedings of the 9th international conference on Mobile systems, applications, and services, 2011, pp. 141–154.
- [11] D. Hanley, A. B. Faustino, S. D. Zelman, D. A. Degenhardt, T. Bretl, Magpie: A dataset for indoor positioning with magnetic anomalies, in: 2017 International Conference on Indoor Positioning and Indoor Navigation (IPIN), IEEE, 2017, pp. 1–8.
- [12] M. Zhang, J. Jia, J. Chen, L. Yang, L. Guo, X. Wang, Real-time indoor localization using smartphone magnetic with lstm networks, Neural Computing and Applications 33 (2021) 10093–10110.
- [13] N. Lee, S. Ahn, D. Han, Amid: Accurate magnetic indoor localization using deep learning, Sensors 18 (2018) 1598.
- [14] Android developers | Sensor, 2019. Available online: https://developer.android.com/guide/topics/sensors/sensors_overview#sensors-intro [Accessed: 01 June 2023].
- [15] P. Patonis, P. Patias, I. N. Tziavos, D. Rossikopoulos, K. G. Margaritis, A fusion method for combining low-cost imu/magnetometer outputs for use in applications on mobile devices, Sensors 18 (2018) 2616.
- [16] Q. Wang, H. Luo, H. Xiong, A. Men, F. Zhao, M. Xia, C. Ou, Pedestrian dead reckoning based on walking pattern recognition and online magnetic fingerprint trajectory calibration, IEEE Internet of Things Journal 8 (2020) 2011–2026.
- [17] L. Antsfeld, B. Chidlovskii, Magnetic field sensing for pedestrian and robot indoor positioning, in: 2021 International Conference on Indoor Positioning and Indoor Navigation (IPIN), IEEE, 2021, pp. 1–8.
- [18] D. Hanley, A. S. D. De Oliveira, X. Zhang, D. H. Kim, Y. Wei, T. Bretl, The impact of height on indoor positioning with magnetic fields, IEEE Transactions on Instrumentation and Measurement 70 (2021) 1–19.
- [19] W. Son, L. Choi, Magnetic vector calibration for real-time indoor positioning, in: ICC 2020-2020 IEEE International Conference on Communications (ICC), IEEE, 2020, pp. 1–7.
- [20] R. Asadi, A. Regan, Clustering of time series data with prior geographical information, arXiv preprint arXiv:2107.01310 (2021).
- [21] T. Vayer, R. Tavenard, L. Chapel, N. Courty, R. Flamary, Y. Soullard, Time series alignment with global invariances, arXiv preprint arXiv:2002.03848 (2020).
- [22] H. Sakoe, S. Chiba, Dynamic programming algorithm optimization for spoken word recognition, IEEE transactions on acoustics, speech, and signal processing 26 (1978) 43–49.
- [23] M. Cuturi, M. Blondel, Soft-dtw: a differentiable loss function for time-series, in: International conference on machine learning, PMLR, 2017, pp. 894–903.