Reasoning About Dynamic Game Models Using Obstruction Logic

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Abstract
Games played within dynamic models have been explored in various domains, including cybersecurity and planning. Our paper introduces Obstruction Logic, a formalism designed for analyzing specific games featuring temporal objectives, which unfold within dynamic models. These games involve players whose actions can impact the underlying game model. We demonstrate how this logic can be employed to express significant properties within the realm of cybersecurity games, particularly those defined on attack graphs. An expanded version of our research has been published in ECAI 2023.

Keywords
Strategic Reasoning, Dynamic games, Attack Graphs

1. Introduction

Multi-agent systems capture the behavior of two or more rational agents engaged in interactions with each other, whether through cooperation or adversarial interactions, all with the aim of achieving a specific objective [2]. Typically, this behavior is represented using a combination of temporal or modal logic and game theory. In this framework, agents are treated as players in games played on directed graphs known as arenas, and their goals are defined using logical formulas. For example, logics like ATL and Strategy Logic [3, 4] provide the means to express the idea that a coalition of players can reach a particular goal through cooperative actions.

In these logics, the game model, within which players participate, is regarded as a fixed entity. While players’ actions affect their positions within the arena, they do not alter the underlying structure of the arena itself. In contrast, dynamic games, where the game model is subject to change, have been examined in various contexts, including cybersecurity and planning [5, 6, 7, 8].

This paper presents a logic designed for the analysis of a specific category of games with temporal objectives played within a dynamic model. These games involve two key players: the Demon and the Traveler, and are conducted on a directed graph. Each edge $e$ in the graph is associated with a deactivation cost $C(e)$. The game unfolds in rounds, with each round...
consisting of a move by the Demon followed by a move by the Traveler. In the Demon’s move, given a node \( v \) in the graph and a natural number \( n \), the Demon deactivates a proper subset \( E \) of the edges incident to \( v \) in such a way that the sum of the deactivation costs of the edges in \( E \) is less or equal to \( n \). Subsequently, the Traveler selects a node \( v' \) connected to \( v \), such that the edge \( \langle v, v' \rangle \) does not belong to \( E \). A new round commences from the last node chosen by the Traveler, and the edges disabled in the previous round are reactivated. The Demon wins the play if the infinite sequence of nodes subsequently chosen by the Traveler satisfies a given temporal property. To reason about the existence of “demonic” strategies for this kind of games, we propose Obstruction Logic (or simply OL). In OL, one can quantify over the existence of demonic strategies that allows the Demon to temporally obstruct some reachable states. OL has direct applications to the cybersecurity field: it can be used to design active security response strategies during an ongoing attack. In fact, these games allow capturing the interactions between an attacker whose possible actions are modeled using an Attack Graph \([9]\), and a defender able to dynamically deploy Moving Target Defense (MTD) mechanisms \([10, 11]\) based on this attack graph.

**Related Work** In the past years, many works focused on the strategic abilities of agents playing in a dynamic game model \([5, 6, 12]\). We compare some of this works to OL.

Sabotage games and Sabotage Modal Logic \([13, 14, 15]\) are a line of research that our work is related to. Sabotage games have been introduced by van Benthem with the aim of studying the computational complexity of a special class of graph-reachability problems in which an agent has the ability to erase edges. To reason about sabotage games, van Benthem introduced Sabotage Modal Logic (SML). Our version of games is incomparable with Sabotage games since we give the ability to temporarily select subsets of edges while in Sabotage games the saboteur can erase only one edge at each turn. On this respect, our work is related to \([8]\) in which the authors use an extended version of Sabotage Modal Logic, called Subset Sabotage Modal Logic (SSML), in which the deactivation of particular subsets of edges of a directed graph is allowed. Furthermore, we recall that SSML is an extension of SML, but it does not include temporal operators as we do. Moreover, neither SML nor SSML take into account quantitative information about the cost of edges as we do.

In \([12]\) the authors introduce NTL a temporal logic to reason about normative systems. A normative system is a Kripke structure in which certain transition are considered illegal, see \([16]\) for a survey. Formally, in NTL one evaluates CTL formulae with respect to a Kripke model in which a set of arcs has been deleted according to a given assignment function. The assignment function on NTL in non-local e non-quantitative: any subset of arcs can be deleted by the assignment, and there is no notion of deletion cost. Moreover, OL model checking is in P while NTL model checking is in NP.

The works in \([17, 18, 19]\) share some ideas with ours on the cybersecurity side. However, the authors do not use dynamic models.

### 2. Syntax, Semantics, and Main Properties

In this section, we introduce the syntax and semantics of our logic. Let \( Ap \) be an at most countable set of atomic formulae (or atoms). Formulae of Obstruction Logic (OL, for short) are
defined by the following grammar:

\[ \varphi ::= \top | p | \neg \varphi | (\downarrow_n) \varphi | (\downarrow_n) (\varphi \cup \psi) | (\downarrow_n) (\varphi R \psi) \]

where \( p \) is an atomic formula and \( n \) is any non-negative integer. The number \( n \) is called the grade of the strategic operator. In what follows we use greek letters \( \varphi \) and \( \psi \) (eventually indexed by natural numbers), to denote arbitrary formulas. The boolean connectives \( \bot, \lor, \land \) and the temporal connectives \( F, W, G \) can be defined as usual. Formulae of OL will be interpreted over obstruction models. The definition follows.

**Definition 1.** An obstruction model \( \mathcal{M} \) (model for short) is given by \( \langle S, R, \mathcal{L}, C \rangle \) where \( \langle S, R, \mathcal{L} \rangle \) is a Kripke structure in which the accessibility relation \( R \) is serial, and \( C : R \rightarrow \mathbb{N} \) is a function assigning to any \( \langle s, s' \rangle \in R \) a positive integer \( n \).

A path \( \pi \) over a model \( \mathcal{M} \) is an infinite sequence of states \( s_1, s_2, \ldots \) such that \( \langle s_i, s_{i+1} \rangle \in R \) for all \( i \in \mathbb{N} \). If \( \pi \) is a path, we write \( \pi_i \) to denote the \( i \)-th element \( s_i \) of \( \pi \), \( \pi \leq i \) to denote the prefix \( s_1, \ldots, s_i \) of \( \pi \), and \( \pi \geq i \) to denote the suffix \( s_i, s_{i+1} \ldots \) of \( \pi \). A history is any finite prefix of some path. We use \( H \) to denote the set of histories.

The intuitive meaning of a formula \( (\downarrow n) \varphi \) with \( \varphi \) temporal formula is “there is a demonic strategy such that all paths of the graphs that are compatible with the strategy satisfy \( \varphi \)” where “demonic strategy” means “a strategy for disabling arcs”. We define an arc-removing strategy as follows.

**Definition 2.** If \( \mathcal{M} \) is a model and \( n \) a natural number, a \( n \)-strategy is a function \( \mathcal{S} : H \rightarrow 2^R \) that given a history \( h \), returns a subset \( E \) of \( R \) such that: (i) \( E \subset R(\text{last}(h)) \) \(^1\), (ii) \( (\sum_{e \in E} C(e)) \leq n \). A memoryless \( n \)-strategy is a \( n \)-strategy \( \mathcal{S} \) such that for all histories \( h \) and \( h' \) if \( \text{last}(h) = \text{last}(h') \) then \( \mathcal{S}(h) = \mathcal{S}(h') \).

A path \( \pi \) is compatible with a \( n \)-strategy \( \mathcal{S} \) if for all \( i \geq 1 \) we have that \( \langle \pi_i, \pi_{i+1} \rangle \notin \mathcal{S}(\pi_{\leq i}) \). Given a state \( s \) and a \( n \)-strategy \( \mathcal{S} \), \( \text{Out}(s, \mathcal{S}) \) denotes the set of paths whose first state is \( s \) and that are compatible with \( \mathcal{S} \).

**Definition 3.** The satisfaction relation between a model \( \mathcal{M} \), a state \( s \) of \( \mathcal{M} \), and a formula \( \varphi \) is defined by induction on the structure of \( \varphi \)\(^2\):

\begin{itemize}
  \item \( \mathcal{M}, s \models (\downarrow_n) \varphi \) iff there is a \( n \)-strategy \( \mathcal{S} \) such that for all \( \pi \in \text{Out}(s, \mathcal{S}) \) we have that \( \mathcal{M}, \pi_2 \models \varphi \);
  \item \( \mathcal{M}, s \models (\downarrow_n) (\varphi \cup \psi) \) iff there is a \( n \)-strategy \( \mathcal{S} \) such that for all \( \pi \in \text{Out}(s, \mathcal{S}) \) there is a \( j \in \mathbb{N} \) such that \( \mathcal{M}, \pi_j \models \psi \) and for all \( 1 \leq k < j \) \( \mathcal{M}, \pi_k \models \varphi \);
  \item \( \mathcal{M}, s \models (\downarrow_n) (\varphi R \psi) \) iff there is a \( n \)-strategy \( \mathcal{S} \) such that for all \( \pi \in \text{Out}(s, \mathcal{S}) \) we have that either \( \mathcal{M}, \pi_1 \models \psi \) for all \( i \in \mathbb{N} \) or there is a \( k \in \mathbb{N} \) such that \( \mathcal{M}, \pi_k \models \varphi \) and \( \mathcal{M}, \pi_i \models \psi \) for all \( 1 \leq i \leq k \).
\end{itemize}

\(^1\)\( \text{last}(h) \) denotes the last element of the finite sequence \( h \).

\(^2\)the clauses for \( \top \), atomic propositions, and boolean connectives are the usual ones and thus omitted.
The memoryless satisfaction relation $\mathcal{M}, s \vdash_r \varphi$ is defined by writing memoryless $n$-strategies instead of $n$-strategies in the above definition. Two formulas $\varphi$ and $\psi$ are semantically equivalent (denoted by $\varphi \equiv \psi$) iff for any model $\mathcal{M}$ and state $s$ of $\mathcal{M}$, $\mathcal{M}, s \vdash \varphi$ iff $\mathcal{M}, s \vdash \psi$. OL enjoys some important theoretical properties.

**Theorem 1.** [20] Formulae that are true under the satisfaction relation and the memoryless satisfaction relation coincides, that is: for any formula $\varphi$, for every model $\mathcal{M}$ and state $s$, we have that $\mathcal{M}, s \vdash_r \varphi$ if and only if $\mathcal{M}, s \vdash \varphi$.

Let $[[\varphi]]^\mathcal{M} = \{s \in S \mid \mathcal{M}, s \vdash \varphi\}$, we can characterize $[[\langle \dagger_n \rangle X \varphi]]^\mathcal{M}$ as follows: given $X \subseteq S$ we write $s \in \dagger(n, X)$ iff the following holds:

$$s \in \text{Pre}(X) \land \left( \sum_{s' \in X} C(s, s') \right) \leq n$$

where $\text{Pre}(X) = \{s \mid \exists s' \in X \land (s, s') \in R\}$. Given the above characterization of $\dagger(\cdot, \cdot)$, we can easily prove that $s \in [[\langle \dagger_n \rangle X \varphi]]^\mathcal{M}$ iff $s \in \dagger(n, [[\varphi]]^\mathcal{M})$. Remark that computing $\dagger(n, X)$ is polynomial in $|S|$ for any $X \subseteq S$. Given this last result, we can prove the following.

**Theorem 2.** [20] Given a model $\mathcal{M}$ and two formulae $\varphi$ and $\psi$ let $U_{\varphi, \psi}^n$ and $R_{\varphi, \psi}^n$ the two monotone functions from $2^S$ into itself defined by:

$$U_{\varphi, \psi}^n(X) = [[\varphi]]^\mathcal{M} \cup ([[\varphi]]^\mathcal{M} \cap \dagger(n, X)) \tag{1}$$

$$R_{\varphi, \psi}^n(X) = [[\psi]]^\mathcal{M} \cap ([[\varphi]]^\mathcal{M} \cup \dagger(n, X)) \tag{2}$$

we have that $[[\langle \dagger_n \rangle (\varphi U \psi)]]^\mathcal{M}$ is the least fix-point of $U_{\varphi, \psi}^n$ and that $[[\langle \dagger_n \rangle (\varphi R \psi)]]^\mathcal{M}$ is the greatest fix-point of $R_{\varphi, \psi}^n$.

Given the above theorem, an algorithm that computes the set of states satisfying a formula $\varphi$ is obtained by slightly modifying the classical labeling algorithm of CTL [21]. Moreover, it exists a simple syntactic embedding from CTL to a proper fragment of OL (see [20] for details). We thus obtain the following.

**Theorem 3.** [20] The model checking problem for Obstruction Logic (OL) is PTIME-complete, that is: given a finite model $\mathcal{M}$ a state $s$ of $\mathcal{M}$, and formula $\varphi$ deciding whether $\mathcal{M}, s \vdash \varphi$ is PTIME-complete and can be computed quadratic time in the number $|S|$ of states of $\mathcal{M}$.

3. Case study

We here give an extended example of how OL can be used to reason about security scenarios modeled by means of attack graphs. An attack graph is a labeled oriented graph, where each node represents both the state of the system (including existing vulnerabilities) and the state of the attacker, and each edge represents an action of the attacker (a scan of the network, the execution of an exploit based on a given vulnerability, access to a device, etc.) that changes the state of the network or the state of the attacker; an edge is labeled with the name of the...
action. Figure 1(i) gives an example of an attack graph. States of the attack graph are denoted as \( s_i \), with \( 0 \leq i \leq 5 \), and atomic attacks as edge labels \( a_j \), with \( 1 \leq j \leq 7 \). A path in the graph corresponds to a sequence of atomic attacks. Attack graphs can be used to perform dynamic analysis to define an optimal security attack/response policy during an ongoing attack scenario, as we do here. We assume that the defender can dynamically prevent an attack using active defense mechanisms such as Moving Target Defense (MTD) mechanisms (see for instance [10] for a survey). Based on some security objectives defined as properties on the attack graph, we would like to be able to check whether there exists for the defender a response strategy based on MTD mechanisms that prevents the attacker from violating the security objectives.

MTD mechanisms use reconfiguration techniques to dynamically shifts the attack surface in order to decrease the success probability of an attack. We consider that for each attack step \( a \) in the attack graph, there exists a corresponding MTD mechanism \( d_a \) able to counter it. During an ongoing attack, when the attacker tries to perform \( a \), if the defender decides to activate \( d_a \), then \( a \) will fail (i.e. the attacker will not reach the corresponding state). However, the effect of \( d_a \) will be temporary: if the attacker tries to launch again \( a \) later and the defender does not activate again \( d_a \), \( a \) will succeed. Regarding the attack graph, it means that the defender is able to temporarily remove an (or a subset of) edge(s). We assign a cost to each MTD, corresponding to the impact on the system due to the reconfiguration phase. Notice that, from the defender’s point of view the cost represents the impact due to the deployment of one (or several) MTD mechanism(s) at a given moment.

In this context, the defender defines Security Objectives based on the attack graph. Let suppose that when reaching state \( s_1 \), \( s_3 \), or \( s_5 \) the attacker has root privilege on a given critical server \( s \). Let suppose that, if the attacker completes attack steps \( a_6 \) or \( a_7 \) (that is, it reaches state \( s_5 \)), then the defender will obtain information on the identity of the attacker. In this example two security objective could be analyzed:

- **\( O_1 \)** the attacker is never able to obtain root privilege on server \( s \) unless the defender is able to obtain information on its identity;
- **\( O_2 \)** while the defender has not obtained information about the attacker identity, the attacker has not root privilege on server \( s \).

Let \( a \) be an atomic proposition that express the fact that the identity of the attacker is known. Let \( r_s \) be an atomic proposition expressing the fact that the attacker has root privilege on server \( s \). The two security objectives \( O_1 \) and \( O_2 \) presented above can be expressed by OL formulae. By using \( t_1 \) as variable for a given threshold, the following OL formula captures \( O_1 \):

\[
\varphi_1 := (\ddagger_{t_1})G(\neg r_s \lor (r_s \rightarrow (\ddagger_{t_1})(F(a))))
\]

Objective \( O_2 \) says that we want \( s_5 \) to be false until we have identified the attacker (\( a \)) if such an identification ever happens. Thus, by using \( t_2 \) as a variable for a given threshold, we can write \( O_2 \) using the weak-until connective:

\[
\varphi_2 := (\ddagger_{t_2})(\neg r_s W a)
\]

Remark that it is fairly easy to see the Attack Graph of Figure 1(i) as a model in the sense of Definition 1. We can simply forget the edge labels, add a loop-edge on \( s_5 \) to grant seriality,
add a cost for each edge (representing the defender’s cost to apply the corresponding MTD countermeasure), and specify the labeling function as showed in Figure 1(ii). If $t_1$ and $t_2$ are respectively 3 and 5, then, in the obtained model $\mathcal{M}$ we have that $\mathcal{M}, s_0 \vDash \varphi_1 \land \varphi_2$.

![Figure 1: (i) An Attack Graph where states $s_1$, $s_3$, and $s_5$ represent the goals of the attacker. (ii) The model $\mathcal{M}$ obtained from (i) where the blue nodes satisfy $r$, the red node satisfies both $a$ and $r$, and the white ones satisfy neither $r$ nor $a$.](image)

4. Conclusions

We presented Obstruction Logic, a logic that allows to reason about two-player games with temporal goals in which one of the players has the power to modify, locally and temporarily, the game structure. In the future, we can explore different directions.

One natural extension, would be to consider many-player games, between a Demon and coalitions of Travelers. At any step a possible continuation of a play would be determined, by the Demon’s deactivation, the synchronous actions of the considered coalition of Travelers $T$, and all possible actions of Travelers not in $T$. Another extension we would like to study is to permit the Demon to permanently deactivate edges of the directed graph. In this case, Demon’s actions could impact the topology of the graph in a non-local fashion: the Demon could choose to erase an edge situated anywhere in the graph. The logic so obtained, would reassemble to a temporal version of the already cited Sabotage Modal Logic [13]. Finally, we would like to study the above scenarios in the context of imperfect information. Unfortunately, this context is in general undecidable [22]. To overcome this problem, we could use an approximation to perfect information [23], a notion of bounded memory [24], or some hybrid technique [25, 26, 27].

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