An approach to modeling elections in bipartisan democracies on the base of the “state-probability of action” model

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Abstract

An approach to constructing the two-level behavioral “state-probability of action” model and to getting appropriate matrices “state-probability of choice” for the case of two competing alternatives has been suggested. The top level is directly connected to probabilities of choice between alternatives. States of the model are connected to grades of pairwise comparisons. For getting rows of the matrix on this basis transitive scales are offered to be applied, but not only. It appears important to distinguish values of preferences themselves and probabilities of choice related to them. For this reason, another parameter standing for decisiveness of agents has been introduced. The bottom level is related to separate criteria influencing a choice.

A way to applying such a model for modeling voting in a bipartisan democracy has been suggested. Within this context, a problem of equilibrium between two alternatives, when no alternative has advantages over the other, is of great importance. Some sufficient conditions for equilibrium between two alternatives have been postulated in the paper, they significantly rely upon properties of symmetry.

The illustrating example of modeling elections in an imaginary country has been provided. Voters in this example are to make a choice between two candidates on the base of comparing them by some given criteria. In the initial example the equilibrium between alternatives holds. Then an issue how agents of influence could change the situation in a desirable direction is discussed.

Keywords

Decision making, agent-based modeling, model “state-probability of action”, equilibrium between alternatives, social modeling, bipartisan democracy, voting, agents of influence

1. Introduction and methodology

Voting procedures and their results obviously are to be acknowledged as an issue of great interest from the point of view of decision making theory, whether it goes about individual or collective decision making. There are many sound results in this field. However, what is needed to be developed is entrenching behavioral aspects of the matter, especially by means of agent-oriented modeling.

MoDaST-2024: 6th International Workshop on Modern Data Science Technologies, May, 31 - June, 1, 2024, Lviv-Shatsk, Ukraine

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In [1] the "state-probability of action" model has been introduced. In general words, the model involves some states, each state determines probabilities of choosing a certain alternative when being in this state. In more details, we should specify a matrix the "state-probability of action" $H = (h_{ij}, i = \overline{1,m}\,, j = \overline{1,n})$, where m is a number of states, n is a number of alternatives, and h_{ij} is the probability that an agent shall choose the i-th alternative when being in the i-th state. In addition to this, a vector of input probabilities $p^{(0)} = \left(p_i^{(0)}, i = \overline{1,m}\right)$, where $p_i^{(0)}$ is the probability that an agent is being in the i-th state at the moment, are to be specified as well.

We are considering the resulting vector of probabilities $p = \left(p_j,\ j = \overline{1,n}\right)$, which means that an agent shall choose the i-th alternative with the probability p_j.

Then, as it has been shown in [1],

$$p = p^{(0)} \cdot H, \tag{1}$$

For better understandability of the point, let's illustrate it with the following numeric example Let there be two alternatives, i.e. $n=2$. Let's take the following matrix H:

$$H = \begin{pmatrix} 1 & 0 \\ 0.75 & 0.25 \\ 0.5 & 0.5 \\ 0.25 & 0.75 \\ 0 & 1 \end{pmatrix}$$

It's very important to mention that the matrix H is centrosymmetric [2, 3]. An $(m \times n)$-matrix is said to be centrosymmetric, if

$$a_{ij} = a_{m-i+1,n-j+1}\ \forall i = \overline{1,m}; j = \overline{1,n}$$

The centrosymmetric matrix H itself poses no preference to any alternative.

Let's take the following vector of input probabilities:

$$p^{(0)} = (0.5,\ 0.3,\ 0.1,\ 0.1,\ 0)$$

(this means that agents tend to choose the first alternative rather than the second one).

Then in accordance with (1) the vector of resulting probabilities equals

$$p = p^{(0)} \cdot H = (0.8,\ 0.2)$$

The first alternative expectedly wins.

A very different situation takes place if the input vector of probabilities is symmetric, for instance as follows:

$$p^{(0)} = (0.1,\ 0.3,\ 0.2,\ 0.3,\ 0{,}1)$$

As it was shown in [2, 3], a product of a symmetric vector and a centrosymmetric matrix yields a symmetric vector. Indeed, in this case

$$p = p^{(0)} \cdot H = (0.5,\ 0.5)$$

Now we have a situation of equilibrium between alternatives when any alternative has no advantages over the other ones. Such a situation is of great importance, this point is going to be explained below in the paper.

Input probabilities $p^{(0)}$ can be specified in different ways, and we must consider their possible changes. A particular case, which is very significant for behavioral simulation, takes place if we can consider a Markov chain of transitions across the states, and input probabilities can be obtained by analyzing this chain. In basic features, such a possibility was illustrated in [1, 4, 5].

Within the initial model described in [1], it was very unclear how to form a basic matrix "state-probability of action", and its states may be very arbitrary. In [4, 5] some modifications for making the approach more structured and understandable were suggested. In general, these modifications referred to how to obtain matrices in a more flexible and understandable way [4] and how to describe different influencing factors by combining different nodes, each of which implements its own model the state-probability of action [5]. In the paper we are going both to develop these approaches and to suggest some others, one of which is related to pairwise comparisons.

It appears that exploring results of voting in bipartisan democracies by means of agent-oriented simulation of them fits quite well into the "state-probability of action" model. We are going to explain this point in the next section.

2. Describing bipartisan democracies within the model

First of all, if we regard a bipartisan democracy, a final choice on elections eventually comes down to the case of two alternatives (*n=2*), whether it goes about a competition between the two main political parties or, even in larger measure, between their leaders. So, the case *n=2* becomes the staple case, and its role becomes very significant. On the other hand, it is just the case, which is the simplest for an analysis.

Situations of equilibrium between alternatives, mentioned and showcased above, acquire a really crucial role for the case of two alternatives. Indeed, it can be shown that if the number of voters becomes large enough, even the slightest advantage of any alternative shall be sufficient for its victory with the probability close to *1*, and the opponent shall have nearly no chances to succeed. This allows us to presume that a real bipartisan system when the two leading parties win in turn and change each other can take place if only there are repeated situations of equilibrium – otherwise one of the parties shall permanently win.

Technically, the matter within the model can be rather complicated. Of course, a distribution of output probabilities p including the equilibrium situations $p = (0.5, 0.5)$ shall be permanent if the background Markov chain is homogenous, i.e. if transitional probabilities across states does not change in the course of time. In fact, it may not be true, transitional probabilities definitely may change. But we can consider some "hidden" chains, "hyper-chains" describing changes of transitional probabilities, and expect at least some of these hyper-chains to turn out homogeneous. Anyway, even if a situation of equilibrium is actually not permanent, for having bipartisan changes of winners such situations definitely are to happen repeatedly. Therefore, the issue of finding equilibrium situations is the issue of great significance.

In social systems there always are agents of influence, who are trying to affect decisions made by other agents, i.e. by voters. Within the framework of our behavioral model, they might try to change transitional probabilities across the states or to change probabilities $p^{(o)}$ directly, but they can also try to affect other parameters of the model. Below we are going to consider some of such parameters.

3. The parametrized model of forming matrices the "state – probability of action"

Firstly, we are developing an approach based on pairwise comparisons typically applied within the well-known Analytic Hierarchy Process (AHP) [6-10]. It appears reasonable to operate not with hardly specified probabilities even if there is no convincing and clear evidence for such specifications, but with much more clear and flexible thinking about grades of preferences between alternatives (for example, *I find John significantly better than Bob* or *John and Bob are equally good for me*). Some grades of preference have more or less clear linguistic meaning (*slight, noticeable, significant* etc.), but ascribing numerical values to them often is in question. The standard grading scale suggested by Saati often yields acceptable results but sometimes not. Many alternative approaches exist. We find reasonable to apply so-called transitive scales [11, 12]. More strictly, we talk about τ-transitive scales, where τ is the parameter specifying how many times the next grade is evaluated bigger than the previous one. Thus, if we postulate that there are q possible grades of preference, then the total number of grades including inverse ones (meaning opposite to *better*, i.e. *worse*) shall be *2q+1*. Namely, they have the keys (indices)

-q, -q+1, …, -1, 0, 1, …, q-1, q

and the value ascribed to the k-th grade within a τ-transitive scale equals τ^k.

It appears natural to connect each state with a certain grade of the scale. More strictly, the consequent states in terms of grade differences for two alternatives may be as follows: *(q, -q), (q-1, -q+1)* and so on. But taking into account that a difference in grades between the alternatives A and B is the same that the difference between B and A with the opposite sign, we may consider the first of them only.

Surely, choosing a specific value of τ significantly depends on the subject domain and of the specific task. Sometimes it can be calculated mathematically. For example, if we want the multiplicative spread (i.e. ratio) between the highest and the lowest values not to exceed the given value ρ, we can obtain τ from the following equation:

$$\tau^{2q} = \rho$$

But more typical is that τ should be evaluated empirically, in an expert way.

We should clearly distinguish values of preferences themselves and probabilities of choice related to them. It is absolutely possible that an agent chooses an alternative with the probability *1* even though its preference is very slight. This depends not only on the strength of preference but also on the agent's decisiveness. So, we have to introduce another parameter named β and to apply it for calculating needed probabilities. Given the preference values $(v_{k1}, \dots, v_{kn})$ for the k-th state, the corresponding probabilities should be obtained as follows [4]:

$$p_{kj} = \frac{e^{\beta v_{kj}}}{\sum_{j=1}^{n} e^{\beta v_{kj}}}$$

The bigger is β, the more decisive is the agent.

To summarize this section, all the described matter can be referred to as the parametrized model $M(q, \tau, \beta)$ for forming a matrix "state-probability of action", where q is the number of preference grades, τ specifies the ratio between the numeric values of two neighboring grade levels, and β is the parameter reflecting the agent's decisiveness.

4. Multicriterial decision making and two-level system of states

Choosing between alternatives is usually carried out on the base of their features, or in other words, on the base of some criteria indicating how good is a certain alternative. This is similar to the classical two-level AHP as well.

Within the framework of the "state-probability of action" model, in addition to the basic node directly connected to probabilities of choice we should introduce separate nodes for each criterion and then combine them likewise this was carried out in [5, 13]. Hence, we consider a two-level state system: the bottom level corresponds to separate criteria, and the top one corresponds to the eventual choice.

Let $R^{(k)}$ be a "state-probability of action" bottom-level matrix associated with the k-th criterion. Levels are connected to each other in the following way: the element r_{ij} stands for the probability that an agent which is currently being in the i-th state for the k-th criterion, is being at the same moment in the j-th state of the top-level node. Therefore, we will name matrices $R^{(k)}$ transition matrices since they describe moving on (transition) from the criteria level to the general one.

In addition to this, instead of top-level input probabilities we should specify bottom-level input probabilities for each criterion. In more details, let $p^{(k)} = \left(p_1^{(k)}, \dots\right)$ be a vector, the i-th component of which is the probability that an agent is being in the i-th state of the system associated with the k-th criterion.

Basically, such a configuration of state systems is somehow relevant to specifying logical rules like *if* R^k *then* L, $k = \overline{1, K}$, K is the total number of criteria. Such rules can be postulated in a different way. As a very simple approach, we can merely stipulate the following rules: *if the alternative A has a preference over the other alternative B by the separate k-th criterion, then A has the overall preference over B*. Certainly, such conclusions made on the base of separate criteria may contradict to each other, and for combining these conclusions we are going to rely upon an idea outlined in [4, 13]. The idea is as follows:

- to obtain separate vectors

$$p^{(0)(k)} = p^{(k)} \cdot R^{(k)}, \tag{2}$$

- to obtain the combined vector

$$p^{(0)} = \sum_{k=1}^{K} \lambda_k \, p^{(0)(k)}, \tag{3}$$

where λ_k are known weighting coefficients.

- then the resulting probabilities can be calculated by the formula (1), we will repeat it here:

$$p = p^{(0)} \cdot H, \tag{4}$$

Let's build a matrix C, the k-th row of which is the vector $p^{(0)(k)}$. Then the formula (3) is equivalent to the following formula:

$$p^{(0)} = \lambda \cdot C$$

where $\lambda = (\lambda_1, \dots, \lambda_K)$.

Now, taking into account basic features of centrosymmetric matrices [2, 3] as well as explanations given in [4, 5, 13], we can formulate the very important statement about one situation of an equilibrium between alternatives for two alternatives:

Statement 1. If both C and H are centrosymmetric matrices, and λ is a symmetric vector, then *p=(0.5, 0.5),* i.e. equilibrium between two alternatives holds.

We will build bottom-level matrices in a way different from described in the previous section. It would be possible to make transition matrices $R^{(k)}$ just unit matrices at all. But we will apply a more flexible approach which allows one-grade step up or down when moving on from the bottom (criteria) level to the top (general) level. Yet we may consider the same transition matrix for all criteria: $R = R^{(1)} = \cdots = R^{(K)}$. Let's also build a matrix D, the k-th row of which is the vector $p^{(k)}$. Such a matrix will be showcased in the illustrative example given below.

Then the algorithm represented by the formulae (2)-(4) obviously can be simplified, and it comes down to the following formula:

$$p = \lambda \cdot D \cdot R \cdot H, \quad (5)$$

In this case, the statement 1, which regards equilibrium of alternatives, can be re-formulated in the following way:

Statement 2. For the case of two alternatives, if *D, R, H* are centrosymmetric matrices, and λ is a symmetric vector, equilibrium between alternatives holds.

5. An illustrative example

Let's consider the following illustrative example.

Let there be an imaginary country, say Somewhereland. Let there be two leading candidates, say John and Bob, who are competing to become the President of Somewhereland. Let John be the representative of a certain party, say the Old-school party, and let Bob be the representative of the Freak party.

Let there be 4 criteria the candidates are compared by (*K=4*), which are as follows:

- personal qualities and merits of candidates
- conservative features of political programs
- innovative features of political programs
- leadership qualities and charisma of candidates.

Probably, in fact such criteria might be very different from the mentioned ones, but this example is very illustrative.

Let's start with the top-level "state-probability of action" model, which is directly related to decisions about voting. We are taking the model $M(q, \tau, \beta)$ described in the Section 3 with the parameters $q = 4, \tau = 1.4, \beta = 4.0$. This results in the following top-level matrix (approximately):

$$H = \begin{pmatrix} 1.0 & 0.0 \\ 0.9999 & 0.0001 \\ 0.9970 & 0.0030 \\ 0.9395 & 0.0605 \\ 0.5 & 0.5 \\ 0.0605 & 0.9395 \\ 0.0030 & 0.9970 \\ 0.0001 & 0.9999 \\ 0.0 & 1.0 \end{pmatrix}$$

Yet such a matrix more or less aligns with the assumption that when an agent is just about to vote, their opinion is usually already established, they typically don't tend to hesitate and to change their minds very much.

Let input probability vectors for criteria, which indicate profiles of social attitude to the candidates and their parties with respect to certain criteria, be as follows:

$$\begin{aligned} p^{(1)} &= (0.1,\ 0.01,\ 0.01,\ 0.3,\ 0.16,\ 0.3,\ 0.01,\ 0.01\ \ 0.1) \\ p^{(2)} &= (0.8,\ 0.13,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01) \\ p^{(3)} &= (0.01, 0.01, 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.13,\ 0.8) \\ p^{(4)} &= (0.1,\ 0.01, 0.01,\ 0.3,\ 0.16,\ 0.3,\ 0.01,\ 0.01,\ 0.1) \end{aligned}$$

Such profiles appear to be quite typical. In the given case, they are indicating more or less homogenous social attitude to both candidates with respect to the 1st and the 4th criteria, but biased and polarized attitude with respect to the 2nd and the 3rd criteria.

We are taking the transition matrix R the same for all criteria. Let it be as follows:

$$R = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.8 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.8 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.8 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0.8 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.8 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

As it was explained before, we can form the matrix D containing vectors $p^{(k)}$ as its rows.

Eventually, we are to specify weighting coefficients λ_k for our criteria. There are many ways to do this. In a simplest case, they may be taken as something ready and known. An approach featured by the AHP presumes building a pairwise comparison matrix across the criteria and getting coefficients from this matrix. Eventually, for getting coefficients of criteria it is possible to apply the approach on the base of the "state-probability of action" model, though this is more complicated as there are usually more than 2 criteria.

For simplicity, let's take the ready weighting coefficients $\lambda = (0.1, 0.4, 0.4, 0.1)$. The vector λ is symmetric, all matrices *D, R, H* are centrosymmetric, and therefore the conditions of the statement 2 hold. Indeed, the formula (5) yields

$$p = \lambda \cdot D \cdot R \cdot H = (0.5, 0.5)$$

So, the equilibrium between alternatives holds, no candidate has regular advantage over their opponent.

6. Breaking the equilibrium

Agents of influence usually tend to influence other agents (voters) in order to make them change their opinions and their behavior. In particular, in a situation of equilibrium like what was described in the previous section they would like to move the situation away from the equilibrium in the direction desirable for an influencer.

Within the framework of our behavioral model, it is reasonable to talk in terms of affecting certain parameters or input data of the model. Some aspects related to this issue have been outlined in [4], now we are going to represent a more extended view.

Among what might be affected by influencers, we are going to consider the following:

- input criteria probabilities
- weighs of criteria
- degrees of decisiveness.

Let's regard these possibilities one by one within our example. It would be sufficient to consider possible actions in favor of one side only, say the Old-school party and John, who is the candidate of this party.

Input criteria probabilities

Let's consider the vector $p^{(2)}$, which is connected to criterion the *conservative features of political programs*. Imagine that influencers managed to change it as follows:

$$p^{(2)} \leftarrow (0.82,\ 0.14,\ 0.,\ 0.,\ 0.01,\ 0.01,\ 0.01,\ 0.01,\ 0.)$$

For this new vector, probabilities of being in some states, which are more beneficial for the Old-school party, have been increased. The formula (5) now gives the following vector of eventual probabilities:

$$p \leftarrow (0.5044,\ 0.4956)$$

Now the Old-school party has got an advantage, and its candidate likely will win.

Weights of criteria

Gaming with weights of criteria, i.e. with measures of their importance, appears to be especially appealing to influencers and manipulators.

Certainly, influencers would like to boost weights of those criteria their team is strong at. In our example, the second criterion is just the cornerstone for the Old-school party. Let's consider the following possible new vector of weighting coefficients:

$$\lambda \leftarrow (0.1,\ 0.5,\ 0.3,\ 0.1)$$

Now the formula (5) gives the vector

$$p \leftarrow (0.5910,\ 0.4090)$$

The Old-school party gained the overwhelming advantage, its candidate definitely wins.

Degrees of decisiveness

Changing degrees of decisiveness directly affects the top-level matrix "state-probability of action" related to eventual decision making. The most interesting issue appears if different states are allowed to have different degrees of decisiveness, and instead of one parameter β we can consider a vector of parameters

$$b = (\beta_1, \dots, \beta_m)$$

Obviously, each party would like to raise decisiveness degrees of its the most convinced proponents. For the Old-school party this technically means increasing those degrees for the states corresponding to the upper rows of the matrix H.

Let influencers have managed to get the new vector

$$b \leftarrow (10., \ 10., \ 10., \ 10., \ 4., \ 4., \ 4., \ 4., \ 4.)$$

It yields the new matrix H as follows:

$$H \leftarrow \begin{pmatrix} 1.0 & 0.0 \\ 1.0 & 0.0 \\ 1.0 & 0.0 \\ 0.9989 & 0.0011 \\ 0.5 & 0.5 \\ 0.0605 & 0.9395 \\ 0.0030 & 0.9970 \\ 0.0001 & 0.9999 \\ 0.0 & 1.0 \end{pmatrix}$$

The interesting fact worth mentioning is that the new top-level matrix H is no longer centrosymmetric. Anyway, by applying (5) we can get the new distribution for probabilities of choice as follows:

$$p \leftarrow (0.5036, \qquad 0.4964)$$

which indicates the advantage of the Old-school party.

7. Conclusions and discussion

In the paper the approach to constructing the two-level behavioral "state-probability of action" model and to getting appropriate matrices "state-probability of choice" for the case of two competing alternatives has been suggested.

The top level is directly connected to probabilities of choice between alternatives like within the initial approach. But the difference is the suggestion that states of the model should be connected to grades of pairwise comparisons (such as equally good, slightly better, significantly better) has been postulated. For getting rows of the matrix on this basis transitive scales are offered to be applied, but not only. It appears important to distinguish values of preferences themselves and probabilities of choice related to them. For this reason, another parameter standing for decisiveness of agents has been introduced.

The bottom level is related to separate criteria affecting a choice. This level is connected to the top level by means of matrices, which were named transition matrices. The way to building such matrices is suggested as well.

A way to applying such a model for modeling voting in a bipartisan democracy has been suggested. Within this context, a problem of equilibrium between two alternatives, when no alternative has advantages over the other, is of great importance. When the number of voters is large enough, even the slightest advantage of any alternative shall be sufficient for its victory with the probability close to *1*, and the opponent shall have nearly no chances to succeed. So, for real bipartisan democracy implying than two parties change each other, situations of an equilibrium between two alternatives should be permanent or at least happen repeatedly. Some sufficient conditions for equilibrium between two alternatives have been postulated in the paper, they significantly rely upon properties of symmetry.

The illustrating example of modeling elections in an imaginary country has been provided. Voters in this example are to make a choice between two candidates on the base of comparing them by 4 given criteria. In the initial example the equilibrium between alternatives holds. Then an issue how agents of influence could change the situation in a desirable direction is discussed. Basically, within the framework of the model they should try to change some its parameters, and such attempts are showcased for the following parameters:

- input criteria probabilities
- weighs of criteria
- degrees of decisiveness.

As regards possible directions of further research, the following ones can be outlined.

Firstly, statements 1 and 2 given in this paper as well as those given in [4, 5, 13] postulate only sufficient conditions for equilibrium between two alternatives, these conditions are not necessary. They presume first of all certain symmetric and centrosymmetric features of involved vectors and matrices. Non-symmetric situations of equilibrium between alternatives do exist, but they are not explored enough so far. On the other hand, the example provided in the paper shows that even for basic top-level matrices the class of centrosymmetric matrices is not representative enough, under some circumstances we have to deal with non-centrosymmetric matrices. The case when the number of alternatives is more than two is not explored enough as well.

When it actually goes about social modelling, an issue concerning adjusting proper parameters of the model so that it would be adequate enough becomes the question of utmost significance. To a certain extent needed information can be obtained by means of statistics, Data Mining, machine learning etc [14]. This may refer, for example, to an information about probabilities of being in certain states, transitional probabilities, or so. To collect raw data of such a kind, various polls might be carried out. The ubiquitous poll question "*Who would you vote for, if the elections took place tomorrow?*" seems to be more or less suitable for achieving this goal, since possible responses directly indicate preferences of one alternative over the other. This may also address the problem of comparing criteria of choice as well as estimating their weighs, and this problem itself is extremely important. However, biased judgments are imminent for such a matter.

In this paper we treated different parameters and criteria as if they were independent, but in fact they are interconnected, probably in large measure. So, techniques of covariance analysis, methods of decorrelation like the principal component analysis appear to be very helpful.

Exploring probabilities of being in certain state referring to comparison between alternatives as well of transitional probabilities across such states within the considered model “state-probability of action” evidently can be significantly supplemented and enriched by exploring connections between people in social networks. It is commonly recognized that judgments of people are largely affected by judgments existing in their neighborhood, whether it goes about face-to-face or network contacts. If somebody expresses a judgment in a social network or changes a judgment and informs other people about this fact (within the model “state-probability of choice” this corresponds to identifying the actual state the person is being at), there is a notable probability that people contacting this person shall take this fact into account and consider something similar. There are many models of spreading information across networks including social ones, tightly connected models of spreading judgments and opinions have been widely and rapidly developing as well [15-22 et.al.]. So, we find it necessary to combine the considered model with such approaches.

We discussed comparisons between alternatives but didn’t pay attention to considering absolute levels of the alternatives’ quality. Both alternatives may be good but may be weak as well. If the latter takes place, the social attitude to them tends to go down, and other forces may go on stage. This aspect should be explored in more details in course of further studies.

The direction aimed at integration with techniques featured by pairwise comparisons and the AHP should get further developed as well. Some approaches within this context are aimed not at evaluating alternatives only but on elaborating recommendations for how to improve positions of certain alternatives as well. One of such systems has been reported in [23, 24]. This appears to be very useful in social modeling.

At last, but not least we should pay attention to game aspects of the matter. Whereas some influencers are striving to change the current situation in a desirable for them direction, their opponents would definitely like to do the same in the opposite direction. So, a certain kind of games arises, and this issue might be explored by means of the game theory.

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