Computer Modeling of Discrete Systems in the Case of Linear and non-Linear Restrictions on the Optimal Speed of the Aircraft Based on the Lagrange Method

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Abstract

The paper of the report represents the study dedicated to the main link of the aircraft flight speed to the minimal time of the air transportation process. The simplest problem of the aviation transportation technologies fundamental factors optimization models the considered process of the delivery and it makes an attempt to the process theoretical description. Two segment air traffic elementary chain of supply is implied at the presented research. The algorithm, which is used for calculating the objective parameters of the aircraft motion, is developed. Approaches to aircraft speed optimization are used. The speed of the delivery by the aircraft at each of the segments have been conditionally optimized. The objective value is the time of delivery. The aircraft speeds are subject to both linear and nonlinear constraints. The influence of the speeds' variations upon the conditionally minimal objective delivery time values are studied. Theoretical contemplations are conducted in the framework of the Lagrange uncertainty multipliers implementation. The hypothetical provisions of the derived mathematical models are illustrated with the help numerical simulation. The part of the computer modeling is conducted on the Mathcad platform at the educational and scientific laboratory "Modeling of transport systems and processes" of National Aviation University. The necessary diagrams are plotted. The obtained results of both theoretical study and computer simulation allows construction of optimal delivery chains with a better determination of the exchange point location.

Keywords

Computer modeling, optimization, simulation, aviation transport technologies, aircraft flight speed, speed optimization, delivery time minimization

1. Introduction

Computer and numerical simulation for searching optimal aircraft speed by the criterion of the minimal time of the delivery is an urgent task. That requires a proper maintenance of aircraft itself [1], as well of the airplane engines and powerplants [2], in order to support the aeronautical components' reliability [3] and risk [4] at the due level.

However, the unsolved part to the general problem there, at references [1 – 4], is the lack of the conditional optimization.

In such context, expected operational efficiency and utility [5] is combined with the choice problems [6].

This means that the transport technologies theoretical constraints, like in reference [7], strengthening learning for intelligent applications [8, 9] are important.

These variants of uncertainty can be estimated using entropy approaches [10].

In conjunction with the economic models [11], the entropy methods resulted in the subjective analysis theory [12] allows solving various types of the applicable problems, similar to the stated in the references of [13 – 16]. Some problems relevant to the air transport management and aviation transport technologies have already been posed in [17 – 20].

According to the presented concepts, a scientific gap that needs to be solved is the development of a reliable mathematical approach to the actual and important problems related to the formulation of the optimal combination of aviation resources, especially in relation to

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supporting the process of analytical decision-making based upon the advantages of the computer modeling.

Thus, in the case with the aircraft transportation speeds, it is necessary to formulate the scientific hypothesis of the conducted research as the speeds' variations, subject to both linear and nonlinear constraints, impact upon the conditional minimal value of the delivery time.

The problem statement of the presented research concerns with the theoretical studies focused on the calculations of the optimal speed of the aircraft as the key element of the aviation transport technologies.

Thus, the goals of the article are a general description of the possible optimization of the aircraft speed for the theoretical and mathematical obtaining of rational solutions.

2. Possibilities of optimization

The simplest problem of the aviation transportation technologies fundamental factors optimization stated here is based upon an elementary two segment supply chain consideration. The speed of the delivery by the aircraft at each of the segments could be conditionally optimized.

This necessitates the further development of the optimization methods of [18] and [19].

2.1. Basic concept

It is going to be considered the theoretical background for calculating aircraft movement parameters and approaches to their optimization.

The simulation of the aircraft motion was conducted with use of the software capabilities of the educational and scientific laboratory "Modeling of transport systems and processes".

2.1.1. A case of a linearly dependable constraint

Taking into account the speed of the aviation transportation delivery

$$T(v_1, v_2) = \frac{|AB|}{v_1 + v_2}, \tag{1}$$

where $T(v_1, v_2)$ is the time of the delivery; v_1 and v_2 are correspondingly the speeds of the first and the second aircraft that fly towards each other the same time $T(v_1, v_2)$; $|AB|$ is the distance covered by both aircraft in the time of $T(v_1, v_2)$ and at the speeds of v_1 and v_2 in respect.

Model (1) imply, for instance, a case when allows you to find an initial reference solution, and then, improving it, get the optimal solution.

Considering the condition of

$$P = f_1\left(v_1 - v_{1_{\min}}\right) + f_2\left(v_2 - v_{2_{\min}}\right) = \mathrm{cost} = \mathrm{idem}, \tag{2}$$

where P is some idempotent (independent upon the parameters of the considered problem model, stable, steady, unchanged, constant) value; f_1 and f_2 are the corresponding speeds coefficients; $v_{1_{\min}}$ and $v_{2_{\min}}$ are the minimal values in respect for the aircraft speeds of v_1 and v_2 possible range of variation. The idea of (2) is close to [18].

The linear dependence between the aircraft speeds of v_1 and v_2 coefficients values of f_1 and f_2 could be derived supposing as from (2).

$$P - f_1\left(v_1 - v_{1_{\min}}\right) = f_2\left(v_2 - v_{2_{\min}}\right). \tag{3}$$

And from (3)

$$v_2(v_1) = \frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}}. \tag{4}$$

Also assuming

$$v_2(v_1) = v_{2_{\max}} - \frac{v_{2_{\max}} - v_{2_{\min}}}{v_{1_{\max}} - v_{1_{\min}}}\left(v_1 - v_{1_{\min}}\right), \tag{5}$$

where $v_{1_{\max}}$ and $v_{2_{\max}}$ are the maximal values in respect for the aircraft speeds of v_1 and v_2 possible range of variation.

Comparing the corresponding members of (4) and (5)

$$\left.\begin{aligned} &\frac{P}{f_2} + v_{2_{\min}} = v_{2_{\max}};\\ &\frac{f_1}{f_2} = \frac{v_{2_{\max}} - v_{2_{\min}}}{v_{1_{\max}} - v_{1_{\min}}}. \end{aligned}\right\} \tag{6}$$

System (6) yields

$$\left.\begin{aligned} &f_2 = \frac{P}{v_{2_{\max}} - v_{2_{\min}}};\\ &f_1 = \frac{P}{v_{1_{\max}} - v_{1_{\min}}}. \end{aligned}\right\} \tag{7}$$

Having determined the coefficients of f_1 and f_2 from (2) – (7), it is possible to consider now the condition of (2) as a constraint to the objective function of (1):

$$\Phi(v_1, v_2) = \frac{v_1 - v_{1_{\min}}}{v_{1_{\max}} - v_{1_{\min}}} + \frac{v_2 - v_{2_{\min}}}{v_{2_{\max}} - v_{2_{\min}}} - 1 \equiv 0\,. \tag{8}$$

Therefore, the problem is becoming a problem of a conditional optimization.

Namely, find the optimal aircraft speeds: v_1 and v_2, (1) – (8), extremizing the time of the delivery by the aviation transportation: $T(v_1, v_2)$, (1), subject to the only constraint of (2) as (8).

Thus, the extended Lagrange function is

$$L(v_1, v_2) = T(v_1, v_2) + \lambda\Phi(v_1, v_2) = \frac{|AB|}{v_1 + v_2} + \lambda\left(\frac{v_1 - v_{1_{\min}}}{v_{1_{\max}} - v_{1_{\min}}} + \frac{v_2 - v_{2_{\min}}}{v_{2_{\max}} - v_{2_{\min}}} - 1\right), \tag{9}$$

where λ is the Lagrange uncertain multiplier.

The necessary conditions for a possible extremum of (9) existence are

$$\left.\begin{aligned} &\frac{\partial L(v_1, v_2)}{\partial v_1} = 0;\\ &\frac{\partial L(v_1, v_2)}{\partial v_2} = 0;\\ &\frac{\partial L(v_1, v_2)}{\partial \lambda} = 0. \end{aligned}\right\} \tag{10}$$

Then

$$\left.\begin{aligned} &\frac{\partial L(v_1, v_2)}{\partial v_1} = -\frac{|AB|}{(v_1 + v_2)^2} + \frac{\lambda}{v_{1_{\max}} - v_{1_{\min}}} = 0;\\ &\frac{\partial L(v_1, v_2)}{\partial v_2} = -\frac{|AB|}{(v_1 + v_2)^2} + \frac{\lambda}{v_{2_{\max}} - v_{2_{\min}}} = 0;\\ &\frac{\partial L(v_1, v_2)}{\partial \lambda} = \frac{v_1 - v_{1_{\min}}}{v_{1_{\max}} - v_{1_{\min}}} + \frac{v_2 - v_{2_{\min}}}{v_{2_{\max}} - v_{2_{\min}}} - 1 = 0. \end{aligned}\right\} \tag{11}$$

The systems of (10) and (11) yield

$$\left.\begin{aligned}
&\frac{\lambda}{v_{1_{\max}}-v_{1_{\min}}}=\frac{|AB|}{(v_1+v_2)^2};\\
&\frac{\lambda}{v_{2_{\max}}-v_{2_{\min}}}=\frac{|AB|}{(v_1+v_2)^2};\\
&\frac{v_1-v_{1_{\min}}}{v_{1_{\max}}-v_{1_{\min}}}+\frac{v_2-v_{2_{\min}}}{v_{2_{\max}}-v_{2_{\min}}}=1.
\end{aligned}\right\} \tag{12}$$

From the first two equations of system (12)

$$v_{1_{\max}}-v_{1_{\min}}=v_{2_{\max}}-v_{2_{\min}}=C. \tag{13}$$

Then, using the third equation of system (12)

$$C=v_1-v_{1_{\min}}+v_2-v_{2_{\min}}. \tag{14}$$

Applying (13) and (14) to the first or second equation of system (12)

$$\frac{\lambda}{v_1-v_{1_{\min}}+v_2-v_{2_{\min}}}=\frac{|AB|}{(v_1+v_2)^2}. \tag{15}$$

From (15)

$$\lambda=\frac{|AB|\left(v_1-v_{1_{\min}}+v_2-v_{2_{\min}}\right)}{(v_1+v_2)^2}. \tag{16}$$

But system (12) might have a solution. It is because its first two equations are the same. Indeed, instead of (12) there is a possibility to write

$$\left.\begin{aligned}
&\frac{\lambda}{C}=\frac{|AB|}{(v_1+v_2)^2};\\
&\frac{\lambda}{C}=\frac{|AB|}{(v_1+v_2)^2};\\
&\frac{v_1-v_{1_{\min}}}{C}+\frac{v_2-v_{2_{\min}}}{C}=1.
\end{aligned}\right\} \tag{17}$$

The rewritten system (12) means

$$\left.\begin{aligned}
&\lambda=\frac{|AB|C}{(v_1+v_2)^2};\\
&v_1-v_{1_{\min}}+v_2-v_{2_{\min}}=C.
\end{aligned}\right\} \tag{18}$$

Then

$$\left.\begin{aligned}
&\lambda=\frac{|AB|C}{\left(C+v_{1_{\min}}+v_{2_{\min}}\right)^2};\\
&v_1+v_2=C+v_{1_{\min}}+v_{2_{\min}}.
\end{aligned}\right\} \tag{19}$$

So, v_1+v_2 has a constant (idempotent) value. One of the speeds can be determined through the other one. It can be resolved with the help of the expressions of (2) – (5), or conditions of (8), (9), the third equations of (11), (12), as well as from (14), the third equation of (17), and the second equations of (18) or (19) too.

Moreover, therefore, the duration of flight has a idempotent (stable, steady, unchanged, constant) value as well. That follows the model expression (1).

Thus, the system of two equations (18) obtained from/of (12) has happened to be a system of two equations with three unknowns. And (19) is actually the one equation with the two unknowns.

The following sections dedicated to simulation and discussion will visualize and dispute upon the case set as (1) – (19).

2.1.2. A variation upon the constraint

This subsection deals with the time: $T(v_1, v_2)$, (1), but subject a nonlinear constraint in the type of the variation to the equation of (2) or to the equation (4).

The necessary conditions for a possible extremum of (9) existence are

Now, it is going to be

$$v_2(v_1) = v_{2_{\max}} - \frac{v_{2_{\max}} - v_{2_{\min}}}{v_{1_{\max}} - v_{1_{\min}}}\left(v_1 - v_{1_{\min}}\right) + \Delta(v_1), \tag{20}$$

where $\Delta(v_1)$ is the variation to the linear dependence of v_2, (2), or the equation (4), both v_2 and $\Delta(v_1)$ being dependent upon v_1.

Suppose a nonlinear variation of $\Delta(v_1)$, (20). The proposed model is

$$\Delta(v_1) = k_{\Delta_1}\left(v_1 - v_{1_{\min}}\right)\left(v_1 - v_{1_{\max}}\right) \quad , \tag{21}$$

where k_{Δ_1} is a coefficient.

On the other hand, it is possible to model the opposite side, of the equation of (2), or to the equation (4), dependence of v_2, (2), upon v_1 variation. That is

$$v_2 = \frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}} + \delta(v_1), \tag{22}$$

where $\delta(v_1)$ is the variation to the linear dependence of v_2, (2) or the equation (4), upon v_1, however this time the variation provides the v_2 values on the contrary to the previous option of (20) and (21).

The variation itself can have formally the mathematically identical expression though:

$$\delta(v_1) = k_{\delta_1}\left(v_1 - v_{1_{\min}}\right)\left(v_1 - v_{1_{\max}}\right) \quad , \tag{23}$$

where k_{δ_1} is a coefficient.

Making allowance for the above option of (20) and (21), just for the certainty of the problem setting, the new constraint will have the view of

$$\Phi(v_1, v_2) = \frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}} + \Delta(v_1) - v_2 \equiv 0 \,. \tag{24}$$

That means

$$L(v_1, v_2) = T(v_1, v_2) + \lambda\Phi(v_1, v_2) = \frac{|AB|}{v_1 + v_2} + \lambda\left[\frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}} + \Delta(v_1) - v_2\right]. \tag{25}$$

Or in the view convenient for differentiating

$$L(v_1, v_2) = \frac{|AB|}{v_1 + v_2} + \lambda\left[\frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}} + k_{\Delta_1}\left(v_1 - v_{1_{\min}}\right)\left(v_1 - v_{1_{\max}}\right) - v_2\right]. \tag{26}$$

After applying the conditions of (10) to (25)

$$\left.\begin{aligned}
&\frac{\partial L(v_1, v_2)}{\partial v_1} = -\frac{|AB|}{(v_1 + v_2)^2} + \lambda\left\{-\frac{f_1}{f_2} + k_{\Delta_1}\left[\left(v_1 - v_{1_{\min}}\right) + \left(v_1 - v_{1_{\max}}\right)\right]\right\} = 0;\\
&\frac{\partial L(v_1, v_2)}{\partial v_2} = -\frac{|AB|}{(v_1 + v_2)^2} - \lambda = 0;\\
&\frac{\partial L(v_1, v_2)}{\partial \lambda} = \frac{P}{f_2} - \frac{f_1}{f_2}\left(v_1 - v_{1_{\min}}\right) + v_{2_{\min}} + k_{\Delta_1}\left(v_1 - v_{1_{\min}}\right)\left(v_1 - v_{1_{\max}}\right) - v_2 = 0.
\end{aligned}\right\} \tag{27}$$

The second equation of (27) immediately meant that

$$\lambda = -\frac{|AB|}{(v_1 + v_2)^2}\,. \tag{28}$$

Then, substituting (28) for its value into the first equation of (27) it yields

$$-\frac{|AB|}{(v_1+v_2)^2}-\frac{|AB|}{(v_1+v_2)^2}\left\{-\frac{f_1}{f_2}+k_{\Delta_1}\left[\left(v_1-v_{1_{\min}}\right)+\left(v_1-v_{1_{\max}}\right)\right]\right\}=0. \tag{29}$$

And

$$-\frac{|AB|}{(v_1+v_2)^2}\left[1-\frac{f_1}{f_2}+k_{\Delta_1}\left(2v_1-v_{1_{\max}}-v_{1_{\min}}\right)\right]=0. \tag{30}$$

Which means

$$1-\frac{f_1}{f_2}+k_{\Delta_1}\left(2v_1-v_{1_{\max}}-v_{1_{\min}}\right)=0\cdot \tag{31}$$

And

$$v_{1_{opt}}=\frac{1}{2}\left(\frac{f_1-f_2}{k_{\Delta_1}f_2}+v_{1_{\max}}+v_{1_{\min}}\right). \tag{32}$$

The solution (32) ensures

$$v_{2_{opt}}=\frac{P}{f_2}-\frac{f_1}{f_2}\left(v_{1_{opt}}-v_{1_{\min}}\right)+v_{2_{\min}}+k_{\Delta_1}\left(v_{1_{opt}}-v_{1_{\min}}\right)\left(v_{1_{opt}}-v_{1_{\max}}\right). \tag{33}$$

2.2. Simulation

Let's consider the results obtained with the help of the theoretical considerations mentioned above using formulas (1) - (20) and calculation procedures. In the interests of achieving the goal of the study, computer modeling of the process of objectivity of the criteria for evaluating the optimization of transport work in the implementation of air transportation was carried out.

2.2.1. Computer modeling with the linearly dependable constraint

In case of (1) – (19), the accepted calculation data are as follows:

$$|AB|=1\cdot 10^4,\quad v_1=600\ldots 1\cdot 10^3 \quad \text{and} \quad v_2=600\ldots 1\cdot 10^3. \tag{34}$$

The results for $T(v_1,v_2)$ obtained by (1) with the use of data (28) are shown in the Figure 1

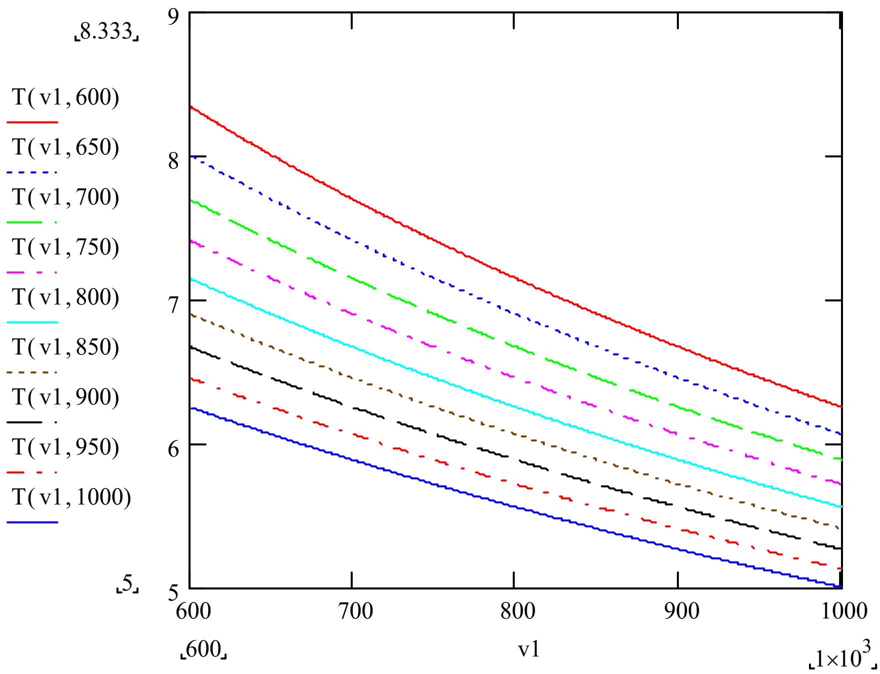


Figure 1: Time of the air transportation delivery

The three-dimensional plot of $T(v_1,v_2)$ by (1) is shown in the Figure 2.

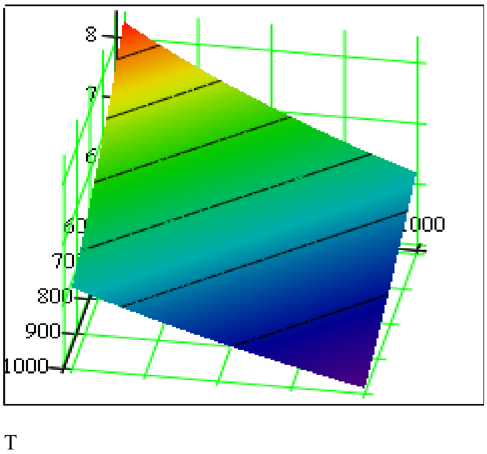


Figure 2: Time of the air transportation delivery

Applying the constraint in the view of (2), or $v_2(v_1)$: (4), or (5), to the duration of the aircraft transportation delivery (flight time), i.e.

$$T[v_1, v_2(v_1)] = \frac{|AB|}{v_1 + v_2(v_1)} = T(v_1). \tag{35}$$

In such case, $T(v_1, v_2(v_1))$: (1), modified to (35), it proves to have no extremum shown in the Figure 3.

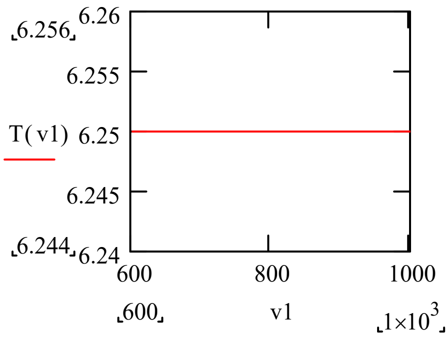


Figure 3: Absence of the extremum of the time of the delivery

The constant (idempotent) value of $T(v_1, v_2(v_1))$: (1), modified to (35), visible in the Figure 3 relates to the equations of (2) – (5), represented in the Figure 4.

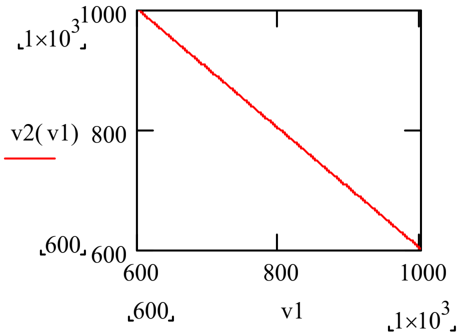


Figure 4: Linearly constrained speeds dependence

Additional data used for plotting diagrams in the Figures 3 and 4 may be relevant to the P value, (2) – (4) and (6), (7), however, it can be canceled or substituted, (8) – (19).

The absence of the extremum is noticeable in the three-dimensional plots of $T(v_1, v_2(v_1))$: (1), and the equations of $v_2(v_1)$: (2) – (5), illustrated for the perceptional ease in the Figure 5.

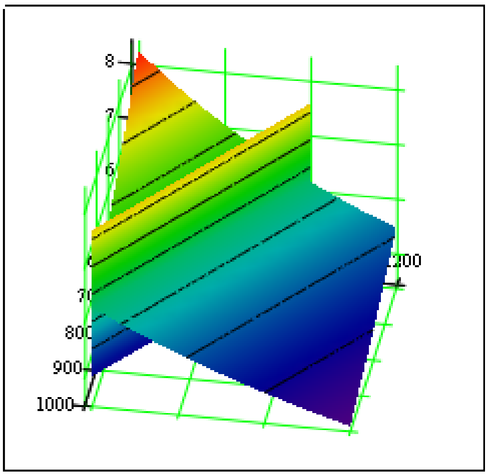


Figure 5: Absence of the extremum of the time of the air transport delivery

X,Y,Z shown in the Figure 5 are the parametric equations of the plain symbolizing the linear constraints (2) – (5).

1. 1. The results of additional experimental studies made it possible to obtain new data regarding the values of the weighting coefficients of the component indicators of the integral indicator and to reveal a significant deviation from the values obtained according to experts' assessments.
2. Local extrema are more inherent in the solution of optimization tasks of the parameters of specific technologies and devices, since, as a rule, they have technical limitations of independent variable objective functions.

2.2.2. Computer modeling with the nonlinearly dependable constraint

In the case with the aircraft transportation speeds variations of (20) – (33), in addition to the data of (34), there is a need to have data for the computer simulations of the variations: $\Delta(v_1)$ and $\delta(v_1)$.

In fact, it was necessary to accept the values for the coefficients of k_{Δ_1} and k_{δ_1}: entering the expressions (21) and (23), and used throughout the modeling (20) – (33). Those data were as the following:

$$k_{\Delta_1} = -5 \cdot 10^{-3} \quad \text{and} \quad k_{\delta_1} = 5 \cdot 10^{-3}. \tag{36}$$

The results are presented in the Figure 6.

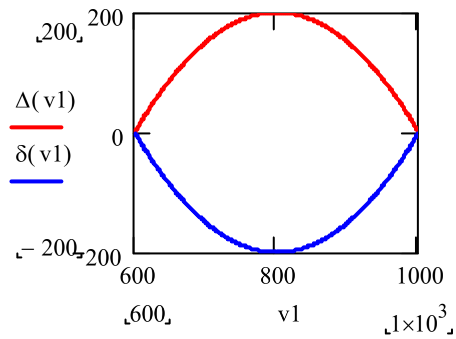


Figure 6: Aircraft speeds variations

The variated speeds with the basic one are shown in the Figure 7.

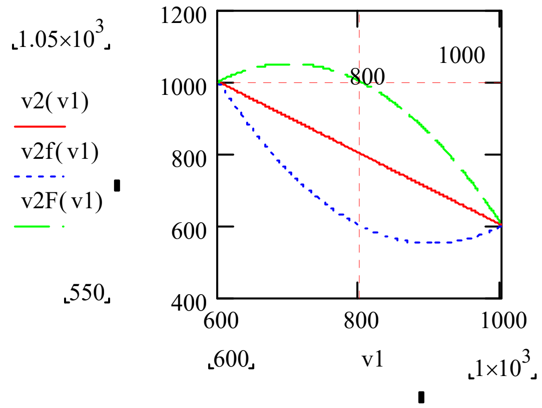


Figure 7: Variated aircraft speeds

Computer modeling for the time of the air transportation delivery is illustrated in the Figure 8.

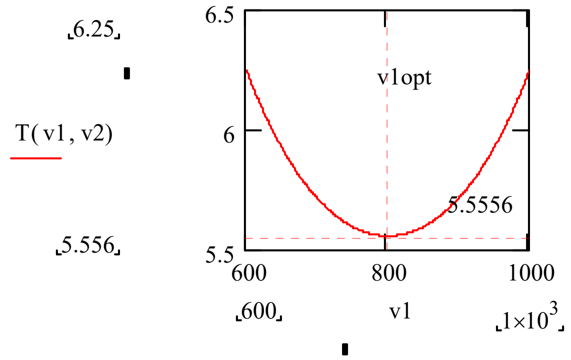
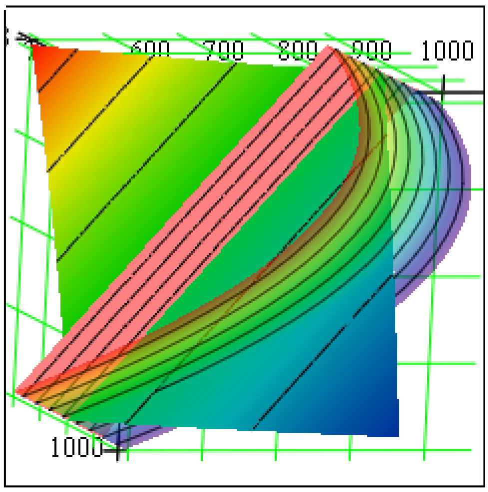


Figure 8: Time of the air transportation delivery constrained by the nonlinearly dependable aircraft speeds

The three-dimensional plots of $T(v_1, v_2)$ by (1) and $v_2(v_1)$ by (2) – (5), as well as $v_2(v_1)$ by (20) are shown in the Figure 9.



T, (X, Y, Z), (X, Ym, Z)

Figure 9: Time of the air transportation delivery subject to constraints upon the aircraft speeds

The extreme values are shown in the Figures 10 and 11 as well.

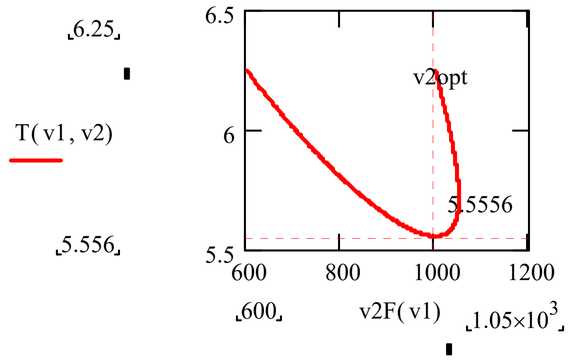


Figure 10: Phase diagram of the time of the air transportation delivery subject to constraints upon the aircraft speeds

The phase portraits in the Figures 10 and 11 demonstrate the minimal time and the optimal speeds combination.

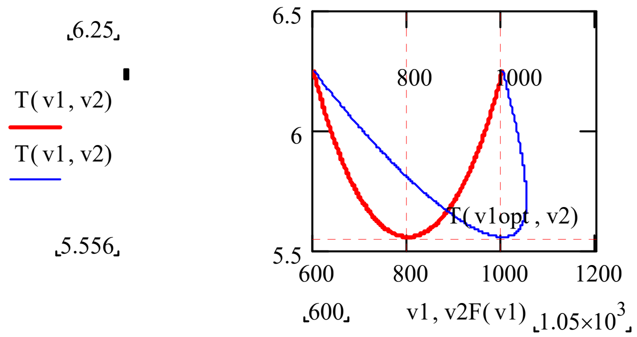


Figure 11: Combined phase portrait of the time of the air transportation delivery subject to constraints upon the aircraft speeds

3. Discussion

As it was shown, the creation of similar formalized models, that is, the relationship of target functions at different levels of the system hierarchy, will allow to maximize adequacy to optimal conditions of air transport operation.

The results of the experiment and the computational experiment based on the mathematical model by successive approximation by appropriate iterative methods are compared.

1. Optimization will always end with the search for local extrema of the objective functions, since the intervals of variation of the independent variables included in the objective functions are set a priori.
2. Phase portraits and trajectories of oscillation forms in the configuration space of the system were constructed and analyzed. The conditions for the localization of the forms of oscillations of the system have been obtained. The stability of the oscillation forms was studied. The presented system and mathematical model can be a source for new modeling approaches.

At the first stage of the research, a problem was identified that lay in the optimization of the aircraft's speed. To achieve this goal and systematize our understanding of the problem and potential ways to solve it, the following was defined:

- The problem that required our attention and what goal we want to achieve through the research.
- Possible ways of solving the problem, various alternatives were considered and the most effective and suitable variant of its mathematical solution was chosen.

The improvement of the criteria for evaluating the transport work in the performance of air transportation is carried out in the direction of expanding the list of factors that are taken into account when determining the relevant indicators, successively - the number (mass) of objects of transportation (passengers and cargo), range (distance between the points of departure and destination) and speed (time) of their spatial movement (delivery) from the point of departure to the destination.

If we draw a parallel between the ratio of optimal speed and the theory of individual risk perception, then this may make it possible to conduct another study regarding the theoretical-mathematical model of the demand for insurance services based on the conditional optimization apparatus. When solving this problem by the method of undetermined Lagrange multipliers, the derivative must equal zero before the necessary extremum condition. Accordingly, in this case, at the optimal point, the budgetary limitation of the insurance cost should be beneficial for the policyholder.

4. Conclusion

The obtained values as the implementation of this study allowed to analytically and graphically determine the region of the optimal solution, taking into account the limitations of the objective function.

The conditionally optimized aircraft speeds ensure minimal time of the air transport delivery, which in turn leads to the improvement of the air transportation technologies.

For further research, it is proposed to investigate the dynamics, of the process of choosing the desired optimal technologies of air transport, and models based on the given calculation conditions that implement operational alternatives and the subjective entropy.

References

[1] M. J. Kroes, W. A. Watkins, F. Delp, R. Sterkenburg, Aircraft Maintenance and Repair, 7th. ed., McGraw-Hill, Education, New York, NY, 2013.

[2] T. W. Wild, M. J. Kroes, Aircraft Powerplants, 8th. ed., McGraw-Hill, Education, New York, NY, 2014.

[3] B. S. Dhillon, Maintainability, Maintenance, and Reliability for Engineers, Taylor & Francis Group, New York, NY, 2006.

[4] D. J. Smith, Reliability, Maintainability and Risk. Practical Methods for Engineers, Elsevier, London, 2005.

[5] M. Amarante, Conditional expected utility, Theory and Decision 83. (2017) 1-19. DOI:10.1007/s11238-017-9597-9, https://www.researchgate.net/publication/315902126_Conditional_expected_utility.

[6] R. D. Luce, Individual Choice Behavior: A theoretical analysis, Dover Publications, Mineola, NY, 2014.

[7] D. Everett, et al., Multisystem Bayesian constraints on the transport coefficients of QCD matter. Physical Review, C 103.5 (2021): 054904. DOI:10.1103/PhysRevC.103.054904, https://journals.aps.org/prc/pdf/10.1103/PhysRevC.103.054904.

[8] A. Coronato, M. Naeem, G. De Pietro, G. Paragliola, Reinforcement learning for intelligent healthcare applications: A survey, Artificial Intelligence in Medicine, Volume 109, 2020, 101964, ISSN 0933-3657, https://doi.org/10.1016/j.artmed.2020.101964. (https://www.sciencedirect.com/science/article/pii/S093336572031229X)

[9] Mehta, Yash, et al., Recent trends in deep learning based personality detection, Artificial Intelligence Review, 53.4 (2020): 2313-2339. https://doi.org/10.1007/s10462-019-09770-z.

[10] F. C. Ma, P. H. Lv, M. Ye, Study on global science and social science entropy research trend, in: Proceedings of the IEEE International Conference on Advanced Computational Intelligence, ICACI, Nanjing, Jiangsu, China, 2012, pp. 238–242. https://ieeexplore.ieee.org/document/6463159. doi:10.1109/ICACI.2012.6463159.

[11] E. Silberberg, W. Suen, The Structure of Economics. A Mathematical Analysis, McGraw-Hill Higher Education, New York, NY, 2001.

[12] V. Kasianov, Subjective Entropy of Preferences. Subjective Analysis, Institute of Aviation Scientific Publications, Warsaw, Poland, 2013.

[13] O. Solomentsev, M. Zaliskyi, T. Herasymenko, O. Kozhokhina, Yu. Petrova, Data processing in case of radio equipment reliability parameters monitoring, in: Proceedings of the International Conference on Advances in Wireless and Optical Communications, RTUWO, Riga, Latvia, 2018, pp. 219–222. https://ieeexplore.ieee.org/abstract/document/8587882. doi:10.1109/RTUWO.2018.8587882.
[14] A. M. B Pavani et al., "VISIR+ Project Follow-up after four years: Educational and research impact," 2023 IEEE Frontiers in Education Conference (FIE), College Station, TX, USA, 2023, pp. 1–8, doi: 10.1109/FIE58773.2023.10343298.
[15] D. Shevchuk, O. Yakushenko, L. Pomytkina, D. Medynskyi, Y. Shevchenko, Neural network model for predicting the performance of a transport task, Lecture Notes in Civil Engineering. 130 LNCE (2021) 271–278. doi:10.1007/978-981-33-6208-6_27.
[16] S. Subbotin, The neuro-fuzzy network synthesis and simplification on precedents in problems of diagnosis and pattern recognition, Opt. Mem. Neural Networks 22 (2013) 97–103. https://doi.org/10.3103/S1060992X13020082.
[17] A. V. Goncharenko, Multi-optional hybridization for UAV maintenance purposes, in: Proceedings of the IEEE International Conference on Actual Problems of UAV Developments, APUAVD, Kyiv, Ukraine, 2019, pp. 48–51. https://ieeexplore.ieee.org/abstract/document/8943902. doi:10.1109/APUAVD47061.2019.8943902.
[18] A. V. Goncharenko, Relative pseudo-entropy functions and variation model theoretically adjusted to an activity splitting, in: Proceedings of the IEEE International Conference on Advanced Computer Information Technologies, ACIT'2019, Ceske Budejovice, Czech Republic, 2019, pp. 52–55. https://ieeexplore.ieee.org/abstract/document/8779876. doi:10.1109/ACITT.2019.8779876.
[19] A. Goncharenko, Development of a theoretical approach to the conditional optimization of aircraft maintenance preference uncertainty, Aviation 22 (2) (2018) 40–44. https://journals.vilniustech.lt/index.php/Aviation/article/view/5929. https://doi.org/10.3846/aviation.2018.5929.
[20] A. V. Goncharenko, Airworthiness support measures analogy to the prospective roundabouts alternatives: theoretical aspects, Journal of Advanced Transportation Article ID 9370597 2018 (2018) 1–7. https://www.hindawi.com/journals/jat/2018/9370597/. https://doi.org/10.1155/2018/9370597.