Method of Binary Detection of Small Unmanned Aerial Vehicles

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Abstract
This research offers a method for detecting Small Unmanned Aerial Vehicles (sUAVs) in binary using state-of-the-art technology and signal processing techniques. The proposed method combines machine learning and signal analysis techniques to reliably determine the presence of sUAVs in a particular airspace. Qualitative characteristics of sUAV signals can be identified by spectral analysis. The system can learn and identify these properties and make judgments regarding the presence or absence of sUAVs thanks to the application of machine learning methods. Furthermore, the system’s ability to recognize common patterns of sUAV activity is improved by the integration of pattern recognition. Processing data in real-time guarantees system responsiveness and lowers the number of false signals. The efficiency of the suggested sUAV detection system is strongly demonstrated by experimental results acquired in a variety of environmental circumstances. This highlights how the system can improve airspace monitoring measures’ effectiveness and safety.

Keywords
Binary detection, signal processing, spectral analysis, detection accuracy, airspace monitoring.

1. Introduction

Let’s consider the task of binary detection of small-sized targets against the background noise of the receiving channel of an active radar system from the perspective of the statistical hypothesis testing theory in the presence of interference. Initially, let’s assume that the movement parameters of the UAV are known [1–3], hence the form of the useful signal is known. Similar tasks were addressed by many authors when developing algorithms to detect signals of a known form, which comprise a bunch of received radio pulses, against the backdrop of additive Gaussian noise [4]. In this section, we examine the task of detecting a small-sized moving target by processing a set of n amplitude beat signals in an FMCW radar, determined for each probing FMCW radio signal over a specific observation interval \( t = [0; T] \).

The amplitude of the \( i \)th beat signal corresponds to the \( i \)th probing FMCW radio pulse in the series [5].

2. Problem Statement

The formulation of this task is determined by the specific nature and technical implementation of receiving channels in typical ground-based short-range active radars with probing FMCW radio signals. In such contemporary digital radars, for each probing FMCW radio signal, signal beat...
samples from the output of the analog-digital converter are processed [6]. This processing involves calculating the Fast Fourier Transform, followed by delineating the amplitude spectrum to determine the distance to relevant objects [7, 8]. Such objects could be a background surface of a particular type. Hence, a distinctive feature of the method discussed in this subsection is the processing of a set of amplitude samples of signals reflected from background surfaces or objects included in a burst of radio pulses.

First, let's determine the probability density of instantaneous signal values at the detector's input both in the presence and absence of UAVs [9].

During the observation time \( t = [0; T] \), such amplitude variation of the radio signal is represented by the time function \( A_e(t) \). This time function \( A_e(t) \) contains information about the presence of UAVs. Therefore, we'll consider \( A_e(t) \) as the useful signal at the detector's input. Let's first address the task of detecting UAVs for a specific set of model parameters, described by expression (1).

Let's assume that the detector's input receives an additive mixture of the useful signal \( A_e(t) \) and the noise of the receiving path, which we will consider as Gaussian and delta-correlated. As inferred from the materials presented earlier, the power of the useful signal exceeds the power of the additive noise of the receiving path by 25–30 dB [6]. Under these conditions, the density distribution of the envelope of the observed radio signal follows a normal law with an average value equal to the instantaneous value of the useful signal envelope. Let's represent the observed signal \( Y(t) \) as the sum of the useful signal \( A_e(t) \) and the receiving path noise \( n(t) \):

\[
Y(t) = A_e(t) + n(t). \tag{1}
\]

In [10], it is demonstrated that the effective scattering surface of the background during the observation of the useful signal acts as a stationary Gaussian random process. The temporal correlation interval of this process is considerably shorter than the duration of the observed useful signal. As a first approximation, we'll describe the correlation function of the Gaussian random process using a delta function. As can be inferred from expression (1), random variations in the background's RCS lead to a multiplicative transformation of the useful signal \( A_e(t) \), meaning they act as multiplicative interference. The background's RCS, \( \sigma_f(\theta_{b2}, \varepsilon_{b2}) \), is nonlinearly incorporated into the amplitude expression of the reflected signal, as described by expression (1) (through the square root).

Let's determine the density distribution of the square root of the RCS and subsequently the density distribution of the amplitude of the reflected signal.

It's well-known that a nonlinear transformation of a random variable results in the alteration of its distribution law in the following way [11]:

\[
W(B) = W[X = \psi(B)] \frac{d\psi(B)}{dB}, \tag{2}
\]

where \( W(X) \) is the normal probability density distribution of the random variable \( X \).

\( W(B) \) is the sought-after probability density distribution of the random variable \( A \).

\( \psi(B) \) is the function inverse to the function \( B = \psi(X) \).

In this context, \( X = \sigma_f(\theta_{b2}, \varepsilon_{b2}) \) represents a stationary random process, \( B = \sqrt{\sigma_f(\theta_{b2}, \varepsilon_{b2})} \), and the density distribution of instantaneous RCS values at a certain point in time is described by a Gaussian law:

\[
W(\sigma_f) = \frac{1}{\sqrt{2\pi}\sigma_{\sigma_f}} e^{-\frac{1}{2}\left(\frac{\sigma_f - m_{\sigma_f}}{\sigma_{\sigma_f}}\right)^2}, \tag{3}
\]

where \( m_{\sigma_f} \) is the average RCS of the background (\( m_{\sigma_f} = \sigma_{f0} \), in square meters; \( \sigma \) represents the root mean square deviation of instantaneous RCS values of the background over the observation time of the useful signal, in square meters [12].

In this expression, to simplify the notation, bi-static angle designations \( \theta_{b2} \) and \( \varepsilon_{b2} \) have
been omitted. In general, they are functions of time during the observation of the useful signal $A_u(t)$. However, in the context discussed, variations of the mentioned bi-static angles do not result in changes to the background RCS as described by expression (2).

The random variations in the background RCS $\sigma_f$ over the observation time of the useful signal can be represented as the sum of the average value $\sigma_{f0}$ and the fluctuation component $\sigma_f$:

$$\sigma_f = \sigma_{f0} + \tilde{\sigma}_f.$$  \hspace{1cm} (4)

The multiplicative nature of interference about the useful signal arises from fluctuations in the RCS (Radar Cross-Section) of the background. However, the average value of the background’s RCS doesn’t distort the shape of the useful signal.

The probability density distribution of the fluctuating component of the background RCS can be written as [13]:

$$W(\sigma_f) = \frac{1}{\sqrt{2\pi}\sigma_f} \cdot e^{-\left(\frac{\sigma_f}{\sigma_f}\right)^2}.$$  \hspace{1cm} (5)

Expressing the random variable $X$ through $B$ and calculating the derivative, we obtain:

$$X = \psi(B) = B^2 \cdot \frac{d\psi(B)}{dB} = [2 \cdot B]^2.$$  \hspace{1cm} (6)

From this, we derive the sought distribution law of the square root from RCS:

$$D_{A_u}(t) = A_{otr}' \cdot \sqrt{1 + k_A(t) + 2 \cdot k_A(t) \cdot \cos[\Delta \phi(t)]} \cdot \int_{-\infty}^{\infty} \frac{\sqrt{\sigma_f}}{\sqrt{2\pi}\sigma_f} \cdot e^{-\left(\frac{\sigma_f}{\sigma_f}\right)^2} d\sigma_f.$$  \hspace{1cm} (7)

The useful signal $A_u(t)$ is the product of the deterministic function $f(t)$ and the square root of the random variable $\sqrt{\sigma_f}$:

$$A_u(t) = \sqrt{\sigma_f} \cdot f(t) = \sqrt{\sigma_f} \cdot A_{otr} \cdot [1 + k_A(t) + 2 \cdot k_A(t) \cdot \cos[\Delta \phi(t)]]$$  \hspace{1cm} (8)

It’s known that multiplying a stationary random process $\sqrt{\sigma_f}$ by a non-random time function $f(t)$ results in a non-stationary random process with the same distribution law $W(\sqrt{\sigma_f})$ over the observation [14] interval of the useful signal. Note that the multiplier $A_{otr}$ also varies over time as the UAV moves due to changes in distances $R_{ab}, R_{bc}, R_{ac}$, and the antenna gain factor. We consider these changes to be insignificant compared to the influence of the oscillating multiplier $f(t)$. In this case, the non-stationary process is the result of the product of the useful signal $A_u(t)$ and the multiplicative interference $\sigma_f$. The non-stationarity of the random process $A_u(t)$ is due to the variability of the variance $D(\sigma_f)$ by $f^2(t)$ times, leading to a change in the scale of the probability density distribution $W(\sqrt{\sigma_f})$.

The change in dispersion over time, described by expression (8), according to the law of the useful signal can be written as follows:

$$D_{A_u}(t) = A_{otr}' \cdot \sqrt{1 + k_A(t) + 2 \cdot k_A(t) \cdot \cos[\Delta \phi(t)]} \cdot \int_{-\infty}^{\infty} \frac{\sqrt{\sigma_f}}{\sqrt{2\pi}\sigma_f} \cdot e^{-\left(\frac{\sigma_f}{\sigma_f}\right)^2} d\sigma_f.$$  \hspace{1cm} (9)

The probability distribution density of instantaneous values of the useful signal over the observation period can be written as follows [15]:

$$W(B) = \frac{2|B|}{\sqrt{2\pi}\sigma_f} \cdot e^{-\frac{1}{2}\left(\frac{B - \Delta \sigma_f}{\sigma_f}\right)^2}.$$  \hspace{1cm} (7)

Regrettably, the integrals in formulas (7)–(9) cannot be expressed in terms of elementary functions and can only be determined by numerical integration.
3. Method of Binary Detection of SUAV

The useful signal \( A_k(t) \), modulated by multiplicative interference, is observed against the backdrop of additive Gaussian noise in the reception path. The noise correlation interval of the reception path does not exceed one microsecond. The duration of the useful signal is in the order of seconds [16]. Hence, disregarding the correlated noise samples \( n(t) \), we regard it as white Gaussian noise with a zero mean value and a probability distribution density of instantaneous values:

\[
W_s(n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{1}{2} \left(\frac{n}{\sigma_n}\right)^2}.
\] (11)

The density function described by formula (13) of the instantaneous values of the observed signal \( \hat{Y}(t) \) at the input of the detector characterizes a non-stationary random process with time-varying variance due to the motion of the target relative to the background surface [16].

It is known that the detection of small targets is based on the processing of the observed signal. In this case, the samples of such a signal are the amplitudes of LFM radio pulse batches observed over a time interval \( t \in [0, T] \). The optimal signal detection algorithm will be sought based on the minimum average risk criterion, taking into account which leads to the determination of a specific expression of the likelihood ratio.

The density distribution described by formula (13) of the instantaneous values of the observed signal \( Y(t) \) at the detector’s input characterizes a non-stationary random process with time-varying variance due to the target’s movement relative to the background surface.

It is known that the detection of small targets is based on the processing of the observed signal. In this case, the samples of such a signal are the amplitudes of LFM radio pulse batches observed over the time interval \( t \in [0, T] \). The optimal algorithm for detecting the useful signal will be sought based on the minimization of the average risk, the consideration of which leads to the determination of a specific expression for the likelihood ratio [17].

\[
L(Y) = \frac{W(Y/H_1)}{W(Y/H_0)}.
\] (14)
Hypothesis $H_1$ corresponds to the case of the UAV's movement in front of the background surface, where the radio signal reflected from the background is modulated by the reflected radio signal from the UAV. The model of the signal observed at the detector's input under the assumption of the validity of the hypothesis $H_1$ is described by the following expression:

$$Y(t) = A_{OTR} \cdot \sqrt{\sigma_f(\theta_{b_2}, \varepsilon_{b_2})} \cdot \sqrt{1+k_A(t)+2 \cdot k_A(t) \cdot \cos[\Delta \varphi(t)]} = A_{OTR} \cdot \sqrt{\sigma_f(\theta_{b_2}, \varepsilon_{b_2})} \cdot f(t) \quad (15)$$

The randomness of the process $Y(t)$ is caused by fluctuations in the Radar Cross-Section (RCS) $\sigma_f(\theta_{b_2}, \varepsilon_{b_2})$ of the background. The modulation law $Y(t)$ is determined by the function $f(t)$, which accounts for the non-stationary nature of the sample distribution density $Y$ under the condition of UAV movement. Assuming that the correlation interval of RCS fluctuations is much smaller than the duration of the useful signal, we can express the density distribution of sample values $Y$ under the presence of a moving UAV as follows:

$$W(Y / H_1, t) = \frac{2}{|A_{OTR}| \cdot \sqrt{2\pi \cdot \sigma_{f}}} \prod_{i=1}^{n} \frac{Y_i}{f(t_i)} \cdot e^{-\frac{1}{2 \cdot A_{OTR} \pi \sigma_{f}^{2}} \sum_{i=1}^{n} \left( \frac{y_i^2 - m_{f(t_i)}}{f(t_i)} \right)^2} \quad (16)$$

Hypothesis $H_0$ corresponds to the case of receiving a radio signal reflected from the background. The model of the signal observed at the detector's input under the assumption of the validity of the hypothesis $H_1$ is described by the following expression:

$$Y(t) = A_{OTR} \cdot \sqrt{\sigma_f(\theta_{b_2}, \varepsilon_{b_2})} = A_{OTR} \cdot \sqrt{\sigma_f(\theta_{b_2}, \varepsilon_{b_2})} \quad (17)$$

The density distribution of the sample $Y$ under the assumption of hypothesis $H_0$, in this case, is stationary, and, following a similar pattern to expressions (10) and (11), it is expressed as follows:

$$W(Y / H_0) = \frac{2}{|A_{OTR}| \cdot \sqrt{2\pi \cdot \sigma_{f}}} \prod_{i=1}^{n} Y_i \cdot e^{-\frac{1}{2 \cdot A_{OTR} \pi \sigma_{f}^{2}} \sum_{i=1}^{n} \left( \frac{y_i^2 - m_{f(t_i)}}{f(t_i)} \right)^2} \quad (18)$$

Substituting the conditional probability density functions (16) and (17) into (18), we obtain:

$$L(Y) = \left\{ \begin{array}{l} \frac{2}{|A_{OTR}| \cdot \sqrt{2\pi \cdot \sigma_{f}}} \prod_{i=1}^{n} \frac{Y_i}{f(t_i)} \cdot e^{-\frac{1}{2 \cdot A_{OTR} \pi \sigma_{f}^{2}} \sum_{i=1}^{n} \left( \frac{y_i^2 - m_{f(t_i)}}{f(t_i)} \right)^2} \\
\frac{2}{|A_{OTR}| \cdot \sqrt{2\pi \cdot \sigma_{f}}} \prod_{i=1}^{n} Y_i \cdot e^{-\frac{1}{2 \cdot A_{OTR} \pi \sigma_{f}^{2}} \sum_{i=1}^{n} \left( \frac{y_i^2 - m_{f(t_i)}}{f(t_i)} \right)^2} \end{array} \right. \quad (19)$$

The obtained expression describes the desired likelihood ratio for the problem of
detecting UAVs in the case of multiplicative interaction between a known useful signal and amplitude fluctuations of the background.

The algorithm for detecting UAVs involves comparing the expression (19), \( L(Y) \), with a certain threshold \( \gamma_0 \). We simplify the optimal detection algorithm described by formula (19) through logarithmization.

\[
\ln[L(Y)] = \sum_{i=1}^{\infty} \left[ \ln[f(t_i)]^{-1} - \frac{\sum_{i=0}^{\infty} \left[ \frac{Y_i^2}{f(t_i)} \right]^2 + \sum_{i=0}^{\infty} \left[ \frac{m_{a_i}}{f(t_i)} \right]^2}{2 \cdot A_{\text{TR}}^2 \cdot \sigma_{a_i}^2} \right. 
\sum_{i=0}^{\infty} \left[ \frac{(Y_i^2)^2}{f(t_i)} \right] + \left. 2 \cdot m_{a_i} \cdot \sum_{i=0}^{\infty} (Y_i^2^2 - f(t_i)) \right] \cdot \left[ 1 - \frac{1}{f(t_i)} \right] \right]
\]

(20)

Therefore, the optimal algorithm for UAV detection based on the Bayesian criterion, considering the multiplicative interaction between a known useful signal and amplitude fluctuations of the background, takes the following form [17]:

\[
z_{\text{II}}(t_i) = \frac{A_i - B_i}{2 \cdot A_{\text{TR}}^2 \cdot \sigma_{a_i}^2} - \sum_{i=0}^{\infty} \ln[f(t_i)]^{-1} + \ln \gamma_0 \quad \text{— the modified threshold of the detector,}
\]

\[
A_i = \sum_{i=0}^{\infty} \left[ \frac{m_{a_i}}{f(t_i)} \right] \]

\[
B_i = n \cdot m_{a_i}^2
\]

(21)

In the case of discrete sampling of observations \( Y(t) \) at the detector's input, it follows from expression (21) that to decide the presence of a signal caused by UAV at the detector's input, a series of operations involving the summation of nonlinearly transformed samples from the observed realization \( Y(t) \) and the multiplication of the square of the realization \( Y(t) \) with a copy of the expected useful signal, followed by summation of the obtained results and comparison with a threshold, should be performed [16].

A distinctive feature of the modified detector threshold \( z_{\text{II}}(t_i) \) is its time dependency proportional to the expected signal due to the non-stationary nature of the random process. When the threshold level is exceeded, the presence of the moving UAV is confirmed; otherwise, a decision is made about its absence [17].

The structure of the optimal detector for detecting a moving UAV under the considered conditions is depicted in Fig. 1.

Figure 1: Structural diagram of the optimal detector for UAV with known motion parameters

The structural diagram of the optimal detector does not show the synchronization device responsible for clocking the detector blocks. Since the additional phase shift during the reflection of the radio signal by the target and the background surface is random, it is necessary to add a second quadrature channel to the
structural diagram presented in Fig. 2.16, where the function $f(t)$ is defined with a phase shift of $\frac{\pi}{2}$ relative to the initial phase of the $f(t)$ function.  
The detection algorithm for UAV (21) is expedient to implement in the digital processing block of the amplitude signals received in the pulse sequence. However, for the implementation of this algorithm, including setting the threshold, precise knowledge of parameters such as the coordinates (angular position) of the phase center (point) of the background surface reflection, the values of current bistatic angles, the three-dimensional shape of the bistatic RCS of the target and background, is required. Furthermore, the start time of the UAV flight relative to the “radar-background” line of sight is unknown. In the conditions of a priori uncertainty about these parameters, the application of known approaches to eliminate this uncertainty significantly complicates the above algorithm and the structural diagram of the optimal detector [18]. To obtain a practically implementable UAV detection algorithm, we will make a series of simplifications relative to the observation model $Y(t)$. These simplifications will lead to the implementation of a quasi-optimal detection algorithm.

For such an observation model, the detector design has been explored by numerous authors [19, 20]. Let’s briefly outline the results of solving this problem. We will assume that the data observation sampling interval is $\Delta t = \frac{1}{2 \cdot f_b}$.

$$W(Y / H_1) = \frac{2}{A_{\text{OTR}}' \cdot \sqrt{2 \pi \cdot \sigma_f}} \cdot \prod_{i=1}^{n} \left( Y_i \cdot e^{\frac{1}{2 \cdot A_{\text{OTR}}' \cdot \sigma_f} \sum_{i}^{n} (Y_i - m_{ei})^2} \right)$$

(23)

$$W_s(n) = \frac{1}{\sqrt{2 \pi \cdot \sigma_n}} e^{-\frac{(n - \mu_n)^2}{2 \cdot \sigma_n^2}}.$$  

(24)

Where $\sigma_n$ is the root mean square deviation of the noise samples in the receiving path.

Under these conditions, the expression for the likelihood ratio will be written in a known manner [19]:

$$L(Y) = e^{-\frac{\Delta t}{N_0} \sum_{i=1}^{\infty} \frac{1}{2 \cdot \sigma_n^2} \sum_{i}^{n} (Y_i - m_{ei})^2}.$$  

(25)

The formula can be alternatively expressed as a likelihood ratio functional, which $N_0 = \sigma_n^2 \cdot f_b$ represents the spectral power density of the noise in the receiving system.

For the likelihood ratio (22), the probability density in the presence of a signal $H_1$ is expressed as follows:

$$z = \frac{2}{N_0} \int_{0}^{T} Y(t) A(t) dt$$

(26)

Instead of comparing it to the threshold of the likelihood ratio or function, we can compare the logarithms of expressions (25) or (26). Thus, we obtain the following decision rule for the considered detection problem [21]:

$$z_{\Omega}(t) = \ln \frac{E_1}{N_0} + \frac{E_1}{N_0}$$

represents the modified threshold. Therefore, the detection device for a moving small-sized target under these conditions corresponds to the well-known correlation receiver scheme depicted in Fig. 2.
The synchronization device ensures coordinated operation between the reference generator and the integrator, facilitating the comparison of its output signal \( z(t) \) with the threshold. To ensure the functionality of the correlation detector, it is necessary to multiply the reference and observed signals at coincident time points. However, the arrival time of the observed signal is unknown.

\[ Y(t) \rightarrow h(\tau) \rightarrow z(t) \rightarrow RC \]

**Figure 2:** Structural diagram of the correlation detector for UAV with known parameters of its motion and initial phase

In this case, the reference signal of the correlation detector should be time-shifted relative to the observed signal, and a procedure for searching and capturing the useful signal should be performed. To simplify this procedure, instead of a correlation detector for the useful signal, its version with matched filters should be used. When the useful signal’s time coincides with the impulse response of the matched filter, the value of the correlation integral will match the amplitude of the output signal of the matched filter. The impulse response of the matched filter \( h(\tau) \) for the useful signal \( A(t) \) is its mirrored copy, shifted in time by \( t_0 \). The structural diagram of the UAV detector with known parameters of its motion using matched filters is shown in Fig. 3.

\[ Y(t) \rightarrow h(\tau) \rightarrow z(t) \rightarrow RC \]

**Figure 3:** Structural diagram of the UAV detector using a matched filter with known parameters of its motion and initial phase

The reference signal \( A(t) \) has a random initial phase due to the reflection of radio waves from the target, underlying surfaces, and background. When radio waves are reflected from these objects, an additional phase shift becomes random.

To eliminate the dependence of the reference signal on the influence of random phase shifts during the reflection of radio waves, we use a structural scheme of a detector for a signal of known shape with a random initial phase. In this scheme, the detector contains two quadrature generators of reference signals, \( A_0(t) \) and \( A'_0(t) \):

\[ A_0(t) = \sqrt{(A_0^2) + (A'_0)^2} = const. \]  

Similarly to how the envelope of a harmonic signal expressed through quadrature components does not depend on time:

\[ E_0(t) = \sqrt{E^2 \cos^2(\phi) + E^2 \sin^2(\phi)} = E. \]  

In this case, processing the observed signal in the small-sized target detection task will involve comparing it to a threshold using the following decision statistic:

\[ z(t) = \sqrt{\int_{-\infty}^{\infty} Y(t)A'_0(t-\tau)d\tau}^2 + \int_{-\infty}^{\infty} Y(t)A_0(t-\tau)d\tau}^2, \]  

where \( Y(t) \) is the observed realization of the signal at the input of the detector.

\( A'_0(t) \) is the sine component of the reference signal for the matched filter \( h_0(t) \).

The structural diagram of the quadrature detector for the UAV with known parameters of its motion using matched filters is shown in Fig. 4 [5].
4. Conclusions

Thus, if the motion parameters of the UAV are known and the background reflection characteristics are sufficiently stable, the detector can be represented by a correlation scheme or a scheme with matched filters.

The structural scheme of the SUAV useful signal detection device for the background radar based on parallel matched filters is developed.

Based on approximations of the widths of functions describing the change in the amplitude of the matched filter response at the mismatch in the parameters of the useful signal, the method and algorithm for calculating the number of matched filters of the IBPLA detector for the background radar are obtained.

References


