Portable Nets: Modeling and Verification of Business Processes with multiple start and end points

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Abstract
Workflow Nets, a subclass of Petri Nets, are suitable to model and analyze business processes formally. However, since Workflow Nets require a single source and sink place, they do not account for business processes with multiple start and end events. To model this behavior with Workflow Nets, it is required to manually reduce the net structure to a single source and sink place. Given this limitation, we propose Portable Nets, a novel class of Petri Nets designed to model business processes containing multiple start and end events. Tailored to Portable Nets, we extend the notion of soundness, a key correctness criterion for Workflow Nets, and provide a corresponding verification approach.

Keywords
Petri Nets, Portable Nets, Soundness

Introduction
Workflow Nets (WF-Nets) [1], a subclass of Petri Nets [2], are an established mathematical framework to model and analyze business processes. The analysis of soundness [1], a crucial behavioral correctness criterion of WF-Nets, requires a static net structure with a dedicated source and sink place representing a unique start and end event. If a business process contains a set of start and end events, one needs to translate this behavior to a WF-Net with a dedicated source and sink place to ensure the required net structure for the verification of soundness [3]. This yields numerous processing steps. To eliminate this overhead, we introduce Portable Nets (P-Nets) as a novel class of Petri Nets. In this paper, we extend soundness for P-Nets and apply the same verification technique used for WF-Nets in [4].

Portable Nets
A Petri Net $N = (P, T, F)$ is a P-Net if:

- there is a set of source places $P_i \subseteq P$, such that $\forall p_i \in P_i : p_i = \emptyset$;
- there is a set of sink places $P_o \subseteq P$, such that $\forall p_o \in P_o : p_o \bullet = \emptyset$;
- the sets $P_i$ and $P_o$ are pairwise disjoint; and
- $\forall x \in P \cup T : x$ is on the path from at least one $p_i \in P_i$ to at least one $p_o \in P_o$.

A P-Net System $\Omega = (N, M_i)$ is a P-Net $N = (P, T, F)$ with an initial marking $M_i := \forall p_i \in P_i : M_i(p_i) = 1 \land \forall p \in P \setminus \{P_i\} : M_i(p) = 0$. Let $\{N, M_f\}$ be the set of reachable markings of $\Omega$. $M_f \in \{N, M_o\}$ is
called final marking of \( \Omega \), if \( \forall p_0 \in P_0 (\subset P) : M_f (p_0) = 1 \) and for all \( p \in P \setminus \{P_0\} : M_f (p) = 0 \). \( \Omega \) is sound, if for any reachable marking \( M_1 \in [N, M_s] \) starting from \( M_i \), it is possible to reach the final marking \( M_f \). Formally, \( (N, M_i) [\sigma] (N, M_f) \), where \( [\sigma] \) denotes a sequence of firing transitions \( t_1 \in T = t_1, t_2, \ldots t_n \) with \( n \in \mathbb{N} \), such that \( [\sigma] \) leads from \( M_i \) over \( M_1 \) to \( M_f \), and there are no dead transitions. To check if \( N \) is sound, we employ liveness and boundedness [2], two standard Petri Net analysis techniques, and define a corresponding P-Net \( \tilde{N} = (\tilde{P}, \tilde{T}, \tilde{F}) \) with an initial marking \( M_1 \). \( \tilde{N} \) is the P-Net that we obtain by adding an extra transition \( t_0 \in T \), which connects \( P_0 \in P \) and \( P_1 \in P \). \( \tilde{N} \) is visualized in Figure 1. For \( N \) and \( \tilde{N} \) we prove the following result: \( N \) is sound, if \( (\tilde{N}, M_i) \) is live and bounded. Therefore, we define three Lemmata.

**Lemma 1.** If \( \tilde{N} \) is live and bounded, then \( N \) is a sound P-Net.

**Proof.** \( \tilde{N} \) is live, i.e. for every reachable state \( M_1 \in [\tilde{N}, M_s] \), there is a firing sequence, which starts from the initial marking \( M_i \) (i.e. \( \forall \tilde{p}_i \in \tilde{T}_1 : M_i (\tilde{p}_i) \geq 1 \)), that leads to a state in which \( t_0 \) is enabled, i.e. \( \forall \tilde{p}_0 \in \tilde{P}_0 : M_f (\tilde{p}_0) \geq 1 \). Since \( \tilde{P}_0 \) is the set of input place of \( t_0 \in \tilde{T} \), for any state \( M_1 \) reachable from the initial state \( M_i \), it is possible to reach a state, where at least all places of \( \tilde{P}_0 \) are marked with at least one token. Suppose that \( M_2 \) and \( M_f \in [\tilde{N}, M_s] \) are reachable states, where \( M_2 \) is an arbitrary reachable marking and \( M_f := \forall \tilde{p}_0 \in \tilde{P} : \tilde{p}_0 \) is marked. Consider that following marking \( M_2 + M_f \). In this state, \( t_0 \) is enabled. If \( t_0 \) fires, then the state \( M_2 + M_f \) is reached. As \( \tilde{N} \) is also bounded, \( M_2 \) should be equal to the empty state. Hence, proper termination, according to [4] is guaranteed. From the fact that \( \tilde{N} \) is live, we can derive that there are no dead transitions. Hence, \( N \) is a sound P-Net. \( \blacksquare \)

**Lemma 2.** If \( N \) is sound, then \( \tilde{N} \) is bounded.

**Proof.** Suppose that \( N \) is sound and \( (N, M_i) \) is not bounded. Since \( N \) is not bounded, there are two markings \( M_1 \) and \( M_2 \) reachable from the initial marking \( M_i \), such that \( (N, M_i) [\sigma] (N, M_1), (N, M_1) [\sigma] (N, M_2) \) and \( M_2 > M_1 \). However, since \( N \) is sound, we know that there is a firing sequence \( \sigma \), which leads to the final marking \( M_f \), such that \( (N, M_i) [\sigma] (N, M_f) \). Therefore, there is a marking \( M_3 \) such that \( (N, M_2) [\sigma] (N, M_3) \) and \( M_3 > M_f \). Hence, it is not possible that \( N \) is both sound and not bounded. So if \( N \) is sound, then \( (N, M_i) \) is bounded. From the fact that \( N \) is sound and \( (N, M_i) \) is bounded, we can deduce that \( (\tilde{N}, M_i) \) is bounded. If transition \( t_0 \in T \) fires, the net returns to the initial marking. \( \blacksquare \)
Lemma 3. If $N$ is sound, then $\overline{N}$ is live.

Proof. Assume $N$ is sound. By Lemma 2 we know that $(\overline{N}, M_i)$ is bounded. Because $N$ is sound, we know that marking $M_i$ is a home-marking of $\overline{N}$, i.e., for every state reachable from $(\overline{N}, M_i)$ it is possible to return to $(\overline{N}, M_i)$. In $(N, M_i)$, it is possible to fire an arbitrary transition $t_i \in T$. This is also the case in $\overline{N}$. Therefore, $(\overline{N}, M_i)$ is live because for every state reachable from $(\overline{N}, M_i)$, it is possible to reach a state, which enables an arbitrary transition. ■

Theorem 1. A P-Net $N$ is sound, if $(\overline{N}, M_i)$ is live and bounded.

Proof. Proof follows from Lemma 1, 2 and 3. ■

Related Work Literature proposes several variants of WF-Nets: Free-choice WF-Nets [4], Resource-constrained WF-Nets [5], and Time Workflow Nets [6]. In [7], business processes with multiple end events are considered, but a dedicated start event is still required. None of them model business processes with a set of source and sink places while also providing a corresponding soundness verification procedure without the need to execute modification steps on the structure of the net.

Conclusion We have presented P-Nets as a novel class of Petri Nets without the structural restrictions of a WF-Net, which requires a single source and sink place to verify soundness. P-Nets facilitate soundness verification of business processes with a set of start and end events. In contrast to WF-Nets, it is not required for P-Nets to perform numerous processing steps to translate such a business process with a set of start and end events into a net structure with a dedicated source and sink place. In future research, we aim to explore the general compositionality of two sound P-Nets while preserving soundness.

References