Regularity of Geotechnological Formation of the Area of Weakened Connections in the Rock Mass

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Abstract

It is offered a mathematical model of propagation of a hydraulic crack which allows to estimate the sizes formed as a result of it in a mountain breed of area of the weakened communications. The results obtained allow to establish the basic laws of influence of mechanical properties of mountain breeds and controllable parameters of hydraulic fracture on the process of formation of an area of weakened connections in the massif during the formation of a crack. Based on the presented methods, appropriate rock mass effect calculation software with a convenient user interface has been developed.

Keywords [1]

Hydraulic fracturing, mathematical model, software, crack, area of weakened connections.

1. Introduction

One of the powerful geotechnologies that is actively used to intensify mineral extraction, and thus to influence and control the physical and mechanical properties of the rock mass, is hydraulic fracturing (fracking). Modeling the process of fracture development, which currently remains the most accurate method of interpreting the entire range of parameters of hydraulic fracturing technology, involves determining its main characteristics - the law of opening and the length of fracture development, which depend on the physical and mechanical properties of rocks, rheological properties of the working agent, and the mode of hydraulic fracturing.

1.1. Related works

Among the known theoretical models, there are two fundamentally different ones that are widely used to predict the geometry of cracks. The first one is the Khristianovich-Zheltov model, and the alternative one is the Perkins-Kern model. These models differ in the principles of describing the crack banks at the crack tip and are the result of applying the methods of the linear theory of elasticity to study the stress-strain state at high deformations, which does not correspond to the actual stress-strain state at the crack tip. This contradiction, noted by Griffiths, became the basis for models in which the crack banks should close smoothly under the influence of large adhesive forces (of the order of theoretical strength). At the same time, all known models of imperfectly brittle bodies are based on the introduction of adhesive forces between the crack banks and differ only in the assumptions regarding these forces, i.e., in such models, unlike Griffiths' model of an ideally brittle body, the end zone is not autonomous [1-7].

1.2. Research objectives

An equally important aspect of research on the description of specific mining situations under conditions of deformation and destruction of the rock environment or some engineering structures is to

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assess the spread of the area of influence of the hydraulic fracturing result beyond the formed fracture. This involves studying the state of the rock mass in the one-sided section of the crack tip in the direction of its propagation.

In this regard, the objective of the study is to build a mathematical model of the process of fracture development of a hydraulic fracturing crack that will allow a more accurate and correct display of the relations between the applied stress and the mode of hydraulic fracturing (parameters controlled at the wellhead: injection pressure, injection rate), the length of the crack, its opening, and, as a result, the size of the formed area of weakened connections in the rock mass.

2. Research methodology

2.1. A solution of the problem of vertical crack propagation

When modeling the propagation of a hydraulic fracture in the underlying rocks of a technogenic deposit, three main types of fundamental relations generally accepted in this case were taken into account:

1) the provisions of linear-elastic fracture mechanics;
2) the laws of motion of a fracture fluid in a narrow slit;
3) the continuity equation.

Let at hydraulic fracturing of the underlying rocks a symmetrical fracture is formed relative to the well, propagating under the directional action of the fracture fluid in the vertical direction to a height and depth sufficient to provide hydraulic connection between the rocks of the technogenic deposit and the protective bottom. In the horizontal direction the fracture is technologically limited and has a constant length 2L.

Following [1-4], we will neglect violations of the continuity of the medium by the borehole, we will consider the elastic constants of the formation and its host rocks to be the same, the fluid pressure p in the fracture to be constant along the length of the fracture ($|x| \leq L$) in each of its horizontal sections ($|z| = const$). Fracture opening $2w$ in each cross-section ($|z| \leq const$), taking into account real time parameters of the pressure pulse and fracture dimensions – $\tau v_p \gg h$, can be considered as a quasi-static process, the parameters of which will be found as a solution of the two-dimensional problem of static elasticity theory, relating the fracture width $2w$ to its propagation height h and the intra-fracture pressure $p(z,t) - q_\infty$ given at its banks, where q_∞ is lateral rock pressure.

With the adopted proposals, we use the Perkins-Kern model for modeling crack propagation when a vertical crack of constant height, strongly extended in the horizontal direction, is formed as a result of fracture. According to the results of [13, 14], in this case of crack propagation, its opening can be represented as

$$w(x,z,t) = \frac{2(1-\nu^2)Lq_\infty}{\pi E} \sqrt{1 - \left(\frac{x}{L}\right)^2} \left(\frac{p(z,t)}{q_\infty} - 1\right), \tag{1}$$

In (1) the variables z and t are parameters; ν – Poisson's coefficient; E – Young's modulus.

Let us limit ourselves to the consideration of impermeable formations or weakly permeable formations at the initial stage of hydraulic fracturing, when leakage into the formation can be neglected.

In this case, the fluid flow in the fracture will be described by the equation of motion generalized to the case of variable fracture width, valid at Reynolds numbers for the flow of viscous fluid in the slit, not exceeding 1000 at relatively small angles of inclination of the slit surface [3]:

$$u = -\frac{w^2}{12\mu} \cdot \frac{\partial p}{\partial z} \tag{2}$$

and the continuity equation:

$$\frac{\partial}{\partial t}\langle w \rangle + \frac{\partial}{\partial z}\langle w \cdot u \rangle = 0, \tag{3}$$

where

$$\langle f \rangle = \frac{1}{2L} \int_{-L}^{L} f(x)dx,$$

and $\langle w\rangle, \langle w\cdot u\rangle$ are values averaged over the crack opening and over the horizontal section of the crack. Here u – velocity of viscous fracture fluid in the crack, μ – viscosity of fracture fluid.

These equations are supplemented by the condition of "no flow" at the ends of the crack for the averaged one-dimensional flow, determining the law of crack propagation

$$\frac{dh(t)}{dt} = \langle u\rangle, \;\; h = h(t), \tag{4}$$

the condition of smooth interlocking of crack surfaces at its ends (analog of Khristianovich's condition)

$$p = q, \qquad z = h(t), \tag{5}$$

initial conditions

$$l(0) = l_0, \qquad p(z,0) = p_0(z), \tag{6}$$

as well as the condition determining the mode of fluid injection into the fracture. The latter will be considered as specified in the form of the flow rate of the fluid to the fracture, which also satisfies the condition of conservation of mass or volume:

$$8L\cdot\int\limits_0^h w(z,t)dz = V_0 + 8L\cdot\int\limits_0^\tau w(0,t)\cdot u(0,t)dt = Q(t), \tag{7}$$

where L is the half-width of the fracture zone, which does not change in the fracture process, V_0 – volume of the initial crack, $Q(t)$ – amount of liquid injected into the crack for time t.

On the basis of formula (1) we will find the average values included in equation (3):

$$\langle w\rangle = \frac{1}{2L}\int\limits_{-L}^{L}\frac{2(1-\nu^2)}{\pi E}Lq\sqrt{1-\left(\frac{x}{L}\right)^2}\left(\frac{p(z,t)}{1}-1\right)dx = \frac{k\pi}{4}Y(z,t),$$
$$k = \frac{2(1-\nu^2)}{\pi E}Lq_\infty, \qquad Y(z,t) = \frac{p(z,t)}{q} - 1. \tag{8}$$
$$\langle w\cdot v\rangle = -\frac{1}{2L}\int\limits_{-L}^{L}\frac{w^3}{12\mu}\frac{\partial p}{\partial z}dx = -\frac{1}{24L\mu}\int\limits_{-L}^{L}\left(1-\left(\frac{x}{L}\right)^2\right)^{\frac{3}{2}}dx.$$

Given that

$$\frac{1}{2L}\int\limits_{-L}^{L}\left(1-\left(\frac{x}{L}\right)^2\right)^{\frac{3}{2}}dx = \frac{1}{L}\int\limits_{0}^{L}\left(1-\left(\frac{x}{L}\right)^2\right)^{\frac{3}{2}}dx = \int\limits_0^1(1-s^2)^{\frac{3}{2}}ds = \frac{1}{2}\frac{\Gamma\left(\frac{1}{2}\right)\cdot\Gamma\left(\frac{5}{2}\right)}{\Gamma(3)} = \frac{3\pi}{16},$$

where $\Gamma(a)$ is a gamma-function, finally, for the average value $\langle w\cdot v\rangle$ we obtain:

$$\langle w\cdot v\rangle = -\frac{\pi k^3 q_\infty}{64\mu}Y^3(z,t)\frac{\partial Y}{\partial z}. \tag{9}$$

Taking into account the obtained mean values (8), (9) for the case of a viscous Newtonian rupture fluid subject to the law of motion in a narrow slit (2), the continuity equation (3) takes the form:

$$\frac{\partial Y}{\partial t} - \frac{k^2 q_\infty}{16\mu}\cdot\frac{\partial}{\partial z}\left(Y^3\cdot\frac{\partial Y}{\partial z}\right) = 0. \tag{10}$$

The resulting equation is fairly well understood in the general form of the notation:

$$\frac{\partial\omega}{\partial t} - a\cdot\frac{\partial}{\partial x}\left(\omega^m\cdot\frac{\partial\omega}{\partial x}\right) = 0. \tag{11}$$

Equation (11) belongs to the class of parabolic partial differential equations with power nonlinearity and is often encountered in nonlinear problems of heat and mass transfer, combustion theory and filtration theory. For example, it describes unsteady heat transfer in a stationary medium when the diffusivity is a stepped function of temperature. General solutions of equation (11) are known [14], one of which for the case (10) is written in the form:

$$Y(z,t) = (Az + A\lambda t + b)^{\frac{1}{3}}, \qquad A = \frac{48\lambda\mu}{k^2 q_\infty}, \tag{12}$$

where λ, B are arbitrary constants. Considering (12), (8) we write the crack opening in the form:

$$w(z,t)=\frac{k\pi}{4}\sqrt[3]{\left(\frac{48\lambda\mu}{k^2q_\infty}z+\frac{48\lambda^2\mu}{k^2q_\infty}t+B\right)}. \tag{13}$$

We determine the values of coefficients λ, B, considering the initial (6), boundary (4), (5) conditions and the condition governing the injection mode (7).

Considering the initial crack opening (6), the value of coefficient B is equal to:

$$B=\left(\frac{4w_0}{k\pi}\right)^3.$$

On the basis of the found solution (13), the condition of smooth closure of the crack surfaces at its ends (5) and the law of propagation (4), the validity of equation:

$$\frac{48\lambda\mu}{k^2q_\infty}h+\frac{48\lambda^2\mu}{k^2q_\infty}t+B=0, \tag{14}$$

Substituting the found form of the solution (12) of equation (10) into the condition (7), we obtain the following equation with respect to the parameters λ, h:

$$\frac{Lqk^3\pi}{32\lambda\mu}\left(\left(\frac{48\lambda\mu}{k^2q_\infty}h+\frac{48\lambda^2\mu}{k^2q_\infty}t+B\right)^{\frac{4}{3}}-\left(\frac{48\lambda^2\mu}{k^2q_\infty}t+B\right)^{\frac{4}{3}}\right)=V_0+Qt. \tag{15}$$

Taking into account the conditions (14), (15) governing the hydraulic fracturing regime, the law of fracture opening (8) and (13), the crack development for each moment of time is found as a solution of a system of nonlinear equations:

$$\begin{cases} \dfrac{48\lambda\mu}{k^2q_\infty}h+\dfrac{48\lambda^2\mu}{k^2q_\infty}t+B=0, \\ \dfrac{-Lq_\infty k^3\pi}{32\lambda\mu}\left(\dfrac{48\lambda^2\mu}{k^2q_\infty}t+B\right)^{\frac{4}{3}}=V_0+Qt, \\ \dfrac{p(z,t)}{q_\infty}-1=\left(\dfrac{48\lambda\mu}{k^2q_\infty}h+\dfrac{48\lambda^2\mu}{k^2q_\infty}t+B\right)^{\frac{1}{3}}. \end{cases} \tag{16}$$

After excluding the unknown parameter λ from the system (16), the formulas for estimating the size of the crack with regard to the hydraulic fracturing parameters (fracture pressure, injection rate) are obtained:

$$w(z,t)=b\sigma_0\sqrt[3]{h-z}, \tag{17}$$

where

$$b=\frac{\pi k^{\frac{2}{3}}}{4}\sqrt[6]{\frac{48\mu}{q_\infty t}},\quad \sigma_0=\sqrt[6]{\left(\frac{p(0,t)}{q_\infty}-1\right)^3-B}.$$

For the case of fracturing fluid flow rates we have

$$w(z,t)=\frac{V_0+Qt}{6hL}\sqrt[3]{\frac{h-z}{h}}, \tag{18}$$

The longitudinal development of the fracture crack is determined by the formula

$$h(t)=\frac{2(V_0+Qt)q_\infty}{3k\pi L(p(0,t)-q_\infty)}. \tag{19}$$

2.2. The area of weakened connections

The found solution of the elastic problem of crack propagation assumes smooth closure of the crack banks, which means a singular stress distribution in the vicinity of the crack tip. On practice, geomaterials under significant external loads in the area usually have a yield strength, which means plastic deformation at stresses above this limit [8]. Thus, in the vicinity of the crack tip, there is always a region where plastic deformations occur, which means that the stresses cannot be singular.

The fundamental difference in the outline of the crack banks at the crack tip is a consequence of the application of linear elasticity theory methods to the study of the stress-strain state of a body in the presence of large deformations, which does not correspond to the real picture of the stress-strain state at the crack tip. This contradiction, noted by Griffiths, led to models in which the crack banks under the influence of large cohesive forces (of the order of theoretical strength) should be washed away smoothly. In this case, all known models of non-ideally brittle bodies are based on the introduction of cohesive forces between the banks of the forming crack and differ only in the assumptions regarding these forces, i.e., in these models, unlike Griffiths' model of an ideally brittle body, the end zone is not autonomous [10-12].

For a more accurate and correct representation of the relations between the applied stresses, the length of the crack, and the size of the area of broken and weakened connections, it is proposed to use the Leonov-Panasyuk brittle crack model, which is formally equivalent to the Dugdale elastic-plastic crack and Barenblatt brittle crack models, although its mechanical content is somewhat different. In this model of a crack, the presence of a finite zone R of the crack is taken into account, where its banks are attracted with constant stress σ if the distance between them does not exceed a certain value δ_{cr}. If $w > \delta_{cr}$, then in accordance with the Leonov-Panasyuk concept of a brittle crack, there is no interaction between the crack banks. The zone of length R is called the area of weakened connections (Fig. 1). In the vicinity of each point of this area, two parameters are set that characterize the places of the beginning and end of fracture and correspond to two criteria of fracture [17]:

1) the condition of finite stresses in the final crack zone, i.e., $K = 0$, where K is the stress intensity factor at the crack tip;

2) the condition of transformation of the adhesion forces to zero at the point of transition from the area of weakened connections to the area of destroyed connections. The value σ is considered to be equal to the brittle strength limit, i.e., the fracture stress in the absence of plastic deformation.

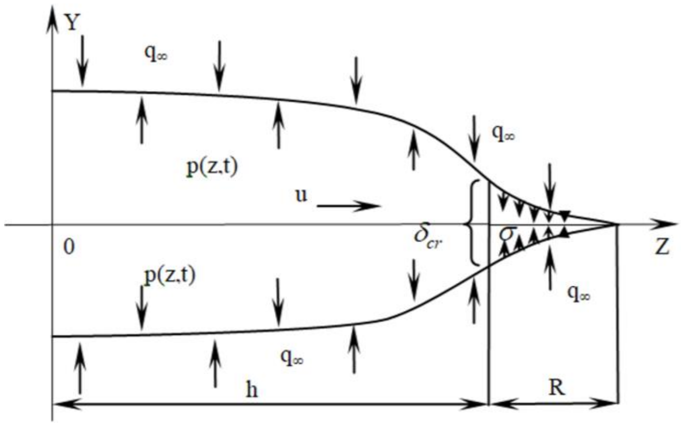

Figure 1: Crack model of hydraulic fracturing with consideration of the area of weakened connections

The critical condition for crack propagation is the equality

$$2w(h,t) = \delta_{cr}, \tag{20}$$

where δ_{cr} is critical crack opening, $w(h,t)$ – crack opening in the direction of the Y-axis.

Condition 2) means that at a certain value of crack bank deflection δ_{cr}, which is a characteristic of the geomaterial, the cohesive forces turn to zero, which leads to the condition $K = 0$ at the point ($x = h$) of transition from the area of weakened connections to the area of broken connections.

2.3. A solution of the problem of crack propagation with consideration of the area of weakened connections

The opening of the fracking crack and the acting load in the absence of an area of weakened connections according to the proposed solution can be found by formulas (17)-(19). In general, for a crack of a certain propagation value due to the acting load, these formulas take the following form:

$$dw = b\varphi(s)\sqrt[3]{s-z}ds, \tag{21}$$

$$dp = q_\infty\left(\frac{4b}{k\pi}\varphi(s)\sqrt[3]{s-z}+1\right)ds. \tag{22}$$

Then, according to the principle of superposition, for a set of cracks of different lengths $s\ (h < s < h + R)$ and taking into account the presence of an area of weakened connections of length R and a small distance δ_{cr} separating the crack banks at point h in accordance with formulas (21), (22), we obtain the following relations:

$$w(z,t) = \begin{cases} b\int\limits_h^{h+R} \varphi(s)\sqrt[3]{s-z}ds \quad \text{for} \quad z < h, \\ b\int\limits_z^{h+R} \varphi(s)\sqrt[3]{s-z}ds \quad \text{for} \quad h < z < h+R, \\ 0 \quad \text{for} \quad z > h+R, \end{cases} \tag{23}$$

$$p(z,t) = q_\infty\int\limits_h^z\left(\frac{4b}{k\pi}\varphi(s)\sqrt[3]{s-z}+1\right)ds \quad \text{for} \quad h < z < h+R. \tag{24}$$

2.4. Estimating the size of the area of weakened connections

It is known [9] that a guaranteed condition for the formation of vertical cracks in hydraulic fracturing is to ensure that the pressure inside the well is such that the following condition is met:

$$p_c = \sigma_p + 2q_\infty, \tag{25}$$

where σ_p is limit value of rock tensile strength.

Taking into account condition (25) and the formulated problem, we can conclude that for the stable development of a fracking crack propagation and in order to overcome the constricting stresses between the crack banks, it is necessary to create an equivalent load (not less than p_c) in the area of weakened connections. Counteraction to the existing constricting loads will be ensured if we set the function $\varphi(s)$ in such way that in the region of weakened connections, for each fracking crack of size $s\ (h < s < h + R)$, the following compensation condition is met:

$$\int\limits_h^z(\sigma_p + 2q_\infty)ds = q_\infty\int\limits_h^z\left(\frac{4b}{k\pi}\varphi(s)\sqrt[3]{z-s}+1\right)ds. \tag{26}$$

From condition (26), to determine the value of the required compensation load $\varphi(z)$, we obtain the following equation:

$$\sigma_p(z-h) = \frac{4bq_\infty}{k\pi}\int\limits_h^z \varphi(s)\sqrt[3]{z-s}ds. \tag{27}$$

Equation (27) is a Volterra integral equation of the first kind of convolution type with a kernel $(z,s) = (z-s)^{\frac{1}{3}}$ [15, 16].

As a result of solving the equation (27), we obtain the following form of compensation load $\varphi(z)$:

$$\varphi(z) = \frac{3\sqrt{3}(\sigma_p + q_\infty)k}{8bq_\infty}(z-h)^{-\frac{1}{3}}. \tag{28}$$

Based on the solution (28) and taking into account the value of the acting load at the wellhead during hydraulic fracturing (injection mode), we estimate the value of the propagation of the area of weakened connections by solving with respect to R the equation

$$\sigma_0 = \frac{3\sqrt{3}(\sigma_p + q_\infty)k}{8bq_\infty}\int\limits_h^{h+R}(s-h)^{-\frac{1}{3}}ds. \tag{29}$$

By finding the integral in the right-hand side of (29), taking into account the value of the characterizing function σ_0 we obtain the size of the region of weakened connections:

$$R = \left(\frac{16\sigma_0 b q_\infty}{9\sqrt{3}(\sigma_p + q_\infty)k}\right)^{\frac{3}{2}}. \tag{30}$$

2.5. Condition for crack opening

According to (23), the development of a fracking crack under the action of a de-claying agent in the area of fractured connections (crack opening) and deformation displacements in the area of weakened connections for each time t are written in the following form:

$$w(z,t) = \begin{cases} \dfrac{3\sqrt{3}k}{8q_\infty}(\sigma_p + q_\infty)\displaystyle\int_h^{h+R} \sqrt[3]{\frac{s-z}{s-h}}\,ds & \text{for} \quad z < h, \\ \dfrac{3\sqrt{3}k}{8q_\infty}(\sigma_p + q_\infty)\displaystyle\int_z^{h+R} \sqrt[3]{\frac{s-z}{s-h}}\,ds & \text{for} \quad h < z < h + R, \\ 0 \quad \text{for} \quad z > h + R, \end{cases} \tag{31}$$

Note that the integral in formula (31) with the new variable $v = \left(\frac{s-z}{s-h}\right)^{\frac{1}{3}}$ is transformed to the integral of a fractional rational function, which in turn is always integrated in finite form and the result is an algebraic sum of elementary functions (fractional rational, natural logarithm and arctangent). However, such an analytical solution for practical application is rather cumbersome and it is reasonable to use numerical integration for formula (31).

Formula (31) allows us to find the critical condition for fracking crack opening, namely, based on condition (20), the adhesive forces turn to zero at the point $z = h$ of transition from the area of weakened connections to the area of fractured connections. Taking into account these relations, we obtain:

$$\delta_{cr} = 2 \cdot \frac{3\sqrt{3}k}{8q_\infty}(\sigma_p + q_\infty)\int_h^{h+R} \sqrt[3]{\frac{s-h}{s-h}}\,ds = \frac{3\sqrt{3}k}{4q_\infty}(\sigma_p + q_\infty)R,$$

so

$$\delta_{cr} = \frac{16}{9\sqrt[4]{3}}\sqrt{\frac{q_\infty}{(\sigma_p + q_\infty)k}} \cdot (\sigma_0 b)^{\frac{3}{2}}.$$

Fig.2 shows the fracture opening profile of the fracturing crack taking into account the disturbed bond zone.

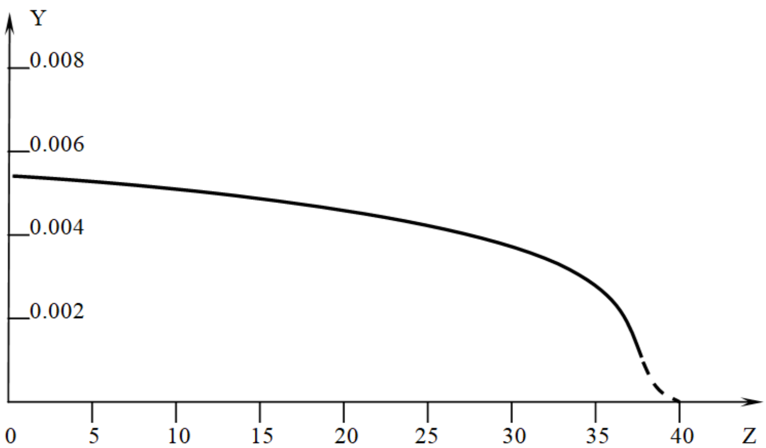

Figure 2: Fracture crack opening profile in the zone of working agent penetration (solid line) and the zone of weakened bonds (dashed line)

2.6 Rock mass effect calculation software

Based on presented methods an appropriate software that allows to calculate the dependence of the propagation of the area of weakened connections on the value σ_p during hydraulic fracturing and the dependence of critical crack opening δ_{cr} on the value σ_p during hydraulic fracturing was developed. Input data consists of type of the rock, maximum propagation, maximum opening, area of weakened connections, critical crack opening and model parameters q_∞, μ, L, z_0, Q, t. The example of the developed software user interface presented on the figure 3. Also, methods that presented in the research [18, 19] can be modified for rock type identification.

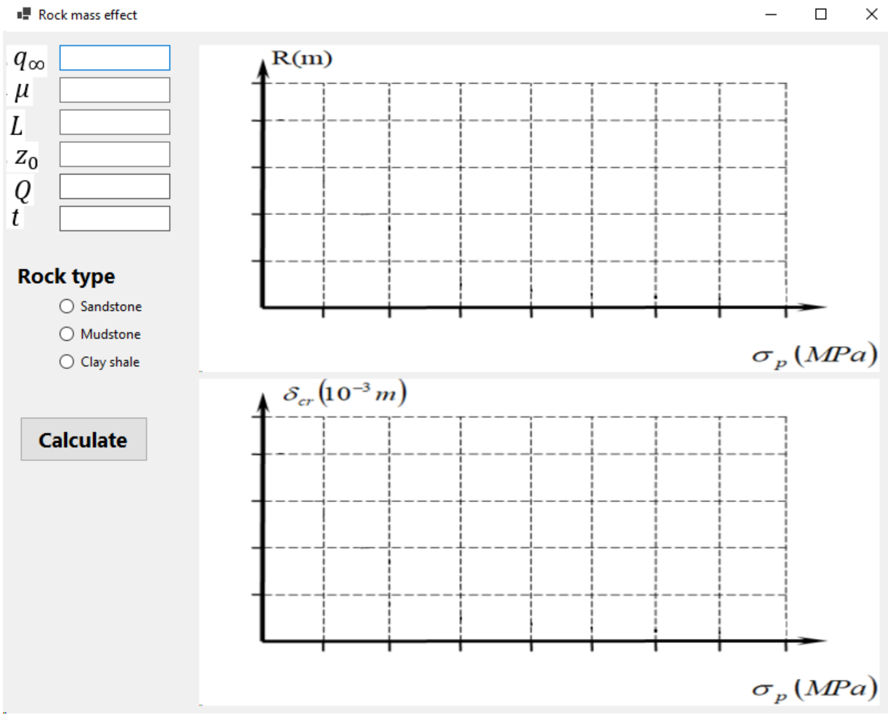

Figure 3: Rock mass effect software user interface

2.7. Results of the numerical experiment

To illustrate the determination of the propagation of the area of weakened connections in the rock mass due to the formation of a fracture crack, a numerical experiment was carried out for the following model parameters of hydraulic fracturing: $q_\infty = 10\ MPa, \mu = 0{,}1\ Pa \cdot s, L = 2\ m,\ z_0 = 0{,}1\ m, Q = 0{,}5\frac{m^3}{s}, t = 3\ s$ and mechanical properties of rocks $E_1 = 5\ GPa, \nu_1 = 0{,}3,$ $E_1 = 5\ GPa, \nu_1 = 0{,}3, \sigma_{p_1} = 30\ MPa, E_2 = 10\ GPa, \nu_2 = 0{,}23, \sigma_{p_2} = 5\ MPa, E_3 = 30\ GPa, \nu_3$=0,2, $\sigma_{p_3} = 10\ MPa$, which correspond to the average mechanical properties of sandstone, mudstone and clay shale.

Table 1 shows the results of calculating the main characteristics of hydraulic fracturing: maximum crack propagation and opening, critical opening, and propagation of the area of weakened connections.

Table 1
Fracking crack parameters taking into account the area of destroyed and weakened connections and critical crack opening

Rock	Maximum propagation $h_{max}\ (m)$	Maximum opening $w_{max}\ (m)$	Area of weakened connections $R\ (m)$	Critical crack opening $\delta_{cr}\ (m)$
Sandstone	19,7	$6{,}34 \cdot 10^{-3}$	0,132	$1{,}6 \cdot 10^{-3}$
Mudstone	27,1	$4{,}58 \cdot 10^{-3}$	0,8	$1{,}8 \cdot 10^{-3}$
Clay shale	46,9	$2{,}65 \cdot 10^{-3}$	0,89	$9{,}4 \cdot 10^{-4}$

In the next series of calculations, we studied the effect of the parameter on the propagation of the region of weakened connections and critical crack opening, which is due to the significant variation of this parameter even for rocks of the same petrographic name, depending on the composition and structure of the rock. Figures 4 and 5 illustrate the obtained results.

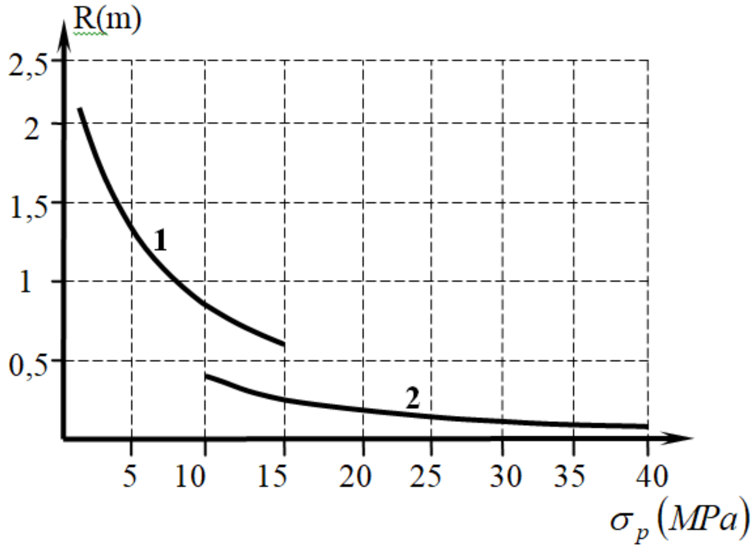

Figure 4: Dependence of the propagation of the area of weakened connections on the value σ_p during hydraulic fracturing

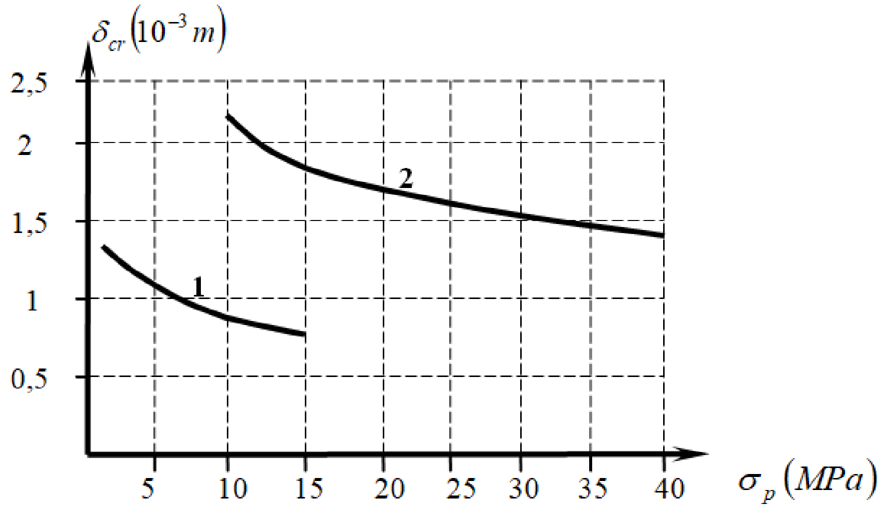

Figure 5: Dependence of critical crack opening δ_{cr} on the value σ_p during hydraulic fracturing

3. Conclusions

On the example of a man-made field, where hydraulic fracturing technology is proposed to be used to improve the filtration properties of the underlying rocks, a solution to the linear-elastic problem of the development of a symmetric crack in the vertical direction is found.

Based on the results of the study, it can be concluded that the extension of the region of weakened connections in the massif containing a fracking crack depends on both the value of the elastic modulus and the ultimate tensile strength σ_p, and is greater for rocks with a lower value of the parameter σ_p at a higher value of the Young's modulus.

For the case of rocks of the same petrographic name, the propagation of the area of weakened connections is larger at lower values of the rock tensile strength. Also, an example of the developed software was presented.

4. References

[1] Y. Petrivskyi. Scientific basis of geotechnological extraction of uranium from technogenic deposits, Volynski Oberegy, Rivne, 2008, 399 pp.

[2] Y. Petrivskyi, V. Petrivskyi, M. Tymchuk, V. Shevchenko. Models and Information Technology Implementation of Contact Interaction of the Edge of a Rock Cutting Tool With Granular Rocks Modeling. Selected Papers of the VIII International Scientific Conference “Information Technology and Implementation". Workshop Proceedings, vol. 3179, no. 2, 2022, pp. 194–204.

[3] T. Anderson. Fracture Mechanics. Fundamentals and Applications, CRC Press, Boca Raton, 2017, 684 pp.

[4] O. Aydan, Rock Mechanics and Rock Engineering, CRC Press, 2019, pp. 394.

[5] A. Carpinteri. Structural Mechanics: A unified approach, CRC Press, Routledge, 2019, 780 pp.

[6] P. Shen, T. Huiming, Y. Nign, D. Xia, A damage mechanics based on the constitutive model for strain-softening rocks, Engineering Fracture Mechanics, Vol. 216, 2019. https://doi.org/10.1016/j.engfracmech.2019.106521.

[7] L.-Y. Zhao, J.-F. Shao, Q.-Z. Zhu, J. A. Nairn, Analysis of localized cracking in quasi-brittle materials with a micro-mechanics based friction-damage approach, Journal of the Mechanics and Physics of Solids, Vol. 119, 2018, pp. 163-187. https://doi.org/10.1016/j.jmps.2018.06.017.

[8] S. Wang, H. Masoumi, J. Oh, S. Zhang, Scale-Size and Structural Effects of Rock Materials, Woodhead Publishing, 648pp. https://doi.org/10.1016/C2019-0-01182-X.

[9] H. Fu, W. Chen, J. Fu, Rock Mechanics and Engineering, Woodhead Publishing Series in Civil and Structural Engineering, 2020, 407pp. https://doi.org/10.1016/C2019-0-04623-7.

[10] M. Supriyono, A. Adabi. Dual boundary element method for elastoplastic fracture mechanics of shear deformable plate, Engineering Analysis with Boundary Elements, Vol. 117, 2020, pp. 132-142. https://doi.org/10.1016/j.enganabound.2020.04.009

[11] Q. Bai, Z. Liu, C. Zhang, F. Wang. Geometry nature of hydraulic fracture propagation from oriented perforations and implications for directional hydraulic fracturing, Computers and Geotechnics, vol. 125, 2020. https://doi.org/10.1016/j.compgeo.2020.103682

[12] A. Khoei, S. Mortazavi, L. Simoni, B. Schrefler. Irregular and stepwise behaviour of hydraulic fracturing: insights from linear cohesive crack modelling with maximum stress criterion, Computers and Geotechnics, vol. 161, 2023. https://doi.org/10.1016/j.compgeo.2023.105570

[13] M. Taylor. Partial Differential Equations III: Nonlinear Equations (Applied Mathematical Sciences, 117) 2nd ed. 2011 Edition. Springer

[14] L. Debnath. Nonlinear Partial Differential Equations for Scientists and Engineers, Springer New York Dordrecht Heidelberg London, Springer Science+Business Media, LLC 2012, 872 pp.

[15] M. Rahman. Integral Equations and their Applications. WIT Press, 2007, 385 pp.

[16] S. Kryvosheia, M. Perestiuk, V. Burym. Differential and Integral Equations. Lybid`, 2004, 408 pp.

[17] V. Panasyuk. Development of investigations on fracture mechanics of materials and structural integrity: achievements and perspectives. Procedia Structural Integrity, no. 16, 2019, pp. 3–10.

[18] O. Bychkov, K. Merkulova, Y. Zhabska, A. Shatyrko. Development of Information Technology for Person Identification in Video Stream. Proceedings of the II International Scientific Symposium “Intelligent Solutions” (IntSol-2021), vol. 3018, 2021, pp. 70–80.

[19] Petrivskyi V., Bychkov O., Martsenyuk V. Proving the Existence of Solutions to the Problems of Minimizing the Energy Consumption of Sensor Networks. Applied Sciences. 2022. Vol. 12, no. 15. P. 7687. DOI: https://doi.org/10.3390/app12157687