A Method of Control and Operational Diagnostics of Data Errors Presented in a Non-positional Number System in Residual Classes

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Abstract

The basis of modern infocommunication systems is computer means of data transmission and processing. Therefore, one of the alternative means of achieving maximum efficiency in the functioning of the infocommunication systems when processing data in real time is to improve, first of all, such characteristics of computer data processing systems (CDPS) as the reliability and performance of information processing, as well as the fault tolerance of its functioning.

Currently, the quality of implementation of information processing procedures is largely determined by the selected mathematical model for organizing the information processing process in the CDPS. Therefore, research and finding ways to solve the problem of increasing the reliability of real-time CDPS, without reducing the productivity of processing large data arrays based on new mathematical models, is an urgent task. One of the possible innovative ways of solving the formulated problem is the usage of a non-positional number system in residual classes (NPNSRC) to create CDPS. The versatility of NPNSRC codes is explained not only by their high correcting abilities, arithmetic and the ability to fight against error packets, but also by their adaptability to flexible changes in correcting properties, without changing the coding method. The article discusses issues related to the control and diagnosis of data errors in the CDPS operating in the NPNSRC. The main attention is paid to the consideration of a method for quickly diagnosing data errors in the NPNSRC. Reducing diagnostic time increases the efficiency of diagnosing solitary errors in a non-positional code structure in the NPNSR. A specific example of the implementation of a method for diagnosing data errors in the NPNSRC is given.

Keywords

Computer data processing system, diagnosing data errors, non-positional code structure, non-positional number system in residual classes, number projection, orthogonal basis.

1. Introduction

An analysis of existing methods for increasing fault tolerance has shown that the best and most widely used in practice today are two methods: redundancy and control, diagnostics with further restoration of the CDPS functionality. When choosing control and diagnostics methods, the main attention should be paid to the ability of this control and diagnostics method to detect errors, as well as the amount of equipment and time spent on control [1-3].

The principles of control of non-positional code structures in the NPNSRC are the same as the principles of control in the positional number system (PNS), while taking into account the principles of formation of the NPNSRC and the influence of properties of the NPNSRC on the structure of the CDPS [4], let us note, in a general way, the principles of data control in non-positional code structures: the principle of reliability of control; the principle of continuity of control; the principle of operational control.

The most effectiveness from the usage of the NPNSRC is accomplished in cases when the realized algorithms comprise arithmetic operations such as addition, multiplication and subtraction [5]. However, in a CDPS operating in the NPNSRC, in addition to the above arithmetic operations, it is necessary to carry out so-called non-modular (positional) operations [6-8]. Such operations include control operations and correction (diagnosis and correction) of

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data errors. Data control, diagnosis and correction operations, compared to arithmetic operations in the NPNSRC, require significant time for their implementation. The need to implement control (monitoring) operations, diagnosis and correction of data reduce the overall effectiveness of the usage of the NPNSRC in real-time CDPS. When processing data in real time, the considerable time required for the implementation of control, diagnostic, and error correction procedures calls into question the feasibility of using NPNSRC as a general-purpose CDPS. The need to ensure the high efficiency of the functioning of the CDPS in the NPNSRC requires the development and implementation of methods of operational data control, diagnostics and correction, other than methods employed in ordinary binary PNS [9].

Within the framework of the concept of development of fast-acting and reliable CDPS presented in the NPNSRC, the urgent task is the development and application of methods and means of operational data control, diagnosis and error correction. In the article, the main focus is devoted to data diagnostics process in the NPNSRC.

The purpose of the research is to develop a method for controlling (monitoring) and operational diagnostics errors in data presented in the NPNSRC, using the orthogonal basis of partial sets of bases (modules).

2. Data correction process in the non-positional number system in residual classes

The correction process (detection and correction) of errors in the information code structure $\tilde{D}$ of data consists of the following main stages [4, 9]:

- data control (monitoring) (the process of detecting the presence of an error in $\tilde{D}=(d_1 \| d_2 \| ... \| \tilde{d}_j \| ... \| d_k)$, presented in the NPNSRC);
- data diagnostics (localization of error locations with a given diagnostic depth);
- error correction in the non-positional code structure (recovery of distorted residues $\{\tilde{d}_j\}$ $(j=\overline{1,k})$ of the incorrect number $\tilde{D}$ and obtaining the correct number D).

The number $D=(d_1 \| d_2 \| ... \| d_{j-1} \| d_j \| d_{j+1} \| ... \| d_k)$ in without redundant NPNSRC is represented by a set of residues $\{d_j\}$ $(j=\overline{1,k})$ according to the selected system of information bases (modules) $\{f_j\}$ in the numerical interval $[0,\ L)$, where $L=\prod_{j=1}^{k} f_j$ is overall amount of information code words [9]. In this case, the greatest common divisor of any two NPNSRC bases is equal to $(d_i, d_j)=1$; $i,j=\overline{1,k}$ $(i \neq j)$.

In order for the non-positional code structure in the NPNSRC to have the necessary corrective abilities, it is required that it contain sufficient information redundancy. First, the extant information redundancy in the original structure of the non-positional code should be determined and quantified [10]. Secondly, when tasked with providing data with additional corrective capabilities, introduce additional (artificial) information redundancy (apply the information redundancy method) by introducing additional (control) bases $\{f_c\}$ NPNSRCS [11].

Without loss of generality of reasoning, when tasked with providing data in the NPNSRCS with additional corrective capabilities, we will assume that only one additional control bases $f_c=f_{k+1}$ is added to the k information bases, which is coprime with any of the k existing information bases [4, 9]. In this case, the non-positional code structure $D=(d_1 \| d_2 \| ... \| d_j \| ... \| d_k \| d_{k+1})$ in the NPNSRC is represented by a set of $\{f_i\}$ $(i=\overline{1,k+1})$ bases in the full (working) numerical interval $[0,\ L_1)$, where $L_1=L \cdot f_{k+1}$ is the overall amount of code words for this NPNSRC with one control base [9].

It is known [4, 12] that for non-positional code structure in the NPNSRC the minimum code distance is defined by the expression $V_{\min}=c+1$, where c is the number of control bases used in the non-positional code structure in the NPNSRC, i.e. minimum code distance depends both on the number of control bases and on the size of each of it.

If for the control bases $\{f_{z_j}\}$ the condition $\prod_{j=1}^{g} f_{z_j} \le f_c$ is satisfied, then the introduction into the system of the NPNSRC bases of one control base $f_c = f_{k+1}$ is equivalent to the presence of g control bases $f_{z_1}, f_{z_2}, ..., f_{z_g}$. Taking into account the fact that all numbers taking part in data processing in the CDPS, along with outcome of the operation are in the interval $[0,\ L)$, then it is clearly that if as outcome of data processing the final outcome D is obtained and at the same time $D \ge L$, this means that the resulting number $\tilde{D}$ is distorted (incorrect). Thus, if $D < L$, then the conclusion is that the number D is correct, and if $D \ge L$, then the number $\tilde{D}$ is incorrect. In this case, only solitary errors (only in one $\{d_j\}$ of the number D) are assumed, or a packet of errors no longer than $s = [\log_2(f_j - 1)] + 1$ binary digits in one residue modulo f_j.

All existing methods for monitoring data in the NPNSRC are based on this principle of comparing the value of the number D with the value $[0,\ L)$ of the information numerical interval. Note that the comparison principle is also used in the development of diagnostic and error correction methods. In the future, in this article, we will consider the method of operational (quick) diagnosis.

3. Control and diagnostics of data in the non-positional number system in residual classes

In [4] there are a number of scientific statements, the outcomes of the proof of which underlie for methods for controlling and diagnosing data errors presented in the NPNSRC. It should be reminded that in what follows only a solitary error is assumed (in one residue d_j $(j = \overline{1, k+1})$ of the number $D = (d_1 \| d_2 \| ... \| d_j \| ... \| d_k \| d_{k+1})$ presented in the NPNSRC).

Let the number $D = (d_1 \| d_2 \| ... \| d_j \| ... \| d_k \| d_{k+1})$ being checked be given in the NPNSRC with informational $\{f_j\}$ $(j = \overline{1, k})$ and one control base $f_c = f_{k+1}$. It is necessary, firstly, to control (determine the correctness) of the number D, and, secondly, to diagnose the residues $\{d_j\}$ $(j = \overline{1, k+1})$ of the number D, i.e. determine distorted (or undistorted) residues.

Data controlling and diagnostics are carried out sequentially in two stages.

First stage. Method for controlling (monitoring) data of non-positional code structure $D = (d_1 \| d_2 \| ... \| d_{j-1} \| d_j \| d_{j+1} \| ... \| d_k \| d_{k+1})$, which from the following algorithm of actions:

1. Determine the values of the orthogonal basis B_j $(j = \overline{1, k+1})$ for the complete system of bases (modules) $\{f_j\}$ NPNSRC:

$$B_j = \frac{e_j \cdot L_1}{f_j}, \tag{1}$$

where e_j is weight of the orthogonal basis B_j.

2. Using the system of orthogonal basis B_j, the original number D in the NPNSRC is represented in the PNS [13]:

$$D_{PNS} = (\sum_{j=1}^{k+1} d_j \cdot B_j) \bmod L_1 . \tag{2}$$

3. Carry out positional comparison operations between the values of D_{PNS} and L. If the comparison result showed that $D_{PNS} < L$, then number D is correct. If $D_{PNS} \ge L$, then the

number $\tilde{D}$ is considered incorrect if only one of the residues $\{d_j\}$ of the number D is distorted.

Second stage. Method for diagnosing the residues $\{d_j\}$ $(j=\overline{1,k+1})$ of the code structure $\tilde{D}$ of data, based on the use of the obtained results of the following statement.

Statement. Let in an ordered $(f_j < f_{j+1})$ NPNSRC with k information and one control base $f_c = f_{k+1}$ the number $D = (d_1 \| d_2 \| \ldots \| d_j \| \ldots \| d_k \| d_{k+1})$ satisfy the following condition:

$$L = \frac{L_1}{f_{k+1}} = L_{k+1} < D < \bar{L}_i, \tag{3}$$

where $L = \prod_{j=1}^{k} f_j$ is overall amount of information code words in the NPNSRC (including only information bases);

$L_1 = \prod_{j=1}^{k+1} f_j$ is overall amount of all code words in the NPNSRC (including information bases and control base);

L_{k+1} is overall amount code words with one control base $f_c = f_{k+1}$;

$\bar{L}_i = \prod_{\substack{p=1,\\ p\neq i.}}^{k+1} f_p$ is overall amount of code words for excluding the base f_i, that is

$\bar{L}_i = f_1 \cdot f_2 \cdot \ldots \cdot f_{i-1} \cdot f_{i+1} \cdot \ldots \cdot f_{k+1}$.

Then the residues of the number D are not distorted (correct) if only a solitary error (in one residue $\tilde{d}_j$) is possible. The second stage of the developed method will be considered in more detail using a specific example.

4. An example of the application of the control and operational diagnostics method

Let's consider an example of using the control and diagnostic method in the NPNSRC for a one-byte (l = 1) machine word (8 binary digits) CDPS. In this case, a complete NPNSRC with one control base is specified by information $f_1 = 3$, $f_2 = 4$, $f_3 = 5$, $f_4 = 7$ and control $f_c = f_{k+1} = f_5 = 11$ bases. At the same time, the requirements for unambiguous representation of code words in a given information numeric $[0,\ L)$ range are ensured.

For a given NPNSRC we can calculate: $L_1 = \prod_{j=1}^{k+1} f_j = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_{k+1} = 3 \cdot 4 \cdot 5 \cdot 7 \cdot 11 = 4620$ – overall amount of code words in this NPNSRC; $L = \prod_{j=1}^{k} f_j = f_1 \cdot f_2 \cdot f_3 \cdot f_4 = 3 \cdot 4 \cdot 5 \cdot 7 = 420$ – overall amount of information code words in this NPNSRC. In this case, the full (working) $[0,\ L_1)$ and informational $[0,\ L)$ numerical ranges of numbers are defined, respectively, as $[0,\ 4620)$ and $[0,\ 420)$. All possible partial sets of the NPNSRC bases for a one-byte (l = 1) CDPS are presented in Table 1, where $\bar{L}_i = \frac{L_1}{f_i}$, f_i is a base that is not included in the given complete system of the NPNSRC bases.

For example, consider the process of finding $\bar{L}_i$, since the complete system of bases of the considered NPNSRC consists of five bases: $f_1 = 3$, $f_2 = 4$, $f_3 = 5$, $f_4 = 7$ and $f_5 = 11$, the base that is not included in the first line $(i=1)$ of Table 1 is $f_i = 3$, so we divide overall amount of code words in this NPNSRC $L_1 = 4620$ by this base: $\bar{L}_1 = \frac{4620}{3} = 1540$ or can get the same result by multiplying all sets of bases NPNSRC in the first line $(i = 1)$ of Table 1: $\bar{L}_1 = 4 \cdot 5 \cdot 7 \cdot 11 = 1540.$

Table 1
Set of partial operating bases NPNSRC (l = 1) [14]

i \ j	f_1	f_2	f_3	f_4	$\bar{L}_i$
1	4	5	7	11	1540
2	3	5	7	11	1155
3	3	4	7	11	924
4	3	4	5	11	660
5	3	4	5	7	420

Let, in the process of data processing, in lieu the correct $D = (1 \| 0 \| 0 \| 2 \| 1)$ $(D_{PNS} = 100 < L = 420)$ result of the operation a number of the form $\tilde{D} = (0 \| 0 \| 0 \| 2 \| 1)$, where $\tilde{D}_{PNS} = 3180 > L = 420.$ It is necessary to verify the correctness of the number $\tilde{D}$ and diagnose its residues $\{d_j\}$ $(j = \overline{1,5})$.

First stage.

1. Let's define all possible orthogonal basis $B_j (j = \overline{1,5})$ using formula (1) for the complete system of bases $f_1 = 3,\ f_2 = 4,\ f_3 = 5,\ f_4 = 7$ and $f_5 = 11$ NPNSRC [14]:

$$\begin{cases} B_1 = (1,0,0,0,0) = 1540,\ e_1 = 1, \\ B_2 = (0,1,0,0,0) = 3465,\ e_2 = 3, \\ B_3 = (0,0,1,0,0) = 3696,\ e_3 = 4, \\ B_4 = (0,0,0,1,0) = 2640,\ e_4 = 4, \\ B_5 = (0,0,0,0,1) = 2520,\ e_5 = 6. \end{cases}$$

2. Using the values of orthogonal basis $B_j (j = \overline{1,5})$, let's determine the value of number $\tilde{D}$ in the NPNSRC is represented in the PNS according to formula (2):

$$\tilde{D}_{PNS} = (0 \cdot 1540 + 0 \cdot 3465 + 0 \cdot 3696 + 2 \cdot 2640 + 1 \cdot 2520) \bmod L_1 = 7800 \bmod 4620 = 3180.$$

3. Let's compare the obtained number of $\tilde{D}_{PNS}$ and the value $L = 420.$ Since $\tilde{D}_{PNS} = 3180 > L = 420$, we make conclusion that the received $\tilde{D}$ is incorrect by $\{d_j\}$ of the correct number $D = (1 \| 0 \| 0 \| 2 \| 1)$.

Second stage.

1. Let's determine the values of partial orthogonal basis B_{ji} for each of the 5 possible sets of the NPNSRC bases [15]. In general, the value of partial orthogonal basis B_{ji} is determined based on the following comparison [16, 17]:

$$B_{ji} = \frac{\bar{L}_i \cdot e_{ji}}{f_j} \equiv 1 (\bmod f_j), \tag{4}$$

where $e_{ji} \equiv \overline{1, f_j - 1}$ – weight of the orthogonal basis B_{ji}.

So, for $j = 4$ and $i = 5$ we have:

$$\begin{cases} B_{1i} = (1,0,0,0), \\ B_{2i} = (0,1,0,0), \\ B_{3i} = (0,0,1,0), \\ B_{4i} = (0,0,0,1). \end{cases}$$

Let's determine the values of B_{j1} for the first $(i=1)$ set of bases: $f_1=4$, $f_2=5$, $f_3=7$ and $f_4=11$ (see Table 1). In this case $\bar{L}_1=\prod_{j=1}^{4} f_j=4\cdot 5\cdot 7\cdot 11=1540$ ($L=420$). We determine the values of the partial orthogonal basis based on the known relationship [18-20].

Determine the value of $B_{11}=\frac{\bar{L}_1\cdot e_{11}}{f_1}$. In this case, we use $f_1=4$. In this case, $\frac{\bar{L}_1}{f_1}=\frac{1540}{4}=385$, $e_{11}\equiv\overline{1,f_1-1}=\overline{1,4-1}=\overline{1,3}$. Let's compose possible values of B_{11} depending on possible e_{11}:

$$\begin{cases} 1\cdot 385\equiv 1(\bmod 4), \\ 2\cdot 385\equiv 2(\bmod 4), \\ 3\cdot 385\equiv 3(\bmod 4). \end{cases}$$

In this case, to fulfill the condition (4) we have that $B_{11}=1\cdot 385=385$.

Let's determine the value of $B_{21}=\frac{\bar{L}_1\cdot e_{21}}{f_2}$. In this case $f_2=5$, $\frac{\bar{L}_1}{f_2}=\frac{1540}{5}=308$, $e_{21}\equiv\overline{1,f_2-1}=\overline{1,5-1}=\overline{1,4}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 308\equiv 3(\bmod 5), \\ 2\cdot 308\equiv 1(\bmod 5), \end{cases} \qquad \begin{cases} 3\cdot 308\equiv 4(\bmod 5), \\ 4\cdot 308\equiv 2(\bmod 5). \end{cases}$$

In this case, $B_{21}=2\cdot 308=616$.

Let's determine the value of $B_{31}=\frac{\bar{L}_1\cdot e_{31}}{f_3}$. In this case $f_3=7$, $\frac{\bar{L}_1}{f_3}=\frac{1540}{7}=220$, $e_{31}\equiv\overline{1,f_3-1}=\overline{1,7-1}=\overline{1,6}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 220\equiv 3(\bmod 7), \\ 2\cdot 220\equiv 6(\bmod 7), \\ 3\cdot 220\equiv 2(\bmod 7), \end{cases} \qquad \begin{cases} 4\cdot 220\equiv 5(\bmod 7), \\ 5\cdot 220\equiv 1(\bmod 7), \\ 6\cdot 220\equiv 4(\bmod 7). \end{cases}$$

In this case, $B_{31}=5\cdot 220=1100$.

Let's determine the value of $B_{41}=\frac{\bar{L}_1\cdot e_{41}}{f_4}$. In this case $f_4=11$, $\frac{\bar{L}_1}{f_4}=\frac{1540}{11}=140$, $e_{41}\equiv\overline{1,f_4-1}=\overline{1,11-1}=\overline{1,10}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 140\equiv 8(\bmod 11), \\ 2\cdot 140\equiv 5(\bmod 11), \\ 3\cdot 140\equiv 2(\bmod 11), \\ 4\cdot 140\equiv 10(\bmod 11), \\ 5\cdot 140\equiv 7(\bmod 11), \end{cases} \qquad \begin{cases} 6\cdot 140\equiv 4(\bmod 11), \\ 7\cdot 140\equiv 1(\bmod 11), \\ 8\cdot 140\equiv 9(\bmod 11), \\ 9\cdot 140\equiv 6(\bmod 11), \\ 10\cdot 140\equiv 3(\bmod 11). \end{cases}$$

In this case, $B_{41}=7\cdot 140=980$.

Let's determine the values of B_{j2} for the second $(i=2)$ set of bases: $f_1=3$, $f_2=5$, $f_3=7$ and $f_4=11$ (see Table 1). In this case $\bar{L}_2=\prod_{j=1}^{4} f_j=3\cdot 5\cdot 7\cdot 11=1155$ ($L=420$).

Let's determine the value of $B_{12}=\frac{\bar{L}_2\cdot e_{12}}{f_1}$. In this case $f_1=3$, $\frac{\bar{L}_2}{f_1}=\frac{1155}{3}=385$, $e_{12}\equiv\overline{1,f_1-1}=\overline{1,3-1}=\overline{1,2}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 385\equiv 1(\bmod 3), \\ 2\cdot 385\equiv 2(\bmod 3). \end{cases}$$

In this case, $B_{12}=1\cdot 385=385$.

Let's determine the value of $B_{22} = \frac{\bar{L}_2 \cdot e_{22}}{f_2}$. In this case $f_2 = 5$, $\frac{\bar{L}_2}{f_2} = \frac{1155}{5} = 231$, $e_{22} \equiv \overline{1, f_2 - 1} = \overline{1, 5-1} = \overline{1, 4}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 231 \equiv 1(\bmod 5), \\ 2 \cdot 231 \equiv 2(\bmod 5), \end{cases} \qquad \begin{cases} 3 \cdot 231 \equiv 3(\bmod 5), \\ 4 \cdot 231 \equiv 4(\bmod 5). \end{cases}$$

In this case, $B_{22} = 1 \cdot 231 = 231$.

Let's determine the value of $B_{32} = \frac{\bar{L}_2 \cdot e_{32}}{f_3}$. In this case $f_3 = 7$, $\frac{\bar{L}_2}{f_3} = \frac{1155}{7} = 165$, $e_{32} \equiv \overline{1, f_3 - 1} = \overline{1, 7-1} = \overline{1, 6}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 165 \equiv 4(\bmod 7), \\ 2 \cdot 165 \equiv 1(\bmod 7), \\ 3 \cdot 165 \equiv 5(\bmod 7), \end{cases} \qquad \begin{cases} 4 \cdot 165 \equiv 2(\bmod 7), \\ 5 \cdot 165 \equiv 6(\bmod 7), \\ 6 \cdot 165 \equiv 3(\bmod 7). \end{cases}$$

In this case, $B_{32} = 2 \cdot 165 = 330$.

Let's determine the value of $B_{42} = \frac{\bar{L}_2 \cdot e_{42}}{f_4}$. In this case $f_4 = 11$, $\frac{\bar{L}_2}{f_4} = \frac{1155}{11} = 105$, $e_{42} \equiv \overline{1, f_4 - 1} = \overline{1,\ 11-1} = \overline{1, 10}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 105 \equiv 6(\bmod 11), \\ 2 \cdot 105 \equiv 1(\bmod 11), \\ 3 \cdot 105 \equiv 7(\bmod 11), \\ 4 \cdot 105 \equiv 2(\bmod 11), \\ 5 \cdot 105 \equiv 8(\bmod 11), \end{cases} \qquad \begin{cases} 6 \cdot 105 \equiv 3(\bmod 11), \\ 7 \cdot 105 \equiv 9(\bmod 11), \\ 8 \cdot 105 \equiv 4(\bmod 11), \\ 9 \cdot 105 \equiv 10(\bmod 11), \\ 10 \cdot 105 \equiv 2(\bmod 11). \end{cases}$$

In this case, $B_{42} = 2 \cdot 105 = 210$.

Let's determine the values of B_{j3} for the third $(i = 3)$ set of bases: $f_1 = 3$, $f_2 = 4$, $f_3 = 7$ and $f_4 = 11$ (see Table 1). In this case $\bar{L}_3 = \prod_{j=1}^{4} f_j = 3 \cdot 4 \cdot 7 \cdot 11 = 924$ ($L = 420$).

Let's determine the value of $B_{13} = \frac{\bar{L}_2 \cdot e_{13}}{f_1}$. In this case $f_1 = 3$, $\frac{\bar{L}_3}{f_1} = \frac{924}{3} = 308$, $e_{13} \equiv \overline{1, f_1 - 1} = \overline{1,\ 3-1} = \overline{1, 2}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 308 \equiv 2(\bmod 3), \\ 2 \cdot 308 \equiv 1(\bmod 3). \end{cases}$$

In this case, $B_{13} = 2 \cdot 308 = 616$.

Let's determine the value of $B_{23} = \frac{\bar{L}_2 \cdot e_{23}}{f_2}$. In this case $f_2 = 4$, $\frac{\bar{L}_3}{f_2} = \frac{924}{4} = 231$, $e_{23} \equiv \overline{1, f_2 - 1} = \overline{1,\ 4-1} = \overline{1, 3}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 231 \equiv 3(\bmod 4), \\ 2 \cdot 231 \equiv 2(\bmod 4), \\ 3 \cdot 231 \equiv 1(\bmod 4). \end{cases}$$

In this case, $B_{23} = 3 \cdot 231 = 693$.

Let's determine the value of $B_{33} = \frac{\bar{L}_2 \cdot e_{33}}{f_3}$. In this case $f_3 = 7$, $\frac{\bar{L}_3}{f_3} = \frac{924}{7} = 132$, $e_{33} \equiv \overline{1, f_3 - 1} = \overline{1,\ 7-1} = \overline{1, 6}$. Let's make a set of comparisons:

$$\begin{cases} 1 \cdot 132 \equiv 6(\bmod 7), \\ 2 \cdot 132 \equiv 5(\bmod 7), \\ 3 \cdot 132 \equiv 4(\bmod 7), \end{cases} \qquad \begin{cases} 4 \cdot 132 \equiv 3(\bmod 7), \\ 5 \cdot 132 \equiv 2(\bmod 7), \\ 6 \cdot 132 \equiv 1(\bmod 7). \end{cases}$$

In this case, $B_{33}=6\cdot 132=792$.

Let's determine the value of $B_{43}=\dfrac{\bar{L}_2\cdot e_{43}}{f_4}$. In this case $f_4=11$, $\dfrac{\bar{L}_3}{f_4}=\dfrac{924}{11}=84$, $e_{43}\equiv\overline{1,f_4-1}=\overline{1,\ 11-1}=\overline{1,10}$. Let's make a set of comparisons:

$$\begin{cases}1\cdot 84\equiv 7(\bmod 11),\\ 2\cdot 84\equiv 3(\bmod 11),\\ 3\cdot 84\equiv 10\bmod 11),\\ 4\cdot 84\equiv 6(\bmod 11),\\ 5\cdot 84\equiv 3(\bmod 11),\end{cases}\qquad \begin{cases}6\cdot 84\equiv 9(\bmod 11),\\ 7\cdot 84\equiv 5(\bmod 11),\\ 8\cdot 84\equiv 1(\bmod 11),\\ 9\cdot 84\equiv 8(\bmod 11),\\ 10\cdot 84\equiv 4(\bmod 11).\end{cases}$$

In this case, $B_{43}=8\cdot 84=672$.

Let's determine the values of B_{j4} for the fourth $(i=4)$ set of bases: $f_1=3$, $f_2=4$, $f_3=5$ and $f_4=11$ (see Table 1). In this case $\bar{L}_4=\prod_{j=1}^{4}f_j=3\cdot4\cdot5\cdot11=660$ $(L=420)$.

Let's determine the value of $B_{14}=\dfrac{\bar{L}_4\cdot e_{14}}{f_1}$. In this case $f_1=3$, $\dfrac{\bar{L}_4}{f_1}=\dfrac{660}{3}=220$, $e_{14}\equiv\overline{1,f_1-1}=\overline{1,\ 3-1}=\overline{1,2}$. Let's make a set of comparisons:

$$\begin{cases}1\cdot 220\equiv 1(\bmod 3),\\ 2\cdot 220\equiv 2(\bmod 3).\end{cases}$$

In this case, $B_{14}=1\cdot 220=220$.

Let's determine the value of $B_{24}=\dfrac{\bar{L}_4\cdot e_{24}}{f_2}$. In this case $f_2=4$, $\dfrac{\bar{L}_4}{f_2}=\dfrac{660}{4}=165$, $e_{24}\equiv\overline{1,f_2-1}=\overline{1,\ 4-1}=\overline{1,3}$. Let's make a set of comparisons:

$$\begin{cases}1\cdot 165\equiv 1(\bmod 4),\\ 2\cdot 165\equiv 2(\bmod 4),\\ 3\cdot 165\equiv 3(\bmod 4).\end{cases}$$

In this case, $B_{24}=1\cdot 165=165$.

Let's determine the value of $B_{34}=\dfrac{\bar{L}_4\cdot e_{34}}{f_3}$. In this case $f_3=5$, $\dfrac{\bar{L}_4}{f_3}=\dfrac{660}{5}=132$, $e_{34}\equiv\overline{1,f_3-1}=\overline{1,\ 5-1}=\overline{1,\ 4}$. Let's make a set of comparisons:

$$\begin{cases}1\cdot 132\equiv 2(\bmod 5),\\ 2\cdot 132\equiv 4(\bmod 5),\end{cases}\qquad \begin{cases}3\cdot 132\equiv 1(\bmod 5),\\ 4\cdot 132\equiv 3(\bmod 5).\end{cases}$$

In this case, $B_{34}=3\cdot 132=396$.

Let's determine the value of $B_{44}=\dfrac{\bar{L}_4\cdot e_{44}}{f_4}$. In this case $f_4=11$, $\dfrac{\bar{L}_4}{f_4}=\dfrac{660}{11}=60$, $e_{44}\equiv\overline{1,f_4-1}=\overline{1,\ 11-1}=\overline{1,\ 10}$. Let's make a set of comparisons:

$$\begin{cases}1\cdot 60\equiv 5(\bmod 11),\\ 2\cdot 60\equiv 10(\bmod 11),\\ 3\cdot 60\equiv 4(\bmod 11),\\ 4\cdot 60\equiv 9(\bmod 11),\\ 5\cdot 60\equiv 3(\bmod 11),\end{cases}\qquad \begin{cases}6\cdot 60\equiv 8(\bmod 11),\\ 7\cdot 60\equiv 2(\bmod 11),\\ 8\cdot 60\equiv 7(\bmod 11),\\ 9\cdot 60\equiv 1(\bmod 11),\\ 10\cdot 60\equiv 6(\bmod 11).\end{cases}$$

In this case, $B_{44}=9\cdot 60=540$.

Let's determine the values of B_{j5} for the fifth $(i=5)$ set of bases: $f_1=3$, $f_2=4$, $f_3=5$ and $f_4=7$ (see Table 1). In this case $\bar{L}_5=\prod_{j=1}^{4}f_j=3\cdot4\cdot5\cdot7=420$ $(L=420)$.

Let's determine the value of $B_{15}=\frac{\bar{L}_5 \cdot e_{15}}{f_1}$. In this case $f_1=3$, $\frac{\bar{L}_5}{f_1}=\frac{420}{3}=140$, $e_{15}\equiv\overline{1, f_1-1}=\overline{1,\ 3-1}=\overline{1, 2}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 140\equiv 2(\bmod 3), \\ 2\cdot 140\equiv 1(\bmod 3). \end{cases}$$

In this case, $B_{15}=2\cdot 140=280$.

Let's determine the value of $B_{25}=\frac{\bar{L}_5 \cdot e_{25}}{f_2}$. In this case $f_2=4$, $\frac{\bar{L}_5}{f_2}=\frac{420}{4}=105$, $e_{25}\equiv\overline{1, f_2-1}=\overline{1,\ 4-1}=\overline{1, 3}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 105\equiv 1(\bmod 4), \\ 2\cdot 105\equiv 2(\bmod 4), \\ 3\cdot 105\equiv 3(\bmod 4). \end{cases}$$

In this case, $B_{25}=1\cdot 105=105$.

Let's determine the value of $B_{35}=\frac{\bar{L}_5 \cdot e_{35}}{f_3}$. In this case $f_3=5$, $\frac{\bar{L}_5}{f_3}=\frac{420}{5}=84$, $e_{35}\equiv\overline{1, f_3-1}=\overline{1,\ 5-1}=\overline{1, 4}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 84\equiv 4(\bmod 5), \\ 2\cdot 84\equiv 3(\bmod 5), \end{cases} \qquad \begin{cases} 3\cdot 84\equiv 2(\bmod 5), \\ 4\cdot 84\equiv 1(\bmod 5). \end{cases}$$

In this case, $B_{35}=4\cdot 84=336$.

Let's determine the value of $B_{45}=\frac{\bar{L}_5 \cdot e_{45}}{f_4}$. In this case $f_4=7$, $\frac{\bar{L}_5}{f_4}=\frac{420}{7}=60$, $e_{45}\equiv\overline{1, f_4-1}=\overline{1,\ 7-1}=\overline{1, 6}$. Let's make a set of comparisons:

$$\begin{cases} 1\cdot 60\equiv 4(\bmod 7), \\ 2\cdot 60\equiv 1(\bmod 7), \\ 3\cdot 60\equiv 5(\bmod 7), \end{cases} \qquad \begin{cases} 4\cdot 60\equiv 2(\bmod 7), \\ 5\cdot 60\equiv 6(\bmod 7), \\ 6\cdot 60\equiv 3(\bmod 7). \end{cases}$$

In this case, $B_{45}=2\cdot 60=120$.

The set of calculated partial orthogonal basis B_{ji} is given in Table 2.

Table 2

Partial orthogonal basis B_{ji} for l = 1 [14]

i \ j	1	2	3	4
1	385	616	1100	980
2	385	231	330	210
3	616	693	792	672
4	220	165	396	540
5	280	105	336	120

2. Let's determine the correctness of the residues of the number $\tilde{D}$. First, let's compose all possible projections $\tilde{D}_i$ of the number $\tilde{D}=(0\|0\|0\|2\|1)$:

$$\begin{cases} \tilde{D}_1=(0, 0, 2, 1), \\ \tilde{D}_2=(0, 0, 2, 1), \\ \tilde{D}_3=(0, 0, 2, 1), \\ \tilde{D}_4=(0, 0, 0, 1), \\ \tilde{D}_5=(0, 0, 0, 2). \end{cases}$$

3. Let's represent the values of the projections $\tilde{D}_i$ in the PNS [14]:

$$\tilde{D}_{1PNS} = (d_1 \cdot B_{11} + d_2 \cdot B_{21} + d_3 \cdot B_{31} + d_4 \cdot B_{41}) \bmod \bar{L}_1 =$$
$$= (0 \cdot 385 + 0 \cdot 616 + 2 \cdot 1100 + 1 \cdot 980) \bmod 1540 = 100 < 420.$$
$$\tilde{D}_{2PNS} = (d_1 \cdot B_{12} + d_2 \cdot B_{22} + d_3 \cdot B_{32} + d_4 \cdot B_{42}) \bmod \bar{L}_2 =$$
$$= (0 \cdot 385 + 0 \cdot 231 + 2 \cdot 330 + 1 \cdot 210) \bmod 1155 = 870 > 420.$$
$$\tilde{D}_{3PNS} = (d_1 \cdot B_{13} + d_2 \cdot B_{23} + d_3 \cdot B_{33} + d_4 \cdot B_{43}) \bmod \bar{L}_3 =$$
$$= (0 \cdot 616 + 0 \cdot 693 + 2 \cdot 792 + 1 \cdot 672) \bmod 924 = 418 < 420.$$
$$\tilde{D}_{4PNS} = (d_1 \cdot B_{14} + d_2 \cdot B_{24} + d_3 \cdot B_{34} + d_4 \cdot B_{44}) \bmod \bar{L}_4 =$$
$$= (0 \cdot 220 + 0 \cdot 165 + 2 \cdot 396 + 1 \cdot 540) \bmod 660 = 540 > 420.$$
$$\tilde{D}_{5PNS} = (d_1 \cdot B_{15} + d_2 \cdot B_{25} + d_3 \cdot B_{35} + d_4 \cdot B_{45}) \bmod \bar{L}_5 =$$
$$= (0 \cdot 280 + 0 \cdot 105 + 2 \cdot 336 + 1 \cdot 120) \bmod 420 = 240 < 420.$$

Thus, of all the received projections $\tilde{D}_i$ of the number $\tilde{D} = (0 \| 0 \| 0 \| 2 \| 1)$ the projections $\tilde{D}_1$, $\tilde{D}_3$, $\tilde{D}_5 < L = 420$ and the projections $\tilde{D}_2$, $\tilde{D}_4 > L = 420$. Therefore, the results of controlling and diagnosing the incorrect $\tilde{D}$ number state that among all five residues of the number it is the residues d_1, d_3 and d_5 may be erroneous, but the residues d_2 and d_4 are definitely not distorted.

5. Conclusions

This article improves the method of control and operational diagnostics of data errors presented in the NPNSRC, using the orthogonal basis B_{ji} of partial sets of bases (modules) NPNSRC. The values of orthogonal basis B_{ji} are formed from the complete system of bases $\{f_i\}$ $(i = \overline{1, k+1})$, and it usage provides an opportunity to organize the parallel processing of projections $\tilde{D}_i$ of numbers D of a non-positional code structure in the NPNSRC. This circumstance makes it possible to enhance efficiency of controlling and diagnosing data in the CDPS operating in the NPNSRC.

Examples of specific realization of the process of controlling (monitoring) and diagnosing errors are given. It is established that the use of the proposed method will improve the efficiency of controlling and diagnosing data errors in the data processing system operating in the NPNSRC, and will also technically simplify the procedure for processing non-positional code structures.

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