Portable Nets: Modeling and Verification of Business Processes with multiple start and end points

Mustafa Ghani[1], Holger Giese[1]

[1]Hasso Plattner Institute, University of Potsdam, Germany

Abstract

Workflow Nets, a subclass of Petri Nets, are suitable to model and analyze business processes formally. However, since Workflow Nets require a single source and sink place, they do not account for business processes with multiple start and end events. To model this behavior with Workflow Nets, it is required to manually reduce the net structure to a single source and sink place. Given this limitation, we propose Portable Nets, a novel class of Petri Nets designed to model business processes containing multiple start and end events. Tailored to Portable Nets, we extend the notion of soundness, a key correctness criterion for Workflow Nets, and provide a corresponding verification approach.

Keywords

Petri Nets, Portable Nets, Soundness

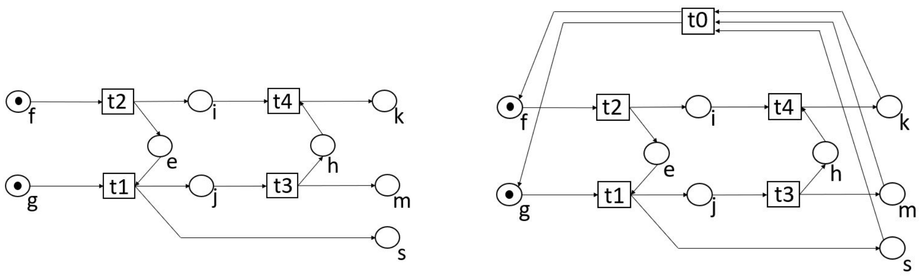
Introduction Workflow Nets (WF-Nets) [1], a subclass of Petri Nets [2], are an established mathematical framework to model and analyze business processes. The analysis of soundness [1], a crucial behavioral correctness criterion of WF-Nets, requires a static net structure with a dedicated source and sink place representing a unique start and end event. If a business process contains a set of start and end events, one needs to translate this behavior to a WF-Net with a dedicated source and sink place to ensure the required net structure for the verification of soundness [3]. This yields numerous processing steps. To eliminate this overhead, we introduce Portable Nets (P-Nets) as a novel class of Petri Nets. In this paper, we extend soundness for P-Nets and apply the same verification technique used for WF-Nets in [4].

Portable Nets A Petri Net N = (P, T, F) is a P-Net if:

- there is a set of source places $P_i \subset P$, such that $\forall\, p_i \in P_i : \bullet\, p_i = \emptyset$;
- there is a set of sink places $P_o \subset P$, such that $\forall\, p_o \in P_o : p_o \bullet = \emptyset$;
- the sets P_i and P_o are pairwise disjoint; and
- $\forall\, x \in P \cup T$: x is on the path from at least one $p_i \in P_i$ to at least one $p_o \in P_0$.

A P-Net System $\Omega = (N, M_i)$ is a P-Net N = (P, T, F) with an initial marking $M_i := \forall\, p_i \in P_i: M_i(p_i) = 1 \land \forall\, p \in P \setminus \{P_i\}: M_i(p) = 0$. Let $[N, M_s\rangle$ be the set of reachable markings of Ω. $M_f \in [N, M_s\rangle$ is

PNSE'24, International Workshop on Petri Nets and Software Engineering, 2024
*Corresponding author.
mustafa.ghani@hpi.de (M. Ghani); holger.giese@hpi.de (H. Giese)
https://hpi.de/giese/people/mustafa-ghani.html (M. Ghani);
https://hpi.de/giese/people/prof-dr-holger-giese.html (H. Giese)
0009-0004-6056-2125 (M. Ghani); 0000-0002-4723-730X (H. Giese)



(a) Portable Net N (b) Portable Net $\overline{N}$

Figure 1: Portable Net $\overline{N}$ = ($\overline{P}$, $\overline{T}$, $\overline{F}$) used to verify soundness of Portable Net N = (P, T, F)

called final marking of Ω, if $\forall\, p_o \in P_o$ ($\subset$ P) : M_f (p_o) = 1 and for all p $\in$ P$\setminus$ {P_o} : M_f (p) = 0. Ω is sound, if for any reachable marking $M_1 \in$ [N, $M_s\rangle$ starting from M_i, it is possible to reach the final marking M_f. Formally, (N, M_i) $[\sigma\rangle$ (N, M_1) $[\sigma\rangle$ (N, M_f), where $[\sigma\rangle$ denotes a sequence of firing transitions $t_i \in T = t_1, t_2, \ldots t_n$ with n $\in \mathbb{N}$, such that $[\sigma\rangle$ leads from M_i over M_1 to M_f, and there are no dead transitions. To check if N is sound, we employ liveness and boundedness [2], two standard Petri Net analysis techniques, and define a corresponding P-Net $\overline{N}$ = ($\overline{P}$, $\overline{T}$, $\overline{F}$) with an initial marking M_1. $\overline{N}$ is the P-Net that we obtain by adding an extra transition t0 $\in \overline{T}$, which connects $\overline{P}_o \in \overline{P}$ and $\overline{P}_i \in \overline{P}$. $\overline{N}$ is visualized in Figure 1. For N and $\overline{N}$ we prove the following result: N is sound, if ($\overline{N}$, M_i) is live and bounded. Therefore, we define three Lemmata.

Lemma 1. *If $\overline{N}$ is live and bounded, then N is a sound P-Net.*

Proof. $\overline{N}$ is live, i.e. for every reachable state $M_1 \in$ [$\overline{N}$, $M_s\rangle$, there is a firing sequence, which starts from the initial marking M_i (i.e. $\forall\, \overline{p}_i \in \overline{P}_i$: M_i ($\overline{p}_i$) $\geq$ 1), that leads to a state in which t0 is enabled, i.e. $\forall\, \overline{p}_o \in \overline{P}_o$: M_f ($\overline{p}_o$) $\geq$ 1. Since $\overline{p}_o$ is the set of input place of t0 $\in \overline{T}$, for any state M_1 reachable from the initial state M_i, it is possible to reach a state, where at least all places of $\overline{P}_o$ are marked with at least one token. Suppose that M_2 and $M_f \in$ [$\overline{N}$, $M_s\rangle$ are reachable states, where M_2 is an arbitrary reachable marking and M_f := $\forall\, \overline{p}_o \in \overline{P}$: $\overline{p}_o$ is marked. Consider that following marking M_2 + M_f. In this state, t0 is enabled. If t0 fires, then the state M_2 + M_i is reached. As $\overline{N}$ is also bounded, M_2 should be equal to the empty state. Hence, proper termination, according to [4] is guaranteed. From the fact that $\overline{N}$ is live, we can derive that there are no dead transitions. Hence, N is a sound P-Net. ■

Lemma 2. *If N is sound, then $\overline{N}$ is bounded.*

Proof. Suppose that N is sound and (N, M_i) is not bounded. Since N is not bounded, there are two markings M_1 and M_2 reachable from the initial marking M_i, such that (N, M_i) $[\sigma\rangle$ (N, M_1), (N, M_1) $[\sigma\rangle$ (N, M_2) and $M_2 > M_1$. However, since N is sound, we know that there is a firing sequence σ, which leads to the final marking M_f, such that (N, M_i) $[\sigma\,\rangle$ (N, M_f). Therefore, there is a marking M_3 such that (N, M_2) $[\sigma\,\rangle$ (N, M_3) and $M_3 > M_f$. Hence, it is not possible that N is both sound and not bound. So if N is sound, then (N, M_i) is bounded. From the fact that N is sound and (N, M_i) is bounded, we can deduce that ($\overline{N}$, M_i) is bounded. If transition t0 $\in \overline{T}$ fires, the net returns to the initial marking. ■

Lemma 3. *If N is sound, then $\overline{N}$ is live.*

Proof. Assume N is sound. By Lemma 2 we know that $(\overline{N}, M_i)$ is bounded. Because N is sound, we know that marking M_i is a home-marking of $\overline{N}$, i.e., for every state reachable from $(\overline{N}, M_i)$ it is possible to return to $(\overline{N}, M_i)$. In (N, M_i), it is possible to fire an arbitrary transition $t_i \in T$. This is also the case in $\overline{N}$. Therefore, $(\overline{N}, M_i)$ is live because for every state reachable from $(\overline{N}, M_i)$, it is possible to reach a state, which enables an arbitrary transition. ■

Theorem 1. *A P-Net N is sound, if $(\overline{N}, M_i)$ is live and bounded.*

Proof. Proof follows from Lemma 1, 2 and 3. ■

Related Work Literature proposes several variants of WF-Nets: Free-choice WF-Nets [4], Resource-constrained WF-Nets [5], and Time Workflow Nets [6]. In [7], business processes with multiple end events are considered, but a dedicated start event is still required. None of them model business processes with a set of source and sink places while also providing a corresponding soundness verification procedure without the need to execute modification steps on the structure of the net.

Conclusion We have presented P-Nets as a novel class of Petri Nets without the structural restrictions of a WF-Net, which requires a single source and sink place to verify soundness. P-Nets facilitate soundness verification of business processes with a set of start and end events. In contrast to WF-Nets, it is not required for P-Nets to perform numerous processing steps to translate such a business process with a set of start and end events into a net structure with a dedicated source and sink place. In future research, we aim to explore the general compositionality of two sound P-Nets while preserving soundness.

References

[1] W. M. Van der Aalst, The application of petri nets to workflow management, Journal of circuits, systems, and computers 8, (1998).

[2] W. Reisig, Understanding Petri Nets, Springer, (2016).

[3] A. A. Kalenkova, W. M. van der Aalst, I. A. Lomazova, V. Rubin, Process mining using bpmn: relating event logs and process models, Software & Systems Modeling 16, (2015).

[4] W. M. Van der Aalst, Verification of workflow nets, Int. Conf. on Application and Theory of Petri Nets, (1997).

[5] K. Van Hee, N. Sidorova, M. Voorhoeve, Resource-constrained workflow nets, Fundamenta Informaticae 71, (2006).

[6] H. Boucheneb, K. Barkaoui, Strongly generalized soundness of time workflow nets, Proceedings of 15th Int. Conference on Application of Concurrency to System Design (2015).

[7] D. Liu, J. Wang, S. C. Chan, J. Sun, L. Zhang, Modeling workflow processes with colored petri nets, Computers in Industry 49, (2002).