Capturing An Optimal Trading Framework Involving Exponential and Second-Order Autoregressive Price Dynamics

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Abstract

In the constantly changing financial markets, investors and traders need to trade using optimal trading strategies. In this article, we optimize the expected cost and the execution risk to develop an optimal trading strategy for risk-averse investors. In this context, we study a second order convex function that incorporates both transient and permanent market impacts in the price path rule of motion. By the use of dynamic programming, we get a closed form solution of the unconstrained problem, in some particular cases for the risk-averse investor.

Keywords

Optimal trading, execution cost, autoregressive, quadratic programming problem

1. Introduction

The optimum trading issue is a significant difficulty faced by academics, algorithmic traders, and market practitioners. Performing all required asset shares in a single transaction is inefficient because of it's potential for considerable price fluctuations, particularly when dealing with large trade volumes. Additionally, privileged knowledge about the asset and the constantly changing nature of the market affect price variations, in addition to the trade order. Every order made in the market provides information on the trading and financial goals of investors, so influencing the viewpoints of other investors over a prolonged duration. As a result, the market expects changes in the stock's fundamental worth or predicts that future prices will deviate from earlier estimates.

The term "no-impact price" refers to the price of an asset when there are no trades or external factors affecting it. Temporary price implications occur due to the urgent need for execution, leading to short-term price fluctuations. On the other hand, the term "permanent price effect" refers to the steady irregularity caused by imbalances in supply and demand, as well as other causes discussed before. The interaction between the price impact factor and the price path without any influence determines the execution price dynamics of an asset. Several models of price dynamics during execution had been suggested in the literature, such as those given by [1];[2];[3];[4];[5];[6].

[1], had proposed three models to explain the dynamics of execution prices: the Linear Price Impact Model, the Linear Price Impact with Information Price Model, and the Linear Percentage Temporary Price Impact Model (LPT model). As compared to the first two models, which represented the no-impact price using an arithmetic random walk, the LPT model models it using a geometric Brownian motion. As a result of the linear price impact model, the price change is directly proportional to the trade magnitude. However, in the linear price impact with information price model, the price impact is directly proportional to both the trade size and the information component. The information component captures the influence of characteristics such as asset information and changing market circumstances on the execution price. The LPT model [1], quantified the price effect by a linear equation that took

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into account the transaction size and an information factor, which was calculated as a percentage of the price without any impact. [2], proposed linear price impact functions and integrated an arithmetic random walk to model the dynamics of execution prices, using the no-effect price as a benchmark. [1], introduced the initial mathematical optimization model for inscribe the optimal trading problem. Further, other researcher, including, [7],[2],[3],[4],[5], explored the inquires related the optimal trading for single asset. [6], introduced the jump diffusion model to capture the unpredictable effects of large transactions on prices during execution. In this model, the purpose is to provide a better understanding of the fluctuations in asset values during significant transactions. However [8]; and [9], proposed the use of stochastic dominance for all types of investors in portfolio optimization.Using the same approach, [5], analyzed optimal trading strategies. In addition, [10], extended the linear price impact with information price model of [1], referred to as the AR(1) price model, to a more general framework. To describe the evolution of asset prices over time, the AR(2) model is commonly used in optimal trading strategies. A second-order autoregressive model is called an AR(2) model.

In this paper, we extend the AR(2) model of [10], to a more general framework. We use the convex combination of temporary and permanent price impacts under the AR(2) model with an AR(2) information factor.

2. Motivation and Contribution of the paper

In existing literature, no-impact price dynamics are often described as arithmetic random walks.According to this assumption, the price of the asset in any given period will be determined by the price of the asset in the previous period, reflecting a first-order autoregressive AR(1) behavior. Using a more comprehensive model for the linear price impact with information, introduced by [1], this paper makes a significant contribution to the literature.In the extension, the second-order autoregressive with exponent behavior is incorporated into the AR(2) price model for both permanent and temporary price impacts by [5]. The information factor indicates that the state in any given time period depends on the state in two previous ones.

3. The optimization problem

Consider an investor with a goal to buy a significant amount of shares, let's call it $\bar{S}$, within a specific time period, say $[0, T]$. This time frame is split into smaller intervals, each with a length of $\Delta t = \frac{T}{N}$, and the t^{th} time frame is equal to $t\Delta t$.

Let's use S_t to represent the shares acquired during the interval t^{th} at a price of P_t, and W_t will be the remaining shares to buy at time t^{th}.

Now, the investor wants to minimize the expected cost of getting $\bar{S}$ shares while keeping a handle on a predetermined risk level. This perspective on risk is inspired by the work of Almgren and Chriss (2000) and Huberman and Stanzl (2005). So, in a nutshell, the investor's challenge is to navigate this buying process in a way that minimizes costs and manages the associated risks effectively. The problem **(P1)**

$$\begin{gathered}\min_{\{S_t\}_{t=1}^{T}} \ E\left(\sum_{t=1}^{T} P_t S_t\right) \\ \text{subject to} \\ \sum_{t=1}^{T} S_t = \overline{S}\end{gathered} \tag{1}$$

where

$$W_1 = \overline{S}, \quad W_t = W_{t-1} - S_{t-1} \quad \forall\, t = 1, 2, ..., T, \quad W_{T+1} = 0.$$

In outlining the dynamics of the stock price for this dynamic programming problem, let's denote $\tilde{P}_t$ as the observed stock price at t^{th} time. To simplify, we'll assume that market prices follow geometric Brownian motion. This allows us to express the market price at t^{th} time as follows:

$$\tilde{P}_t = \alpha \exp(Z_t) P_{t-1} + (1-\alpha) u_1 P_{t-1} + (1-\alpha) u_2 P_{t-2} \qquad ; \alpha \in [0,1] \tag{2}$$

A trader's decision-making involves not only the current stock price but also factors in the broader market conditions. We represent this by introducing the market information variable, denoted as X_t. This variable captures the potential influence of evolving market conditions, encompassing aspects like variations in buy-sell volume or private information about the security.

To model this variable, we adopt an autoregressive process of order 2 that is (AR(2)). In simpler terms, the market information variable at t^{th} time, denoted as X_t, is represented by its value at the previous time step.

$$X_t = \rho_1 X_{t-1} + \rho_2 X_{t-2} + \eta_t \tag{3}$$

Here, η_t is a white noise process with zero mean and variance σ_η^2. At t^{th} time, the trader observes the price $\tilde{P}_t$ and market information X_t and decides to trade S_t shares. We assume that the price impact of trading is a linear function of S_t and X_t. Therefore, the average execution price P_t of trading S_t shares at t^{th} time is given by:

$$P_t = \tilde{P}_t + \theta S_t + \gamma X_t + \epsilon_t \tag{4}$$

The average execution price P_t of trading S_t shares at t^{th} is determined by the observed market price $\tilde{P}_t$, the quantity of shares traded S_t, and the market information X_t. Additionally, there is an uncertainty component ϵ_t denoting the noise in the acquisition price, characterized by a white noise process with zero mean and variance σ_ϵ^2.

The motion of price expressed in equ. $(2)-(4)$, is the generalization of [1] linear temporary price impact. Once the order placed at time t_{th} is executed, the resulting stock price will be influenced not only by the actions of other market participants but also by the lasting impact of executing S_t shares.

4. Optimal trading under AR(2) price model with exponents: dynamic programming method

In solving the given problem using dynamic programming (DP) method, denoted as **(P1)**, we leverage the insight presented in (Bertsimas and Lo (1998)), According to this approach, the portion of the optimal trading strategy $(S_t, S_{t+1}, ..., S_T)$ during any time period t, corresponding to time periods t to T, must be optimal for these time intervals.

In the context of an auto regressive model of order 2 (AR(2)), specifically considering the last two trading periods P_{t-1} and P_{t-2}, the relevant information factors include X_t and X_{t-1}. Additionally, the factor W_t represents the number of shares that remain to be traded after the $(t-1)^{th}$time period. The control variable for the t^{th} time period is denoted by the number of shares S_t to be traded during that specific period.

Consider the problem **(P1)**, to be solved by DP method.

DP Problem

$$\begin{aligned} &\min_{\{S_t\}_{t=1}^T} \; E(\sum_{t=1}^T P_t S_t) \\ &\text{subject to} \\ &\sum_{t=1}^T S_t = \overline{S} \\ &W_1 = \overline{S}, \quad W_t = W_{t-1} - S_{t-1} \qquad \forall\, t = 1,2,...,T \quad W_{T+1} = 0. \end{aligned} \tag{5}$$

Let

$$V_t(\tilde{P}_t, P_{t-1}, P_{t-2}, X_t, X_{t-1}, W_t) = E_t(\sum_{k=1}^{T} P_t S_t) \quad \forall\, t = 1, 2, ..., T. \tag{6}$$

be the optimal cost of execution corresponding to trades into periods $t\ to\ T$.
During the final trading period T, the optimal strategy is to execute all the remaining shares W_T to finish the trade. i.e. $S_T = W_T$. Thus V_T is given as follows:

$$\begin{aligned} V_T(\tilde{P_T}, P_{T-1}, P_{T-2}, X_T, X_{T-1}, W_T) &= \min_{S_t} E_T[P_T S_T] \\ &= (\tilde{P_T} + \theta W_T + \gamma X_T) W_t \end{aligned} \tag{7}$$

The Bellman Equation relating $V_{T-t}\ \ to\ \ V_{T-t+1}$, is given as follows:

$$\begin{aligned} &V_{T-t}(\tilde{P_{T-t}}, P_{T-t-1}, P_{T-t-2}, X_{T-t}, X_{T-t-1}, W_{T-t}) \\ &\quad = \min_{S_{T-t}} E_{T-t}[P_{T-t} S_{T-t} + V_{T-t+1}(\tilde{P_{T-t+1}}, P_{T-t}, P_{T-t-1}, X_{T-t+1}, X_{T-t}, W_{T-t+1})] \end{aligned} \tag{8}$$

Proposition 1. *Under the AR(2), price model Under the Execution price dynamics P_t, Eq. (2) and (3) with information factor (4)*

$$\begin{aligned} &P_t = \tilde{P}_t + \theta S_t + \gamma X_t + \epsilon_t \\ &\tilde{P}_t = \alpha \exp(Z_t) P_{t-1} + (1-\alpha) u_1 P_{t-1} + (1-\alpha) u_2 P_{t-2} \\ &X_t = \rho_1 X_{t-1} + \rho_2 X_{t-2} + \eta_t \end{aligned} \tag{9}$$

In each time period corresponding to the DP problem, the following equation gives the optimal trade and the optimal cost:

$$S_{T-r} = A_{r-1} \tilde{P}_{T-r} + B_{r-1} X_{T-r} + C_{r-1} X_{T-r-1} + D_{r-1} P_{T-r-1} + E_{r-1} W_{t-r} \tag{10}$$

$$\begin{aligned} V_{T-r}(\tilde{P}_{T-r}, P_{T-r-1}, P_{T-r-2}, X_{T-r}, X_{T-r-1}, W_{T-r}) &= a_r \tilde{P}^2_{T-r} + b_r \tilde{P}_{T-r} X_{t-r} \\ + c_r \tilde{P}_{T-r} X_{T-r-1} + d_r \tilde{P}_{T-r} P_{T-r-1} &+ e_r \tilde{P}_{T-r} W_{T-r} + f_r X^2_{T-r} + g_r X^2_{T-r-1} \\ + h_r X_{T-r} W_{T-r} + i_r X_{T-r-1} X_{T-r} &+ j_r X_{T-r-1} W_{T-r} + k_r W^2_{T-r} \\ + l_r P_{T-r-1} X_{T-r} + m_r P^2_{T-r-1} &+ n_r P_{T-r-1} X_{T-r-1} + o_r P_{T-r-1} W_{T-r} \end{aligned} \tag{11}$$

where all the value should be put at initial.

$$\begin{aligned} m_r &= 2\theta + 2a_{r-1}(1-\alpha)^2 u_1^2 \theta^2 + 2d_{r-1}(1-\alpha) u_1 \theta^2 - 2e_{r-1}(1-\alpha) u_1 \theta \\ &\quad + 2k_{r-1} + 2m_{r-1}\theta - 2o_{r-1}\theta \\ \delta_{1,r} &= o_{r-1} - 2m_{r-1}\theta + (\alpha q + (1-\alpha) u_1)\, e_{r-1} - 2(1-\alpha) u_1 \theta \left((1-\alpha) u_1 - \alpha q\right) a_{r-1} \\ &\quad - \theta \left(\alpha q + 2(1-\alpha) u_1\right) d_{r-1} - 1 \\ \delta_{2,r} &= o_{r-1}\gamma - n_{r-1}\theta - 2m_{r-1}\theta\gamma - l_{r-1}\rho_1\theta + j_{r-1} + h_{r-1}\rho_1 - e_{r-1}(1-\alpha) u_1 \gamma \\ &\quad - 2d_{r-1}(1-\alpha) u_1 \theta\gamma - c_{r-1}(1-\alpha) u_1 \theta - b_{r-1}\rho_1 (1-\alpha) u_1 \theta \\ &\quad - 2a_{r-1}(1-\alpha)^2 u_1^2 \theta\gamma - \gamma \\ \delta_{3,r} &= h_{r-1}\rho_2 - b_{r-1}\rho_2 (1-\alpha) u_1 \theta - l_{r-1}\rho_2\theta \\ \delta_{4,r} &= e_{r-1} u_2 - 2a_{r-1}(1-\alpha)^2 u_1 u_2 \theta - d_3 (1-\alpha) u_2 \theta \\ \delta_{5,r} &= 2k_{r-1} - o_{r-1}\theta - e_3 (1-\alpha) u_1 \theta \end{aligned} \tag{12}$$

From the above value we, calculate S_{T-r} as follows:

$$A_{r-1} = \frac{\delta_{1,r}}{m_r}, \quad B_{r-1} = \frac{\delta_{2,r}}{m_r}, \quad C_{r-1} = \frac{\delta_{3,r}}{m_r}, \quad D_{r-1} = \frac{\delta_{4,r}}{m_r}, \quad E_{r-1} = \frac{\delta_{5,r}}{m_r}$$

where

$$\begin{aligned} a_r = {} & \theta + a_{r-1}(1-\alpha)^2 u_2^1 \theta + d_{r-1}(1-\alpha)u_1\theta^2 - e_{r-1}(1-\alpha)u_1\theta + k_{r-1}^2 \\ & + m_r\theta^2 - o_{r-1})A_r^2 + (1 + 2a_{r-1}(1-\alpha)^2 u_1^2\theta + m_{r-1} + (2a_{r-1}\alpha q - e_{r-1}\alpha q - \\ & e_{r-1}(1-\alpha)u_1)\theta + a_{r-1}\alpha^2 q^2 + (d_{r-1}\alpha q + 2d_{r-1}(1-\alpha)u_1)\theta - o_{r-1}) A_r \\ & + 2\alpha q(1-\alpha)u_1 a_{r-1} + d_{r-1}\alpha q(1-\alpha)u_1 d_{r-1} \end{aligned} \tag{13}$$

$$\begin{aligned} b_r = {} & B_{r-1}(1 + 2\theta A_{r-1} + \gamma A_{r-1} + 2a_{r-1}(1-\alpha)^2 u_1^2\theta^2 A_{r-1} + 2a_{r-1}(1-\alpha)^2 u_1^2\theta \\ & + 2a_{r-1}\alpha q(1-\alpha)u_1\theta) + A_{r-1}(2k_{r-1}B_{r-1} + \rho_1 l_{r-1} + \theta l_{r-1}\rho_1 + 2m_{r-1}(\theta^2 B_{r-1} \\ & + \theta + \gamma + \theta\gamma) + n_{r-1}(1+\theta) - o_{r-1}(B_{r-1} + 2\theta B_{r-1} + \gamma)) + \rho_1 l_{r-1}\theta A_{r-1} \\ & - j_{r-1}A_{r-1} - h_{r-1}\rho_1 + c_{r-1}(\alpha q - (1-\alpha)u_1 + (1-\alpha)u_1\theta) + d_{r-1}(\alpha q\theta \\ & + \alpha q\gamma + (1-\alpha)u_1\theta^2 + 2B_{r-1} + 2(1-\alpha)u_1\theta + 2(1-\alpha)u_1\gamma + 2(1-\alpha)u_1\theta\gamma) \\ & - e_{r-1}(\alpha q + (1-\alpha)u_1 + 2(1-\alpha)u_1\theta + (1-\alpha)u_1\gamma) + b_{r-1}\rho_1(\alpha q + (1-\alpha)u_1 \\ & + (1-\alpha)u_1\theta) + 2a_{r-1}((1-\alpha)^2u_1^2\theta^2 + (1-\alpha)^2u_1^2\theta + \alpha q(1-\alpha)u_1\theta + \alpha q \\ & (1-\alpha)u_1\gamma + c_{r-1}(\alpha q - (1-\alpha)u_1 + (1-\alpha)u_1\theta) + 2d_{r-1}((1-\alpha)u_1\theta^2 \\ & + (1-\alpha)u_1\theta + \alpha q\theta + \gamma) - e_{r-1}(\alpha q + (1-\alpha)u_1) + \rho_1 l_{r-1}\theta \\ & + o_{r-1}(2\theta A_{r-1} + \gamma A_{r-1}) \end{aligned} \tag{14}$$

$$\begin{aligned} c_r = {} & c_{r-1} + 2\theta A_{r-1}C_{r-1} + C_{r-1}\theta[2a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1 + 2a_{r-1}\alpha q \\ & (1-\alpha)u_1] + \rho_2[b_{r-1}(\alpha q + (1-\alpha)u_1 + (1-\alpha)u_1\theta A_{r-1}) + l_{r-1}(1 + \theta A_{r-1})] \\ & + \theta C_{r-1}[2d_{r-1}(1-\alpha)u_1 - e_{r-1}(2a_{r-1}(1-\alpha)u_1 - 2(1-\alpha)u_1)] - o_{r-1}C_{r-1} \end{aligned} \tag{15}$$

$$\begin{aligned} d_r = {} & D_{r-1} + 2\theta A_{r-1}D_{r-1} + D_{r-1}\theta[2a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1 + 2a_{r-1} \\ & \alpha q(1-\alpha)u_1] + \theta D_{r-1}[2a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1 + 2a_{r-1}\alpha q(1-\alpha)u_1] \\ & + \alpha q D_{r-1}[d_{r-1} - e_{r-1}] + \theta D_{r-1}[2d_{r-1}(1-\alpha)u_1 - e_{r-1}(2a_{r-1}(1-\alpha)u_1 \\ & - 2(1-\alpha)u_1)] + d_{r-1}(1-\alpha)u_2 + \theta A_{r-1}D_{r-1}[2a_{r-1}(1-\alpha)u_1 - 2(1-\alpha)u_1 \\ & - e_{r-1}(2o_{r-1})] + \theta D_{r-1}[2m_{r-1}\theta + 2m_{r-1}] - o_{r-1}D_{r-1} \end{aligned} \tag{16}$$

$$\begin{aligned} e_r = {} & E_{r-1} + 2\theta A_{r-1}E_{r-1} + E_{r-1}\theta[2a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1 + 2a_{r-1}\alpha q \\ & (1-\alpha)u_1] + \theta E_{r-1}[2a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1 + 2a_{r-1}\alpha q(1-\alpha)u_1] \\ & + \theta E_{r-1}[2d_{r-1}(1-\alpha)u_1 - e_{r-1}(2a_{r-1}(1-\alpha)u_1 - 2(1-\alpha)u_1)] + e_{r-1}(1-\alpha)u_1 \\ & + 2k_{r-1}A_{r-1}E_{r-1} + m_{r-1}\theta^2 + 2m_{r-1}\theta E_{r-1} + o_{r-1} + o_{r-1}\theta A_{r-1} \\ & - o_{r-1}E_{r-1} - 2o_{r-1}\theta A_{r-1}E_{r-1} \end{aligned} \tag{17}$$

$$\begin{aligned} f_r = {} & \theta B_{r-1}^2[1 + a_{r-1}(1-\alpha)^2u_1^2 + d_{r-1}(1-\alpha)u_1\theta + k_{r-1} + m_{r-1}\theta^2 - o_{r-1}\theta] \\ & + \gamma[B_{r-1} + a_{r-1}(1-\alpha)^2u_1^2 + 2a_{r-1}(1-\alpha)^2u_1^2\theta B_{r-1} + b_{r-1}\rho_1(1-\alpha)u_1 \\ & + d_{r-1}(1-\alpha)u_1\gamma + l_{r-1}\rho_1\gamma + n_{r-1}\theta + n_{r-1}\theta\gamma - o_{r-1}\gamma] + f_{r-1}\rho_1^2 \\ & + c_{r-1}(1-\alpha)u_1\theta B_{r-1} + g_{r-1} - h_{r-1}\rho_1 B_{r-1} + i_{r-1}\rho_1 - j_{r-1}B_{r-1}. \end{aligned} \tag{18}$$

$$
\begin{aligned}
g_r = {} & \theta C_{r-1}^2[1 + a_{r-1}(1-\alpha)^2u_1^2\theta + d_{r-1}(1-\alpha)u_1\theta + k_{r-1} + m_{r-1}\theta^2 - o_{r-1}\theta] + \rho_2^2 f_{r-1} \\
& + \theta C_{r-1}b_{r-1}\rho_2(1-\alpha)u_1 - \rho_2 C_{r-1}h_{r-1} + \theta C_{r-1}l_{r-1}\rho_2 - \theta C_{r-1}e_{r-1}(1-\alpha)u_1.
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
h_r = {} & \theta[2B_{r-1}C_{r-1} + \gamma E_{r-1} + 2a_{r-1}(1-\alpha)^2u_1^2\theta B_{r-1}E_{r-1} + b_{r-1}\rho_1(1-\alpha)u_1E_{r-1} \\
& + 2a_{r-1}(1-\alpha)^2u_1^2\gamma E_{r-1}] + c_{r-1}(1-\alpha)u_1E_{r-1} + 2d_{r-1}(1-\alpha)u_1\theta B_{r-1}E_{r-1} \\
& + 2d_{r-1}(1-\alpha)u_1\gamma E_{r-1} + e_{r-1}(1-\alpha)u_1B_{r-1} + e_{r-1}(1-\alpha)u_1\gamma - e_{r-1}(1-\alpha)u_1 \\
& \gamma E_{r-1} - 2e_{r-1}(1-\alpha)u_1\theta B_{r-1}E_{r-1} + h_{r-1}\rho_1 - h_{r-1}\rho_1E_{r-1} + j_{r-1} - j_{r-1}E_{r-1} \\
& + 2k_{r-1}B_{r-1}E_{r-1} - 2k_{r-1}B_{r-1} + l_{r-1}\rho_1\theta E_{r-1} + 2m_{r-1}\theta^2B_{r-1}E_{r-1} + 2m_{r-1}\theta\gamma E_{r-1} \\
& + n_{r-1}\theta E_{r-1} + o_{r-1}\theta B_{r-1} + o_{r-1}\gamma - 2o_{r-1}\theta B_{r-1}E_{r-1} - o_{r-1}\gamma E_{r-1}]
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
i_r = {} & \theta C_{r-1}[2\theta B_{r-1} + \gamma + c_{r-1}(1-\alpha)u_1 + n_{r-1}\theta - j_{r-1}] + \theta B_{r-1}[2a_{r-1}(1-\alpha)^2u_1^2\theta \\
& + b_{r-1}\rho_2(1-\alpha)u_1 + l_{r-1}\rho_2\theta + 2m_{r-1}\theta^2] + \gamma[b_{r-1}\rho_2(1-\alpha)u_1 + l_{r-1}\rho_2 + o_{r-1}] \\
& + 2f_{r-1}\rho_1\rho_2 - h_{r-1}\rho_1C_{r-1} - h_{r-1}\rho_2B_{r-1} + i_{r-1}\rho_2 + 2k_{r-1}B_{r-1}C_{r-1} + l_{r-1}\rho_1\theta C_{r-1} \\
& + 2d_{r-1}(1-\alpha)u_1\theta[\theta B_{r-1} + \gamma C_{r-1}] - e_{r-1}(1-\alpha)u_1\left(\theta B_{r-1}C_{r-1} + \gamma C_{r-1}\right) \\
& - 2o_{r-1}\theta[B_{r-1}C_{r-1} + \gamma C_{r-1}]
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
j_r = {} & \theta C_{r-1}[2 + 2a_{r-1}(1-\alpha)^2u_1^2\theta + b_{r-1}\rho_2(1-\alpha)u_1 + e_{r-1}(1-\alpha)u_1 + l_{r-1}\rho_2\theta E_{r-1} \\
& - 2e_{r-1}(1-\alpha)u_1E_{r-1} + 2d_{r-1}(1-\alpha)u_1\theta^2] + h_{r-1}\rho_2[1 - E_{r-1}] - 2k_{r-1}C_{r-1} \\
& [1 - E_{r-1}] + 2m_{r-1}\theta^2C_{r-1}E_{r-1} + o_{r-1}\theta C_{r-1}\left(1 - 2E_{r-1}\right).
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
k_r = {} & \theta E_{r-1}^2[1 + a_{r-1}(1-\alpha)^2u_1^2 + d_{r-1}(1-\alpha)u_1\theta + m_{r-1}\theta - o_{r-1}\theta] + e_{r-1}(1-\alpha)u_1 \\
& \theta e_{r-1}^2 + k_{r-1}\left(1 + E_{r-1}^2 - 2E_{r-1}\right).
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
l_r = {} & \theta D_{r-1}[2B_{r-1} + \gamma + 2a_{r-1}(1-\alpha)^2u_1^2\theta + 2a_{r-1}(1-\alpha)^2u_1\gamma + 2a_{r-1}(1-\alpha)^2 \\
& u_1u_2\gamma + b_{r-1}\rho_1(1-\alpha)u_1\theta + c_{r-1}(1-\alpha)u_1\theta] + 2d_{r-1}(1-\alpha)u_1\theta^2B_{r-1} \\
& + 2d_{r-1}(1-\alpha)u_1\theta\gamma + d_{r-1}(1-\alpha)u_2\gamma - 2e_{r-1}(1-\alpha)u_1\theta B_{r-1} - e_{r-1}(1-\alpha) \\
& u_1\gamma - e_{r-1}(1-\alpha)u_2B_{r-1} - h_{r-1}\rho_1 - j_{r-1} + 2k_{r-1}B_{r-1} + l_{r-1}\rho_1\theta + 2m_{r-1}\theta^2B_{r-1} \\
& + 2m_{r-1}\theta\gamma + n_{r-1}\theta - 2o_{r-1}\theta B_{r-1} - o_{r-1}\gamma
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
m_r = {} & \theta D_{r-1}^2[1 + a_{r-1}(1-\alpha)^2u_1^2 + k_{r-1} + m_{r-1}\theta^2 - o_{r-1}\theta] + a_{r-1}(1-\alpha)^2u_2^2 + d_{r-1} \\
& (1-\alpha)u_2\theta - e_{r-1}(1-\alpha)u_2D_{r-1}.
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
n_r = {} & \theta D_{r-1}[2C_{r-1} + 2a_{r-1}(1-\alpha)^2u_1^2\theta C_{r-1} + b_{r-1}\rho_2(1-\alpha)u_2 + 2d_{r-1}(1-\alpha)u_1\theta^2C_{r-1}] \\
& + d_{r-1}(1-\alpha)u_2\theta - 2c_{r-1}(1-\alpha)u_1\theta C_{r-1} - e_{r-1}(1-\alpha)u_2 - h_{r-1}\rho_2 + 2k_{r-1}C_{r-1} \\
& + l_{r-1}\rho_2\theta + 2m_{r-1}\theta^2C_{r-1} - 2o_{r-1}\theta C_{r-1}
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
o_r = {} & 2\theta D_{r-1}E_{r-1}[1 - e_{r-1}(1-\alpha)u_1\theta + a_{r-1}(1-\alpha)^2u_1^2\theta^2 - 2o_{r-1}] + d_{r-1}(1-\alpha)u_2 \\
& \theta E_{r-1} + e_{r-1}(1-\alpha)u_2 - 2e_{r-1}(1-\alpha)u_1\theta D_{r-1}E_{r-1} - e_{r-1}(1-\alpha)u_2E_{r-1} \\
& + 2k_{r-1}D_{r-1}E_{r-1} - 2k_{r-1}D_{r-1} + 2m_{r-1}\theta^2D_{r-1}E_{r-1} + o_{r-1}\theta D_{r-1}
\end{aligned} \tag{27}
$$

Examining the influence of a change in α on the strategy is another noteworthy aspect. In the price path, α determines the degree to which our trade has a lasting impact on the market price. To illustrate this, we conducted a numerical example involving the purchase of 1000 shares, using simulation parameters from [10].

$$T = 20, \quad \overline{S} = 1000, \quad P_0 = 50, \quad X_0 = 0, \quad \theta = 5 \times 10^{-5},$$
$$\rho_1 = 0.5, \quad \rho_2 = 0.5, \quad u_1 = 0.5, \quad u_2 = 0.5, \quad \sigma_\eta = \sqrt{0.001}$$

Price process and market information have been considered as $\epsilon_t \sim N(0, \sigma_\eta^2)$. Figure 1 represent the optimal trading strategy with α. We have also investigated how strategies change based on whether the market is expected to be bullish or bearish. We consider the three market categories.

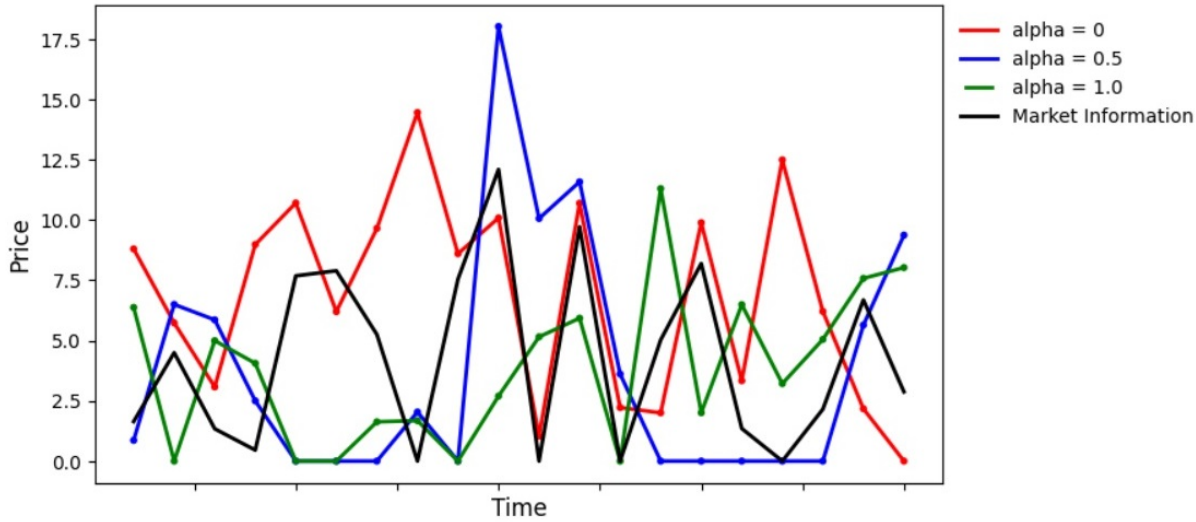

Figure 1: The optimal execution plans for investors with different levels of risk aversion. The market information is shown by the black curve. For $\alpha = 0$ (only permanent market impact), the red curve exhibits the same trend as the available market data.

In addition, we analyzed how strategies differ when market expectations are bullish or bearish.we examined three types of market situations by taking different value of α. There are different market scenario mention as follows in figure 3 and figure 4 as follows:
Additionally, we have examined how trading strategies adapt to different market expectations, whether bullish or bearish. We are considering three types of market scenario.

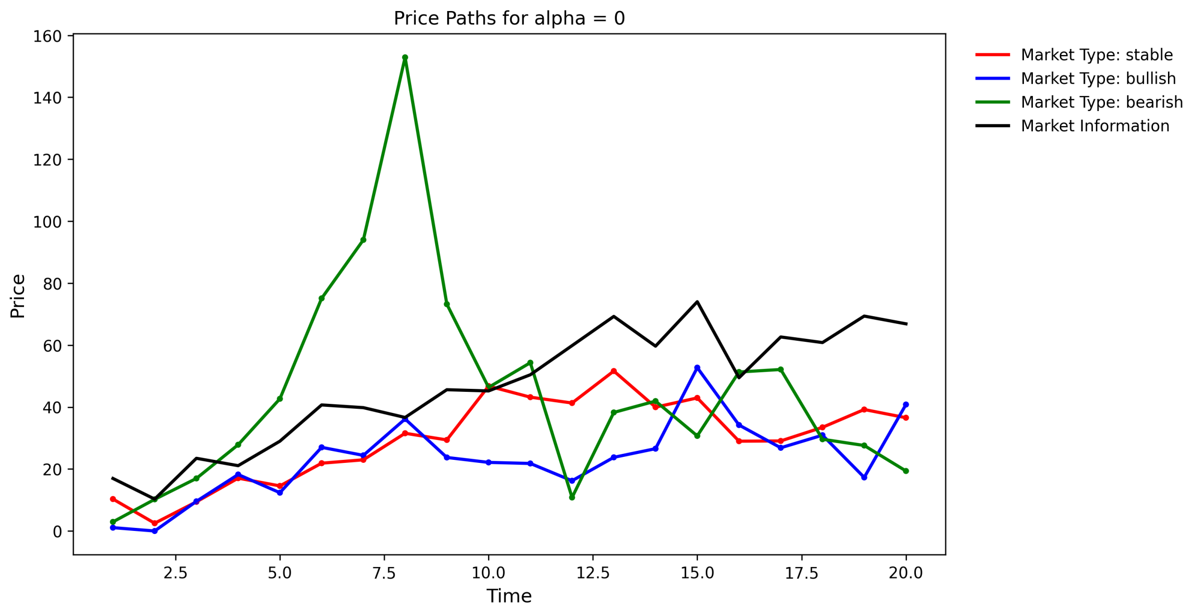

Figure 2: The optimal execution plans for investors with different levels of risk aversion. The market information is shown by the black curve. For $\alpha = 0$ (only permanent market impact), the red curve exhibits the same trend as the available market data.

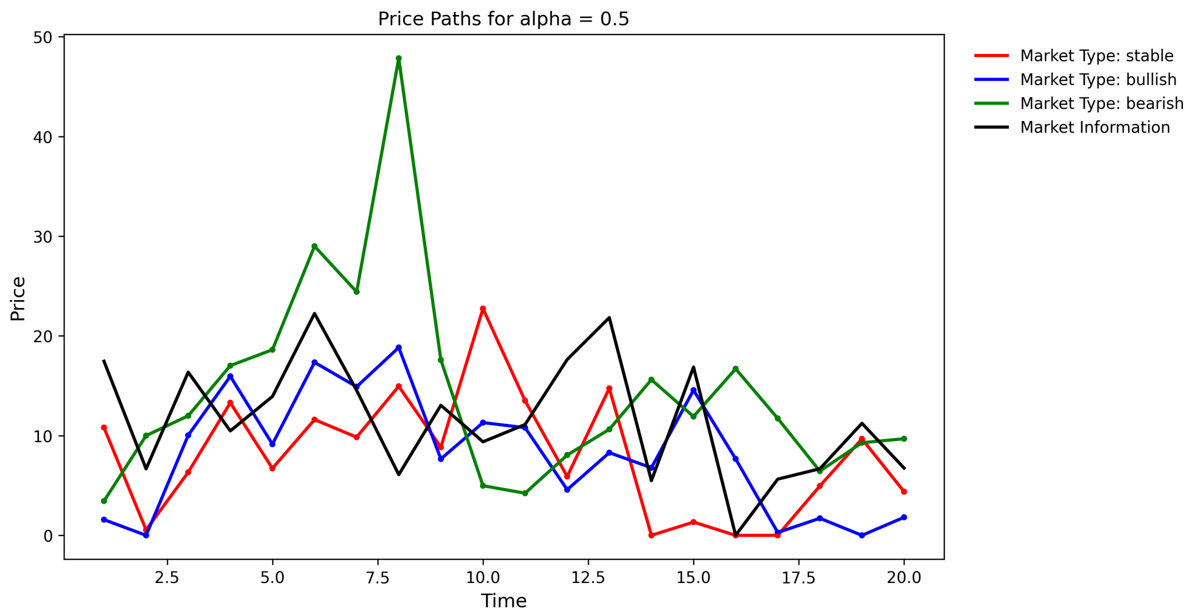

Figure 3: The optimal execution plans for investors with different levels of risk aversion. The market information is shown by the black curve. For $\alpha < 0$ (strong permanent market impact), the red curve exhibits the same trend as the available market data.

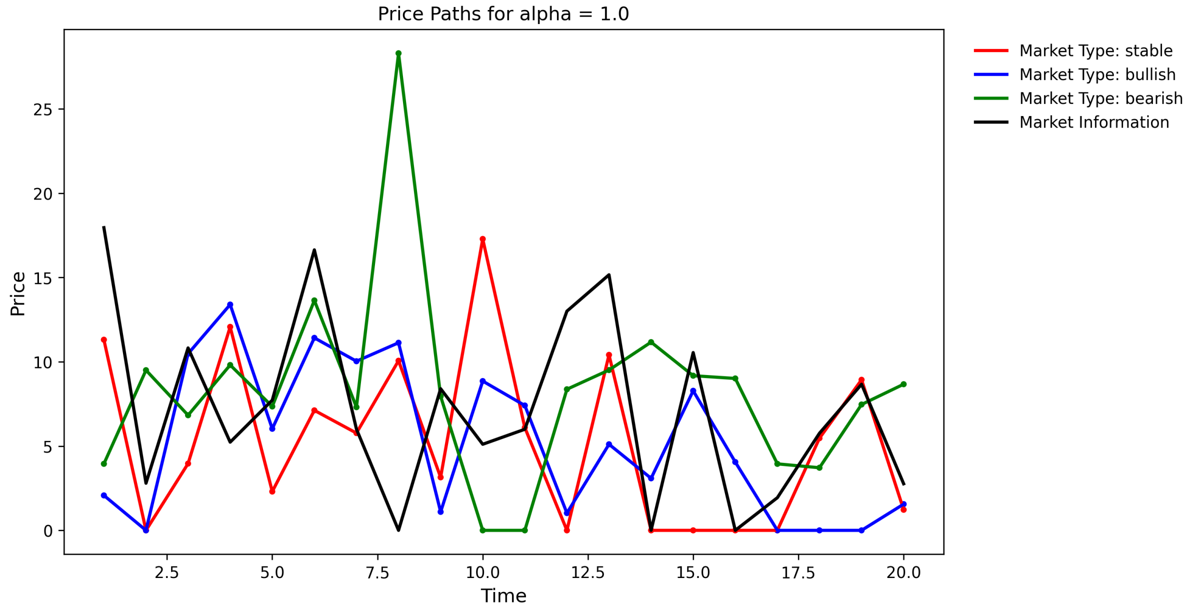

Figure 4: The optimal execution plans for investors with different levels of risk aversion. The market information is shown by the black curve. For $\alpha > 0$ (weak permanent market impact), the red curve exhibits the same trend as the available market data.

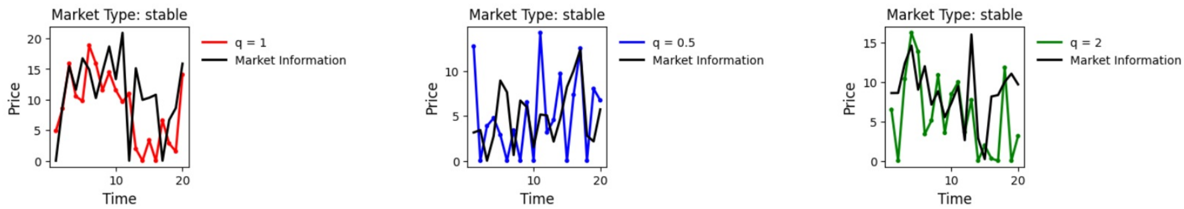

Figure 5: The market type with different levels of risk aversion. The market information is shown by the black curve. For $\alpha > 0$ (weak permanent market impact).

In figure 5, the graph represents the market behaviour of different value of q for various type of market.

5. Conclusion

To develop complex methods for trading in financial markets optimally, this research proposes a novel nonlinear programming approach. In a geometric Brownian motion under AR (2) model, our model includes a second-order information element based on dynamic programming concepts. Trading and portfolio managers are able to make more informed and flexible decisions by utilizing advanced mathematical techniques, especially when navigating unpredictable and volatile market conditions.
We present a novel nonlinear programming approach to the optimal trading problem in the context of developing sophisticated financial market trading methods. Utilizing dynamic programming concepts, our model incorporates a second-order information element within the context of geometric Brownian motion under AR (2). As part of our dynamic programming methodology, we explain its mathematical foundation and its ability to manage second-order information elements under AR (2). Simulations and empirical validations presented in the research demonstrate the effectiveness of our proposed solution. We demonstrate the adaptability of our model across various market conditions, consistently outperforming other methods.
The research presents simulations and empirical validations that show the effectiveness of our suggested

solution. In a variety of market conditions, the model consistently outperforms other trading strategy optimization techniques.However, we also acknowledge that more empirical the market information's are necessary to validate and improve our model in a variety of asset classes and market situations given the complex of financial markets.

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