Connect++: A New Automated Theorem Prover Based on the Connection Calculus

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Abstract

CONNECT++ is an automated theorem prover for first-order logic with equality, based on the clausal connection calculus and designed with three primary goals. The first was to produce a carefully coded system in a compiled language (C++). The second was to allow the system to support my own research agenda involving the addition of machine learning to connection provers. The third, somewhat inspired by the success of the MINISAT solver for Boolean satisfiability, was to provide an implementation sufficiently modifiable as to provide a common basis for experiments by others. In addition to these aims I wanted to exploit the opportunities inherent in the connection calculus to explore the production of readable and certified proofs. This paper describes the system as it stands; development is ongoing and some plans for the future are also outlined.

Keywords

Connection Prover, C++, Machine Learning, Certified Proof, Automated Theorem Prover, System description, First-Order Logic, Clausal Connection Calculus

1. Introduction

CONNECT++ is a prover for first-order logic, implemented in C++ and based on clausal connection calculus. The advantages of connection provers, with respect to their goal-oriented search and ability to produce readable proofs, are well-known, and have led to great interest in their use for applying machine learning (ML) to automated theorem proving (ATP). This paper introduces CONNECT++, and gives a high-level description of the system; it also motivates its development. The paper falls a little outside the usual form of discourse for such papers, combining three related perspectives. First, it is a 'system description'. Second, it argues for design decisions based on practical experience in conducting large-scale experiments; a pursuit often mandating use of preferred technologies for these two, often disparate research areas. Third, it presents motivations underlying the system's design decisions, based purely on accommodating the author's ongoing interest in applying ML in the field of ATP.

CONNECT++ is open source under the GNU General Public License (GPL) Version 3, and can be downloaded from www.cl.cam.ac.uk/~sbh11/connect++.html. The work in this paper is based on the current development version.

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One motivation for CONNECT++ arose while writing a review of ML applied to *satisfiability* (*SAT*) *solvers* [1]. It was apparent that there are many SAT solvers, but an *enabler of research* over the last two decades was MINISAT [2], which had a transformative effect. MINISAT was fast enough to be competitive,¹ while being sufficiently easy to modify that it provided a tool for other researchers. Much of the research leading to today's best solvers relied on MINISAT. One aim of CONNECT++ is to provide a similar basis for experiments with connection provers. (And this is true regardless of any ML.)

At present, several connection calculus provers are available, implemented in Prolog [5, 6], C [7], OCaml [8, 9], Rust [10], Python [11] and C++ [12]. This has led to a difficulty which in the SAT world MINISAT helped to defray: if two researchers conduct experiments using different solvers, how do we decouple the effect of any proposed technical contribution from the characteristics of the solvers? For one to re-implement the other's work on their own system would be a solution; but the use of a common solver, in the manner of MINISAT, would benefit both sides.

Further motivation to develop CONNECT++ arose from two observations made while working on ML applied to ATP more generally:

- 1. The ATP world has two main currencies: proving *more things* and proving *known things faster*. This motivates the development of ATPs that are inherently fast. It is also in conflict with the use of *lean* ATPs for connection calculus, which tend to rely on languages known not to be the fastest.
- 2. The ML world is heavily invested in Python, with the most prevalent libraries using it to implement their API. To produce research in ML for ATP, one needs to run large numbers of experiments. My own groups' experience in continuing the work presented in [13] was that there are significant difficulties in combining Python (for ML), Prolog (for leanCoP [5, 6]), and one or more compiled languages supporting the overall process, into an experiment on a high-performance computing facility.

These points suggested the development of a connection prover in a fast, compiled language such as C++, or in Python. C++ emphasizes the speed requirement and is the approach taken here, addressing Point 1.² Python is also the subject of current work [11]. Both approaches address Point 2, but in complementary ways.³

Further motivations appeared while I worked on the development of CONNECT++, and these are some of the most central issues, both discussed further in Section 2:

1. The lean approach to ATP, exploiting the inherent strengths of Prolog, leads to beautiful, compact implementations. However Prolog's model of computation based on backtracking search is not always amenable to the use of ML. (See also Point 2 above.) There are two

¹I will not be claiming that CONNECT++ currently competes with the fastest solvers such as E [3] and Vampire [4]; any comparison is among connection solvers.

²While speed was a motivating factor in the choice of language for CONNECT++, I make no claim in this paper regarding its speed in comparison to existing provers—an experimental comparison remains a subject for a future paper; here the aim is to provide a general description of the system.

³A possible future development that might be of interest would be to incorporate either of these systems into the FLoP system [14]. This system employs Python and the C foreign function interface, along with a connection prover implemented in OCaml, and hence represents a good target for the kind of simplification envisaged.

$$\frac{C', \mathbb{M}, \{\}, \{\}}{\epsilon, \mathbb{M}, \epsilon, \epsilon} \operatorname{Start} \qquad \frac{C, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L'\}}{C \cup \{L\}, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L'\}} \operatorname{Lemma} \\ \text{where } C' \text{ is a copy of } C \in \mathbb{M} \qquad \frac{C, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L'\}}{W \text{ where } \sigma(L) = \sigma(L')} \text{Lemma} \\ \frac{C, \mathbb{M}, \mathbb{P} \cup \{L'\}, \mathbb{L} \cup \{L\}}{C \cup \{L\}, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L'\}} \operatorname{Reduction} \\ \frac{C' \setminus L', \mathbb{M}, \mathbb{P} \cup \{L\}, \mathbb{L} - C, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L\}}{C \cup \{L\}, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L\}} \text{Extension} \\ \text{where } \sigma(L) = \sigma(\overline{L'}) \qquad \frac{C' \setminus L', \mathbb{M}, \mathbb{P} \cup \{L\}, \mathbb{L} - C, \mathbb{M}, \mathbb{P}, \mathbb{L} \cup \{L\}}{C \cup \{L\}, \mathbb{M}, \mathbb{P}} \text{Extension} \\ \text{where } C' \text{ is a copy of a clause in } \mathbb{M}, \\ L' \in C' \text{ and } \sigma(L) = \sigma(\overline{L'}) \end{array}$$

Figure 1: Rules for constructing a clausal connection calculus proof. See Figure 3 for an example proof.

issues here: APIs allowing ML methods to interact with a running Prolog program may present a barrier, and Prolog's model of computation itself can present a limitation.

2. In particular, the Prolog cut is perfect for leanCop's backtracking restriction [6]. However it is a blunt instrument, and more subtle backtracking heuristics might profitably be explored, in turn providing new ways for ML to control the proof search. However the implementation of such alternatives in Prolog potentially becomes cumbersome.

While the realisation that the use of C++ supported these points was serendipitous, the overall design of CONNECT++ was informed by them as described in Section 4, with the overall aim of making a system capable of supporting new research.

A final motivation for CONNECT++ was to explore the production of readable proofs and proof certificates.

2. ML for connection calculus: why use C++?

Prolog provides compact implementations for connection provers, but to facilitate ongoing research on improving connection provers there are good reasons to consider alternatives. In this section I attempt to explain my reasoning.

CONNECT++ implements the clausal connection calculus with lemmata and regularity, as in Figure 1. The *matrix* \mathbb{M} is the set of clauses for the problem. The *path* \mathbb{P} is a set of literals, as is the set \mathbb{L} of lemmata. A copy of an item has a fresh set of unused variables. An overline denotes the complement of a literal. Regularity adds the condition

$$\forall L' \in C \cup \{L\} . \sigma(L') \notin \sigma(\mathbb{P})$$

to the Reduction and Extension rules. When applying the Reduction and Extension rules the substitution is applied to the entire proof. In the case of Regularity and Lemmata we test for equality using the substitution as it stands, but no further unification is applied.

The rules in Figure 1 are the basis for a classical backtracking search problem. leanCoP, for example, arranges this search using Prolog's underlying search algorithm. It is well-known that the use of heuristics to order such a search is critical for good performance. For the calculus

described, there is ample scope for learning such heuristics; for example, at any point in the search there may be multiple ways to extend a partially complete proof, possibly including multiple applicable uses of Reduction, Extension, or Lemmata rules, and we might aim to find improved ways of choosing of the next to try. Researchers therefore need to be able to influence such choices, and the extent to which this is feasible will depend on the details of the implementation of a prover. This is a particular area where we need a solver implementation facilitating fine control over search heuristics.

Some of the most successful heuristics for connection provers go further, relying on the restriction of backtracking during the proof search [6]. In the Reduction, Extension and Lemmata rules, L is called the *principal literal* when the rule is selected as a candidate for extending a proof. For the Reduction and Lemmata, L is considered solved; for an Extension it is considered solved if the left sub-tree of the Extension is completed. After L is solved, no other options for solving it will be considered on backtracking. This heuristic is remarkably effective and the Prolog cut makes its implementation extremely convenient. However, by moving beyond the use of cut we potentially open a wide design space of backtracking heuristics. For example, having solved a literal using Extension, should we backtrack within the left subtree or discard it completely? (leanCoP, and CONNECT++ under its default settings do the latter; but use of the --explore-left-trees flag forces CONNECT++ to adopt the former behaviour.) Should we stop backtracking for a principal literal after it has been solved once, or should we limit the maximum number of ways we try to solve it? Should we limit backtracking just for this principal literal, or remove other possibilities once some literal has been solved? There is considerable opportunity for new work here, and facilitating such research is a key motivation for CONNECT++. (See [15] which provides some results in this direction.)

Other successful heuristics include restriction of backtracking on start clauses, forms of reordering and randomization [16], and others. Ideally a solver should support all forms of heuristic, and allow researchers to experiment with them freely. However we should also consider a further potential need: to modify the operation of the proof search *dynamically*. For example, much research on ML for ATP addresses the tuning of heuristics that are then *fixed* when attempting new proofs. There is ample evidence from work on SAT solvers that learning on a *per-proof* basis is extremely effective. For example, *variable selection heuristics* are light-weight learning algorithms adjusting variable choice using feedback obtained *while* the proof search progresses. This type of learning also deserves attention in ATP, and the need to modify heuristics *during proof search* might be better served by a move away from Prolog.

A final consideration relates to the way in which the proof search is structured. Some systems search recursively, trying the left branches of Extensions first. There has been considerable interest in applying *reinforcement learning* [17] and *Monte-Carlo tree search* [18] to connection provers, and in these cases it makes sense to allow proofs to be constructed in a less constrained manner. This is the subject of complementary work on representing the process as a *Markov Decision Process* [11].

```
2 --pos-neg-start --complete 7 ;
60 --conjecture-start ;
20 --pos-neg-start --restrict-start ;
2 --conjecture-start --reorder 23 ;
2 --pos-neg-start --restrict-start --reorder 29 ;
2 --conjecture-start --reorder 37 ;
2 --pos-neg-start --reorder 37 ;
2 --conjecture-start --reorder 47 ;
0 --pos-neg-start --reorder 47 ;
```

Figure 2: Default schedule file for CONNECT++.

3. Connect++

CONNECT++ supports most of the functionality provided by leanCoP, and has some additional facilities. It supports restricted backtracking, restriction of start clauses and reordering. It deals with equality by detecting its use in the input file and adding the necessary axioms; more sophisticated approaches, such as described in [19, 20], are a subject for future extension. It assumes negative (CNF) representation by default but can use positive (DNF) representation—this changes the equality axioms added, and the behaviour for some of the start clause selection options. It supports iterative deepening by path length or tree depth; switching to complete search after a given depth; and specified start depth and depth increment. It accepts input in the conjunctive normal form (CNF) format of the *Thousands of Problems for Theorem Provers (TPTP)* [21] library. Support for problems in first-order form is a work in progress, and subsequently it does not at present support *definitional clausal form (DCF)* [6]; this is perhaps the first priority for further development as leanCoP's standard schedule employs different settings related to DCF.

A lesson learned applying ML to SAT is that solvers should avoid hiding within their implementation, parameters that are potentially important for tuning performance. Instead, such parameters should be exposed, preferably at the command line, so that they can be adapted to suit a particular domain of application. Automated systems for this task, such as ParamILS [22], SMAC [23] and GGA [24] have been used with great success, as has Bayesian optimisation [25]. CONNECT++ exposes a large, and growing, number of such parameters.

Some connection provers run with a hard-coded schedule optimized through experiments. A schedule is a sequence of sets of parameter settings, each element of the sequence being assigned a percentage of the run time. The prover might start with a schedule line stating that it should run with full backtracking for 10% of the time, then switch to restricted backtracking for 5% of the time, then switch to a different configuration and so on. Figure 2 shows the current default schedule used by CONNECT++, which is similar to that of leanCoP version 2.0. The differences are necessary as CONNECT++ does not at present implement definitional clausal form. This is clearly an effective method, but there is evidence that learning of schedules can be beneficial [26]. CONNECT++ supports the use of arbitrary schedules and can read these from a file in a simple format, making it easy to incorporate the results of ML applied to schedule choice, and potentially to support the ML process for learning schedules. Referring again to Figure 2, each line of a schedule file begins with an integer specifying the percentage of the time available (which can be set using the --timeout command line option) for which to run the

$$(1 \leftarrow c) \xrightarrow{\{\}, M, [\neg p, \neg q(_1)], [p]}{\{p\}, M, [\neg p, \neg q(_1)], []} \operatorname{Red} \xrightarrow{\{\}, M, [\neg p], [\neg q(_1)]}{\{\}, M, [\neg p], [\neg q(_1)]\}} \operatorname{Axiom}_{\mathsf{Ext}} (1 \leftarrow c) \xrightarrow{\{\{\}, M, [\neg r], [\neg p]\}} \operatorname{Lem} \xrightarrow{\{\}, M, [], [\neg p, \neg r]\}}_{\{\neg r\}, M, [], [\neg p]} \operatorname{Lem} \xrightarrow{\{\}, M, [], [\neg p, \neg r]\}}_{\{\neg r\}, M, [], [\neg p]} \operatorname{Ext} \operatorname{Ext} (1 \leftarrow c) \xrightarrow{\{\{\neg p, \neg r\}, M, [], [\neg p]\}}_{\{\neg p, \neg r\}, M, [], [\neg p]} \operatorname{Ext}} \operatorname{Start}$$

Figure 3: ET_{EX} output for Example 1. Variables of the form 1, 2, ... are fresh variables introduced by the Start or Extension rules. Annotations to the left of a rule, such as $(1 \leftarrow c)$, denote substitutions applied to the entire proof.

<pre>matrix(1, matrix(2, matrix(3,</pre>	[-p, -r]). [p, q(c)]). [p, -q(X)]). [-p, r]). [-p, q(a)]).	
--	--	--

```
proof_stack([
    start(0),
    left_branch(0, 2, 0, 2),
    left_branch(0, 1, 1, 3),
    reduction(0, 0),
    right_branch(3),
    right_branch(2),
    left_branch(0, 3, 1, 3),
    lemmata(0, 0),
    right_branch(3)
]).
```

(a) Prolog-readable representation of the matrix.

Figure 4: Certificate for the proof of Example 1.

```
(b) Stack representation of the proof.
```

following settings (or 0 in the last line denoting "all time remaining"). By default, CONNECT++ runs in essentially the same way as the underlying leanCoP search, choosing proof steps on the basis of the first available literal, and restricting backtracking in the same way. This default behaviour is set at each line of the schedule, and the schedule line then modifies it. The flags in the schedule line are named in the same way as the command line options; all command line options that affect the search are available to a schedule file.

CONNECT++ can produce a readable proof via LTEX. Example 1 is a problem from [6].

Example 1. $\mathbb{M} = \{\{\neg P, \neg R\}, \{P, Q(c)\}, \{P, \neg Q(x)\}, \{\neg P, R\}, \{\neg P, Q(a)\}\}$.

Figure 3 shows the typeset output for a proof of Example 1.

CONNECT++ can output a simply-formatted proof certificate. While there is currently no consensus on what format a certificate should take, suggestions have appeared [27, 8, 28] and a proposal for a standard is given in [29]. Figure 4 shows a certificate for the proof shown in Figure 3. It consists of Prolog-readable summaries of the matrix and a stack representation of the proof (see Section 4). Each element of the stack describes the proof rule employed, with left_branch and right_branch denoting the left and right premises of the Extensions. Numbers after an item identify the element(s) used in applying a rule; clauses are indexed from 0 in the matrix and literals are indexed from 0 within clauses. For example, referring again to Figure 1, left_branch(0, 2, 0, 2) denotes that there is an Extension rule. The first index denotes that $L = \neg P$ (the 0-indexed literal in the clause $C \cup \{L\}$).⁴ The second index

⁴Readers familiar with leanCoP might find this unusual. leanCoP always uses the leftmost literal in $C \cup \{L\}$ as the

denotes that C' is a copy of $\{P, \neg Q(x)\}$ (the 2-indexed clause in the matrix). The third index denotes that L' = P (the 0-indexed literal in the copy of C'). The fourth number denotes the depth within the proof. A short Prolog program reads this certificate and verifies that the rule has been correctly applied at each stage, using the built-in unification mechanism to build the substitution.

4. Implementation: some technical details

The requirements for compiling CONNECT++ are light. Aside from a C++ compiler, it uses the Boost libraries⁵ for parsing TPTP and schedule files, and dealing with command-line parameters. LATEX is needed for readable proofs and SWI Prolog [30] is needed for verifying proofs.

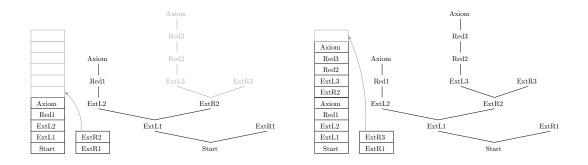
CONNECT++ exploits the structure of the connection calculus by using an optimized representation for variables and terms. As substitutions apply to an entire proof, any variable is only represented once, and terms using that variable do so via a pointer. Any substitution applied to a variable therefore takes effect everywhere the variable appears, in constant time, and backtracking (removing a substitution) is equally fast and straightforward. Fresh variables are recycled to preserve memory. Terms are constructed using pointers to subterms; subterms are shared and no copy of an existing subterm is ever made. This is supported by an index: new (sub)terms are added to the index and only stored if not already present; if already present then a pointer to the existing copy is provided. Literals are straightforward identifiers paired with a list of pointers to terms. Clauses are lists of literals. The matrix is indexed to allow fast lookup of which clauses contain a literal, and the position of that literal in each relevant clause. This aids fast identification of possible Extensions.

As one aim of CONNECT++ is to provide flexibility in the proof search, it avoids the use of recursion in favour of an iterative approach using a pair of stacks. Figure 5 illustrates this. The proof is built in the left stack, and as left premises of Extensions are explored first, the right stack is used to store the currently outstanding right premises. Stack items store C, \mathbb{P} and \mathbb{L} , a substitution, and a list of actions (applications of the proof rules) that can be used to further extend the proof at this point. This structure is manipulated iteratively, and as at any point there is direct access to the list of possible actions stored in each stack item, there is great flexibility in directing the search—the action list in each stack item can arbitrarily be re-ordered, added to (increasing the degree of backtracking) or reduced (restricting the degree of backtracking) *while* a proof is in progress.

Planned developments to CONNECT++ include: (1) completion of clause translation, including definitional clausal form, from TPTP first-order format; (2) enumeration of different proofs for a single problem as a means of generating training data; (3) addition of other heuristics for connection provers; (4) experiments with new heuristics; (5) addition of further command-line options, and where appropriate inclusion of these in the system for handling schedules; (6) implementation of readable proofs similar to those produced by leanCoP version 2.1; (7)

principal literal, and at each stage only uses that literal to generate possible new steps for the proof tree. CONNECT++ does the same in its default settings, but future development is planned allowing the choice of this literal to be subject to one or more heuristics, with the aim of ordering the search more effectively. The format of the certificate supports this planned extension.

⁵https://www.boost.org/



(a) Early in a proof search—the shaded area is not (b) After generating the shaded area, explore yet explored. ExtR3.

Figure 5: Proof search arranged around two stacks. The proof itself is built on the left-hand stack, while the right-hand stack maintains details of the currently outstanding right premises for Extensions.

implementation of any forthcoming standard for proof certificates; and (8) implementation of better approaches to equality.

5. Evaluation

While a large-scale study of the power of CONNECT++ in comparison with other connection provers, and across a large collection of problems, remains for future work, we provide some brief, initial experimental results to demonstrate the prover in its current state.

A selection of 52 AGT problems from TPTP version 8.0.0⁶ were converted to a format readable by CONNECT++.⁷ CONNECT++ was run⁸ on these problems using its default schedule (Figure 2) and a 10 second timeout, on two systems: a MacBook Air M1 with 16GB memory running OS X Sonoma, and the University of Cambridge High-Performance Computing (HPC) Facility.⁹ Figure 6 shows the running times for the problems solved by CONNECT++ within the 10 second timeout.

For comparison, running¹⁰ leanCoP version 2.1 with ECLiPSe version 5.10 #147 on the HPC, again with the timeout set to 10 seconds and with the problems in first-order TPTP format¹¹, it is clear that leanCoP can solve the same problems, and some others. However at this stage further comparison is complicated as the systems have significant differences. In particular, leanCoP has a significantly more detailed schedule employing various forms of clause translation, most significantly definitional clausal form.

⁶Selected using: tptp2T -pp Domains AGT Form FOF Status Theorem Equality.

⁷tptp2X with options -tclausify:tptp -f tptp followed by a second run with options -tadd_equality -f tptp.

⁸./connect++ --schedule default -v0 --no-equality --timeout 10 file.tptp

⁹Technical details for the Ice Lake CPUs employed can be found at https://www.hpc.cam.ac.uk/.

¹⁰./leancop.sh file.tptp 10

¹¹tptp2X with option -f tptp

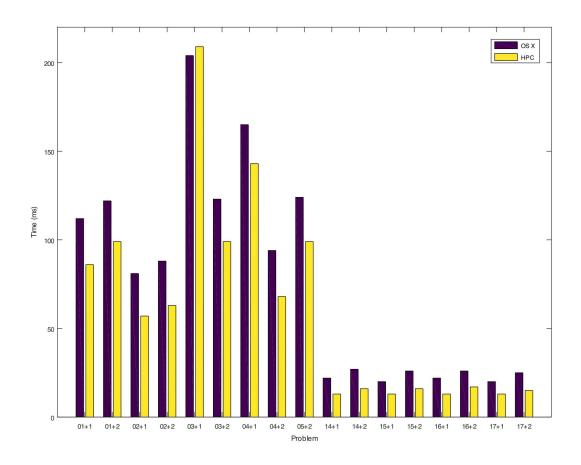


Figure 6: Running times for CONNECT++ on two architectures for the AGT problems solved with a 10 second timeout and the default schedule. Names on the *x*-axis identify AGT problems when appended to the prefix 'AGT0'.

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