Optimal multifactorial planning of experiments based on the combinatorial configurations MCDM

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Abstract

The paper is devoted to MCDM supporting multifactor combinatory analysis using optimal planning of experiments based on the remarkable properties of the proposed combinatorial configurations, namely the concept of "Ideal Ring Bundles" (IRBs). These combinatorial structures are ring-ordered positive integers that form a finite set of integers from 1 to the sum of all these numbers using both these numbers and all their consecutive terms. The application of Ideal Ring Bundles provides for finding optimal solution problems by reducing the volume of experiments in fuzzy decision analysis while maintaining on validity of the analysis. It is possible to use a simple algorithm to design an optimized multifactor combinatory analysis for MCDM support.

Keywords

Optimal multifactor planning of experiments, combinatorial configuration, attribute, decision analysis

1. Introduction

Modern methods of Multi-Criteria Decision Making (MCDM), as well as Multi-Criteria Decision Analysis (MCDA), are the most accurate methods of decision-making, and they can be known as a revolution in this field [1,2]. The underlying methods differ from each other in some aspects which were regarded and discussed in [3]. Publications in this field concern many decision-making problems not only in differing branches of science and technology but for everyday problems in human lives also, for example, to find the best solution if the price and quality of the processes are among the most common criteria in many differ variants for decision-making [4]. These methods are related to the complexity level of algorithms, the way of representing preferences evaluation criteria, weighting methods for criteria, uncertain data possibility, and other factors [5]. To interpret solving an MCDM problem can be selecting different ways. The process there is the most preferred way for solution choosing the best alternative from a set of other alternatives. If there are manifold preference sets of grouping alternatives then opt for a small set from them. These are feasible possibilities to define the alternatives that are efficient in rejecting non-dominated ones. A comprehensive review of the application of different MCDM methods is presented [6], where we can see a list of application fields and appropriate

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examples of the application focus, including education with contextual learner modeling in personalized and ubiquitous learning, E-learning, career and job, supply chain management, and other fields and examples (Table 1).

Table 1
List of application fields and appropriate examples of the application focus

Application fields	Examples of the application focus
Education	Contextual learner modeling in personalized and ubiquitous learning, E-learning
Career and job	Occupational stressors among firefighters, personnel selection problems, job choice
Supply chain management	Supporting sustainable supplier selection, green supplier evaluation, and selection
Civil engineering	Flood disaster risk analysis
Finance/economics	Project portfolio management
Energy sector	Ranking renewable energy sources, techniques for energy policy
Engineering and production	Engineering, material selection for optimal design, optimum process parameters
Organizations and corporates	The system selection process in enterprises, corporate sustainability
Transportation	Urban passenger transport systems, integrated transportation systems
Healthcare	The assessment of COVID-19 regional safety, occupational health, and safety risk assessment

The main decision-making methods that consider more than one criterion in the decision-making process are regarded in [7]. The authors give you an idea about using MCDMs in different fields and are one of the most common decision-making methods, as well as propose to classify considering them for different aspects. This paper aims to discuss the important concepts, applications, and types of MCDM methods. Based on the results of investigating the popularity of MCDM methods in different subject areas this paper was focused on many complementary studies.

2. Ideal Ring Bundles

Let us regard a numerical n-stage chain sequence of distinct positive integers {k1, k2,…,kn} as being cyclic so that kn is followed by k1. We call this a ring sequence. A sum of consecutive terms in the ring sequence can have any of the n terms as its starting point as well as any number of terms from 1 to n-1 (Table 2).

Table 2
Sums of consecutive terms in the ring sequence

pj	*qj*				
	1	*2*	...	*n-1*	*n*
1	*k1*	$\sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n-1} k_i$	$\sum_{i=1}^{n} k_i$
2	$\sum_{i=1}^{n} k_i$	*k2*	...	$\sum_{i=2}^{n-1} k_i$	$\sum_{i=2}^{n} k_i$
...	...	...	...	...	...
n-1	$\sum_{i=n-1}^{n} k_i + k_1$	$\sum_{i=n-1}^{n} k_i + \sum_{i=1}^{2} k_i$	...	*kn-1*	$\sum_{i=n-1}^{n} k_i$
n	*kn+k1*	$k_n + \sum_{i=1}^{2} k_i$	...	$\sum_{i=1}^{n} k_i$	*kn*

Each numerical pair (*pj, qj*), *pj* ,*qj*∈{1, 2,..., *n*}, corresponds to sum *Sj = S (pj,qj),* and can be calculated by equation (1), *pj ≤ qj.*

$$S_j = S\,(p_j\,,q_j) = \sum_{i=p_j}^{q_j} k_i \qquad (1)$$

In case pj > qj a ring sum is

$$S_j = S\,(p_j\,,q_j) = \sum_{i=1}^{q_j} k_i + \sum_{i=p_j}^{n} k_i \qquad (2)$$

Easy to see from table 2, that the maximum number of distinct sums Sn of consecutive terms of the ring sequence is

$$S_n = n(n-1) + 1 \qquad (2)$$

An n-stage ring sequence Kn= {k1, k2, ... ,ki, ... kn} of natural numbers for which the set of all Sn sums of consecutive terms in the ring sequence consists of the numbers from 1 to Sn= n(n−1) +1, each number occurring exactly once is called "Ideal Ring Bundle" (IRB) [8].
Here is a graphical representation of an Ideal Ring Bundle containing five (n=5) elements {1, 3, 10, 2, 5} (Figure 1).

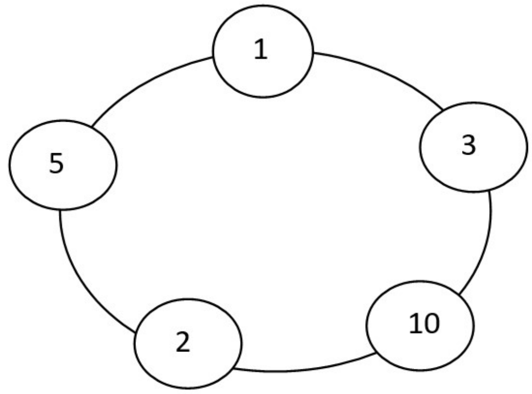

Figure 1: A graph of Ideal Ring Bundle containing five (n=5) elements {1, 3, 10, 2, 5}.

Consecutive terms in the numerical ring sequence {1,3,10,2,5} with parameters n= 5, Sn= $n(n-1)$ +1 = 21 presents in Table 3.

Table 3
Consecutive terms in the ring sequence {1,3,10,2,5}

pj	qj				
	1	2	3	4	5
1	1	4	14	16	21
2	21	3	13	15	20
3	18	21	10	12	17
4	8	11	21	2	7
5	6	9	19	21	5

Table 3 was calculated in a similar way to the above, using equations (1) − (3). Table 3 contains the set of all Sn= n(n−1)+1= 5(5−1)+1 = 21 sums to be consecutive elements of the 5-stage (n=5) ring sequence {1,3,10,2,5}, and each circular sum from 1 to Sn −1= 20 occurs exactly once. So, the numerical ring sequence {1,3,10,2,5} is the Ideal Ring Bundle (IRB) with information parameters n= 5, and Sn= 21.

2.1. IRB structure as a finite field

The study of IRB structure as a finite field uses modern mathematical methods for optimization of systems that exist in the theory of combinatorial configurations [8], and algebraic constructions based on cyclic groups in extensions of Galois fields [8]. A finite field exists for any prime power q, namely GF(q). The multiplicative group of GF(q) is cyclic; thus it is generated by any of its φ (q − 1) elements of order q − 1. These generating elements are primitive roots and for prime p, the residues 0,1,..., p- 1 form a field concerning addition and multiplication modulo p. GF(qm) is represented by the set of all m-tuples with entries from GF(q). In this representation, addition is performed component-wise wise but multiplication is more complicated. Associate with the m-tuple am-1 , am-2 , ..., a1 , a0 the polynomial am-1 xm-

1 +…+ a1x + a0. Then, to multiply two m-tuples, multiply instead their associated polynomials and reduce the result modulo any fixed mth degree polynomial f(x) irreducible over GF(q). The coefficients of the resulting polynomials constitute the m- tuple which is the product of the original two. For multiplicative purposes it is more convenient to represent GF(qm) in terms of a primitive root α; in which case, GF(qm) consists of 0, α 0 , α1, α2, α3,…, αr-2 where r = qm .

Multiplication then becomes a simple matter of reducing exponents modulo qm − 1, but addition is more complicated. Both these representations of GF(qm) are used in the proof of Singer's theorem. Singer discovered a large class of difference sets related to finite projective geometries [written PG (N, q)]. These have parameters υ = S = (qN+1 − 1)/(q − 1), k = n = (qN − 1)/(q − 1), λ = R = (qN-1 − 1)/(q − 1), for N ≥ 1 and they exist whenever q is a prime power.

Example 1. This represents the set of elements for finite projective geometry PG (2,3), of dimension N=2 over GF(3), with f(x) = x3 − x − 1:

$$\left.\begin{aligned}
x^0 &\equiv 1\\
x^1 &\equiv x\\
x^2 &\equiv x^2\\
x^3 &\equiv x+1\\
x^4 &\equiv x^2+x\\
x^5 &\equiv x^2+x+1\\
x^6 &\equiv x^2+2x+1\\
x^7 &\equiv 2x^2+2x+1\\
x^8 &\equiv 2x^2+2\\
x^9 &\equiv x+2\\
x^{10} &\equiv x^2+2x\\
x^{11} &\equiv 2x^2+x+1\\
x^{12} &\equiv x^2+2
\end{aligned}\right\}\ (\mathrm{mod}d\ 3,\ x^3-x-1) \tag{4}$$

Easy to see, that xi ↔ i, i = 0,1,…, S - 1= n(n − 1) = 12, where n = 4. Here a sequence of all fixed elements of zero coefficients by x2 are as follows: 1, x, x+1, x+2. We regard the PG(2,3) as a central symmetrical figure {0, 1, …,12}of order S = 13, where elements 1, x, x+1, x+2 generate a ring sequence of positive integers {1, 2, 6, 4} as a set of angular distances between diverged lines of projective geometry PG (2,3).

3. Application of Ideal Ring Bundles for optimal multifactorial planning of experiments

An optimal combinatorial plan of an experiment is known and can be developed using so-called Latin squares and their complete sets [9]. A Latin square of order p forms p×p - matrix, which contains each symbol in each row as well as in each column exactly once. And two Latin squares of order p are called "mutually orthogonal Latin squares" [9].

If putting one square on the other of it provides each symbol from one square occurs with each symbol from the other square exactly once. A combination of numbers a row, a column, and a symbol form the sub-set of factor levels for the first, the second, and the third factors of the optimal combinatorial measure plan. So, a single Latin square measure model of order p consists of three factors with p its levels. However, it is considerable only p2 variety combinations of the factor levels instead of p3 combinations. Thanks to it this plan of experimental measures allows us to cut the volume of work for p times. For the development of a factors measure plan that is well applicable to a system by two orthogonal squares, for five or more factors plans can apply to several orthogonal Latin squares accordingly.

The maximum number of factors under study F, the maximum number of levels R for each of these factors, and the maximum number N of pairwise orthogonal Latin squares of the M-th order, which forms the matrix M×M, chosen to build an optimal plan for a multivariate experiment, are interconnected by a system of simple equations:

$$F = R = n \tag{5}$$

$$N = M - 1 \tag{6}$$

$$M = F - 2 \tag{7}$$

where n is the order of IRB.

From equations (5)-(7) it is easy to see that to draw up optimal multifactor experimental plans, it is necessary to construct a system of pairwise-orthogonal Latin squares of the appropriate order n.

The optimal combinatorial measurement plan based on IRB elements can be generated using the next calculating actions.

1. Form based on n - sequence of numbers k1, k2, k3, ..., kn the IRB of order n, build an auxiliary matrix of numerical symbols P for numbering rows of Latin squares:

$$p_{ij} \equiv \left\{ \begin{array}{ll} 1 + \sum_{l=i+j}^{i+j} k_l, & \text{if} \quad i + j \le n \\ 1 + \sum_{l=i+1}^{n} k_l + \sum_{l=1}^{i+j-n} k_l, & \text{if} \quad i + j > n \end{array} \right\} (\text{mod } S_n), \tag{8}$$

   where i, j – row and column number of the set P, respectively; i, j = 1, 2, . . , p − 1= 4; Sn= n(n−1)+1.

2. Build a supporting matrix C for numbering columns of Latin squares:

$$c_{ij} \equiv \left\{ \begin{array}{ll} 1 + k_1 + \sum_{l=i}^{i+j-1} k_l, & \text{if}\, i + j \le n+1 \\ 1 + k_1 + \sum_{l=i}^{n} k_l + \sum_{l=1}^{i+j-n-1} k_l, & \text{if} \;\, i + j > n+1 \end{array} \right\} (\text{mod} S_n), \tag{9}$$

Construction to continue until the requirement is satisfied

$$c_{ij} \not\equiv 1 + \sum_{l=1}^{r} k_l (\bmod S_n), r \in \{1,2,...,n\} \tag{10}$$

If requirement (7) is not satisfied, continue the construction of matrix C using the formulas

$$c_{ij} \equiv \begin{cases} 1 + k_1 + \sum_{l=i+1}^{i+j} k_l, & \text{if} \quad i + j \le n \\ 1 + k_1 + \sum_{l=i+1}^{n} k_l + \sum_{l=1}^{i+j-n} k_l, & \text{if} \quad i + j > n \end{cases} (\bmod S_n) \tag{11}$$

Similarly to the construction of the matrix C, all other matrices can be found, and the formulas for calculating the z-th (z = 1, 2, ... , n − 1) of additional auxiliary matrices M(z) take the following form:

$$m_{ij}(z) \equiv \begin{cases} 1 + k_1 + \sum_{l=2}^{z+1} k_l + \sum_{l=i+b}^{i+j-a} k_l, & \text{if} \quad i + j \le n + a \\ 1 + k_1 + \sum_{l=2}^{z+1} k_l + \sum_{l=i+b}^{n} k_l + \sum_{l=1}^{i+j-n-a} k_l, & \text{if} \quad i + j > n + a \end{cases} (\bmod S_n), \tag{12}$$

here $a = 1, b = 0$ if $m_{ij} \not\equiv 1 + \sum_{l=1}^{r} k_l (\bmod S_n)$;

$a = 0, b = 1$ if $m_{ij} \equiv 1 + \sum_{l=1}^{r} k_l (\bmod S_n)$; $z = 1,2,..., n-2$; $r \in \{1,2,..., n\}$.

4. Synthesis of optimal multifactorial plans of experiments

4.1. Constructing a system of pairwise orthogonal Latin squares

As an example of the implementation of the above algorithm, we construct a system of pairwise orthogonal Latin squares based on IRB (1, 3, 10, 2, 5), where $k1$=1, $k2$=3, $k3$=10, $k4$= 2, $k5$= 5; n = 5.

1. Calculate a set of numbers P for numeration of Latin squares rows by elements k1, k2, k3, ..., kn

The set *P* for the numeration of Latin squares rows forms the *P-matrix* (Table 4).

Table 4
The set P for numeration of Latin squares rows (P-matrix)

4	14	16	21
11	13	18	19
3	8	9	12

6	7	10	20

2. Calculate a set of numbers *C* for the numeration of Latin squares columns of elements in the C-matrix.

The set *C* for the numeration of Latin squares columns forms the *C*-matrix (Table 5).

Table 5
The set C for numeration of Latin squares columns (C-matrix)

3	6	16	18
12	14	19	20
4	9	10	13
7	8	11	21

3. To calculate the rest three (n–2= 3) of numerical sets *M*(z) for finding the complete set of Latin squares in the manner that above.

The family *M*(z) of numerical sets, z = 1,2,3, for finding the complete set of Latin squares represents Tables 6, 7, and 8.

Table 6
The set M(1) for finding the complete set of Latin squares

6	9	19	21
8	18	20	4
7	12	13	16
10	11	14	3

Table 7
The set M(2) for finding the complete set of Latin squares

16	19	8	10
18	7	9	14
4	6	11	12
20	21	3	13

Table 8
The set M(3) for finding the complete set of Latin squares

18	21	10	12
20	9	11	16
6	8	13	14
19	3	4	7

4. According to the coordinates of the sets P and C, we construct Latin squares Q1, Q2, Q3. To obtain the first line in each of these squares in a normalized form (1,2,3,4), we carry out the corresponding renumbering of sets M(z). As a result, we get the pairwise orthogonal Latin squares:

Q_1

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

Q_2

1	2	3	4
2	1	4	3
4	3	2	1
3	4	1	2

Q_3

1	2	3	4
4	3	2	1
3	4	1	2
2	1	4	3

These squares form a complete set (all family) of the matrix to design n-factors (n=5) optimal measure plan, each factor can have four (n−1= 4) levels. So, a plan built on 3 squares can be applied in a 5-factor experiment, where the levels of the first factor correspond to the column numbers, the second to the row numbers, and the levels of the remaining three factors to the symbols of the first, second and third squares. An example of the application of the mentioned algorithm for composing the 5-factor optimal measure plan for the fuzzy process is below.

Let fuzzy process is characterized by the following physical parameters A, B, C, D, and E, each of which makes some mutual correlation as well as influence on static and dynamic behavior observed too. The range of parameter changing is subdivided into elected parts, and each of the ranges is described by four (n−1=4) characteristic levels. Of course, it is possible to regard measurable parameters as well as non-measurable too.

We have built a system of three orthogonal squares. Now the optimal plan of experiments with five (n=5) influence factors (for example, A, B, C, D, E) and four (n−1=4) discrete levels for each of the factors can be generated simply by choice of the levels of the first, the second and the third factors as correspond symbols (numbers) which are the same cell's co-ordinates in each Latin squares. Just the numbers of the fixed coordinates give us the levels of the fourth and the fifth factors. For example, we can make the first experiment selecting the first discrete level (symbol 1) of the factors A, B, C, D, E; the second experiment can be fulfilled taking the first level of the factor A while the second level (symbol 2) of the factors B, C, D, E; the third experiment involves the 1st level of A and the 3-rd of B, C, D, E; the fourth - 1st of A and 4th of B, C, D, E; the next experiment includes in the action the combination 2- A, 1- B, 3- C, 2- D, 4-E, and so on. So, this plan provides the fulfillment only (n−1)2 = 42 = 16 experiments (Table 9) instead of (n−1)3 = 64 in comparison with the standard plan of multifactorial experiments.

Table 9

The optimal plan of experiments with five (n=5) influence factors

Factor No	Experiment No															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
B	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
C	1	2	3	4	3	4	1	2	2	1	4	3	4	3	2	1
D	1	2	3	4	2	1	4	3	4	3	2	1	3	4	1	2

E	1	2	3	4	4	3	2	1	3	4	1	2	2	1	4	3

In general case each of 16 observed results can be considered measurable as well as any non-measurable factor or linguistic values.

The system consists of three Latin squares. A plan built from as many squares as can be used in a five-factor experiment, where the levels of the first factor correspond, for example, to column numbers, the second factor - to row numbers, and the levels of the remaining three factors correspond to the symbols of the first, second, and third squares. The described algorithm is developed based on the technique of constructing a finite projective plane in affine form and the existing relationship between IRB and the theory of combinatorial configurations. According to the algorithm, a program has been compiled that allows generation of multifactor optimal experimental plans on a computer with the exclusion of undesirable effects in static studies and creating application packages for machine synthesis of plans with specified properties.

4.2. Results of the pairwise orthogonal Latin squares applications

Variance analysis is also used to identify promising or the best combinations of levels of qualitative factors in the study of multifactor systems. Here, the experiment is set to optimize, which consists of finding the optimal qualitative composition of the system in the early stages of research, very often a large number of factors have to be included in the experiment so as not to miss any of the potentially significant ones, since further experiments may lose all meaning if some strongly influential factor is not included in the research program. Here is a need to conduct a screening experiment, the purpose of which is to isolate a group of essential factors and weed out insignificant ones. At the next stage of the study, the influence of significant factors can be studied in more detail. To build multi-level plans during the screening of experiments, combinatorial configurations, such as hypercubes, Latin squares, and Greco-Latin squares, are widely used, which can significantly reduce the enumeration of options. For any real experiment, the presence of various kinds of heterogeneities is very typical, the influence of which is desirable to exclude when comparing the levels of the main factors. If we are talking about planned experiments, then a variety of plans are proposed here, allowing the processing of data to exclude what distorts the influence of inhomogeneities. Here is a need to conduct a screening experiment, the purpose of which is to isolate a group of essential factors and weed out insignificant ones. At the next stage of the study, the influence of significant factors can be studied in more detail. To build multi-level plans during the screening of experiments, combinatorial configurations, such as hypercubes, Latin squares, and Greco-Latin squares, are widely used, which can significantly reduce the enumeration of options. For any real experiment, the presence of various kinds of heterogeneities is very typical, the influence of which is desirable to exclude when comparing the levels of the main factors. If we are talking about planned experiments, then a variety of plans are proposed here, allowing the processing of data to exclude what distorts the influence of inhomogeneities.

An experimental plan designed to investigate the effect on the effective attribute of four factors, each of which has levels. The plan of this type allows several times to reduce the number of observations compared to a four-factor analysis of variance. This assumes the absence of the influence of the interaction of factors on the effective attribute. It is obtained by superimposing on the Latin square another Latin square of the same dimension and "orthogonal" first. In this case, orthogonality means that each letter of both the Latin squares appeared only once in each row and each column. Usually in the second Latin square Greek letters are used, hence the name. For example, to construct an optimal 5-factor (F = 5) experimental plan with five (R = 5) levels for each of these factors, one should find three (N = 3) pairwise orthogonal Latin squares of the four (M = 4) order using the IRB of the five (n = 5) order. An example of the pairwise orthogonal Latin squares applications of material selection for optimal design combinatory analysis with five (n=5) influence factors is illustrated in Table 10.

Table 10

The table shows the results of experiments with an assessment of the level of achieved indicators according to various optimality criteria.

Factor No	Experiment No															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4
B	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
C	1	2	3	4	3	4	1	2	2	1	4	3	4	3	2	1
D	1	2	3	4	2	1	4	3	4	3	2	1	3	4	1	2
E	1	2	3	4	4	3	2	1	3	4	1	2	2	1	4	3
	Results of experiments															
Criterion X	5,1	4,4	4,5	3,8	3,4	5,1	4,4	4,5	3,8	3,4	4,2	5,1	4,4	4,5	3,8	5,1
Criterion Y	2,3	3,2	3,8	3,4	2,5	3,4	1,9	3,8	3,4	4,2	2,8	3,8	3,4	2,7	4,3	2,6
	- - -															
Criterion Z	3,2	2,3	3,5	4,3	3,4	3,4	3,3	3,6	4,4	2,7	3,1	2,9	4,4	2,7	3,1	2,9

Conclusion

The application of IRBs in fuzzy decision analysis provides for the minimizing of experiments while maintaining on validity of the analysis. It is possible to use a simple algorithm to design the optimal multifactorial plan of experiments in matrix form. The proposed algorithm can be well applicable for analysis of the influence of a lot of physical parameters as well as some other factors on the behavior of the analyzed fuzzy object or its model. This approach makes it possible to provide sufficiently less of computing in fuzzy decision analysis while maintaining on validity of the analysis. The application of Ideal Ring Bundles provides for finding optimal solution problems by reducing the volume of experiments in fuzzy decision analysis while maintaining on validity of the analysis. It is possible to use a simple algorithm to design of optimized multifactor combinatory analysis for MCDM support.

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