

# A Principle-based Framework for Repair Selection in Inconsistent Ontologies (Extended Abstract)

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## Abstract

This paper investigates a general principle-based framework for retrieving preferred repairs from inconsistent ontologies under a broad family of strategies. To begin with, we define a set of principles that ensure rational behaviours of repair selection strategies. Then, we classify the strategies into two basic categories: (i) ranking repairs without requiring formula information; and (ii) ranking repairs based on formula information. Based on this classification, we introduce several novel repair selection strategies and show that our framework encompasses a range of popular existing strategies. Additionally, through a systematical analysis of these selection strategies using the proposed principles, we conclude that our principles allow for effective discrimination among the strategies. Finally, preliminary experimental results are presented to show the feasibility of our proposed framework.

## Keywords

Ontologies, Inconsistency handling, Preferred repairs, Rationality principle

## 1. Introduction


The study of preferences has a long tradition in various disciplines. It has been recently applied in query answering over databases [1], propositional logic knowledge bases (KBs) [2, 3], description logic KBs [4, 5, 6], and the existential rule language [7]. In this context, these existing proposals rely on the concept of *repairs*, that is, the  $\subseteq$ -maximal consistent subsets of formulas.

The need of retrieving preferred repairs is justified by numerous issues associated with using all repairs. The primary drawback of reasoning with all repairs stems from their typically large numbers in real-world applications [8]. Moreover, repairs are often not equally important in practice. For example, when one data source is more reliable than another or when new information is preferred over earlier ones [9]. We may also prefer one repair over another if it contains less problematic information [10, 11, 12, 13, 14, 15, 16]. Therefore, in this paper, we turn our attention to how to choose the most preferred repairs among all the potential repairs of an inconsistent KB, while filtering out undesired repairs. Unlike [5], and in line with [4, 6, 17, 7], we aim to guarantee that the retrieved consistent subsets are still  $\subseteq$ -maximal.

Despite their success, the aforementioned proposals on preferred repairs-based query answering have been generally developed using ad hoc repair selection strategies. A basic strategy is based on the cardinality of repairs. More advanced strategies often use some aggregation functions of formula information (e.g., weight, priority level, inconsistency measure), either provided as system inputs or computed from the given KB. Without being limited to aggregation functions, this paper provides a complementary principle-based framework that allows us to define different repair selection strategies to retrieve preferred repairs from inconsistent KBs. This paper makes the following concrete contributions:


- We propose a set of logical principles that guarantee rational behaviours of repair selection strategies.

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- We present various repair selection strategies that allow for comparing repairs using solely the internal structure of a KB.
- We evaluate the different repair selection strategies against the proposed principles (Table 1), showing that these principles allow for an effective discrimination among different strategies.

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## 2. Principle-based framework for repair selection

Our framework is not restricted to a particular logic language, but we consider here Description Logics (DL) and write  $\mathbb{K}$  for the set of all DL-based KBs.

**Definition 1.** For a DL KB  $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ , an ABox **repair** is a  $\subseteq$ -maximal subset  $\mathcal{A}'$  of  $\mathcal{A}$  such that  $\mathcal{T} \cup \mathcal{A}'$  is consistent. An ABox **conflict** is a  $\subseteq$ -minimal subset  $\mathcal{A}''$  of  $\mathcal{A}$  s.t.  $\langle \mathcal{T}, \mathcal{A}'' \rangle \models \perp$ . We denote by  $\mathcal{R}(\mathcal{K})$  and  $\mathcal{C}(\mathcal{K})$  the set of all possible repairs and conflicts of  $\mathcal{K}$ , respectively.

In what follows, the repairs of a given KB will be ordered according to some criteria to determine only the most desired ones. More formally, our framework returns the set of *preferred repairs* among the (large) candidate set of repairs by taking as input three basic elements:

- a knowledge base  $\mathcal{K} \in \mathbb{K}$ ,
- a *formula characterisation*  $ch$  over formulas in  $\mathcal{L}$ :  $ch$  can be a binary function or relation over  $\text{Form}(\mathcal{L})$ , e.g., a weight, a distance, or a priority relation between two formulas. Though  $ch$  could be arbitrary, the following property is natural for  $ch$  to satisfy: If  $\alpha \equiv \alpha'$  and  $\beta \equiv \beta'$ , then  $ch(\alpha, \beta) = ch(\alpha', \beta')$ .
- a *repair comparison strategy*, written  $\succ_s \subseteq \mathcal{R}(\mathcal{K}) \times \mathcal{R}(\mathcal{K})$ , to compare the repairs of  $\mathcal{K}$ .

**Definition 2.** Let  $\mathcal{K} \in \mathbb{K}$  and  $R, R' \in \mathcal{R}(\mathcal{K})$ . A **repair comparison strategy**  $\succ_s$  is an acyclic preference relation over  $\mathcal{R}(\mathcal{K})$  with  $R \succ_s R'$ , meaning that  $R$  is preferred over  $R'$ . Given a relation  $ch$ , a **repair selection function** is a mapping  $\mathcal{F} : \mathcal{K} \times ch \times \succ_s \rightarrow \Xi$  s.t.  $\Xi \in 2^{\mathcal{R}(\mathcal{K})}$ .

An **optimal repair selection function** is a repair selection function  $\mathcal{F}$  such that there is no  $R \in \mathcal{F}(\mathcal{K}, ch, \succ_s)$  and  $R' \in (\mathcal{R}(\mathcal{K}) \setminus \mathcal{F}(\mathcal{K}, ch, \succ_s))$  such that  $R' \succ_s R$ .

**Rationality Principles** Definition 2 is general enough regarding the repair comparison strategy  $\succ_s$  and the formula characterisation  $ch$ . To restrict the possible candidates, we first establish a set of desiderata that a suitable optimal repair selection function  $\mathcal{F}$  should fulfil. Such formal principles are important for defining, characterizing, and comparing selection functions. We refer the long version of the paper for the intuitions underlying these principles: **Abstraction:** For any  $\mathcal{K} \in \mathbb{K}$  and any isomorphism<sup>1</sup>  $\gamma$ , we have that  $\forall R \in \mathcal{R}(\mathcal{K}), R \in \mathcal{F}(\mathcal{K}, ch, \succ_s)$  iff  $\gamma(R) \in \mathcal{F}(\gamma(\mathcal{K}), ch, \succ_s)$ .

**Equivalence Invariance:** For  $\mathcal{K}_1, \mathcal{K}_2 \in \mathbb{K}$ , if  $\mathcal{K}_1 \cong \mathcal{K}_2$ , then for each  $R \in \mathcal{F}(\mathcal{K}_1, ch, \succ_s)$ , there exists  $R' \in \mathcal{F}(\mathcal{K}_2, ch, \succ_s)$  s.t.  $R \cong R'$ .

**Coverness:** For any KB  $\mathcal{K}$ ,  $\bigcup_{R \in \mathcal{F}(\mathcal{K}, ch, \succ_s)} R = \mathcal{K}$ .

**Non-Emptiness:**  $\mathcal{F}(\mathcal{K}, ch, \succ_s) \neq \emptyset$ , if  $\mathcal{R}(\mathcal{K}) \neq \emptyset$ .

**Monotonicity:** If  $ch_1, ch_2$  are two relations s.t.  $ch_2 \subseteq ch_1$ , then  $\mathcal{F}(\mathcal{K}, ch_1, \succ_s) \subseteq \mathcal{F}(\mathcal{K}, ch_2, \succ_s)$ .

**Non-Discrimination:**  $\mathcal{F}(\mathcal{K}, ch, \emptyset) = \mathcal{R}(\mathcal{K})$ .

**Reducibility:** If  $\mathcal{K} \vdash \perp$ , then  $\mathcal{F}(\mathcal{K}, ch, \succ_s) \subseteq \mathcal{R}(\mathcal{K})$ .

**Consistency:** If  $\mathcal{K} \not\vdash \perp$ , then  $\mathcal{F}(\mathcal{K}, ch, \succ_s) = \{\mathcal{K}\}$ .

**Improvement:** Given  $\mathcal{K}, \mathcal{K}' \in \mathbb{K}$ , if  $\mathcal{K} \subseteq \mathcal{K}'$ , then for any  $R \in \mathcal{F}(\mathcal{K}, ch, \succ_s)$  and  $R' \in \mathcal{F}(\mathcal{K}', ch, \succ_s)$ ,  $R \not\succeq_s R'$ .

**Stability:** Given  $\mathcal{K}, \mathcal{K}' \in \mathbb{K}$ , if  $\mathcal{K} \subseteq \mathcal{K}'$ , for any  $R \in \mathcal{F}(\mathcal{K}, ch, \succ_s)$ , there exists  $R' \in \mathcal{F}(\mathcal{K}', ch, \succ_s)$

<sup>1</sup>The isomorphism  $\gamma$  renames all the variables (i.e., concept, role names, and instances). We apply  $\gamma$  to formulas and knowledge bases.

such that  $R \subseteq R'$ .

**Uniqueness:**  $|\mathcal{F}(\mathcal{K}, ch, \succ_s)| = 1$ .

**Non-Triviality:** There is  $\mathcal{K} \in \mathbb{K}$  such that  $\mathcal{F}(\mathcal{K}, ch, \succ_s) \subset \mathcal{R}(\mathcal{K})$ .

The intuition about the principles is as follows: The *Equivalence Invariance* concerns the main intuition behind the logical equivalence between two sets of formulas. It ensures that two equivalent KBs exhibit the same behaviour according to the function  $\mathcal{F}$ . The *Coverness* principle requires that a KB should be covered by its preferred repairs, i.e., each formula in  $\mathcal{K}$  must belong to at least one preferred repair. The *Non-Emptiness* principle says that an optimal repair selection strategy  $\mathcal{F}$  must return at least one preferred repair. *Monotonicity* implies that looking at richer preferences among formulas can only narrow down the set of preferred repairs. The *Non-Discrimination* principle states that the preferred repairs are the set of all repairs when no repair comparison strategy is expressed. *Reducibility* claims that there always exists a manner to remove certain repairs from being the selected preferred ones. *Consistency* states that if a KB is consistent, then the only preferred repair is the base itself. The *Improvement* principle says that the expansion of a KB can only result in better preferred repairs according to  $\succ_s$ . The *Stability* principle states that the expansion of a KB  $\mathcal{K}$  preserves all the formulas involved in the preferred repairs of  $\mathcal{K}$ . *Uniqueness* says that an optimal repair selection strategy should return one unique preferred repair. Finally, the *Non-Triviality* principle is to avoid repair selection functions that do not discriminate between repairs and return all repairs for any KB  $\mathcal{K}$ .

Table 1 details the principles satisfied by different repair selection functions discussed below. In the table, we highlight DL-Lite, for which ABox conflict sets are always binary, i.e.,  $|\mathcal{C}(\mathcal{K})| = 2$ .

### 3. Categories of selection strategies

We distinguish two categories of selection repair strategies: the ones defined on a given  $ch$  as in many existing work; and the ones independent of  $ch$  for the setting where no formula information is available. The former includes improvement-based strategies [1, 17] and the scoring function-based strategy  $\succeq_{\text{score}}$  [2]. We focus on presenting the other category here.

**Table 1**

Principles vs. repair selection strategies:  $\checkmark$  (resp.  $\checkmark^*$ ) means that the strategy satisfies the property (resp. under certain conditions),  $\otimes$  stands for unsatisfaction, and NA means inapplicability.

Principles	$\succ_{\text{card}}$	$\succ_{\text{comp}}^{\#}$	$\succ_{\text{comp}}^{\supseteq}$	$\succ_{\text{cover}}$
Abstraction	$\checkmark$		$\checkmark$	$\checkmark$
Equivalence Invariance	$\otimes$		$\checkmark$	$\checkmark$
Coverness	$\otimes$		$\otimes$	$\checkmark$
Non-Emptiness	$\checkmark$		$\checkmark$	$\checkmark$
Monotonicity	$\checkmark$		$\checkmark$	$\checkmark$
Non-Discrimination	$\checkmark$		$\checkmark$	$\checkmark$
Reducibility	$\otimes$		$\otimes$	$\otimes$
Consistency	$\checkmark$		$\checkmark$	$\checkmark$
Improvement	$\checkmark$	$\checkmark$ (DL-Lite), $\otimes$ (in general)		$\otimes$
Stability	$\otimes$	$\checkmark$ (DL-Lite), $\otimes$ (in general)		$\otimes$
Uniqueness	$\otimes$		$\otimes$	$\otimes$
Non-Triviality	$\checkmark$	$\otimes$ (DL-Lite), $\checkmark$ (in general)		$\checkmark$

***ch*-Free Selection Strategies** A straightforward way for filtering preferred repairs without using  $ch$  involves employing a cardinality-based criterion [4] that can be formalized in our notation as follows. Given two repairs  $R, R' \in \mathcal{R}(\mathcal{K})$ ,  $R \succ_{\text{card}} R'$  iff.  $|R| \geq |R'|$ . The key idea underlying the cardinality-based method is to retain as many information as possible. A problem with  $\succ_{\text{card}}$  is that no account is taken of the possible interaction between repairs of a given KB.

So we propose a strategy called *compatibility-based strategy* that compares all pairs of repairs based on the next criterion: We prefer the repairs having more compatibility with other repairs, i.e., opposed

by less repairs by ignoring their intersection. Formally,

**Definition 3.** Let  $\mathcal{K} \in \mathbb{K}$  and  $R \in \mathcal{R}(\mathcal{K})$ . The **compatible set** of  $R$  w.r.t.  $\mathcal{R}(\mathcal{K})$ , written  $\text{comp}(R, \mathcal{K})$ , is defined as  $\text{comp}(R, \mathcal{K}) = \{R' \in \mathcal{R}(\mathcal{K}) \mid (R \setminus R') \cup (R' \setminus R) \not\vdash \perp\}$ . Then for  $R, R' \in \mathcal{R}(\mathcal{K})$ ,  $R$  is preferred to  $R'$  by cardinality (resp. set-inclusion), written  $R \succ_{\text{comp}}^{\#} R'$  (resp.  $R \succ_{\text{comp}}^{\supseteq} R'$ ), iff  $|\text{comp}(R, \mathcal{K})| \geq |\text{comp}(R', \mathcal{K})|$  (resp.  $\text{comp}(R, \mathcal{K}) \supseteq \text{comp}(R', \mathcal{K})$ ).

Unfortunately, the compatibility-based strategies are not specifically geared toward DL-Lite. In fact, the notion of compatible set of a repair is trivial because it becomes a singleton and only contains the repair itself. This result also explains the violation of Non-Triviality and satisfaction of Stability of  $\succ_{\text{comp}}^x$  ( $x \in \{\#, \subseteq\}$ ) for DL-Lite.

Another strategy without using *ch* is called *cover-based strategy* that aims to selecting among the repairs the ones covering the KB. The coverness in KBs is defined as follows: Given a KB  $\mathcal{K}$ ,  $\Gamma = \{\mathcal{K}_1, \dots, \mathcal{K}_n \mid \mathcal{K}_i \subseteq \mathcal{K}, 1 \leq i \leq n\}$  is a *cover* of  $\mathcal{K}$  iff  $\bigcup_{1 \leq i \leq n} \mathcal{K}_i = \mathcal{K}$ . A cover  $\Gamma$  of  $\mathcal{K}$  is minimal if there exists no other cover  $\Gamma'$  of  $\mathcal{K}$  s.t.  $\Gamma' \subset \Gamma$ . We call  $\Gamma$  a **minimal repair cover** of  $\mathcal{K}$ , if  $\mathcal{K}_i \in \mathcal{R}(\mathcal{K})$  for  $1 \leq i \leq n$ . We write  $\text{cover}(\mathcal{K})$  for the set of minimal repair covers of  $\mathcal{K}$ .

**Definition 4.** Let  $\mathcal{K} \in \mathbb{K}$ . For two repairs  $R, R' \in \mathcal{R}(\mathcal{K})$ , we say that  $R \succ_{\text{cover}} R'$  if there is  $\Gamma \in \text{cover}(\mathcal{K})$  such that  $R \in \Gamma$ .

This definition implies that the set of the preferred repairs will be the union of minimal covers, i.e.,  $\mathcal{F}(\mathcal{K}, \emptyset, \succ_{\text{cover}}) = \bigcup_{\Gamma \in \text{cover}(\mathcal{K})} \Gamma$ . Moreover, we have examples showing that even for the language with binary conflict property, which makes  $R \succ_{\text{comp}}^{\{\#, \subseteq\}} R'$  strategy trivial, the  $\succ_{\text{cover}}$  strategy is still able to give non-trivial results.

**Feasibility study** We presented above several novel strategies for selecting preferred repairs. Table 1 summarises the repair selection strategies and their satisfaction of a rich set of rationality principles. It shows that our principles give an effective way to discriminate among these optimal repair selection strategies.

A preliminary empirical study compares the compatibility-based and the cardinality-based strategies for a real-world KB constructed using the National Downloadable File given by the Centers for Medicare and Medicaid Services<sup>2</sup>, as outlined in [19]. We conclude that computing preferred repairs is a feasible task.

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<sup>2</sup><https://data.cms.gov/provider-data/>

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