

# Verification of Bayesian Mechanisms with Strategy Logic

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## Abstract

The design of mechanisms for aggregating preferences while achieving a socially desirable outcome is a central problem in Multi-Agent Systems. In this paper, we motivate a recent approach [1] for formally verifying Bayesian mechanisms using a logic for strategic reasoning, namely Probabilistic Strategy Logic. This approach has been used to encode classic notions from Mechanism Design, including, Bayesian-Nash equilibrium and incentive compatibility.

## Keywords

Strategic Reasoning, Formal Methods, Bayesian Mechanisms

## 1. Introduction

The design of mechanisms for aggregating preferences while achieving a socially desirable outcome is a central problem in Multi-Agent Systems (MAS). In recent years, there has been a growing effort at designing novel mechanisms for a wide variety of problems and settings, including peer selection [3], hedonic coalition formation games [4, 5], sponsored search auctions [6], and diffusion auctions [7]. In Automated Mechanism Design (AMD) [8], designing mechanisms is seen as a computational optimization problem. Different techniques may be used such as neural networks [9], statistical machine learning [10], black-box optimization algorithms [11], as well as evolutionary search methods [12]. All these techniques treat AMD by solving it for a specific setting: no general perspective is considered and consequently, typical properties are always defined in terms of the specific problem.

In line with the well established logical approach to system verification [13], the work presented in [14] advocates the use of Alternating-time Temporal Logic (ATL) [15] to reason about mechanism design. Due to ATL's limitations regarding the expression of solution concepts (such as Nash equilibria) as well as handling quantitative aspects, recent works proposed the use of variants of Quantitative Strategy Logic (SL[ $\mathcal{F}$ ]) [16] for reasoning about mechanisms. In [17], the authors demonstrate how to represent and verify knowledge-based benchmarks

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and properties such as efficiency and strategyproofness in Epistemic  $SL[\mathcal{F}]$ . Similarly,  $SL[\mathcal{F}]$  with natural strategies have been considered for reasoning with bounded recall [18]. Finally, the automated design of deterministic mechanisms was reduced to  $SL[\mathcal{F}]$ -synthesis in [19]. However,  $SL[\mathcal{F}]$  semantics is deterministic and thus the logic is unable to express probabilistic features, which are essential when considering Bayesian and randomized mechanisms.

**Related Work** In probabilistic model checking, specifications are given in probabilistic logics, and their validity is evaluated w.r.t. a system. For instance, the problem has been considered for Probabilistic ATL Chen and Lu [20], Probabilistic Alternating-Time  $\mu$ -Calculus [21], and Probabilistic Strategy Logic [22]. Probabilistic ATL has been also studied in the setting of imperfect information and memoryless strategies [23], and with accumulated costs/rewards [24]. In algorithmic mechanism design, probabilistic verification refers to the use of statistical tests to evaluate mechanisms [25]. It has been considered, for instance, in the standard mechanism design setting Ball and Kattwinkel [26] and for obviously strategy-proof mechanisms [27].

## 2. Contribution and Discussion

The approach recently proposed in [1] makes use of Probabilistic Strategy Logic (PSL) [22] for AMD. Randomness and imperfect information are foundational and must be addressed by any formal verification technique for Bayesian mechanisms. Generalizing from the deterministic to the probabilistic setting is challenging due to several aspects. First, the wide and heterogeneous range of settings considered in the literature obscures the path for a general and formal approach to verification. The setting may consider deterministic or randomized mechanisms, incomplete information about agents' types (Bayesian mechanisms), mixed or pure strategies, and direct or indirect mechanisms (iterative protocols). Second, considering Bayesian mechanisms brings out different methods for evaluating a mechanism according to the time-line for revealing the incomplete information as the mechanism is executed. The work in [1] considers a very general Bayesian framework for mechanism design and show how to capture it with PSL. This allows for automatic verification of a wide class of Bayesian mechanisms through PSL model checking, and motivates further research on applications of logic-based approaches for AMD.

Unlike previous proposals, the automated verification of Bayesian mechanisms using Probabilistic Strategy Logic (PSL) is able to take into account a wide range of settings (e.g. randomized, indirect, and Bayesian mechanisms). Furthermore, thanks to the great expressiveness of the specification language, PSL, the verification *ex ante*, *interim* and *ex post* of complex solution concepts and properties is fully automated through model checking of logical formulas.

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