Stable Model Semantics for Description Logic Terminologies (Extended Abstract)

Federica Di Stefano[1], Mantas Šimkus[1,2]

[1]*Institute of Logic and Computation, Vienna University of Technology, Austria*
[2]*Department of Computing Science, Umeå University, Sweden*

Abstract

This paper is an extended abstract of our recent work on proposing a *stable model semantics* for Description Logic terminologies. Our semantics is based on Quantified Equilibrium Logic and overcomes some limitations of previous approaches. We prove that standard reasoning problems w.r.t. terminologies in $\mathcal{ALCI}$ under the stable model semantics are decidable in deterministic single exponential time. In contrast, we show that concept satisfiability w.r.t. general KBs in $\mathcal{ALCI}$ under the stable model semantics is undecidable.

Keywords

Stable Models, Quantified Equilibrium Logic, Terminologies, Computational Complexity

1. Introduction and Motivation

Description Logics are often seen as fragments of the classical first-order logic, equipped with a syntax that is more convenient for knowledge representation [1]. In particular, this means that most DLs nowadays make the *open-world assumption (OWA)*, in which, intuitively, everything that is not forbidden is considered possible. However, it is acknowledged that supporting the *closed-world assumption (CWA)* is also important in order to enable commonsense reasoning in DLs, e.g., by means of *default negation* (see, e.g., the overview and the references in [2]). In [3], we consider Description Logics under the stable model semantics of Quantified Equilibrium Logic (QEL) [4] which provides an elegant logical formalization of the stable model semantics of logic programs with default negation [5]. The stable model semantics of QEL turns out to be an appropriate semantics for *terminologies* in $\mathcal{ALCI}$. A terminology $\mathcal{T}$ consists of statements of the form $A := C$, where a concept name A is *defined* using a complex concept expression C. A terminology may contain *terminological cycles*, where a definition of some concept name may be recursive, as in the following terminology:

$$\text{BasicUser} := User \sqcap \neg\text{PrivilegedUser}$$
$$\text{PrivilegedUser} := Admin \sqcup \exists promotedBy.\text{PrivilegedUser}$$

Here BasicUser and PrivilegedUser are (intensional or *defined*) concept names defined using (extensional) base predicates $User$, $Admin$, and $promotedBy$. A concrete semantics for terminologies tells us how to interpret the defined concept names given an extension for the base predicates. Suppose the base predicates correspond to the following facts:

$$User(a) \quad User(b) \quad promotedBy(a,b) \quad promotedBy(b,a)$$

In the last three decades, essentially three semantics have been proposed to deal with terminologies: the *least fixpoint semantics*, the *greatest fixpoint semantics*, and the *descriptive semantics*. The *descriptive semantics* [6] is nowadays the classical semantics for DLs, deemed as the most natural semantics and it sees ":=" as a logical equivalence; in our example, it produces two possible extensions for the defined concept names:

(i) $\text{BasicUser}(a), \text{BasicUser}(b)$

(ii) PrivilegedUser(a), PrivilegedUser(b)

While the extension (i) is natural and expected, the extension (ii) is questionable: the membership of a and b in PrivilegedUser is not well-founded (there is only a self-supported justification). Baader [7] advocates fixpoint semantics to overcome some weaknesses of the descriptive semantics. However, the semantics of Baader is too strong in this example: it rejects both (i) and (ii). The *stable model semantics* based on QEL proposed in [3] is stronger than the classical semantics, in the sense that some classical models of a KB or terminology will be rejected as implausible (like (ii) in our example above), and accepts (i). Enabling default negation, the stable model semantics of QEL allows to combine the OWA and the CWA in a natural way. The reconciliation of OWA and CWA specifically in DL *terminologies* is a problem whose relevance is boosted by the new W3C SHACL standard for expressing constraints over RDF graphs [8]. SHACL is syntactically very close to DL terminologies, but its semantics has not been fully established yet (but it clearly leans towards CWA). The main contributions of [3] can be summarized as follows:

- We define a semantics for general DL knowledge bases using *Quantified Equilibrium Logic (QEL)* [4]. The use of QEL in defining the semantics allows us to elegantly support *fixed predicates*, which are needed for modeling (extensional) base predicates in terminologies.
- We provide a strong undecidability results for reasoning in $\mathcal{ALCIO}$ and $\mathcal{ALCI}$ in the presence of role minimization. In particular, for $\mathcal{ALCIO}$ the proof is given for KBs without negation, which means that it carries over and applies to *circumscribed* $\mathcal{ALCIO}$ KBs where *all* predicates are set to be minimized. This complements the negative results in [9], which rely on the use of *varying* predicates.
- We define a stable model semantics for DL terminologies. To achieve this, we instantiate our stable model semantics for general KBs. Intuitively, for a given terminology $\mathcal{T}$, we require all base concept and role names of $\mathcal{T}$ to be interpreted as fixed predicates, i.e. they are not subject to minimization. In addition, we provide two alternative definitions of stable models, based on *level mappings* and *fixpoint computation*. We prove that the semantics are equivalent.
- We study the case of $\mathcal{ALCI}$ terminologies and prove ExpTime-completeness for the standard reasoning tasks. This is achieved by proving a *tree-model property* and employing 2-way alternating tree automata [10].

2. Stable Model Semantics and Contributions

QEL is based on Here-and-There Logic (HT) with an additional minimality requirement. In contrast to the classical case, an interpretation in the logic HT consists of a pair of structures $(\mathcal{I}, \mathcal{J})$ sharing the same domain and interpreting individuals in the same way. The interpretation $\mathcal{I}$ is the 'here' world while $\mathcal{J}$ is the 'there' world. We call *assumed* everything that is true 'there', and *founded* everything that is true 'here'. The two 'worlds' are related by the inclusion relation, i.e. $\mathcal{I} \subseteq \mathcal{J}$: 'here' is included in 'there', i.e. $p^{\mathcal{I}} \subseteq p^{\mathcal{J}}$, for all predicates p. Fixed predicates can be easily expressed by requiring that $\mathcal{I}$ and $\mathcal{J}$ agree on a set F of predicates. In this case, we write $\mathcal{I} \subseteq_F \mathcal{J}$.

Definition 1. *A Here-and-There (HT) interpretation is a pair $(\mathcal{I}, \mathcal{J})$ of interpretations with $\mathcal{I} \subseteq \mathcal{J}$. We define an* interpretation function $\cdot^{(\mathcal{I},\mathcal{J})}$ *using the equations in Figure 1.*

In the HT logic, the implication is intuitionistic: in jargon it needs to be 'founded', meaning that the HT interpretation must model it, and 'assumed', meaning that the 'there' world must model it. In DLs, the universally quantified concept of the form $\forall r.C$ can be translated in FOL as $\forall y((r(x,y) \rightarrow C(y))$. Thus the interpretation must align with the interpretation of implication in quantified HT. As a matter of fact, concept inclusions are also affected by this double nature of implication, as they are 'explicit' implications in DLs.

Definition 2. *Assume a KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ and an HT interpretation $(\mathcal{I}, \mathcal{J})$. We write:*

$$a^{(\mathcal{I},\mathcal{J})} = a^{\mathcal{I}} \qquad A^{(\mathcal{I},\mathcal{J})} = A^{\mathcal{I}} \qquad r^{(\mathcal{I},\mathcal{J})} = r^{\mathcal{I}} \qquad \top^{(\mathcal{I},\mathcal{J})} = \Delta^{\mathcal{I}} \qquad \bot^{(\mathcal{I},\mathcal{J})} = \emptyset$$

$$(r^-)^{(\mathcal{I},\mathcal{J})} = \{(e,e') \mid (e',e) \in r^{\mathcal{I}}\} \qquad\qquad (\neg C)^{(\mathcal{I},\mathcal{J})} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{J}}$$

$$(C_1 \sqcap C_2)^{(\mathcal{I},\mathcal{J})} = C_1^{(\mathcal{I},\mathcal{J})} \cap C_2^{(\mathcal{I},\mathcal{J})} \qquad\qquad (C_1 \sqcup C_2)^{(\mathcal{I},\mathcal{J})} = C_1^{(\mathcal{I},\mathcal{J})} \cup C_2^{(\mathcal{I},\mathcal{J})}$$

$$(\exists R.C)^{(\mathcal{I},\mathcal{J})} = \{e \in \Delta^{\mathcal{I}} \mid \exists e' : (e,e') \in R^{(\mathcal{I},\mathcal{J})} \wedge e' \in C^{(\mathcal{I},\mathcal{J})}\}$$

$$(\forall R.C)^{(\mathcal{I},\mathcal{J})} = \left\{ e \in \Delta^{\mathcal{I}} \mid \forall e' : \begin{array}{l} (e,e') \in R^{(\mathcal{I},\mathcal{J})} \text{ implies } e' \in C^{(\mathcal{I},\mathcal{J})} \text{ and} \\ (e,e') \in R^{\mathcal{J}} \text{ implies } e' \in C^{\mathcal{J}} \end{array} \right\}$$

Figure 1: HT semantics for DLs.

- $(\mathcal{I},\mathcal{J}) \models C \sqsubseteq D$, if $C^{(\mathcal{I},\mathcal{J})} \subseteq D^{(\mathcal{I},\mathcal{J})}$ and $C^{\mathcal{J}} \subseteq D^{\mathcal{J}}$;

- $(\mathcal{I},\mathcal{J}) \models \mathcal{K}$, if $\mathcal{I} \models \mathcal{A}$ and $(\mathcal{I},\mathcal{J}) \models C \sqsubseteq D$, for all $C \sqsubseteq D \in \mathcal{T}$.

Definition 3 (Stable model). *Given $F \subseteq N_C \cup N_R$, an interpretation $\mathcal{J}$ is a* stable model *of a KB $\mathcal{K}$ under fixed predicates F, if*

(i) *the HT interpretation $(\mathcal{J},\mathcal{J})$ is a model of $\mathcal{K}$, and*

(ii) *there is no $\mathcal{I}$ s.t. $(\mathcal{I},\mathcal{J})$ is a model of $\mathcal{K}$ and $\mathcal{I} \subset_F \mathcal{J}$.*

Intuitively, a model is stable if it cannot be further improved in a better 'here' world. In the semantics introduced above, the negation $\neg$ behaves as *negation as failure* or *default negation* in logic programs. Given an HT model, the 'there' is a classical model and a concept A true at some domain element in the 'there' can be thought of as 'to be justified'. An HT model is not stable if the truth of an atom in the 'there' cannot be proved. Intuitively, the truth of $\neg A$ at a domain element d in a stable model amounts to 'we cannot justify A at d'.

The standard reasoning tasks of *satisfiability*, *concept satisfiability*, *subsumption* and *instance checking* are adapted to stable models in the obvious way [3]. Identifying a stable model of a KB requires checking minimality (see (ii) in Definition 3), which is computationally difficult.

Theorem 1. *Concept satisfiability in $\mathcal{ALCIO}$ under the stable model semantics is undecidable.*

The result above is based on a reduction from the *domino problem*. The constructed TBox is negation-free, thus the undecidability result applies to *circumscribed $\mathcal{ALCIO}$* with all predicates minimized, extending the results of [9] relying on the use of varying predicates. We prove that under the stable model semantics reasoning w.r.t. KBs in $\mathcal{ALCIO}$ can be reduced to reasoning w.r.t. KBs in $\mathcal{ALCI}$. Thus standard reasoning problems in $\mathcal{ALCI}$ are undecidable.

Terminologies. A key feature of terminologies is the separation of the predicates in a terminology $\mathcal{T}$ into two sets: $\mathsf{def}(\mathcal{T})$ are the *intensional* predicates that are defined using concept definitions based on the *extensional* predicates in $\mathsf{base}(\mathcal{T})$. In the context of the stable model semantics, it is thus natural to not require minimization of the predicates in $\mathsf{base}(\mathcal{T})$, i.e. the extensions of these predicates should remain fixed during the minimization process. Based on this observation, a stable model semantics for terminologies $\mathcal{T}$ can be immediately obtained by instantiating Definition 3, which covers general KBs: (a) view every concept definition $A := C \in \mathcal{T}$ as an inclusion $C \sqsubseteq A$, and (b) use $F = \mathsf{base}(\mathcal{T})$ as the set of fixed predicates.

In [3], we define an equivalent stable model semantics for terminologies based on *level mappings*, similarly to [11]. Level mappings allow tracking the *justifications* of defined predicates. Roughly speaking, a model of a terminology is stable if all occurrences of *defined concepts* are *well-founded*, meaning that they are supported by a finite justification. We use level mapping to guide the unraveling of stable models, preserving the well-foundedness.

Theorem 2. *If a terminology $\mathcal{T}$ has a stable model, then it has a tree-shaped stable model.*

Given a terminology $\mathcal{T}$, we can construct a 2ATA **A** (with Büchi acceptance condition) whose number of states is polynomial in the size of $\mathcal{T}$ and such that $\mathcal{T}$ has a tree-shaped stable model iff **A** is not empty, i.e. **A** accepts a tree. The automaton **A** is the intersection of two automata on labeled trees. The first automaton $\mathbf{A}_M$ checks that the input tree is a classical model of the terminology, it can be constructed in the usual way [12]. The second automaton $\mathbf{A}_F$ checks the well-foundedness of defined concepts and can be constructed in a similar way of [13].

Theorem 3. *For $\mathcal{ALCI}$ terminologies under the stable model semantics, the problems of satisfiability, concept subsumption, and concept satisfiability are ExpTime-complete.*

3. Conclusions

We expect that our result for terminologies in $\mathcal{ALCI}$ can be extended to $\mathcal{ALCIO}$ following the techniques of [13]. *Finite model reasoning* in terminologies under the stable model semantics is also a relevant open problem: it has the potential to provide new insights into, e.g., the complexity of static analysis problems for SHACL. Another natural direction (also relevant for SHACL) is to study terminologies that support *regular expressions* over roles, which enable recursive navigation of paths in an interpretation.

Acknowledgments

References

[1] F. Baader, I. Horrocks, C. Lutz, U. Sattler, An Introduction to Description Logic, Cambridge University Press, 2017. doi:10.1017/9781139025355.

[2] T. Schneider, M. Šimkus, Ontologies and data management: A brief survey, Künstliche Intell. 34 (2020) 329–353. URL: https://doi.org/10.1007/s13218-020-00686-3. doi:10.1007/S13218-020-00686-3.

[3] F. Di Stefano, M. Šimkus, Stable model semantics for description logic terminologies, in: AAAI, AAAI Press, 2024, pp. 10484–10492.

[4] D. Pearce, A. Valverde, Quantified equilibrium logic and foundations for answer set programs, in: Proc. of ICLP 2008, volume 5366 of *Lecture Notes in Computer Science*, Springer, 2008, pp. 546–560.

[5] D. Pearce, Equilibrium logic, Ann. Math. Artif. Intell. 47 (2006) 3–41.

[6] B. Nebel, Terminological cycles: Semantics and computational properties, in: J. F. Sowa (Ed.), Principles of Semantic Networks - Explorations in the Representation of Knowledge, The Morgan Kaufmann Series in representation and reasoning, Morgan Kaufmann, 1991, pp. 331–361. URL: https://doi.org/10.1016/B978-1-4832-0771-1.50018-7. doi:10.1016/B978-1-4832-0771-1.50018-7.

[7] F. Baader, Terminological cycles in KL-ONE-based knowledge representation languages, in: Proc. of AAAI 1990, AAAI Press / The MIT Press, 1990, pp. 621–626.

[8] H. Knublauch, D. Kontokostas, Shapes constraint language (W3C SHACL), https://www.w3.org/TR/shacl/, 2017. Accessed: 2024-01-24.

[9] P. A. Bonatti, C. Lutz, F. Wolter, The complexity of circumscription in description logic, J. Artif. Intell. Res. 35 (2009) 717–773.

[10] M. Y. Vardi, Reasoning about the past with two-way automata, in: Proc. of ICALP 1998, volume 1443 of *Lecture Notes in Computer Science*, Springer, 1998, pp. 628–641.

[11] M. Andresel, J. Corman, M. Ortiz, J. L. Reutter, O. Savkovic, M. Šimkus, Stable model semantics for recursive SHACL, in: Proc. of WWW 2020, ACM / IW3C2, 2020, pp. 1570–1580.

[12] D. Calvanese, T. Eiter, M. Ortiz, Answering regular path queries in expressive description logics: An automata-theoretic approach, in: Proc. of AAAI 2007, AAAI Press, 2007, pp. 391–396.

[13] U. Sattler, M. Y. Vardi, The hybrid μ-calculus, in: Proc. of IJCAR 2001, volume 2083 of *Lecture Notes in Computer Science*, Springer, 2001, pp. 76–91. URL: https://doi.org/10.1007/3-540-45744-5_7. doi:10.1007/3-540-45744-5_7.