Mathematical Modeling of COVID-19 Pandemic and its Impact on Economy

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Abstract

A model for the spread of the COVID-19 pandemic and its induced changes in individual sectors of the economy has been developed. An approach to studying the interconnection of the food production system, food transportation, and the socio-economic sphere during the pandemic conditions using a three-sector Lorenz model has been proposed. Research has been conducted on the impact of the pandemic on changes in the supply-demand balance in these sectors of the economy. A model of the T-cell immune response has been developed, considering the peculiarities of its course in COVID-19, when excessive inflammation leads to cytokine release syndrome (cytokine storm), causing damage to vital organs and disruption of the immune system. The model considers the influence of cellular energetics on the regulation of the immune response and the amplification of inflammation.

Keywords

Mathematical modeling, COVID-19, economic processes

1. Introduction

In the 2021 Davos Forum report [1], it was noted that infectious diseases rank first among the highest risks influencing the next decade. On April 7, 2024, a total of 774,699,366 people was infected worldwide, and 7,033,430 people died [2]. The global changes of the past decades have so closely intertwined various spheres of social organization that the global COVID-19 pandemic has significantly impacted many processes in the world economy. This has initiated a chain of changes leading to disruptions in socio-economic connections at the local, regional, and global levels [3[.

In an effort to control the epidemic process, governments have implemented lockdowns and other restrictions on economic activity, which have closed many businesses, limited domestic travel, closed borders to the movement of labor and certain food products, and introduced requirements for social distancing and curfews. Anti-epidemic measures are becoming a heavy burden on the economy as they begin to affect a complex set of ecological, economic, political, and social processes, which have enormous consequences for the lives of individuals, societal well-being, economic activity, and food security. In formulating effective strategies to normalize the economic situation amidst the pandemic, many governments are faced with the difficult problem of how to provide support to the economy in the short term while avoiding unwanted inflationary consequences and risks that could threaten financial stability in the medium term.

The uniqueness of the COVID-19 virus is associated with the lack of a single viewpoint on the nature of its origin and dissemination pathways. This leads to a situation of uncertainty that requires special mathematical methods used to investigate improbable or unique events, where the stochastic nature of the object under study, or the incompleteness of the sample, reduces the effectiveness of traditional statistical methods. Therefore, the task of developing models that consider the impact of random disturbances becomes particularly relevant in forecasting the development of the pandemic, as these disturbances become additional factors increasing the level of structural disruptions in the epidemic system.

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At the same time, it becomes clear that forecasting the development of pandemics is impossible without the development of mathematical models to study the negative impact of global epidemics on economic development. These models should consider both epidemic and economic factors and allow the determination of effective strategies to minimize the number of casualties and economic losses from the epidemic under various scenarios of its development. They should also enable a systematic analysis of responses to challenges for sustainable development in the face of new bio-threats. The aim of this work is to develop precisely such mathematical models for studying the negative impact of global pandemics on economic development and to conduct a system analysis of responses to challenges for sustainable development in the face of new bio-threats.

2. Modeling the impact of COVID-19 on economy

In [4], an approach to studying the interconnection of water, food, energy resources, food transportation, and medical consequences of a pandemic was proposed using a multisectoral Lorenz model. This model unifies sectors of the economy that are similarly described in a single structure, with each sector considered in terms of productivity levels, the number of jobs, and the level of structural disruptions. Through modeling, conditions for the emergence of deterministic chaos were identified, and trajectories of changes in socio-economic factors were calculated, allowing for the reduction of the number of structural disruptions. This reduction is achieved by altering the balance between supply and demand in creating jobs and production in relevant sectors of the economy. The study of the model allowed tracing how changes in the balance of supply and demand in interconnected systems lead to the emergence of a chaotic stable attractor.

The SIR epidemic model used frequently to study the spread of COVID-19. It describes the interaction of three population groups: the healthy (S), the infected (I), and those who recovered (R) [5]. To investigate the interrelationship between changes in the economy and the epidemic process, these two approaches are combined. This allows connecting the SIR model with parameters whose dynamics are determined using a model of interconnections in the food production, its transportation, and the socio-economic sphere. It is assumed that the level of disruptions is proportional to the level of disruptions in the operation of production systems in different sectors of the economy.

2.1. Mathematical model of COVID-19 spread

Let's consider three manufacturing systems (MS) related to food production, its transportation, and medical sector. We will use the Lorenz model to study interrelated sectors of the economy [4]. Let (X_i), (Y_i) and (Z_i) be normalized levels of productivity, employment quantity, and level of structural disruptions respectively for the food production system (i = 1), food transportation (i = 2), and the socio-economic sphere (medical infrastructure) (i = 3). Let's assume that different sectors of the economy compete with each other for labor, while changes in climatic conditions, financial, and political instability introduce random disturbances $w_{ij}(t)$, becoming additional factors increasing the level of structural disruptions in the socioeconomic system [4]. Since processes in different sectors of the economy proceed at different rates, we will scale time in them by introducing parameters ε_i . The model has the following form:

$$\varepsilon_i \frac{dX_i}{dt} = \sigma_i(Y_i - X_i) + \delta_i \dot{w}_{ij}, \qquad \varepsilon_i \frac{dY_i}{dt} = [r_i(X_1, X_2, X_3) - Z_i]X_i - Y_i + \delta_i \dot{w}_{ij}, \qquad (1)$$

$$\varepsilon_i \frac{dZ_i}{dt} = X_i Y_i - b_i Z_i + \delta_i \dot{w}_{ij},$$

where $r_i = r_{i0}(1 - \sum a_{ik} X_i)$; δ_i– perturbation intensities; σ_i, r_i, b_i – parameters of Lorenz model; a_{ik} – parameters that characterize competition in labor markets $(i \neq k)$; $w_{ij}(t)$ – independent standard Wiener processes with parameters $E\left(w_{ij}(t) - w_{ij}(s)\right) = 0$, $E\,(w_{ij}(t) - w_{ij}(s))^2 = |t - s|$[4].

Research on the canonical Lorenz model shows that increasing the parameters r_i leads to the emergence of turbulence in the model (1). Since in model (1) r_i are functions of the variables X_i, in fact we have a Lorentz model with variable parameters r_i. Threats to food resources, according to [1], are considered long-term, while threats to disruptions in food transportation and the socio-medical sphere are considered medium-term. Therefore, the following parameter values are chosen: $\varepsilon_1 = 1$, $\varepsilon_2 = \varepsilon_3 = 2.5$. The coefficients a_{ik} are selected considering the weight of threat factors. Their values are shown in Table 1.

Table 1
Ranking of threats factors

Threats	Rank of influence	Value of parameters
Crisis of food resources	43.9	$a_{12} = 0.012, a_{13} = 0.013$
Disruption of food supply chains	38.3	$a_{21} = a_{23} = 0.013$
Socio-economic risks	43.4	$a_{31} = 0.013, a_{32} = 0.012$

The results obtained in [4] allow us to relate the parameters σ_i, r_i, b_i with the characteristics of the sectors of the economy in the following way:

$$\sigma_i = (\alpha_{1i}\beta_{2i})/(\alpha_{2i}\gamma_{2i}),\ r_i = (\beta_{1i}\gamma_{1i})/(\beta_{2i}\gamma_{2i}),\quad b_i = \zeta_i/(\alpha_{2i}\gamma_{2i})\,, \tag{2}$$

where α_{1i} and α_{2i} are parameters characterizing adaptive capabilities; β_{1i} are demands for the activity of the i-th MS, normalized per unit of the material production system, considering the workplace in the corresponding industry of production Y_i; β_{2i} are supplies, normalized per unit of function of the i-th MS X_i; γ_{1i} are demands for an increase in number of jobs, normalized per unit of X_i; γ_{2i} are supplies of jobs involved in providing X_i, normalized per unit of Y_i; ζ_i are the specific rate of growth in the number of disruptions. For the study of the epidemic process, we will use the modified SIR model, which has the following form:

$$\frac{dy_1}{dt} = -\frac{R(Z_3)y_1y_2}{NT_{inf}} + \alpha(t)y_3 + \beta_1\dot{w}_1,\quad \frac{dy_2}{dt} = \frac{R(Z_3)y_1y_2}{NT_{inf}} - \frac{U_1(X_3)y_2}{NT_m} + \beta_2\dot{w}_2, \tag{3}$$

$$\frac{dy_3}{dt} = (1-a_0)\frac{U_1(X_3)y_2}{NT_m} - \alpha(t)y_3 + \beta_3\dot{w}_3,\quad \frac{dy_4}{dt} = a_0\frac{U_1(X_3)y_2}{NT_m} + \beta_4\dot{w}_3,$$

where y_1 - susceptible individuals, y_2 - infected patients, y_3 - recovered patients, y_4 - deceased patients; β_k (k=1,4) - disruption intensity parameters; $w_k(t)$ has the same meaning as it had in model (1) and corresponds to the same conditions; a_0- the fraction of infected individuals who died; T_{inf} - the active period during which the infected individual is contagious; T_m - average recovery period of an infected individual; N– total population. We will consider that the effectiveness of the medical system's operation is determined by the level of disruptions within it Z_3, the dynamics of which is calculated using model (1), and the reproduction rate (average number of infections caused by one infected individual) depends on the effectiveness of the medical system's operation and is determined by the function $R(Z_3)$. We will also assume that the recovery rate depends on the productivity level of the medical system, which is determined by the function $U_1(X_3)$. The dynamics of variable X_3 is also calculated using model (1). Additionally, we will consider that immunity to the virus decreases over time for those who have recovered from COVID-19, i.e., a person can get sick again. The rate of this process is described by function $\alpha(t)$ in model (4). The negative impact of the epidemic on economic processes will be investigated by replacing the parameters r_i in (2) with function $r_iU_{2i}(y_2)$. An increase in the number of infected individuals will increase the parameters r_{10}, r_{20} and r_{30} in (2) and when they reach bifurcation values, this will lead to the emergence of stochastic regimes.

2.2. Investigation of the epidemic dynamics

Let us consider the initial wave of the epidemic, when there are no individuals with immunity to the virus. Let $\alpha(t) = 0$, and let the functions $R(Z_3), U_1(X_3)$ and $U_2(y_2)$ have the following form:

$R(Z_3) = R(1 + Z_3/k_1)$,

$U_1(X_3) = 1 + k_2 {X_3}^2/(k_3 + {X_3}^2)$, $U_{2i}(y_2) = 1 + l_i y_2/N$, (4)

where l_i and k_i are parameters of the model $(i = \overline{1,3})$.

In figures 1-2, the results of simulating the initial wave of the epidemic are shown for various initial numbers of infected individuals and varying recovery rates k_2.

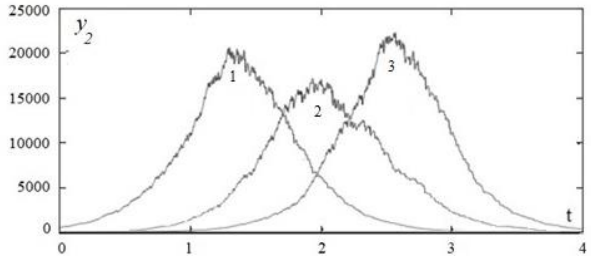

Figure 1: Dynamics of the number of infected individuals y_2 under different initial conditions: $1 - y_2(0) = 500;\ 2 - y_2(0) = 50;\ 3 - y_2(0) = 5$.

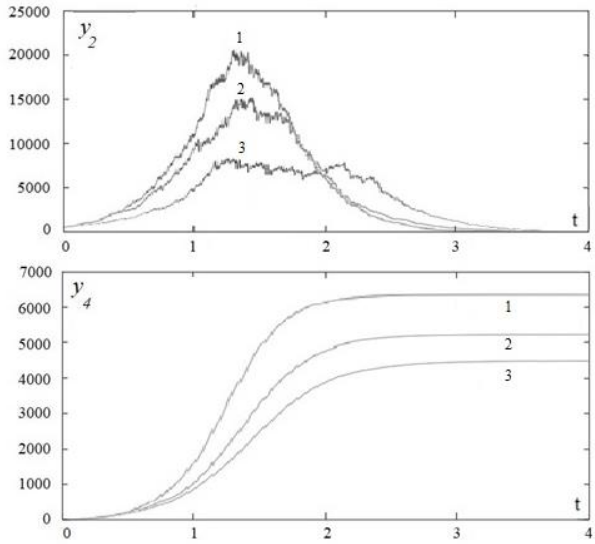

Figure 2: Dynamics of the number of y_2 and deceased individuals y_4 with variation in k_2: $1 - k_2 = 27;\ 2 - k_2 = 30;\ 3 - k_2 = 34.5$

Figure 3 displays the results of modeling when a portion of the sick who died (a_0) is a function of the level of disruptions in the medical sector (Z_3) and is defined as follows:
$a_0(Z_3) = a_0[1 + k_4 Z_3/(k_5 + Z_3)]$.

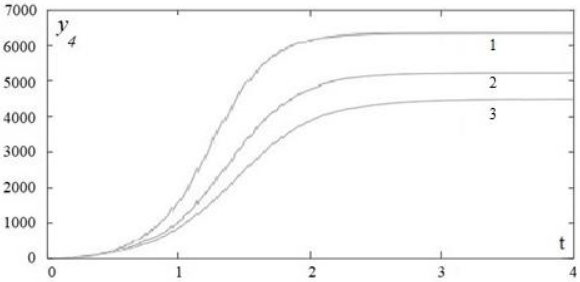

Figure 3: Dependency of the dynamics of deceased individuals y_4 on the number of disruptions Z_3: $1 - k_4 = 0;\ 2 - k_4 = 0.01;\ 3 - k_4 = 0.02;\ 4 - k_4 = 0.025$.

y_i, in all figures are measured by the number of people, Z_i are measured by the number of disruptions, time is measured in months.

In [4], the investigation focused on how changes in the parameter r affect the formation of system (1) regimes and the emergence of a stable strange attractor. Let's examine how the

emergence of a pandemic can alter the functioning of interrelated sectors of the economy from deterministic to chaotic regimes. The of the Lorenz model shows that increasing the parameter r is crucial for the onset of turbulence. The operating modes in model (1) change at the following bifurcation values of r: $r = 13.926, r = 24.06,\ r = 24.74$. When r>24.74, the system transitions to a regime of metastable chaos - a strange attractor emerges [6]. For modeling purposes, the value of parameter $r = 16$ was chosen. The increase in the variable y_2 will raises r and lead to a change in the operating mode.

In figure 4, it is shown how the increase in the parameter l_3, which characterizes the level of impact of the pandemic on the functioning of the medical sector, contributes to the emergence of stochastic regimes (time measured in months). In the absence of such influence $(l_3 = 0)$, damped oscillations of the variable Z_3 are observed.

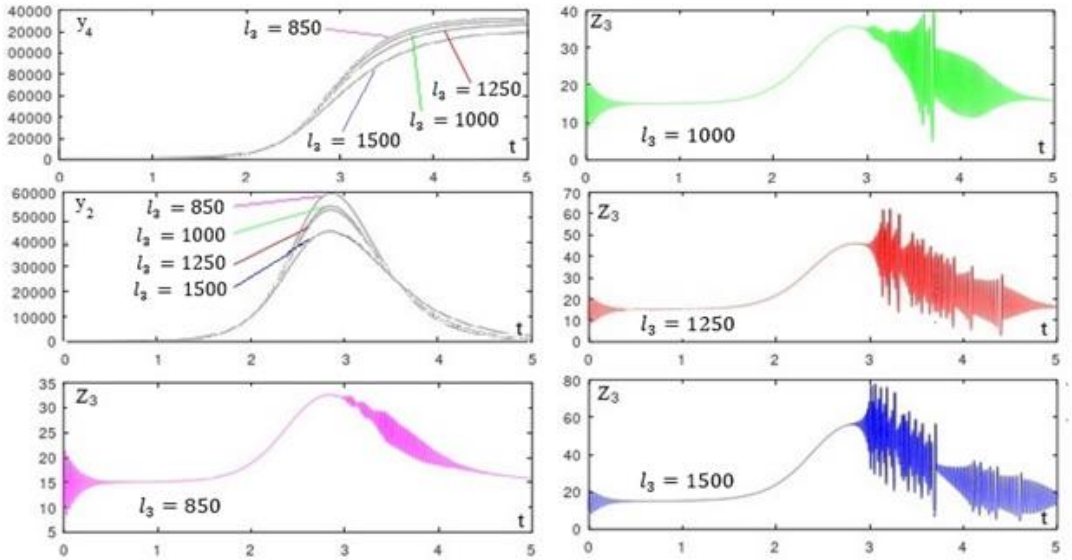

Figure 4: Modeling the impact of pandemic on structured disruptions of medical sector.

At $l_3 = 100$, the variable Z_3 reaches a steady level. Then it only deviates from it, depending on the changes in y_2. At $l_3 = 850$ and $l_3 = 1000$, cyclic oscillations begin, which dampen when the level of infected individuals decreases to level that was before the beginning of the epidemic. At $l_3 = 1250$ and $l_3 = 1500$, stable chaotic oscillations arise.

In figure 5 the results of modeling changes in the levels of disruptions in the food production system Z_1, food transportation Z_2, and medical infrastructure Z_3 during the epidemic are shown. During the simulation, the total population was chosen to be 34 million people. With increasing values of the parameters r_i, the number of infected individuals also increases. Periodic trajectories transition to chaotic ones upon reaching the bifurcation value of this parameter.

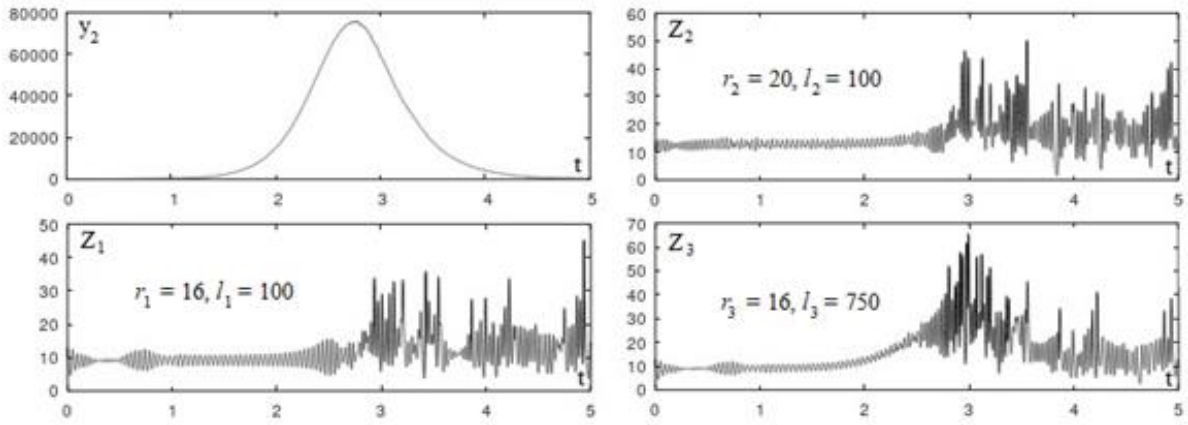

Figure 5: Modeling the emergency of chaotic operating regimes in various sectors of economy.

There exists a mutual negative influence. On one hand, the state of the economy affects the ability to implement effective measures against the pandemic. On the other hand, the infection rate influences the state of the economy. Figure 6 presents the results of modeling with variations in parameters b_i. The increase in these parameters simulates a decrease in job supply levels in sectors of the economy related to food production, its transportation, and medical infrastructure. We considered the case where these parameters are equal to each other. The growth of b_i leads to a significant change in the phase portrait of the system, resulting in a reduction in the levels of structural disruptions in various sectors of the economy Z_i and a decrease in the peak number of infected individuals y_2 in the chosen initial modeling interval. With b_i growing the overall number of infected individuals Σy_2 initially decreases and then begins to rise again.

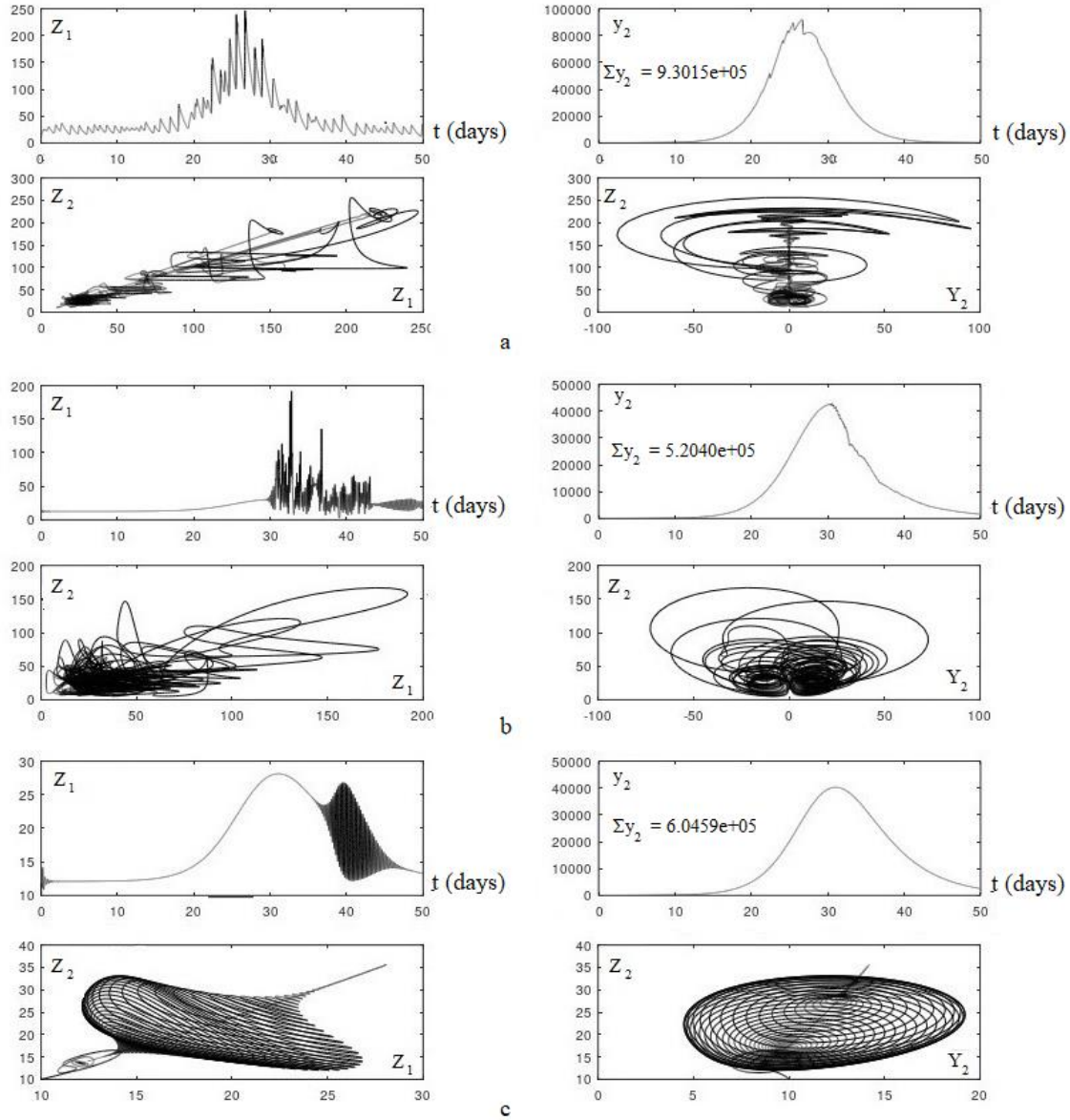

Figure 6: Changes in the level of infected individuals and structural disruption with parameters b_i variations: a) $b_i = 1$; b) $b_i = 15$; c) $b_i = 17$.

When modeling, we used a standard package for solving differential equations - the GNU Octave environment version 4.4.1, which is licensed under the GNU GPL and can run on Linux, macOS, BSD and Windows operating systems. Identification of model parameters was carried out using data [2]. According to these data, the peak of the epidemic in Ukraine was reached after 3 months, with the number of deaths reaching about 100,000 after 5 months when the first wave of the epidemic subsided. The total number of infected individuals during this time was over 3.5 million. These data align well with the results of modeling the first wave of the pandemic's development in Ukraine. To estimate the parameters of model (1), more detailed models [7, 8]

were used, developed as part of the joint project of the National Academy of Sciences of Ukraine and the International Institute for Applied Systems Analysis (Austria) "Complex modeling of the management of the safe use of food, water, and energy resources for sustainable social, economic, and environmental development."

3. Modeling the T-cell immune response in COVID-19

One of the most serious complications of COVID-19 is an excessive immune response, in which the level of pro-inflammatory cytokines responsible for regulating intercellular and inter-systemic interactions sharply increases (a "cytokine storm").This leads to the disruption of the immune system, an increase in the level of free radicals, causing multiple damages to internal organs (lungs, heart, kidneys, blood vessels, liver, gastrointestinal tract, brain), severe multiorgan failure, and alters the balance of synthesis and expenditure of energy in cells, posing a lethal risk. Unfortunately, the specific mechanism of the cytokine storm remains undefined, making research on the impact of the immune system on the course of COVID-19 highly relevant.

Typically, when modeling the epidemic process, the key parameters determining the dynamics of epidemics include: 1) the average incubation period of the disease; 2) the average active period when the patient is contagious; 3) the average recovery period; 4) the average period until death. It is important to note that these constants in epidemiological models are functions of the state of the patient's immune system, which plays a crucial role in fighting infection. However, there are currently no epidemiological models that allow for the consideration of the influence of the immune system's state on the course of the disease and the selection of therapy. Therefore, the task arises to develop a mathematical model of the immune response in COVID-19 and analyze, through it, possible mechanisms of imbalance between different components of the immune system leading to a cytokine storm. This task will be addressed in the following section.

3.1. Mathematical model of immune system

In mathematical modeling of the immune system response to antigenic determinants of various natures (viruses, allergens, tumor antigens, etc.), typically, the components facilitating antigen presentation, its destruction, regulation of proliferative processes, and modulation of the immune response are considered. These include helper, killer, macrophage, and suppressor cells [9].

In each of these links, there is a group of precursor cells, including cytotoxic T-cells - Y_{pK}, helper T-cells - Y_{pH} , normal macrophages - Y_{NM} , suppressor T-cells - Y_{pS} ; as well as a group of mature cells forming the immune response, including effector T-cells - Y_K, helper T-cells - Y_H , activated macrophages - Y_{AM} and suppressor T-cells - Y_S. The listed cellular populations are responsible for the recognition and destruction virus-infected cells (VIC) - Y_{VC} . VIC debris - Y_D contribute to the emergence of antigen-presenting cells (AK).

When infected with the coronavirus, the infection affects gene expression and protein synthesis. The virus imposes its protein synthesis algorithms on cells, necessary for its replication. Within hours, it neutralizes the cell's antiviral signaling, delaying and confusing the immune response. The virus reduces the ability of infected cells to translate genes into proteins, thereby decreasing overall protein synthesis. Additionally, it actively degrades cellular messenger RNA (mRNA), while its own mRNA remains protected. Finally, the virus can also prevent the export of mRNA from the cell nucleus, where they are synthesized. Based on the analysis of data [10, 11] on the peculiarities of the immune response in COVID-19, the following assumptions can be made, which should be considered in the model.

1. VIC accelerates the transition of precursor cytotoxic T-cells to a mature form and enhances the proliferation of effector cells. Moreover, they promote the production of lymphoid factors (F) by helper cells: interleukins, growth factors, and others that activate the proliferation of mature T-cells, the transition of normal macrophages into the activated form, and the inflammatory process.

2. Increased inflammation (I) enhances the proliferation rate of mature effector, helper, suppressor T cells, and activated macrophages, as well as accelerates the influx of all precursors.

3. *VIC* are recognized and destroyed by cytotoxic T-lymphocytes and activated macrophages.

4. The proliferation of helper cells is activated after contact with antigen-presenting cells (*AK*).

5. The increase in the level of effector cells activates the process of maturation of suppressors. Suppressor cells inhibit the processes of proliferation and maturation of helper and effector cells, as well as the processes of *VIC* destruction by effectors and macrophages.

6. There is a temporary hierarchy that allows identifying a group of fast variables that manage to reach a stationary state: antigen-presenting bed, lymphoid factors, and the inflammatory reaction.

7. The coronavirus can evade immune system recognition through suppression at the precursor cell level, thereby reducing the levels of killer and helper T-cells. Additionally, it may affect the efficiency of killer T-cells and activated macrophages in destroying cells infected with the coronavirus.

8. The coronavirus can also infect activated macrophages. This increases the level of *VIC* due to the action of virus-infected macrophages Y_{VM}.

9. One of the possible paths of COVID-19 immunopathogenesis is the induction of damage and death to vessel endothelium due to virus replication in the infection focus. Uncontrolled inflammatory reactions lead to significant tissue damage and the development of severe forms of the disease.

10. Tissue damage results in an energy imbalance, disrupting the interrelationships between energy function Y_{CF} and mitochondrial activity Y_{CM} for energy synthesis in the cell. A significant indicator of this imbalance, which increases with infection growth, is the number of structural disruptions Y_{CD} in the cellular energy system. An increase in Y_{CD} leads to a decrease in the activity of effector cells to destroy *VIC*. In [12], the Lorenz model was used to investigate these relationships. We will use these results to describe the energy block of the model for cells affected by the coronavirus. Taking these assumptions into account, the model has the following form:

$$
\begin{aligned}
&\frac{dY_{pK}}{dt} = a_1(1 + a_2 I) - \frac{a_3 Y_{pK} Y_{VC}}{(c_1 + c_2 Y_S)(d_1 + d_2 Y_{VC})} - a_4 Y_{pK},\\
&\frac{dY_{pH}}{dt} = a_5(1 + a_6 I) - \frac{a_7 Y_{pH} AK}{(c_3 + c_4 Y_S)(d_3 + d_4 Y_{VC})} - a_8\, Y_{pH},\\
&\frac{dY_{NM}}{dt} = a_9(1 + a_{10} I) - \ a_{11} Y_{NM}\mathrm{F} - a_{12}\, Y_{NM},\\
&\frac{dY_{pS}}{dt} = a_{13}(1 + a_{14} I) - a_{15} Y_K Y_{pS} - a_{16}\, Y_{pS},\\
&\frac{dY_K}{dt} = \frac{a_3 Y_{pK} Y_{VC}}{(c_1 + c_2 Y_S)(d_1 + d_2 Y_{VC})} - a_{17} Y_K + \frac{a_{18} Y_K I}{(c_5 + c_6 Y_S)},\\
&\frac{dY_H}{dt} = \frac{a_7 Y_{pH} AK}{(c_3 + c_4 Y_S)(d_3 + d_4 Y_{VC})} - a_{19}\frac{a_{20} Y_H F}{(c_7 + c_8 Y_S)},\\
&\frac{dY_{AM}}{dt} = \ a_{11} Y_{NM}\mathrm{F} - a_{21}\, Y_{AM} - a_{22}\, Y_{AM} Y_{VC}\ ,\\
&\frac{dY_S}{dt} = a_{15} Y_K Y_{pS} - a_{23}\, Y_S + \ a_{24}\, Y_S I,\\
&\frac{dY_{VM}}{dt} = \ a_{22}\, Y_{AM} Y_{VC} - \ a_{25}\, Y_{VM},\\
&\frac{dY_{VC}}{dt} = \frac{a_{26} Y_{VC}}{(c_9 + c_{10} Y_{VC})} - \frac{a_{27} Y_{VC}(Y_K + Y_{AM})}{c_{10} + d_5 Y_{CD} + Y_{VC}} + \frac{a_{28} Y_{VM} Y_{VC}(1 + Y_{VC})^3}{L + (1 + Y_{VC})^4},\\
&\frac{dY_D}{dt} = \ \frac{a_{27} Y_{VC}(Y_K + Y_{AM})}{c_{10} + d_5 Y_{CD} + Y_{VC}} - a_{29}\, Y_D,\\
&\frac{dY_{CF}}{dt} = \ \sigma(Y_{CM} - Y_{CF}), \qquad \frac{dY_{CM}}{dt} = \ Y_{CF}(r + \ d_6 I - Y_{CD}\ ) - Y_{CM} \qquad \frac{dY_{CD}}{dt} = \ Y_{CF} Y_{CM} - b Y_{CD},
\end{aligned}
\tag{5}
$$

$$AK = \frac{Y_D(Y_{NM} + Y_{AM})}{c_{11} + Y_D}, \qquad F = \frac{Y_H Y_{VC}}{c_{12} + Y_{VC}}, \qquad I = \frac{c_{13} F}{c_{14} + F},$$

where a_i are constants that characterize: i = 1, 5, 9, 13 - inflows of $Y_{pK}, Y_{pH}, Y_{NM}, Y_{pS}$, respectively; i = 2, 6, 10, 14 - delay mode for activation of $Y_{pK}, Y_{pH}, Y_{NM}, Y_{pS}$ inflows, respectively; i = 3, 7, 11, 15 - speed of Y_K, Y_H, Y_M, Y_S, cells maturation, respectively; i = 4, 8, 12, 16, 17, 19, 21, 23, 25 – death rates of $Y_{pK}, Y_{pH}, Y_{NM}, Y_{pS}, Y_K, Y_H, Y_M, Y_S$, respectively; i = 18, 20, 24, 26 - population reproduction rates of Y_K, Y_H, Y_S, Y_{VC}, respectively; i = 27, 29 - destruction rates of Y_{CV} and Y_D, respectively; i =22 - this rate quantifies how quickly viruses infect macrophages Y_{VM} ; i =28 - this rate represent how infected macrophages stimulate the reproduction or replication of viruses; $c_k (k = \overline{1,14})$– are constants that characterize the nonlinear effects of the interaction between different chains of the immune system; $d_l\left(l = \overline{1,4}\right)$- are constants characterizing immunosuppression from the coronavirus; d_5 – is a constant characterizing a decrease in the efficiency of virus destruction due to structural disruptions in the energy system Y_{CD} ; d_6 - is a constant characterizing the influence of inflammation on the energy system; L – is a constant characterizing the activity of stimulating virus reproduction from the side of macrophages Y_{VM}; r, σ, b are parameters of the Lorenz model, which are functions of energy synthesis and expenditure processes in the cell [12].

3.2. Model investigation of immune system response in COVID-19

The mathematical formulation of the problem is as follows: given the model (4), the Cauchy problem needs to be solved over the time interval [0, T] for the specified intervals of model parameters. Consider the immune response in conditions where there are no interconnections between the immune and energy systems ($d_1 = d_3 = 1;\ d_2 = d_4 = d_5 = d_6 = 0$), and there is no infection of macrophages ($a_{22} = 0$).

Figure 7 illustrates how the dynamics of the immune response change with an increase in the parameter c_{13}, which characterizes the influence of lymphoid factors on the level of inflammation. $Y_{VC}, Y_D, Y_{AM}, F, AK$ in all figures are measured by the number of cells, Y_{CD} is measured by the number of violations, I is dimensionless variable.

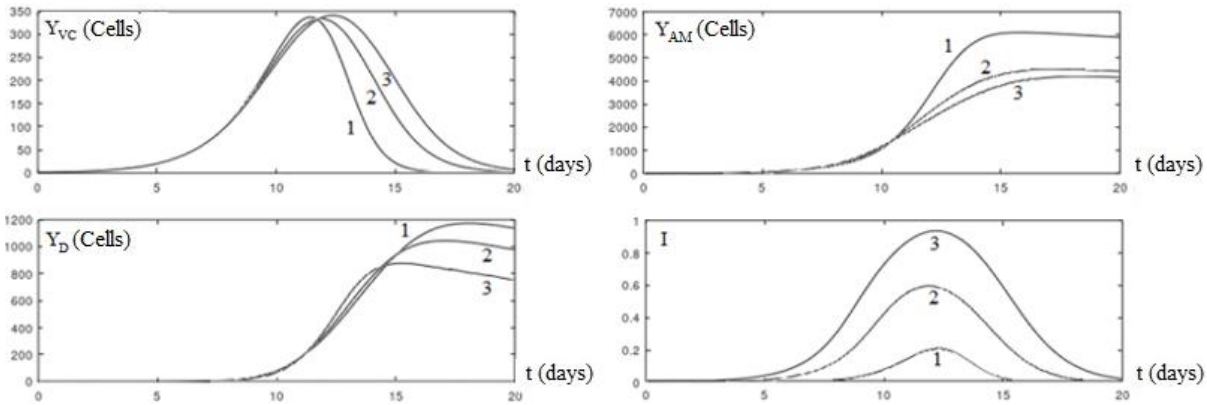

Figure 7: Changes in the dynamics of the immune response when parameter c_{13} increases: $1 - c_{13} = 9; 2 - c_{13} = 50; 3 - c_{13} = 100$.

Figure 8 shows how the dynamics of the immune response changes when the initial number of Y_{VC} increases. As follows from the simulation results, at relatively high initial levels of Y_{VC} , the intensity of the immune response is sufficient for rapid destruction of Y_{VC} . However, with small initial values of viruses (curve 4), the immune response is delayed, so viruses have time for reproduction and their complete destruction requires more time.

Let's explore what changes in the immune response will result from considering the interactions between the immune and energy systems. In figure 9, the results of modeling are presented with variations in the parameter d_5. This parameter characterizes the degree of inhibition of the effector function of killer cells and activated macrophages by the coronavirus.

The modeling results indicate that an increase in the degree of inhibition (increasing of d_5) leads to an increase in the time to reach the peak in lymphoid factors, inflammation, and Y_{VC} . The level of disruptions in the cellular energy system Y_{CD} also increases. (S - the area under the curve of disruptions).

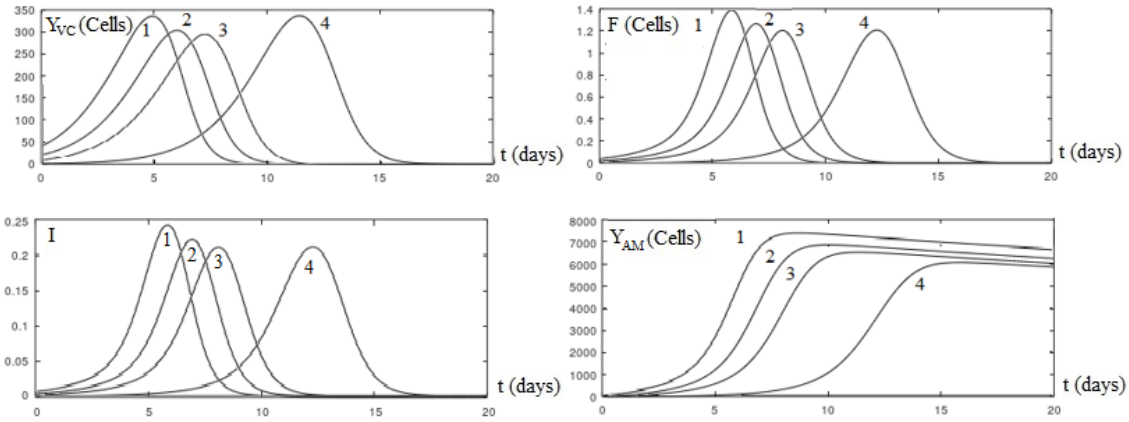

Figure 8: Changes in the dynamics of the immune response when initial level of Y_{VC} increases; $1 - Y_{VC}(0) = 40; 2 - Y_{VC}(0) = 20; 3 - Y_{VC}(0) = 10;\ 4 - Y_{VC}(0) = 1.$

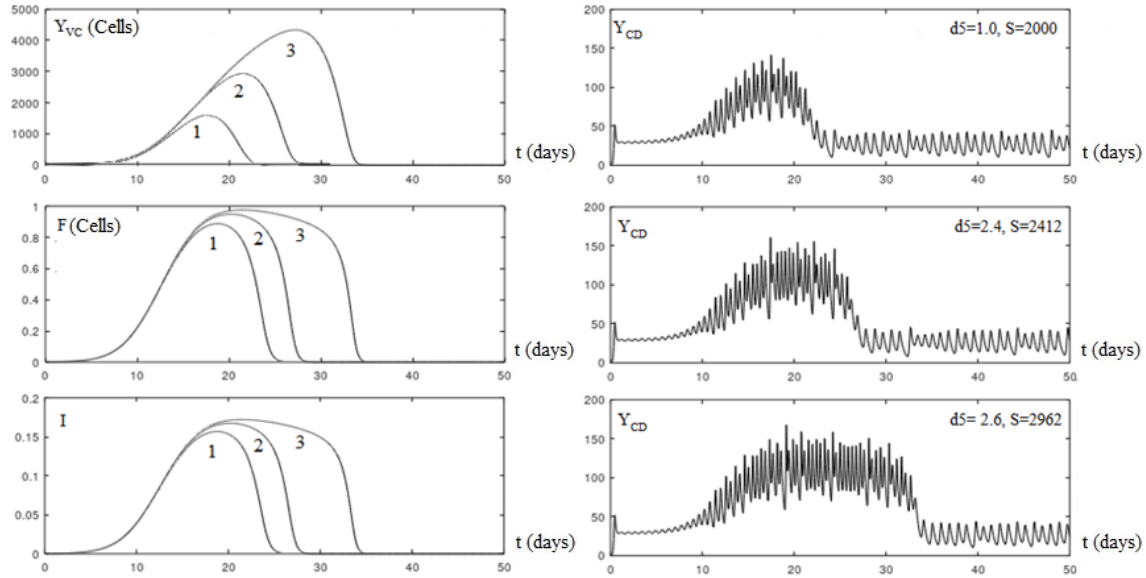

Figure 9: Dynamic of model variables with variation of parameter d_5:
$1 - d_5 = 1; 2 - d_5 = 2.4; 3 - d_5 = 2.6.$

The results of the simulation for the case where the VIC infects macrophages are shown in figures 10 and 11. As evident from figure 10, when the value of parameter a_{28} is reaches 0.07, the total amounts of the viruses, lymphoid factors, inflammation, and disruptions sharply increase, corresponding to the cytokine release syndrome.

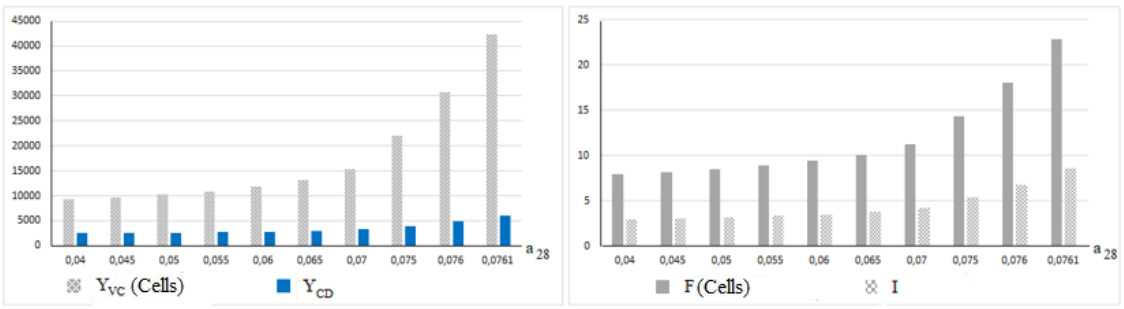

Figure 10: The dependency of the total quantity of VIC, lymphoid factors, inflammation, and disruptions on the parameter a_{28}.

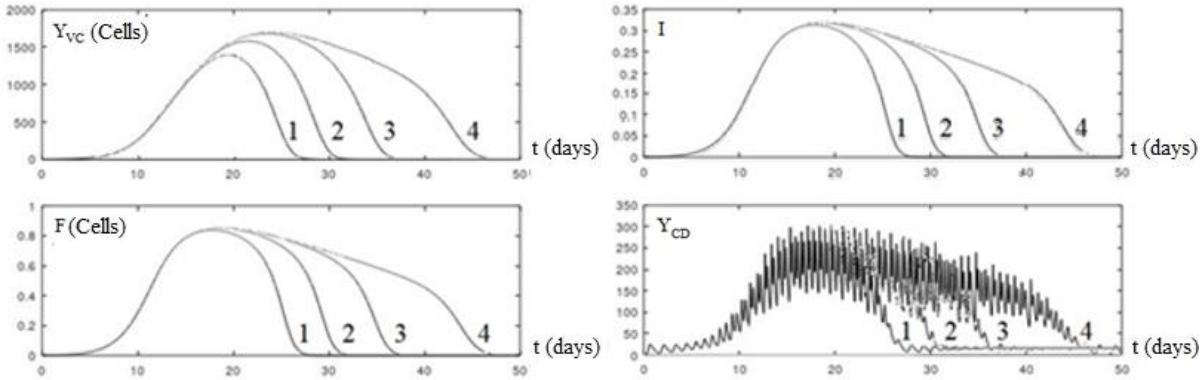

Figure 11: The dynamics of VIC, lymphoid factors, inflammation, and disturbances with an increase in the a_{28}: $1 - a_{28} = 0.07; 2 - a_{28} = 0.075; 3 - a_{28} = 0.076; 4 - a_{28} = 0.0761$.

Further growth leads to uncontrolled reproduction of the virus, which is incompatible with life. As seen from this figure, the system is highly sensitive to small changes in the parameter a_{28} within the interval [0.07, 0.0761].

If we consider the variables of model (5) as the mean values across a selected population, then the indicator of the number of structural disruptions Y_{CD} could characterize the collective immunity of this population. It is possible to examine the dependence of the parameters of model (3) on Y_{CD}. The relationships between (3) and (5) will be defined through the parameters a_0, T_{inf}, T_m. Parameters k_1 and k_2 from (4) also may be examined as functions of Y_{CD}. This will be considered in the further development of this work.

4. Conclusions

A model of the COVID-19 pandemic spread and associated changes in individual economic sectors has been developed. An approach to studying the interrelation within the food production system, food transportation, and the socio-economic sphere has been proposed using the three-sector Lorenz model. The model integrates economic sectors, uniformly described within a single framework, each of which is considered in terms of productivity, employment, and structural disruptions. The model studies were conducted with aim of: 1) assessing the influence of initial conditions and the state of economic sectors on the dynamics of the epidemic; 2) analyzing the impact of the epidemic process on interconnected sectors of the economy. The existence of deterministic and stochastic operational modes of interconnected economic sectors during a pandemic has been demonstrated. The transition from one mode to another is accompanied by an increase in the level of structural disruptions. The dynamics of these disruptions have been determined, along with the dependence of their total quantity on the epidemic process.

A mathematical model of T-cell immune response has been developed considering the peculiarities of its course in COVID-19, where excessive inflammation and increased levels of pro-inflammatory cytokines lead to cytokine release syndrome, causing damage to vital organs and disruption of immune system. The model considers the influence of cellular energetics, particularly mitochondria, on the regulation of the immune response and the amplification of inflammation. The simulation results have allowed investigation of the following: 1) how increases in the initial levels of virus-infected cells affect the average recovery period of severe symptomatic patients; 2) how inhibition of effector functions of killer cells and the level of inflammation affect the dynamics of the immune response; 3) how changes in the balance of energy synthesis and expenditure within the cell affect the occurrence of periodic and turbulent trajectories of the immune response.

The scientific novelty of the work lies in the following: 1) a proposed approach to the development of complex systems based on the use of multi-sector Lorenz models, allowing the exploration of interrelationships between the state of economy and the dynamics of epidemic; 2) the development of a mathematical model that accounts for the role of herd immunity in the dynamics of pandemic spread; 3) the developed models consider the superposition of two types

of random processes - "deterministic" Lorenz stochasticity and traditional Wiener stochasticity. Further development of the work will be associated with solving optimization tasks for immune response management aimed at minimizing virus-induced damage to the organism and substantiating various therapeutic regimens used in treatment.

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