Post-press product quality assessment models for the IIoT system

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Abstract
This study proposes a method for building a model of assembly quality factors by implementing a factor ranking approach. The method considers the relationships between factors, their types, and the expert weights assigned to each type. The reliability of the obtained models confirmed, supported by the implementation of a universal set of values and corresponding linguistic terms for each linguistic variable of the assembly stage. A model of logical derivation constructed, reflecting factor classification and the process of forming forecasted quality indicators for small-volume editions production at the VSHRA. The study also involves calculating membership functions of linguistic variables and constructing graphs to illustrate the relationships between parameters of the linguistic variables and values of the membership functions. Additionally, a fuzzy knowledge base designed, and mathematical models for forecasting assembly quality were developed using expert evaluations of fuzzy logical statements. Fuzzy logic equations derived to establish relationships between input and output data membership functions. These findings provide a foundation for developing a module to assess the production process quality within the industrial Internet of Things system of a printing company.

Keywords
factor ranking, assembly quality, linguistic variables, fuzzy knowledge base, fuzzy logic, industrial Internet of Things, printing company, printing machine.

1. Introduction

Trends in the development of the printing industry indicate that the circulation of publications is constantly decreasing with a simultaneous increase in their nomenclature. Production time reduced, product quality requirements are increasing, thinner and lighter paper is used while reducing its costs, and the finishing of printed products is becoming more complicated and improved. Global manufacturers of post-press equipment, such as Muller Martini, Heidelberg, Hohner, Osaka, Purlux and others, work in such realities, creating modern automated flow lines, which include insert-sewing-cutting units (VSHRA). They satisfy the needs of the market by
strictly performing the necessary operations and shortening the time intervals between them as much as possible. The most effective is the application of VSHRA for the release of small-volume editions complete with attachments.

The technological process of the production of small-volume publications (TPVMV) at VSHRA is equipped with post-operational machines that perform systematic operations, and the quality of production of small-volume publications (ML) depends on many heterogeneous factors. The properties of the materials used, modes of operation, design features of post-operative machines of the VSHRA and their individual elements determine them.

The integration of the entire complex of printing equipment into a single information system of control and management allows for continuous production of orders and assessment of their quality at all stages of the technological process.

Based on the above, there is a need to develop models for assessing the quality of the insertion-sewing-cutting unit in IIoT, which will allow not only to ensure quality control, but also to forecast the technological process when planning production tasks.

2. Researching the state of the problem and setting a task

To correctly solve the problem of quality prediction, it is necessary to study a large number of factors and take into account the variety of their characteristics, since the factors can be quantitative (block thickness, paper moisture, paper density), qualitative (type of wire material, quality of notebook preparation, knife sharpness) and binary. Under such circumstances, the relevance of the problem of building expert systems designed to solve informal problems arising at various stages of scientific research and engineering and technical activity becomes obvious [1-3]. Based on the analyzed literature [1, 2], it was established that quality control mainly takes place visually without taking into account the human factor. In addition, apply the described requirements to various types of printing products to a limited extent [4, 5].

The above data indicate the absence of research results focused on forecasting and quality management of the production [6] of small-volume publications at the VSHRA, based on the establishment of factors and prioritization of their influence on the studied processes, the design of alternative options for their implementation, a priori calculation of the prognostic integral quality indicator.

3. Theoretical information about the research object

The market offers VSHRA (Saddle-Stitch Binding Machines) with varying productivity and configurations. Different models of VSHRA have different formats for incoming booklets, finished product formats, and operating speeds [7, 8, 9].

The productivity of modern machines exceeds 20,000 copies of finished products per hour. For example, Heidelberg offers several options. The Stitchmaster ST 300 operates at speeds up to 13,000 cycles per hour with a maximum of 16 feeder sections. Another model, the Stitchmaster ST 100, has slightly lower productivity (up to 9,000 cycles per hour) and can be equipped with four double feeder sections. In 2000, a new model, the Stitchmaster ST 400, was introduced. This VSHRA has six stitching heads that can stitch blocks up to 12 mm thick. The machine features a high degree of automation, including a mechanism for automatic format adjustment. The feeder sections are modular, and the operating speed has been increased to 14,000 cycles per hour. Additionally, when stackers (pile forming modules) and a packing
machine are connected to the VSHRA, the finished products can be immediately sent to the warehouse or the customer. One drawback of VSHRA is that it is only used for assembling blocks by insertion, so the volume of publications, even printed on thin paper, does not exceed 128 pages [10].

In addition to the main operations—automatic block assembly by insertion, cover folding, wire stitching (2, 4, 6, 8 stitching heads), and three-side trimming of finished brochures—VSHRA machines offer additional options such as central knife cutting of double brochures, card pasting, CD insertion, and other products [11, 12].

Modern VSHRA machines can have up to 40 automatic booklet feeding sections (feeders), as seen in models like Muller Martini Tempo 22 and Muller Martini Primera 160. However, typically, lines consist of 4-8 automatic feeders. Feeders can supply pre-prepared booklets with a positive (or negative) shingling—opened by hooks, and non-shingled booklets—opened by a pair of vacuum suckers. When preparing for operation, feeders must be adjusted according to the format and thickness of the booklets. Feeders can be horizontal or vertical in design, with varying formats and speeds. Vertical feeders have a larger capacity and are much more convenient for loading booklets. To increase productivity and ensure continuous operation, they can be connected to continuous feeding devices, such as cascade feeders, for example, Muller Martini 3736 and Muller Martini 3738 [13].

Horizontal feeders work more efficiently with single-fold booklets on thin paper (including LWC). If a publication needs to include thin inserts (such as tracing paper or designer papers with advertising modules), both horizontal and vertical feeding stations can be installed in the line, in the sequence required by the brochure manufacturing technology.

It is not always justified to have a VSHRA with 8 feeders if 6 are sufficient for 98% of the jobs. However, with a machine equipped with 4-6 feeders, it is sometimes necessary to add 1-2 booklets or folded sheets with advertising inserts made of plastic, cardboard, tracing paper, or papyrus paper of a completely different format. For this, so-called manual feeding sections and various auxiliary devices (such as a plastic card gluing device) are used, which have an additional spot on the transport chain where the operator can insert the needed booklet [14]. These additional devices significantly expand the capabilities of VSHRA, which is very important given the high market interest in various types of advertising inserts, discs, samples, cards, and souvenirs [15, 16].

Many brochures, newspapers, magazines, and other printed products have a cover. Typically, this is a sheet offset printed and cut to format from material thicker than the inner pages of the future publication. The cover feeder separates a single sheet from the stack, folds it (some models also feature pre-scoring for inner/outer creasing), giving it the shape of a cover. It then places it onto the transporter, thereby reducing one technological operation—the folding of the cover on a folding machine. Therefore, such feeders are also called "folding feeders." When preparing for operation, feeders must be adjusted according to the format and thickness of the cover.

Each manufacturer offers cover feeders in their product range. For instance, Muller Martini has types 1528 and 1529, Heidelberg Stitchmaster has type UFA, Osako has model ORC-305, and Purlux has ZYDY440E, etc.

After the block is assembled, it enters the stitching station. Here, the block is stitched with wire using stitching heads (ranging from 2 to 8 heads). The heads can be standard (e.g., HK 75, Hohner Universal 52/8, Deluxe G8) or designed for loop stitching. For binding blocks, printing
wire or low-carbon steel wire of general use is applied. Technological instructions recommend using wire with a diameter of 0.4 to 0.7 mm for stitching blocks with a thickness of 0.5 to 5 mm and a diameter of 0.8 mm for greater thickness. Stitching is done saddle-stitch style [9, 12].

The three-knife trimmer station performs the final operation in the VSHRA, giving the brochure, magazine, or book its final appearance. Therefore, it is very important for this module to operate as efficiently as possible without defects according to the required format of the finished publication.

In addition to trimming the finished brochure on three sides, three-knife trimmer stations can be equipped with additional knives for cutting double brochures [16] or perforation. Each manufacturer has several models of three-knife trimmer stations with which they equip their VSHRA. For Muller Martini, these include models 890, 1522, and more modern models 0304, 0449, 0459. For Heidelberg, these are the TR models (TR 100, TR 300, etc.).

New-generation VSHRA machines are equipped with automatic setup systems. For example, Muller Martini equips the VSHRA BravoPlus with the AMRYS system. The AMRYS automatic setup system controls the thickness of the book block and the feed speed. The parameters of the adjusted three-side trimming module are stored in the computer memory. The system manages: synchronization of book blocks, format parameters on the feeder, thickness, feed for trimming, width, and length. For the feeder, this includes controlling the length, width, and offset for the booklet opening device with air flow regulation, booklet stop, and synchronization [13, 14].

Depending on the manufacturer, model, and configuration, VSHRA machines are equipped with sensors and control systems [8-9]: booklet feed/non-feed sensor; block passage sensor; block completeness sensor; side block thickness gauge; block positioning/misalignment control; staple presence sensor; automatic quality control system; multiprocessor integrated control system over an optical network [15, 16].

All VSHRA machines are equipped with transport systems that ensure the passage of the future publication block from module to module. Booklet transportation in VSHRA is carried out by thin belts made of modern, completely smooth, anti-static materials with specially treated edges, preventing the accumulation of paper dust and ink, protecting the product from unwanted marks and mechanical damage [10].

Table 1 shows the technical characteristics of VSHRA machines popular in the Ukrainian market from various manufacturers [17, 18, 19].

4. Presentation of the IIoT model of a printing enterprise.

The Industrial Internet of Things integrates information technologies, user data with equipment data into production and allows machines to communicate with each other (Fig. 1) [20, 21]. As a result of managing things, devices and machines, printing production becomes autonomous, flexible, efficient and resource-saving [22].

The traditional model of "supplier-consumer" interaction of the new concept is radically changing due to the following factors: automation of the process of monitoring and management of the product life cycle; the organization of effective logical structures that self-optimize from enterprises - suppliers to enterprises - end consumers, etc.

Thus, the Industrial Internet of Things allows for the organizational and technological transformation of production, which in turn makes it possible to integrate material, transport, human, engineering and other resources and to scale software-controlled virtual pools (shared
economy) almost without limits and to provide the user with more than the devices themselves
cut, and the results of their use (device functions) due to the implementation of cross-functional
production and business processes.
Therefore, the need to create models for assessing the quality of work of the insert-sewing-
cutting unit is one of the necessary factors for the implementation of end-to-end production
and business processes.

Table 1
Technical characteristics of VSHRA machines popular in the Ukrainian market from various
manufacturers

<table>
<thead>
<tr>
<th>Company, country</th>
<th>Model</th>
<th>Format, mm max.</th>
<th>Format, mm min.</th>
<th>The maximum thickness of the brochure</th>
<th>Quantity outlays, max</th>
<th>Number of sewing machines, max</th>
<th>Speed, cycles/h. (cycles/min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hohner, Germany</td>
<td>HBS 7000</td>
<td>390×350</td>
<td>105×130</td>
<td>8</td>
<td>8</td>
<td>4</td>
<td>7000</td>
</tr>
<tr>
<td>HBS 10000</td>
<td>400×350</td>
<td>130×105</td>
<td>285×220</td>
<td>12</td>
<td>12</td>
<td>4</td>
<td>10000</td>
</tr>
<tr>
<td>ST 300</td>
<td>480×320</td>
<td>128×92</td>
<td>15</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>9000</td>
</tr>
<tr>
<td>Muller-Martini,</td>
<td>ST 500</td>
<td>500×330</td>
<td>128×80</td>
<td>16</td>
<td>16</td>
<td>6</td>
<td>13000</td>
</tr>
<tr>
<td>Switzerland</td>
<td>Presto II</td>
<td>365×300</td>
<td>93×60</td>
<td>10</td>
<td>10</td>
<td>6</td>
<td>9000</td>
</tr>
<tr>
<td>Primera II</td>
<td>480×320</td>
<td>90×80</td>
<td>13</td>
<td>16</td>
<td>16</td>
<td>8</td>
<td>14000</td>
</tr>
<tr>
<td>Primera 160</td>
<td>480×320</td>
<td>105×74</td>
<td>13</td>
<td>40</td>
<td>40</td>
<td>8</td>
<td>16000</td>
</tr>
<tr>
<td>Tempo E220</td>
<td>340×260</td>
<td>115×90</td>
<td>13</td>
<td>40</td>
<td>40</td>
<td>4</td>
<td>22000</td>
</tr>
<tr>
<td>Muller-Martini,</td>
<td>Supra</td>
<td>340×260</td>
<td>115×90</td>
<td>13</td>
<td>30</td>
<td>4</td>
<td>30000</td>
</tr>
<tr>
<td>Switzerland</td>
<td>NOVA 10</td>
<td>450×311</td>
<td>158×100</td>
<td>8</td>
<td>6</td>
<td>14</td>
<td>10000</td>
</tr>
<tr>
<td>PURLUX, China</td>
<td>NOVA 12</td>
<td>480×320</td>
<td>153×108</td>
<td>10</td>
<td>6</td>
<td>14</td>
<td>12000</td>
</tr>
</tbody>
</table>

Figure 1: Basic IIoT model of a printing enterprise.
5. Factors influencing the quality of production of publications at VSHRA

The technological process for manufacturing small-volume publications (TPMVP) on VSHRA (Saddle-Stitch Binding Machines) is implemented using operational machines that perform sequential operations, namely: block assembly 1, cover folding 2, block stitching with wire 3, three-side trimming 4, which can be represented by a structural diagram (see Fig. 2). Thus, TPMVP and operational machines are complex systems where the quality of small-volume publications depends on many factors. These factors are determined by the properties of the materials used, operating modes, the design features of the VSHRA operational machines, and their individual elements, among others. The importance of each factor determines the weight of the factor or the degree of its influence on the final result and allows for the prediction of product quality.

Figure 2: Structural Diagram of the Main Operations for TPMVP on VSHRA.

The task of predicting the quality of TPMVP at the output of VSHRA is solved based on the evaluation of defined factors and is considered as finding the mapping

\[ X^* = (x_1^*, x_2^*, \ldots, x_n^*) \rightarrow d \in D = (d_1, d_2, \ldots, d_n), \]

where \( X^* \) - is the set of factors affecting the quality of small-volume publications (SVP), and D is the set of predicted quality outcomes for SVP.

To correctly solve the quality prediction task, it is necessary to study a large number of factors and consider the variety of their characteristics, as the factors can be quantitative (e.g., block thickness, paper humidity, paper density), qualitative (e.g., type of wire material, quality of booklet preparation, knife sharpness), and binary.

Under these circumstances, the relevance of building expert systems to solve unstructured problems that arise at various stages of scientific research and engineering activities becomes evident [23, 24].

5.1. Calculation of the membership functions of the selection factors of small volume editions.

The problem of any technological process remains the numerical prediction of the set values of the parameters of the same process, which a priori would ensure the proper quality of the investigated process. A well-argued answer to this task can be obtained by using the methods and tools of the theory of fuzzy sets and linguistic variables [1, 9, 16] for its solution, the main component of which are membership functions constructed using the term set meanings and linguistic terms of factors. Among the many methods of determining membership functions, Saati’s method of pairwise comparisons has become the most popular [14, 24]. This method involves certain difficulties, which are due to the need to find the eigenvector of the matrix of pairwise comparisons, which is specified using a specially proposed scale. The more the
universal set on which the linguistic term is defined increases, the more difficult the research becomes.

The method of constructing membership functions [10, 14], which is used in the work, also uses the matrix of pairwise comparisons of the universal set. However, it differs in that it does not require the calculation of the eigenvector of the matrix, unlike Saati’s method [9].

A method of constructing the membership functions of the factorization factors [8] of fuzzy sets is proposed. A method based on which linguistic evaluations are formalized not only of all factors affecting the quality of the process of assembly of small-volume publications, but also of factors influencing the subsequent stages of production of small-volume publications at the VSHRA, such as the process of folding the cover, the process of sewing with wire and the process of trimming with three sides [14]. The hierarchical set of the above stages corresponds to the model [18] presented in fig. 3.

Let is a linguistic variable (LZ) that describes the quality of assembly of small-volume editions at the VSHRA. The quality of the indicator depends on a number of linguistic variables, which are presented in table 2.14 and can be written in the form of the following ratio:

\[ K = f_K(k_1, k_2, k_3, k_4, k_5, k_6), \]

(2)

where \( k_1 \) - LZ "number of loaded stations"; \( k_2 \) - LZ "notebook format"; \( k_3 \) - LZ "number of complex notebooks in a block"; \( k_4 \) - LZ "speed of operation of the VSHRA"; \( k_5 \) - LZ "quality of preparation of notebooks"; \( k_6 \) - LZ "operator qualification".

6. Modeling of the technological process of assembly of small-volume editions at VSHRA

The main goal of this section is to create mathematical models based on the theory of fuzzy logic [55, 56, 68, 70], which allow predicting the level of quality of the assembly of small-volume publications based on the known values of the influencing factors. Fuzzy logic equations of
different levels of input and output variables shown in fig. 3. Each fuzzy equation corresponds to a knowledge base, which is an expert statement about the relationship between fuzzy terms of input and output linguistic variables in relation (2).

Consider the relation (2). To evaluate the linguistic variables that combine the quality of the assembly of small-volume editions on the VSHRA (\(K\)) with the number of loaded stations (\(k_1\)), the format of the notebooks (\(k_2\)), the presence of complex notebooks in the block (\(k_3\)), the speed of the VSHRA work (\(k_4\)), the quality of the preparation of notebooks (\(k_5\)) and qualification of the operator (\(k_6\)), we suggest using the following system of term sets:

\[
T(K) = \langle\text{low, below, average, average above, average high}\rangle, \quad (3)
\]

\[
T(k_1) = \langle\text{minimum, average, maximum}\rangle,
\]

\[
T(k_2) = \langle\text{small, medium, large}\rangle,
\]

\[
T(k_3) = \langle\text{few, moderate, many}\rangle,
\]

\[
T(k_4) = \langle\text{low, medium, high}\rangle,
\]

\[
T(k_5) = \langle\text{low, medium, high}\rangle,
\]

\[
T(k_6) = \langle\text{low, below average, average, above average, high}\rangle.
\]

The fuzzy knowledge base corresponding to relation (2) has the form:

\[
\text{IF } (k_1 = \text{maximum}) \text{AND} (k_2 = \text{large}) \text{AND} (k_3 = \text{many}) \text{AND} (k_4 = \text{high}) \text{AND} (k_5 = \text{low}) \text{AND} (k_6 = \text{low}) \text{OR} (k_1 = \text{maximum}) \text{AND} (k_2 = \text{large}) \text{AND} (k_3 = \text{many}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{low}) \text{AND} (k_6 = \text{below average}) \text{THEN} (K = \text{low}), \quad (4)
\]

\[
\text{IF } (k_1 = \text{average}) \text{AND} (k_2 = \text{medium}) \text{AND} (k_3 = \text{moderate}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{average}) \text{AND} (k_6 = \text{average}) \text{OR} (k_1 = \text{average}) \text{AND} (k_2 = \text{small}) \text{AND} (k_3 = \text{moderate}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{high}) \text{AND} (k_6 = \text{average}) \text{THEN} (K = \text{average}), \quad (5)
\]

\[
\text{IF } (k_1 = \text{maximum}) \text{AND} (k_2 = \text{large}) \text{AND} (k_3 = \text{many}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{average}) \text{AND} (k_6 = \text{below average}) \text{OR} (k_1 = \text{average}) \text{AND} (k_2 = \text{average}) \text{AND} (k_3 = \text{many}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{low}) \text{AND} (k_6 = \text{below average}) \text{THEN} (K = \text{below average}), \quad (6)
\]

\[
\text{IF } (k_1 = \text{average}) \text{AND} (k_2 = \text{small}) \text{AND} (k_3 = \text{moderate}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{average}) \text{AND} (k_6 = \text{above average}) \text{OR} (k_1 = \text{minimum}) \text{AND} (k_2 = \text{small}) \text{AND} (k_3 = \text{few}) \text{AND} (k_4 = \text{average}) \text{AND} (k_5 = \text{high}) \text{AND} (k_6 = \text{above average}) \text{THEN} (K = \text{above average}), \quad (7)
\]

\[
\text{IF } (k_1 = \text{minimum}) \text{AND} (k_2 = \text{small}) \text{AND} (k_3 = \text{few}) \text{AND} (k_4 = \text{low}) \text{AND} (k_5 = \text{high}) \text{AND} (k_6 = \text{above average}) \text{OR} (k_1 = \text{minimum}) \text{AND} (k_2 = \text{small}) \text{AND} (k_3 = \text{few}) \text{AND} (k_4 = \text{low}) \text{AND} (k_5 = \text{high}) \text{THEN} (K = \text{above average}), \quad (8)
\]
AND ($k_6 = \text{high}$) THEN ($K = \text{high}$),

The logical expressions (4) - (8), which describe the fuzzy matrix of values of the assembly process in table 2, correspond to the following fuzzy logical equations (9) - (13).

Table 2

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$k_5$</th>
<th>$k_6$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum</td>
<td>big</td>
<td>many</td>
<td>high</td>
<td>low</td>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>maximum</td>
<td>big</td>
<td>many</td>
<td>average</td>
<td>low</td>
<td>below average</td>
<td>below average</td>
</tr>
<tr>
<td>maximum</td>
<td>big</td>
<td>many</td>
<td>average</td>
<td>average</td>
<td>below average</td>
<td>below average</td>
</tr>
<tr>
<td>average</td>
<td>average</td>
<td>many</td>
<td>average</td>
<td>low</td>
<td>below average</td>
<td>average</td>
</tr>
<tr>
<td>average</td>
<td>average</td>
<td>norm</td>
<td>average</td>
<td>average</td>
<td>average</td>
<td>average</td>
</tr>
<tr>
<td>average</td>
<td>small</td>
<td>norm</td>
<td>average</td>
<td>high</td>
<td>average</td>
<td>above average</td>
</tr>
<tr>
<td>average</td>
<td>small</td>
<td>few</td>
<td>average</td>
<td>high</td>
<td>above average</td>
<td>high</td>
</tr>
<tr>
<td>minimal</td>
<td>small</td>
<td>few</td>
<td>low</td>
<td>high</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>minimal</td>
<td>small</td>
<td>few</td>
<td>low</td>
<td>high</td>
<td>above average</td>
<td>high</td>
</tr>
</tbody>
</table>

\[
\mu^{\text{low}}(K) = \mu^{\text{maximum}}(k_1)\mu^{\text{big}}(k_2)\mu^{\text{many}}(k_3)\mu^{\text{high}}(k_4)\mu^{\text{low}}(k_5)\mu^{\text{low}}(k_6) \quad (9)
\]

\[
\mu^{\text{below average}}(K) = \mu^{\text{maximum}}(k_1)\mu^{\text{big}}(k_2)\mu^{\text{many}}(k_3)\mu^{\text{high}}(k_4)\mu^{\text{low}}(k_5)\mu^{\text{below average}}(k_6) \quad (10)
\]

\[
\mu^{\text{average}}(K) = \mu^{\text{average}}(k_1)\mu^{\text{average}}(k_2)\mu^{\text{norm}}(k_3)\mu^{\text{average}}(k_4)\mu^{\text{average}}(k_5)\mu^{\text{average}}(k_6) \quad (11)
\]

\[
\mu^{\text{above average}}(K) = \mu^{\text{average}}(k_1)\mu^{\text{small}}(k_2)\mu^{\text{norm}}(k_3)\mu^{\text{average}}(k_4)\mu^{\text{average}}(k_5)\mu^{\text{above average}}(k_6) \quad (12)
\]

\[
\mu^{\text{high}}(K) = \mu^{\text{average}}(k_1)\mu^{\text{small}}(k_2)\mu^{\text{few}}(k_3)\mu^{\text{low}}(k_4)\mu^{\text{above average}}(k_5)\mu^{\text{high}}(k_6) \quad (13)
\]
To utilize logical equations (9) - (19), it is necessary to define the membership functions for all fuzzy terms.

The membership functions cannot be used if the input variable varies continuously, meaning it can take not only the values, but also intermediate ones between them. To overcome this limitation, we will employ linear interpolation [15].

\[
f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_1 - x_0),
\]

Thus, we obtain:

\[
\mu(k_1) = \mu_i + \frac{\mu_{i-1} - \mu_i}{u_{i-1} - u_i}(k_1 - u_i),
\]

The membership functions of the variables for the assembly stage are presented in Appendix B. Based on the obtained term sets with normalized values of membership functions at five division points of the universal set, we will form tables (3 - 8) of linguistic variables for the defuzzification stage.

### Table 3
Membership Functions of the Term Set \(T(k_1)\) - Number of Loaded Stations

<table>
<thead>
<tr>
<th>(u_i), pcs</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>24</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^{\text{min}}(u_i))</td>
<td>1</td>
<td>0.78</td>
<td>0.56</td>
<td>0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>(\mu^{\text{average}}(u_i))</td>
<td>0.56</td>
<td>0.81</td>
<td>1</td>
<td>0.81</td>
<td>0.56</td>
</tr>
<tr>
<td>(\mu^{\text{max}}(u_i))</td>
<td>0.11</td>
<td>0.33</td>
<td>0.56</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 4
Term-set membership functions \(T(k_2)\) - notebook format

<table>
<thead>
<tr>
<th>(u_i), m²</th>
<th>0.004</th>
<th>0.012</th>
<th>0.044</th>
<th>0.088</th>
<th>0.165</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^{\text{small}}(u_i))</td>
<td>1</td>
<td>0.78</td>
<td>0.44</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td>(\mu^{\text{average}}(u_i))</td>
<td>0.14</td>
<td>0.36</td>
<td>1</td>
<td>0.36</td>
<td>0.14</td>
</tr>
<tr>
<td>(\mu^{\text{big}}(u_i))</td>
<td>0.11</td>
<td>0.22</td>
<td>0.44</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 5
The membership functions of the term set are the number of complex notebooks \(T(k_3)\)

<table>
<thead>
<tr>
<th>(u_i), pcs</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu^{\text{low}}(u_i))</td>
<td>1</td>
<td>0.89</td>
<td>0.67</td>
<td>0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>(\mu^{\text{norm}}(u_i))</td>
<td>0.66</td>
<td>0.89</td>
<td>1</td>
<td>0.89</td>
<td>0.66</td>
</tr>
<tr>
<td>(\mu^{\text{many}}(u_i))</td>
<td>0.11</td>
<td>0.33</td>
<td>0.67</td>
<td>0.89</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 6
The membership functions of the term set are the operating speed of the VSHRA $T(k_4)$

<table>
<thead>
<tr>
<th>$u_i$, cycle/hour</th>
<th>1000</th>
<th>5000</th>
<th>15000</th>
<th>25000</th>
<th>30000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{low}}(u_i)$</td>
<td>1</td>
<td>0.83</td>
<td>0.54</td>
<td>0.23</td>
<td>0.1</td>
</tr>
<tr>
<td>$\mu_{\text{average}}(u_i)$</td>
<td>0.48</td>
<td>0.83</td>
<td>1</td>
<td>0.83</td>
<td>0.48</td>
</tr>
<tr>
<td>$\mu_{\text{high}}(u_i)$</td>
<td>0.27</td>
<td>0.48</td>
<td>0.72</td>
<td>0.92</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 7
Membership functions of the term set $T(k_5)$ - the quality of the preparation of notebooks

<table>
<thead>
<tr>
<th>$u_i$, points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{low}}(u_i)$</td>
<td>1</td>
<td>0.78</td>
<td>0.56</td>
<td>0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>$\mu_{\text{average}}(u_i)$</td>
<td>0.56</td>
<td>0.81</td>
<td>1</td>
<td>0.81</td>
<td>0.56</td>
</tr>
<tr>
<td>$\mu_{\text{high}}(u_i)$</td>
<td>0.11</td>
<td>0.33</td>
<td>0.56</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

As a result of substituting degrees of membership into the system of fuzzy logic equations (9) – (13), the equation of the membership functions is obtained:

$$\mu_{\text{low}}(K) = 0.33\alpha_{0,22}\alpha_{0,33}\alpha_{0,72}\alpha_{0,11}\alpha_{0,33}v_{0.11\alpha_{0,11}} = 0.11,$$

$$\mu_{\text{average}}(K) = 0.81\alpha_{0,36}\alpha_{0,89}\alpha_{0,92}\alpha_{0,56}\alpha_{0,81}v_{0.36\alpha_{0,22}} = 0.36,$$

$$\mu_{\text{high}}(K) = 0.78\alpha_{0,78}\alpha_{0,89}\alpha_{0,54}\alpha_{0,89}\alpha_{0,78}v_{0.54\alpha_{0,54}} = 0.54.$$  

Table 8
Term-set membership functions $T(k_6)$ – operator qualification

<table>
<thead>
<tr>
<th>$u_i$, points</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{low}}(u_i)$</td>
<td>1</td>
<td>0.78</td>
<td>0.56</td>
<td>0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>$\mu_{\text{average}}(u_i)$</td>
<td>0.56</td>
<td>0.81</td>
<td>1</td>
<td>0.81</td>
<td>0.56</td>
</tr>
<tr>
<td>$\mu_{\text{high}}(u_i)$</td>
<td>0.11</td>
<td>0.33</td>
<td>0.56</td>
<td>0.78</td>
<td>1</td>
</tr>
</tbody>
</table>

Thus, the numerical values of the membership functions for the linguistic variable $K$ were obtained, which will be used to calculate the quantitative value of the assessment of the quality of the TPVMV at the VSHRA.
Conclusions

As a result of the study, a factor ranking method was implemented to build a model of assembly quality factors, which takes into account the number and types of relationships between factors and the different expert weight of each of these types. The obtained result confirms the reliability of the obtained models. A universal set of values, as well as corresponding linguistic terms for each linguistic variable of the assembly stage, was formed. A model of logical derivation was built, the structure of which reflects the classification of factors and the process of forming a forecasted indicator of the quality of production of small-volume editions at the VSHRA.

The values of the membership functions of linguistic variables were calculated through the construction and calculation of pairwise comparison matrices for a set of linguistic terms relative to the dividing points of the value intervals of the universal set. Graphs were constructed showing the relationships between the parameters of the LZ from the universal set and the values of the membership functions of the corresponding linguistic terms.

A fuzzy knowledge base was designed and mathematical models for forecasting the quality of assembly were developed based on the known values of influencing factors, for the construction of which expert evaluations of fuzzy logical statements of the "IF-TH" type were used. Fuzzy logic equations are built that determine the relationship between the membership functions of input and output data.

The obtained results serve as a basis for creating a module for assessing the quality of the production process in the industrial Internet of Things system of a printing company.

References


