

# An Update on Non-Rigid Designators in Modalised Description Logics (Extended Abstract)

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## Abstract

We investigate decidability and complexity of the satisfaction problem for modal free description logics with non-rigid designators, which have recently been introduced as a powerful extension of standard modalised description logics. Our three main contributions are as follows. First, we systematically link the satisfiability problem for the one-variable fragment of first-order modal logic with counting to modal description logics with non-rigid designators. This enables us to transfer both negative and positive results from logics with counting to logics with non-rigid designators. Second, we prove a promising NEXPTIME upper bound for concept satisfiability for the fundamental epistemic multi-agent logic,  $\mathbf{S5}^n$ , and various neighbours. Finally, we conduct a fine-grained analysis of the decidability of temporal logics with non-rigid designators.

## Keywords


Epistemic and temporal description logics, Definite descriptions, Non-rigid designators


## 1. Introduction

Definite descriptions and individual names that are not rigid across worlds or time points have been one of the main research topics in first-order modal and temporal logics [1, 2, 3, 4, 5, 6, 7, 8, 9]. Recently, also a modal description logic (DL) formalism admitting non-rigid definite descriptions has been introduced [10]. While for first-order modal logics with rigid designators and no counting the restriction to *monodic* formulas (in which modal operators are only applied to formulas with a single free variable) very often ensures decidability, this is no longer the case if non-rigid designators and/or some counting are admitted [11]. For modal DLs, this implies that the standard recipe for designing decidable languages – apply modal operators only to concepts – does not always work anymore. Here, we explore in detail when this recipe still works, and when it does not. This paper is an extended abstract of [12]; see [13] for full details and proofs.

Our first contribution closely links the two main sources of bad computational behaviour: non-rigid designators and counting. This enables us to use the results and machinery introduced for logics with counting [14, 15, 16]. We emphasise that the non-rigidity of symbols is *not*, by itself, the main source of difficulty. For instance, rigid roles are known to often cause an increase in the hardness of the satisfiability problem compared with the case of non-rigid roles only [11]. What makes non-rigid designators computationally harder is their ability to count in an unbounded way across worlds. On the other hand, we prove that, rather surprisingly, for some fundamental modal epistemic logics, non-rigid designators come for free: concept satisfiability for DLs based on  $\mathbf{K}^n$  and  $\mathbf{S5}^n$  is in NEXPTIME and thus not harder than without nominals at all. Finally, we show that undecidability is a relatively widespread phenomenon in the temporal setting: most combinations are undecidable (or even  $\Sigma_1^1$ -complete), and concept satisfiability is decidable only in fragments with the ‘next time’ operator, where we obtain an EXPTIME upper complexity bound, or in the expanding domain case with finite time, where the problem is actually Ackermann-hard.

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## 2. Preliminaries

The  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  language is a modalised extension of the free DL  $\mathcal{ALCO}_u$  [17].  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  terms and concepts are defined by the following grammar:

$$\tau ::= a \mid \iota C, \quad C ::= A \mid \{\tau\} \mid \neg C \mid (C \sqcap C) \mid \exists r.C \mid \exists u.C \mid \diamond_i C,$$

where  $a$  ranges over individual names,  $A$  over concept names,  $r$  over role names,  $\diamond_i$  over a finite set  $I = \{1, \dots, n\}$  of modalities, and  $u$  is the universal role. A term of the form  $\iota C$  is called a *definite description*. An  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  *concept inclusion (CI)* is an expression of the form  $C \sqsubseteq D$ , for concepts  $C, D$ . An  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  *ontology*  $\mathcal{O}$  is a finite set of CIs.

A *frame* is a pair  $\mathfrak{F} = (W, \{R_i\}_{i \in I})$ , where  $W$  is a non-empty set of *worlds* (or *states*) and each  $R_i \subseteq W \times W$ , for  $i \in I$ , is a binary *accessibility relation* on  $W$ . A *partial interpretation with expanding domains* based on  $\mathfrak{F}$  is a triple  $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$ , where  $\Delta$  is a function associating with every  $w \in W$  a non-empty set,  $\Delta^w$ , called the *domain of  $w$  in  $\mathfrak{M}$* , such that  $\Delta^v \subseteq \Delta^u$ , whenever  $vR_i u$ , for some  $i \in I$ ; and  $\mathcal{I}$  is a function associating with every  $w \in W$  a *partial DL interpretation*  $\mathcal{I}_w = (\Delta^w, \cdot^{\mathcal{I}_w})$  that maps every  $A$  to a subset  $A^{\mathcal{I}_w}$  of  $\Delta^w$ , every  $r$  to a subset  $r^{\mathcal{I}_w}$  of  $\Delta^w \times \Delta^w$ , and a subset of the individual names  $a$  to elements  $a^{\mathcal{I}_w}$  in  $\Delta^w$ . Hence, every  $\mathcal{I}_w$  is a total function on concept and role names, but a partial function on individual names. If  $\mathcal{I}_w$  is defined on  $a$ , then we say that  $a$  *designates at  $w$* . We say that  $\mathfrak{M}$  is a *total interpretation* if every  $a$  designates at every  $w \in W$ . Note that we do *not* assume that  $a^{\mathcal{I}_w} = a^{\mathcal{I}_v}$ , for  $w, v \in W$ , and thus do *not* make the rigid designator assumption (RDA). An *interpretation with constant domains* is such that  $\Delta^w = \Delta^v$ , for all  $w, v \in W$ .

We define the *value*  $\tau^{\mathcal{I}_w}$  of a term  $\tau$  at  $w \in W$  as  $a^{\mathcal{I}_w}$ , for  $\tau = a$ , and as follows, for  $\tau = \iota C$ :

$$(\iota C)^{\mathcal{I}_w} = \begin{cases} d, & \text{if } C^{\mathcal{I}_w} = \{d\}, \text{ for some } d \in \Delta^w; \\ \text{undefined,} & \text{otherwise.} \end{cases}$$

A term  $\tau$  is said to *designate at  $w$*  if  $\tau^{\mathcal{I}_w} = d$ , for some  $d \in \Delta^w$ . The *extension*  $C^{\mathcal{I}_w}$  of a concept  $C$  in  $w \in W$  is defined as usual, with the following variant:

$$\{\tau\}^{\mathcal{I}_w} = \begin{cases} \{\tau^{\mathcal{I}_w}\}, & \text{if } \tau \text{ designates at } w, \\ \emptyset, & \text{otherwise.} \end{cases}$$

A concept  $C$  is *satisfied at  $w \in W$  in  $\mathfrak{M}$*  if  $C^{\mathcal{I}_w} \neq \emptyset$ ;  $C$  is *satisfied in  $\mathfrak{M}$*  if it is satisfied at some  $w \in W$  in  $\mathfrak{M}$ . A CI  $C \sqsubseteq D$  is *satisfied in  $\mathfrak{M}$*  if  $C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}$ , for every  $w \in W$ . An ontology  $\mathcal{O}$  is *satisfied in  $\mathfrak{M}$*  if every CI in  $\mathcal{O}$  is satisfied in  $\mathfrak{M}$ ; we also say a concept  $C$  is *satisfied in  $\mathfrak{M}$  under an ontology  $\mathcal{O}$*  if  $\mathfrak{M} \models \mathcal{O}$  and  $C^{\mathcal{I}_w} \neq \emptyset$ , for some  $w \in W$ .

Let  $\mathcal{C}$  be a class of frames (e.g., with equivalence relations for  $\mathbf{S5}^n$ ). We consider the following two main reasoning problems.

**Concept  $\mathcal{C}$ -Satisfiability:** Given an  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$ -concept  $C$ , is there an interpretation  $\mathfrak{M}$  based on a frame from  $\mathcal{C}$  such that  $C$  is satisfied in  $\mathfrak{M}$ ?

**Concept  $\mathcal{C}$ -Satisfiability under Global Ontology:** Given an  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$ -concept  $C$  and an  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$ -ontology  $\mathcal{O}$ , is there an interpretation  $\mathfrak{M}$  based on a frame from  $\mathcal{C}$  such that  $C$  is satisfied in  $\mathfrak{M}$  under  $\mathcal{O}$ ?

We begin with some simple observations on the reductions between the satisfiability problems for different semantic conditions and languages.

**Proposition 1.** *In  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$ , concept  $\mathcal{C}$ -satisfiability (under global ontology) in total interpretations is polytime-reducible to concept  $\mathcal{C}$ -satisfiability (under global ontology, respectively) in partial interpretations, and the other way round. The reductions work both with constant and with expanding domains.*

**Proposition 2.** *Concept  $\mathcal{C}$ -satisfiability (under global ontology) in  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  is polytime-reducible to concept  $\mathcal{C}$ -satisfiability (under global ontology, respectively) in  $\mathcal{ML}_{\mathcal{ALCO}_u}^n$  (i.e., the fragment without  $\iota$ ), both with constant and with expanding domains.*

**Table 1**  
Concept satisfiability (under global ontology) for  $L_{\mathcal{ALCCO}_u^t}$

modal logic $L$	concept satisfiability		concept sat. under global ontology	
	const. domain	expanding domains	const. domain	expanding domains
$\mathbf{K}^n, n \geq 1$	NEXP-complete	NEXP-complete	undecidable	?
$\mathbf{S5}$		NEXP-complete		NEXP-complete
$\mathbf{S5}^n, n \geq 2$		NEXP-complete		undecidable
$\mathbf{K}^{*n}, n \geq 1$	$\Sigma_1^1$ -complete	undecidable	$\Sigma_1^1$ -complete	undecidable
$\mathbf{Kf}^{*n}, n \geq 1$	undecidable	decidable, Ackermann-hard	undecidable	decidable, Ackermann-hard

### 3. Main Results

**Non-Rigid Designators and Counting** We prove a strong link between non-rigid designators and the first-order one-variable modal logic enriched with the ‘elsewhere’ quantifier,  $\mathcal{ML}_{\text{Diff}}^n$  [14, 15, 16], which can be introduced using DL-style syntax:

$$C ::= A \mid \neg C \mid (C \sqcap C) \mid \exists u.C \mid \exists^{\neq} u.C \mid \diamond_i C,$$

where  $i \in I$ . Note that the language has no terms and no roles apart from the universal role  $u$ . All constructs are interpreted as before and  $(\exists^{\neq} u.C)^{\mathcal{I}_w} = \{d \in \Delta^w \mid C^{\mathcal{I}_w} \setminus \{d\} \neq \emptyset\}$ . Observe that  $\mathcal{ML}_{\text{Diff}}^n$  can be regarded as a basic first-order modal logic with counting because the counting quantifier  $\exists^=1 u.C$ , with  $(\exists^=1 u.C)^{\mathcal{I}_w} = \{d \in \Delta^w \mid |C^{\mathcal{I}_w}| = 1\}$ , is equivalent to  $\exists u.(C \sqcap \neg \exists^{\neq} u.C)$  and, conversely,  $\exists^{\neq} u.C$  is equivalent to  $\exists u.C \sqcap (C \Rightarrow \neg \exists^=1 u.C)$ .

**Theorem 3.**  *$\mathcal{C}$ -satisfiability of  $\mathcal{ML}_{\mathcal{ALCCO}_u^t}^n$ -concepts (under global ontology) can be reduced in double exponential time to  $\mathcal{C}$ -satisfiability of  $\mathcal{ML}_{\text{Diff}}^n$ -concepts (under global ontology, respectively), both with constant and with expanding domains.*

*Conversely,  $\mathcal{C}$ -satisfiability of  $\mathcal{ML}_{\text{Diff}}^n$ -concepts (under global ontology) is polytime-reducible to  $\mathcal{C}$ -satisfiability of  $\mathcal{ML}_{\mathcal{ALCCO}_u^t}^n$ -concepts (under global ontology, respectively), both with constant and with expanding domains.*

**Reasoning in Modal Free Description Logics** Given a propositional modal logic  $L$  with  $n$  operators and the class  $\mathcal{C}_L$  of frames validating  $L$ , we define  $L_{\mathcal{ALCCO}_u^t}$  concept satisfiability (under global ontology) as the problem of deciding  $\mathcal{C}_L$ -satisfiability of  $\mathcal{ML}_{\mathcal{ALCCO}_u^t}^n$ -concepts (under global ontology, respectively). For  $L = \mathbf{K}^n$ ,  $\mathcal{C}_L$  is the class of all frames with  $n$  relations; for  $L = \mathbf{S5}^n$ ,  $\mathcal{C}_L$  is the class of frames with  $n$  equivalence relations; for  $\mathbf{K}^{*n}$ ,  $\mathcal{C}_L$  is the class of all frames  $(W, R_1, \dots, R_n, R)$  such that  $R$  is the transitive closure of  $R_1 \cup \dots \cup R_n$ ; and for  $\mathbf{Kf}^{*n}$ ,  $\mathcal{C}_L$  is as for  $\mathbf{K}^{*n}$ , with in addition  $W$  finite and  $R$  irreflexive. (i.e., there is no chain  $w_0 R_{i_1} w_1 \dots R_{i_n} w_n$  with  $w_0 = w_n$ ). We drop superscript 1 from  $L^1$ .

Table 1 presents our main results for modal logics relevant in the epistemic context. The NEXPTIME membership for concept satisfiability in  $\mathbf{K}_{\mathcal{ALCCO}_u^t}^n$  and  $\mathbf{S5}_{\mathcal{ALCCO}_u^t}^n$  is shown using the quasimodel technique: we prove that a concept is satisfiable iff there is a quasimodel of exponential size, which gives us the exponential finite model property and an exponential-time non-deterministic algorithm for concept satisfiability. Decidability of concept satisfiability under global ontology in  $\mathbf{Kf}_{\mathcal{ALCCO}_u^t}^{*n}$  is also shown with quasimodels. In this case, we count the number of times a type occurs in a world and represent quasistates as vectors with elements in  $\mathbb{N} \cup \{\infty\}$ . Then Dickson’s Lemma is used to obtain a computable bound on the size of a satisfying interpretation. Note, however, that Ackermann-hardness, which follows by a non-trivial reduction from a result on  $\mathcal{ML}_{\text{Diff}}^n$  [15], shows that the interpretation size is not bounded by a primitive recursive function.

Many challenging open problems remain, in particular, decidability of  $\mathbf{K}^n$  under global ontology and expanding domains, as well as decidability of logics of transitive frames, e.g.,  $\mathbf{K4}$ . As a first step, we show decidability with expanding domains for the Gödel-Löb provability logic  $\mathbf{GL}$  (whose transitive and irreflexive frames have no infinite ascending chains) and its reflexive companion Grzegorzcyk ( $\mathbf{Grz}$ ), using reductions to expanding-domains products [18].

**Table 2**  
Concept satisfiability (under global ontology) for  $L_{\mathcal{ALCCO}_u^t}$

temporal logic $L$	concept satisfiability		concept sat. under global ontology	
	const. domain	expanding domains	const. domain	expanding domains
$\mathbf{LTL}^\diamond, \mathbf{LTL}$	$\Sigma_1^1$ -complete	undecidable	$\Sigma_1^1$ -complete	undecidable
$\mathbf{LTLf}^\diamond, \mathbf{LTLf}$	undecidable	decidable, Ackermann-hard	undecidable	decidable, Ackermann-hard
$\mathbf{LTL}^\circ$	Exp-complete	Exp-complete	undecidable	?
$\mathbf{LTLf}^\circ$	Exp-complete	Exp-complete	undecidable	decidable

**Reasoning in Temporal Free Description Logics** For the temporal DL language  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}$ , we build  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}$  terms, concepts, concept inclusions and ontologies as in the  $\mathcal{ML}_{\mathcal{ALCCO}_u^t}^n$  case, with  $n = 2$ : the language has two modalities – temporal operators ‘sometime in the future’,  $\diamond$ , and ‘at the next moment’,  $\circ$ . In particular, the  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}$  concepts are defined by the following grammar:

$$C ::= A \mid \{\tau\} \mid \neg C \mid (C \sqcap C) \mid \exists r.C \mid \exists u.C \mid \diamond C \mid \circ C.$$

A *flow of time*  $\mathfrak{F}$  is a pair  $(T, <)$ , where  $T$  is either the set  $\mathbb{N}$  of non-negative integers or a subset of  $\mathbb{N}$  of the form  $[0, n]$ , for  $n \in \mathbb{N}$ , and  $<$  is the strict linear order on  $T$ , which naturally gives rise to interpretations  $\mathfrak{M}$  based on corresponding frames. Given  $\mathfrak{M}$ , the *value* of a term  $\tau$  at  $t \in T$  and the *extension* of a concept  $C$  at  $t \in T$  are defined as in the modal case for  $n = 2$ : for example,  $(\circ D)^{\mathfrak{I}_t} = \{d \in \Delta^t \mid t+1 \in T \text{ and } d \in D^{\mathfrak{I}_{t+1}}\}$ . In particular, the extension of  $\circ D$  is empty in the last instant of a finite flow of time. Note that  $\diamond$  is interpreted by  $<$  and thus does not include the current instant, but we can easily define  $\diamond^+ C = \diamond C \sqcup C$ , which includes the current time instant. We also use standard abbreviations such as  $\square C$  (‘always in the future’) and  $\square^+ C$  (‘from now on’).

Fragments  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}^\diamond$  and  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}^\circ$  are obtained from  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}$  by disallowing  $\circ$  and  $\diamond$  operators, respectively; they correspond to  $\mathcal{ML}_{\mathcal{ALCCO}_u^t}^1$ , but with different accessibility relations. We refer, for instance, to the satisfiability problem for  $\mathcal{TL}_{\mathcal{ALCCO}_u^t}^\diamond$  concepts in interpretations with finite flows of time ( $f$ ) as  $\mathbf{LTLf}_{\mathcal{ALCCO}_u^t}^\diamond$  concept satisfiability.

Table 2 summarises our results. Concept satisfiability under global ontology for languages with the  $\diamond$  operator is undecidable over  $(\mathbb{N}, <)$  in both constant and expanding domains and over finite flows of time in constant domains. Positive results, however, can be obtained by combining finite flows of time with expanding domains, or by restricting to concept satisfiability in fragments with only the  $\circ$  operator. An interesting open problem is decidability of  $\mathbf{LTL}_{\mathcal{ALCCO}_u^t}^\circ$  concepts under global ontology in expanding domains.

## 4. Discussion and Future Work

In this work, we have made first steps towards understanding the computational behaviour of non-rigid designators and definite descriptions in epistemic and temporal DLs. Potential applications include business process management where formalisms for representing the dynamic behaviour of data and information are crucial [19, 20, 21] and context, knowledge, or standpoint dependent reasoning for which possible worlds semantics is needed [22, 23]. Future research directions include the extension of our results to more expressive monodic fragments [11, 24], automated support for the construction of definite descriptions and referring expressions [17, 25], the design of ‘practical’ reasoning algorithms for the languages considered here, and the extension of our results to non-normal modal DLs [26].

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