# Vector data systems machine learning with combinatorial optimization 

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#### Abstract

The article examines the present-day vector data systems machine learning with combinatorial technologies based on rotational symmetry and "perfect" combinatorial constructions with remarkable properties and perfection of one- and multidimensional Ideal Ring Bundles (IRBs). Numerical IRBs are cyclic sequences of positive integers that form perfect partitions of a finite interval $[1, S]$ of integers. The sums of connected sub-sequences of an Ideal Ring Bundle enumerate the set of integers exactly $R$-times. Two- and multidimensional IRBs are made available to configure vector data information and telecommunication systems that can reproduce the maximum number of combinatorial varieties in the system with limited elements and bonds. The favorable qualities of the combinatorial structures provide many opportunities to apply them to vector data coding and compression, signal reconstruction and system security, and other branches of science and advanced technology.


## Keywords

information technology, symmetry-asymmetry, harmony, perfection, performance, torus coordinate system, star-code ensemble, big data, optimum vector data coding system, IRB

## 1. Introduction

The main goal of modern systems engineering is the development of effective data processing for finding optimal solutions to wide classes of problems, including intelligent information technology and systems focused on international academicians, scientists, and practitioners to exchange new ideas for future collaboration. Big data information technology, which is known, can be defined as a software utility that is designed to analyze process, and extract data from extremely complex and large data sets that the traditional data processing software could never deal with [1]. In recent times, many new models, conceptions, parallel algorithms, platforms, applications, and processing gears have been developed to improve the value of multidimensional systems theory [2] and big data technology [1], [3-11]. The big data sets again involve many indexes of infrastructure, such as economic, national defense, and other factors, which have led to difficulties. The papers [3-5] present prospects and problems of big vector data for distant sensing. A technique for composing a map procedure, which performs filtering, sorting, and summary operations of big data presented at the IEEE International Conferences on Data Engineering [6]. Developing a reversible rapid coordinate transformation big vector data model for the cylindrical projection we see in the paper [7]. The paper [8] contains fast multidimensional ensemble empirical mode decomposition for the analysis of big spatial-temporal datasets.

[^0]The geometric computing algorithms are always very complex and time-consuming, which makes big spatial data processing very slow, even impossible [9]. A framework that couples cloud and highperformance computing for the parallel map projection of vector-based big spatial data is regarded in [10]. In the United States, large payer data amassed to explore large data to advance knowledge discovery in nursing methodologies, clinical trials, and lab research [12]. The idea of topological coordinates for torus chemical structures is in agreement with describing the physics of torusconfined plasmas [13]. The research works provide large-scale spatial modeling of big vector data under a common coordinate system. Still, the algorithmic complexity of the map projections represents a pressing computational challenge.

The present theory of vector combinatorial optimization includes such spatial structures as perfect difference sets [14], algebraic constructions based on cyclic groups in extensions of Galois fields [15], manifolds [16], and structures connecting algebra through geometry [17]. In general case it was possible to take in consideration a new conceptual model of the data processing based on the laws of worldwide harmony, such as the Golden ratio [18] and Optimum Cyclic Relationships [19]. The problem to be of very important for configure intelligent information technologies and systems of information security with improved quality indices of the system concerning performance reliability, data protection, and speed transformation content.

This machine learning involves novel techniques based on combinatorial configurations such as cyclic difference sets [17] and "Ideal Ring Bundles" [19]. These design techniques will make it possible to configure systems with fewer structural elements and bonds than at present while maintaining or improving computer power, data protection, and the other operating characteristics of the system.

## 2. The intelligent symmetry and asymmetry ensembles

"Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection." H. Weyl

### 2.1. Symmetry and asymmetry relation as geometric structure

Symmetry and asymmetry relation in geometric structure is the most familiar type of them. The more general meaning of symmetry-asymmetry is in combinatorial configurations as a whole. In this context, symmetries and asymmetries underlie some of the most profound results found in modern physics, including aspects of space and time [20]. Finally, discusses interpenetrating symmetry and asymmetry in the humanities, covering its rich and varied use in architecture, philosophy, and art. Space-time symmetries are features of space-time that can be described as exhibiting some form of symmetry [20]. The role of symmetry in physics is important in simplifying solutions to many problems, e.g. the study of isometrics in two or three-dimensional Euclidian space [21]. Only one angular interval in one-fold rotational symmetry enumerates the set $\{1\}$ exactly once $(R=1)$ is a singleton, known as a unit set [22].

Let us regard a sketch of $S$-fold rotational symmetry joined on two complementary asymmetries of the symmetry, where we require all angular distances between straight lines to emanate from a common point in each of the complementary asymmetries enumerated in the set of angles fixed number of times. An example of such rotational symmetry of order seven $(S=7)$ is given in Figure 1.


Figure 1: A sketch of rotational symmetry of order seven ( $S=7$ ) joined on two complementary asymmetries of the symmetry represented by three $\left(n_{1}=3\right)$ solid $(H, A, N)$ and four $\left(n_{2}=4\right)$ dash lines $(R, M, O, Y)$

If we let go round seven $(S=7)$ lines moving clockwise, we can obtain a set of angular distances $[\alpha$, $6 \alpha$ ] between distinct pairs of three ( $n_{1}=3$ ) solid lines $(H, A, N)$ as cyclic numerical relationship $\{1,4$, $2\}$, whereas between of distinct pairs of four ( $n_{2}=4$ ) dash ones $(R, M, O, Y)$ as cyclic link $\{1,1,2,3\}$. The sequence $\{1,4,2\}$ allows optimal partition of a ring in three ( $n_{1}=3$ ) parts to obtain the set of harmonious proportions from $1 / 7$ to $6 / 7$ by spatial interval $\alpha=360^{\circ} / 7$ exactly once, while $\{1,1,2,3\}$ as an optimal partition of a ring in four ( $n_{2}=4$ ) parts for finding the same proportions exactly twice. Note, this sequence begins Golden ratio [18].

Easy-to-interpret sketch of the intelligent system based on a 7 -fold rotational symmetry and asymmetry ensemble depicted in Figure 2.


Figure 2: A sketch of the intelligent system based on 7-fold rotational symmetry and asymmetry ensemble

Hence, the ring scale reading intelligent system based on 7-fold ( $S=7$ ) rotational symmetry allows a partition of planar space perfectly for the minimum number of intersections relative to the reading point by spatial interval $\alpha=360^{\circ} / 7$. We call this "intelligent symmetry and asymmetry ensemble" of order $S=7$, which provides an ability to reproduce the maximum number of combinatorial varieties in the systems with a limited number of intersections.

### 2.2. Comparative analysis of ordered chain and ring topology sequences

The ordered chain approach to the study of systems is known to be of widespread applicability and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in distributed technological systems.

Let us calculate all $S_{\mathrm{n}}$ sums of the terms in a numerical $n$-stage chain sequence of distinct positive integers $K_{\mathrm{n}}=\left\{k_{1}, k_{2}, \ldots, k_{\mathrm{n}}\right\}$, where we require all terms in each sum to be consecutive elements of the sequence. The maximum such sum is the sum $S_{n}$ of all $n$ elements:

$$
\begin{equation*}
S_{\mathrm{n}}=1+2+\ldots+n=n(n-1) / 2 \tag{1}
\end{equation*}
$$

If we regard a numerical $n$-stage sequence of distinct positive integers $K_{1 \mathrm{D}}=\left\{k_{1}, k_{2}, \ldots, k_{\mathrm{n}}\right\}$, where we require all terms in each sum to be consecutive elements of the sequence as being cyclic, so that $k_{n}$ is followed by $k_{1}$, we call this a ring sequence. A sum of terms in the ring sequence can have any of the $n$ terms as its starting point and can be of any number of terms from 1 to $n-1$. In addition, there is the sum of all $n$ terms, which is the same independent of the starting point. Hence the maximum number of distinct sums $S$ of consecutive terms of the ring sequence is given by

$$
\begin{equation*}
S=n(n-1)+1 \tag{2}
\end{equation*}
$$

An $n$-stage sequence $C_{\mathrm{n}}=\left\{k_{1}, k_{2}, \ldots, k_{\mathrm{n}}\right\}$ of natural numbers for which the set of all $S$ circular sums consists of the numbers $S=n(n-1)+1$ (each number occurs exactly $R$-times) is called an "Ideal Ring Bundle" (IRB) [19]. Here is an example of an $\operatorname{IRB}$ with $n=4$ and $S=13$, namely $\{1,2,6,4\}$. To see this, we observe:

| $1=1$ | $5=4+1$ | $9=1+2+6$ |
| :--- | :--- | ---: |
| $2=2$ | $6=6$ | $10=6+4$ |
| $3=1+2$ | $7=4+1+2$ | $11=6+4+1$ |
| $4=4$ | $8=2+6$ | $12=2+6+4$ |
|  |  | $13=1+2+6+4$ |

Note that if we allow summing over more than one complete revolution around the ring, we can obtain all positive integers as such sums. Thus: $14=1+2+6+4+1, \quad 15=2+6+4+1+2$, etc.

Next, we consider a more general type of IRB, where the $S$ ring-sums of consecutive terms give us each integer value from 1 to $N$, for some integer $N$, exactly $R$ times, as well as the value $N+1$ (the sum of all $n$ terms) exactly once. Here we see that:

$$
\begin{equation*}
N=n(n-1) / R \tag{3}
\end{equation*}
$$

An example with $n=4$ and $R=2$, so that $N=6$, is the ring sequence $\{1,1,2,3\}$, for which the sums of consecutive term are:

$$
1,1,2,3
$$

$$
\begin{array}{lll}
1+1=2, & 1+2=3, & 2+3=5, \\
1+1+2=4, & 1+2+3=6, & 2+3+1=6,
\end{array}
$$

$$
1+1+2+3=7
$$

We see that each "circular sum" from 1 to 6 occurs exactly twice $(R=2)$. This IRB has the parameters $n=4, R=2$.

Comparing the equations (1) and (2), we see that the number of sums $S$ for consecutive terms in the ring topology is nearly double the number of sums $S_{n}$ in the daisy-chain topology, for the same sequence $C_{n}$ of $n$ terms.

### 2.3. Vector Ideal Ring Bundles

Let us calculate all $S$ sums of the terms in the $n$-stage chain sequence of non-negative integer 2stage sub-sequences (2D vectors) of the sequence $K_{2 \mathrm{D}}=\left\{\left(k_{11}, k_{12}\right),\left(k_{21}, k_{22}\right), \ldots,\left(k_{\mathrm{i} 1}, k_{\mathrm{i} 2}\right), \ldots,\left(k_{\mathrm{n} 1}, k_{\mathrm{n} 2}\right)\right\}$ as being cyclic, so that ( $k_{\mathrm{n} 1}, k_{\mathrm{n} 2}$ ) is followed by ( $k_{11}, k_{12}$ ). We require all terms in each modular 2D vector sum to be consecutive elements of the cyclic sequence, and a modulo sum $m_{1}$ of $k_{i 2}$ and a modulo sum $m_{2}$ of $k_{i 2}$ are taken, respectively. A modular $\left(\bmod m_{1}, \bmod m_{2}\right) 2 \mathrm{D}$ vector sum of consecutive terms in this sequence can have any of the $n$ terms as its starting point and can be of any length (number of terms) from 1 to $n-1$.
An $n$-stage ring sequence $K_{2 \mathrm{D}}$, for which the set of all two-modular vector-sums $\left(\bmod m_{1}, \bmod m_{2}\right)$ forms a two-dimensional grid over torus $m_{1} \times m_{2}$, where each node of the grid occurs exactly $R$ times, is named two-dimensional Ideal Ring Bundle (2D IRB) with parameters $n, R$, and $m_{1}, m_{2}$ [19].

Example. Cyclic sequence $\{(0,1),(1,3),(0,2),(2,3)\}$ containing four $(n=4)$ two-dimensional $(t=2)$ vectors $k_{1}=(0,1), k_{2}=(1,3), k_{3}=(0,2), k_{4}=(2,3)$ generates ring vector-sums, taking complex modulo $m_{1}=3$, and $m_{2}=4$ as follows:

$$
\begin{aligned}
& (1,0) \equiv(0,1)+(1,3) ; \\
& (1,1) \equiv(1,3)+(0,2) ; \\
& (2,1) \equiv(0,2)+(2,3) ; \\
& (2,0) \equiv(2,3)+(0,1) ; \\
& \\
& (0,0) \equiv(1,3)+(0,2)+(2,3) ; \\
& (1,2) \equiv(0,1)+(1,3)+(0,2) ; \\
& (2,2) \equiv(0,2)+(2,3)+(0,1) ; \\
& (0,3) \equiv(2,3)+(0,1)+(1,3) .
\end{aligned}
$$

So long as the vectors $(0,1),(1,3),(0,2),(2,3)$ of the ring sequence themselves are circular 2D vectorsums too, the complete set of these vector sums:

| $(0,0)$ | $(0,1)$ | $(0,2)$ | $(0,3)$ |
| :---: | :---: | :---: | :---: |
| $(1,0)$ | $(1,1)$ | $(1,2)$ | $(1,3)$ |
| $(2,0)$ | $(2,1)$ | $(2,2)$ | $(2,3)$ |

The result of the calculation forms a two-dimensional $(t=2)$ grid over a torus surface of sizes $3 \times$ 4 , where 2D modular coordinates of each node of the grid occur exactly once $(R=1)$. Hence, the ring sequence $\{(0,1),(1,3),(0,2),(2,3)\}$ is a two-dimensional $(t=2)$ Ideal Ring Bundle with parameters $S=$ $13, n=4, R=1, m_{1}=3, m_{2}=4$, where $S=n(n-1) / R+1, m_{1}=n(n-1)=3, m_{2}=n=4$.

## 3. Torus coordinate system based on 2D vector Ideal Ring Bundles

Creation of a torus coordinate system based on the 2D vector Ideal Ring Bundle $\{(0,1),(1,3),(0,2),(2,3)\}$ with parameters $S=13, n=4, R=1, m_{1}=3, m_{2}=4$, where $S=n(n-1) / R+1, m_{1}=n(n-1)=3, m_{2}=n=4$ is given in Table 1.

## Table 1

Node points of reference coordinate grid $3 \times 4$ created by modular vector sums ( $m_{1}=3, m_{2}=4$ ) using 2D Ideal Ring Bundle $\{(0,1),(1,3),(0,2),(2,3)\}$

| No | Node point | The first vector | The second vector | The third vector |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $(0,0)$ | $(1,3)$ | $(0,2)$ | $(2,3)$ |
| 2 | $(0,1)$ | $(0,1)$ | - | - |
| 3 | $(0,2)$ | $(0,2)$ | - | $(1,3)$ |
| 4 | $(0,3)$ | $(2,3)$ | $(0,1)$ |  |


| 5 | $(1,0)$ | $(0,1)$ | $(1,3)$ | - |
| :--- | :--- | :--- | :--- | :--- |
| 6 | $(1,1)$ | $(1,3)$ | $(0,2)$ | - |
| 7 | $(1,2)$ | $(0,1)$ | $(1,3)$ | - |
| 8 | $(1,3)$ | $(1,3)$ | - | - |
| 9 | $(2,0)$ | $(2,3)$ | $(0,1)$ | - |
| 10 | $(2,1)$ | $(0,2)$ | $(2,3)$ | $(0,1)$ |
| 11 | $(2,2)$ | $(0,2)$ | $(2,3)$ | - |
| 12 | $(2,3)$ | $(2,3)$ | - | - |

Table 1 contains $n(n-1)=12$ node points of reference coordinate grid created by modular vector sums ( $m_{1}=3, m_{2}=4$ ) using 2D Ideal Ring Bundle $\{(0,1),(1,3),(0,2),(2,3)\}$.

The result of the calculation forms a two-dimensional $(t=2)$ grid over torus surface grid $3 \times 4$, where 2D modular coordinates of each node of the grid occur exactly once $(R=1)$.

Planar projection of spatially disjointed ring axes $m_{1}=2$ and $m_{2}=3$ coordinate points of twodimensional $(t=2)$ torus reference grid $m_{1} \times m_{2}=3 \times 4$ with common reference point $(0,0)$ given in Figure 3.


Figure 3: Planar projection of spatially disjointed ring axes $m_{1}=2$ and $m_{2}=3$ coordinate points of two-dimensional $(t=2)$ torus reference grid $m_{1} \times m_{2}=3 \times 4$ with common reference point $(0,0)$

Schematic model of torus coordinate system given as the simplest and well useful presentation of combinatorial optimization of vector data coding and processing based on remarkable properties of two- and multidimensional Ideal Ring Bundles. A more general model of the $t$-dimensional intelligent toroidal coordinate system for vector data coding made from multidimensional combinatorial configurations that provide an ability to reproduce the maximum number of vectors in the system with IRB of appropriate dimensionality.

In the Figure 4 shows a symbolic view of the two-dimensional projection of the annular axes of the $t$-dimensional toroid coordinate system $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}}$ with a common point " + " to count the identified values of $t$-dimensional discrete signals on non-intersecting axes. Each reference point in a system with $t$ coordinates mutually unambiguously corresponds to a certain set of features of a $t$ dimensional discrete signal, and the set of reference points corresponds to a set of sets of signal signs. $t$-counting is carried out on $t$ ring axes of the coordinate system according to the ordered sets of which mutually unambiguously correspond to a combination of $t$-dimensional binary code [19].


Figure 4: A symbolic view of the two-dimensional projection of the annular axes of the $t$ dimensional toroid coordinate system $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}}$ with a common point "+"

Consider the spatial model of the toroidal coordinate system with $t$ annular axes $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}}$ and common point of reference, where $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$ - the reference points on the corresponding axis that do not intersect with each other. The optimal $t$-dimensional coordinate system is described by the parameters $n, S, R, m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$, where $n$ is the number of basic $t$-tuples, the set of annular vector-sums of which covers the set of nodal coordinates of the grid $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}} t$-measurable torus surface by adding the corresponding $t$-tuples on the complex module ( $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$ ), $S$ - the order of rotational symmetry of the coordinate grid, $R$ - the number of different ways of forming identical annular $t$-dimensional vector sums on basic vectors, $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$ - the values of the modules that set the dimensions of the coordinate system:

$$
\begin{equation*}
\left(\left(k_{11}, k_{12}, \ldots, k_{1 t}\right),\left(k_{21}, k_{22}, \ldots, k_{2 t}\right), \ldots,\left(k_{i 1}, k_{i 2}, \ldots, k_{i t}\right), \ldots,\left(k_{\mathrm{n} 1}, k_{\mathrm{n} 2}, \ldots, k_{\mathrm{nt}}\right)\right), \tag{4}
\end{equation*}
$$

this $k_{i 1} \equiv k_{i}\left(\bmod m_{1}\right), k_{i 2} \equiv k_{i}\left(\bmod m_{2}\right), \ldots, k_{i t} \equiv k_{i}\left(\bmod m_{\mathrm{t}}\right)$

Thus, a $t$-measurable coordinate grid of $t$-torus is formed on the set of annular vector-sums of $n$ basis $t$-tuples, where $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$ are the numerical values of the modules.

## 4. Optimized vector data encoding system

### 4.1. Binary 2D optimized vector data code

The binary 2 D vector code based on the $\operatorname{IRB}\{(0,1),(1,3),(0,2),(2,3)\}$ in intelligent torus coordinate system $m_{1} \times m_{2}=3 \times 4$ presented in Table 2.

## Table 2

Binary 2D vector code based on the Ideal Ring Bundle $\{(0,1),(1,3),(0,2),(2,3)\}$ in intelligent torus coordinate system of sizes $3 \times 4$

| No | Vector | Digit 1 | Digit 2 | Digit 3 | Digit 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $(0,1)$ | $(1,3)$ | $(0,2)$ | $(2,3)$ |  |


| 1 | $(0,0)$ | 0 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | $(0,1)$ | 1 | 0 | 0 | 0 |
| 3 | $(0,2)$ | 0 | 0 | 1 | 0 |
| 4 | $(0,3)$ | 1 | 1 | 0 | 1 |
| 5 | $(1,0)$ | 1 | 1 | 0 | 0 |
| 6 | $(1,1)$ | 0 | 1 | 1 | 0 |
| 7 | $(1,2)$ | 1 | 1 | 0 | 0 |
| 8 | $(1,3)$ | 0 | 1 | 0 | 1 |
| 9 | $(2,0)$ | 1 | 0 | 1 | 1 |
| 10 | $(2,1)$ | 0 | 0 | 1 | 1 |
| 11 | $(2,2)$ | 1 | 0 | 0 | 1 |
| 12 | $(2,3)$ | 0 |  |  |  |

The vector code based on the $\operatorname{IRB}\{(0,1),(1,3),(0,2),(2,3)\}$ provides 2 D vector data coding design in the intelligent toroidal coordinate system in the $3 \times 4$ grid.

Table 2 contains $n(n-1)=12$ binary four-digit $(n=4)$ combinations for coding two attributes $(t=$ 2) both with three ( $m_{1}=3$ ) categories of the first, and four $\left(m_{2}=4\right)$ - the second attribute concurrently.

In general case a $t$-dimensional coordinate system formed by $t$-dimensional IRB is spatial coordinate grid of fixed sizes $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}}$, which covers surface of a spatial manifold. A set of all node point's grid of the system created by summing minimized sub-set of the set taking complex modulo $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$ to complete the $t$-dimensional coordinate system [20].

A more general model of the $t$-dimensional intelligent torus coordinate system for vector data coding made from multidimensional combinatorial configurations that provide an ability to reproduce the maximum number of vectors in the system with a limited number $n$ of appropriate IRB for needed sets of attributes and categories in an optimized basis.

A $t$ - dimensional torus coordinate system designed for vector data coding $t$ attributes and $m_{i}$ categories for an indexed attribute $i=1,2, \ldots, t$ requires $t$ concurrent disjointed axes $m_{1}, m_{2}, \ldots, m_{i}, \ldots$, $m_{t}$ with a common reference point for forming a $t$ - dimensional coordinate grid of the system with sizes $m_{1} \times m_{2} \times \ldots \times m_{\mathrm{t}}$.

The $t$-dimensional combinatorial configurations with torus reference grid $m_{1} \times m_{2} \ldots \times m_{\mathrm{t}}$ form $t$ dimensional vector codes, whose code sizes equal to a number of the node points grids of the torus coordinate system.

A "perfect" $t$-dimensional torus coordinate system based on vector IRB has information parameters $S, n, R$, and $m_{1}, m_{2}, \ldots, m_{\mathrm{t}}$.

The perfect torus code is weighed binary vector code forms a set of $t$-stage $n$-digital code combinations as $t$-modular ( $m_{1}, m_{2} \ldots, m_{t}$ ) sums of connected digit weights that allow an enumeration of a set of $t$-dimensional torus coordinate grid $m_{1} \times m_{2} \ldots \times m_{t}$, using minimum number $n$ of code binary digits.

A chart of multidimensional combinatorial configuration for constructing a $t$-dimensional big vector data coding system is a cyclic set of $n t$-dimensional sequence $\left\{K_{1}, K_{2}, \ldots, K_{\mathrm{i}}, \ldots, K_{\mathrm{n}}\right\}$ (Figure 5).


Figure 5: A chart of multidimensional combinatorial configuration for constructing $t$-dimensional big vector data coding system as a cyclic set of $n t$-dimensional sequence $\left\{K_{1}, K_{2}, . ., K_{\mathrm{i}}, .,, K_{\mathrm{n}}\right\}$

In the chart (Figure 5) each $t$-dimensional vector $K_{\mathrm{i}}=\left(k_{\mathrm{i} 1}, k_{\mathrm{i} 2}, \ldots, k_{\mathrm{it}}\right)$ of cyclic $t$-dimensional $n$ sequence $\left\{K_{1}, K_{2}, \ldots, K_{\mathrm{i}}, \ldots, K_{\mathrm{n}}\right\}$ is a $t$-stage sequence of non-negative integers, a set of ordered vector data attributes are indicated accordingly to $t$ categories of the big vector data processing.

### 4.2. Star-code Ensembles

The star-code ensembles follow from the intelligent sets of IRB combinatorial configurations. These codes are well appreciated for configuring vector data systems security. For example, the star codes based on the ensemble of two $\operatorname{IRBs}\{(1,1),(1,0),(1,4),(1,3),(1,2)\}$, and $\{(1,1),(1,3),(1,0),(1,2),(1,4)\}$ for encoding design in torus coordinate system $m_{1} \times m_{2}=4 \times 5$ presented in Tables 3 , and 4 .

Table 3
Binary 2D star-code based on the $\operatorname{IRB}\{(1,1),(1,0),(1,4),(1,3),(1,2)\}$

| No | Vector | Digit 1 | Digit 2 | Digit 3 | Digit 4 | Digit 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $(1,1)$ | $(1,0)$ | $(1,4)$ | $(1,3)$ | $(1,2)$ |
| 1 | $(0,0)$ | 1 | 0 | 1 | 1 | 1 |
| 2 | $(0,1)$ | 1 | 1 | 0 | 1 | 1 |
| 3 | $(0,2)$ | 1 | 1 | 1 | 0 | 1 |
| 4 | $(0,3)$ | 1 | 1 | 1 | 1 | 0 |
| 5 | $(0,4)$ | 0 | 1 | 1 | 1 | 1 |
| 6 | $(1,0)$ | 0 | 1 | 0 | 0 | 0 |
| 7 | $(1,1)$ | 1 | 0 | 0 | 0 | 0 |
| 8 | $(1,2)$ | 0 | 0 | 0 | 0 | 1 |
| 9 | $(1,3)$ | 0 | 0 | 0 | 1 | 0 |
| 10 | $(1,4)$ | 0 | 0 | 1 | 0 | 0 |
| 11 | $(2,0)$ | 0 | 0 | 0 | 1 | 1 |
| 12 | $(2,1)$ | 1 | 1 | 0 | 0 | 0 |
| 13 | $(2,2)$ | 0 | 0 | 1 | 1 | 0 |
| 14 | $(2,3)$ | 1 | 1 | 1 | 0 | 1 |
| 15 | $(2,4)$ | 0 | 1 | 1 | 0 | 0 |
| 16 | $(3,0)$ | 1 | 0 | 0 | 1 | 0 |
| 17 | $(3,1)$ | 1 | 1 | 1 | 1 | 1 |
| 18 | $(3,2)$ | 0 | 0 | 1 | 0 | 0 |
| 19 | $(3,3)$ | 0 | 0 | 1 | 1 | 1 |

Table 4
Binary 2D star-code based on the $\operatorname{IRB}\{(1,1),(1,3),(1,0),(1,2),(1,4)\}$

| No | Vector | Digit 1 | Digit 2 | Digit 3 | Digit 4 | Digit 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $(1,1)$ | $(1,3)$ | $(1,0)$ | $(1,2)$ | $(1,4)$ |
| 1 | $(0,0)$ | 1 | 1 | 0 | 1 | 1 |
| 2 | $(0,1)$ | 1 | 1 | 1 | 1 | 0 |
| 3 | $(0,2)$ | 1 | 0 | 1 | 1 | 1 |
| 4 | $(0,3)$ | 1 | 1 | 1 | 0 | 1 |
| 5 | $(0,4)$ | 0 | 1 | 1 | 1 | 1 |
| 6 | $(1,0)$ | 0 | 0 | 1 | 0 | 0 |
| 7 | $(1,1)$ | 1 | 0 | 0 | 0 | 0 |


| 8 | $(1,2)$ | 0 | 0 | 0 | 1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | $(1,3)$ | 0 | 1 | 0 | 0 | 0 |
| 10 | $(1,4)$ | 0 | 0 | 0 | 0 | 1 |
| 11 | $(2,0)$ | 1 | 0 | 0 | 0 | 1 |
| 12 | $(2,1)$ | 0 | 0 | 0 | 1 | 1 |
| 13 | $(2,2)$ | 0 | 1 | 1 | 0 |  |
| 14 | $(2,3)$ | 0 | 1 | 0 | 0 | 0 |
| 15 | $(2,4)$ | 1 | 1 | 1 | 0 | 0 |
| 16 | $(3,0)$ | 0 | 0 | 1 | 1 | 0 |
| 17 | $(3,1)$ | 0 | 0 | 0 | 1 | 1 |
| 18 | $(3,2)$ | 1 | 1 | 0 | 0 | 1 |
| 19 | $(3,3)$ | 1 | 1 | 1 | 0 | 1 |
| 20 | $(3,4)$ | 1 |  |  | 0 |  |

Comparing Tables 3 and 4, you can see that the violation of ordering bits within the star-code combinations retaining properties of this intelligent configuration.

Vector $t$-dimensional star-codes form ensembles of cyclic multiplicative groups with unique properties of fine structure to rebuild the scheme of cyclic permutations of elements according to the laws of mirror symmetry. The star-codes are formed on these groups acquire advantages over traditionally cyclic codes, expanding the possibilities of combined encoding and encryption of vector data. Optimal $t$-dimensional star-codes and $t$-dimensional coordinate systems of torus are described by the same type of parameters.

Two main methods of optimal coding can be defined - non-random or monolithic-group selfcorrective $t$-dimensional star-codes. The first method involves the use of vector binary code. The second is based on the encoding of $t$-measurable signals by annular monolithic-group code, where any allowed ring code combination allows the presence of no more than one block of characters of the same name. This allows you to instantly detect false combinations based on group distribution, and the code acquires self-corrective properties.

A significant advantage of monolithic-group tor codes over other noise-resistant codes is the ease of detection and automatic correction on the receiving side of inauthentic combinations. Since in $t-$ measurable monolithic-group code, each binary combination mutually unambiguously corresponds to a certain $t$ - set of features of a discrete signal, such a code allows you to correct part or all of the packet of code signals with erroneous signs according to the majority decoding principle, which greatly simplifies the correction procedure.

According to their properties, $t$-measurable star-codes compare favorably with the classic analogues of this class in the following indexes:

- a non-rigid star-code can control the behavior of objects in the spatial field of the $t$-torus coordinate grid of any large size $m_{1} \times m_{2} \ldots \times m_{\mathrm{t}}=S$, using only $\log _{2} S$ vector weight discharges;
- due to the higher information power, the star-code makes it possible to transmit more messages in wired and non-conductive communication systems than the standard code;
- in monolithic-group star-codes, the number of energy transitions during the coding of phaselatitudinal code signals is minimized, reducing the impact of the phenomenon of "competition", which makes it possible to increase the upper limit of the clock frequency, and, accordingly, the speed of transmission of $t$-dimensional signals.
Theorem. The power of the method of converting the form of information with $t$-measurable star- code is greater than in classical binary codes.

Proof. With the increase in the number of $t$ measurements of vector weight digits of the starcode, the total number of transformations on the set of basic vectors of weight digits as multiplicative groups increases accordingly, supplemented by options for mutual rearrangements of digits in the structure of "stellar" ensembles and corresponding permutations of numbers within the base vectors, which makes it possible to obtain more invariants of code combinations than standard code.

The theorem is proved.
Table 5 illustrates fragment of forming 4-digit 2D star-code $\{(1,2),(2,4),(1,3),(2,1)\}$ over intelligent torus coordinate system $3 \times 5$.

Table 5
Binary 2D vector star-code based on the Ideal Ring Bundle $\{(1,2),(2,4),(1,3),(2,1)\}$ over intelligent torus coordinate system of sizes $3 \times 5$

| No | Vector | Digit 1 | Digit 2 | Digit 3 | Digit 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $(1,2)$ | $(2,4)$ | $(1,3)$ | $(2,1)$ |
| 1 | $(0,0)$ | 1 | 1 | 1 | 1 |
| 2 | $(0,1)$ | 1 | 1 | 0 | 0 |
| 3 | $(0,2)$ | 0 | 1 | 1 | 0 |
| 4 | $(0,3)$ | 1 | 0 | 0 | 1 |
| 5 | $(0,4)$ | 0 | 0 | 1 | 1 |
| 6 | $(1,0)$ | 0 | 1 | 0 | 1 |
| 7 | $(1,1)$ | 1 | 0 | 1 | 1 |
| 8 | $(1,2)$ | 1 | 0 | 0 | 0 |
| 9 | $(1,3)$ | 0 | 0 | 1 | 0 |
| 10 | $(1,4)$ | 1 | 1 | 1 | 0 |
| 11 | $(2,0)$ | 1 | 0 | 1 | 0 |
| 12 | $(2,1)$ | 0 | 0 | 0 | 1 |
| 13 | $(2,2)$ | 1 | 1 | 1 | 1 |
| 14 | $(2,3)$ | 0 | 1 | 0 | 1 |
| 15 | $(2,4)$ | 0 | 1 | 0 |  |

Table 5 illustrates that 4 -digit ( $n=4$ ) binary 2D $(t=2)$ star-code created under the IRB $\{(1,2),(2,4),(1,3),(2,1)\}$ is non-redundant code with parameters $n=4, R=1$, and code size $P(n)=15$.

## 5. Vector data processing in intelligent spatial coordinate systems

The basic ideas of vector data processing in intelligent spatial coordinate systems are as follows:

- determine sizes of the intelligent spatial coordinate system and its dimensionality according to entity-attribute-value of vector data list;
- make a digital indexing entity-attribute-value list in the intelligent coordinate system;
- fetch from an information base applicable vector code to computer power and processing program;
- make vector data processing in the intelligent coordinate system.

The underlying methods provide opportunities to apply them to configure suitable relation vector data models, for example, using three categories: entities, characteristics, and its associations.

Surface topology is superior to geometry relating the torus as a "perfect" shape that is useful to visualize objects as a mathematical model of multidimensional systems for big vector data processing under torus reference systems based on IRB combinatorial configurations. Since there is a priory infinite set of such configurations the underlying techniques can be used for easy-to-grasp representation and multidimensional big vector data processing under the systems. Remarkable combinatorial properties and structural perfection of the combinatorial configurations provide highperformance vector computing technologies for effective big vector data processing. Advanced intelligent big vector data information technologies based on the concept of IRB combinatorial configurations provide competitive advantages of the information technologies concerning processing speed and storage capacity due to vector star-coding of compound attributes for two or more their categories simultaneously. Theoretically, there are infinitely many intelligent IRB ensembles, the number of which increases bit depth and dimension of optimized $t$-dimensional torus codes, which can be processed and sent by communication channels for the same time a greater amount of information compared to the capabilities of classical analogues.

## Conclusion

Combinatorial optimization of vector information technologies focuses on intelligent vector information technologies and systems security based on the Ideal Ring Bundles (IRB)s concept for the development of new directions in fundamental and applied research in system engineering using the underlying theory. Vector $t$-dimensional star-codes form ensembles of cyclic multiplicative groups with unique properties of fine structure to rebuild the scheme of cyclic permutations of elements according to the laws of mirror symmetry. The star codes formed on these groups acquire advantages over traditionally cyclic codes, expanding the possibilities of combined encoding and encryption of vector data. Star-codes open up new prospects for the use of combinatorial methods of optimized encoding of multidimensional signals in the problems of radio engineering, information and communication technologies and vector computer engineering. The favorable properties of IRBs make it possible to reproduce the maximum number of combinatorial varieties in information systems with a limited number of elements and operations for designing advanced information technologies. The essence of the vector data processing under a $t$-dimensional coordinate system is a smaller set of basic coordinates than the total number of coordinates, which generates them by adding the latter. This technology allows reducing the total processing time of information flows by encoding data on lists of two or more attribute categories at the same time. Therefore, vector IRBs improve the structure of databases with minimal memory and computing resources, supporting complexes of standard libraries of programming languages. Implementation of intelligent multidimensional optimum encoding systems planned in laboratory works provides learning for graduate students studying computer sciences and information technologies at Lviv Polytechnic National University.

Vector data systems machine learning with combinatory optimization focused on all technical and practical aspects of the latest research and results of international academicians, scientists and practitioners related to intelligent information technologies and systems of information security in Ukraine.

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