The Perceptron and the Tooth

A New Family of Logics for Cognitive Modelling

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Abstract

We present the key motivations and technical results of a family of weighted descriptions logics, called tooth (or perceptron) logics, which expand on ideas from the paradigm of prototype theory, and which were conceived and designed to provide a cognitively more adequate formal modelling of concepts and concept combinations.

Keywords

Prototype Theory, Cognitive Modelling, Concept Combination, Concept Learning, Tooth Logic, Perceptron

The notion of ‘concept’ has been an elusive one both in cognitive science and psychology as well as in the formal sciences, particularly within logic and knowledge representation. From the cognitive-psychological perspective, a number of empirical phenomena and theoretical perspectives have led to a range of approaches to the definition and modelling of concepts, including most prominently exemplar theory, prototype theory, and theory theory (see [1] for a survey and detailed references).

In parallel, the engineering of formal logical approaches to model adequately some of the key phenomena in human reasoning with concepts still remains one of the central challenges in AI, and indeed receives renewed urgency in the era of deep learning with issues such as biases and hallucinations.

We here sketch the motivation, core theoretical development, and applications, of a family of logics, ‘Tooth Logics’, derived from the core idea of marrying the notion of a prototype description with the notion of the perceptron as a threshold operator.

In classical logic, the meaning of a concept is normally defined by giving strict necessary and sufficient conditions for concept membership. In contrast, the core idea of approaches in the prototype-theoretic paradigm, roughly speaking, is to consider selected relevant features and their relative importance and to devise ways of accumulating ‘evidence’ to determine concept membership.

Based on this idea, Masolo and Porello [2] proposed a first logical treatment of this idea, based on first-order logic, proposing a simple aggregation function summing up weights of features that are satisfied by an individual in a specific situation (model). Once a certain numerical threshold is reached, concept membership is answered affirmatively. The similarities to the basic perceptron model are evident [3, 4].

In subsequent work, it was quickly realised that the idea of a threshold operation can be turned into a logical sentence forming operator in the sense of modal logic. This kind of operator can in fact be build into different kinds of logics. When added to standard description logic $\mathcal{ALC}$, the resulting grammar, adding only the ‘basic tooth’ $\nabla\!\!\!\!\nabla$, looks as follows:

$$C ::= A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \exists R.C \mid \forall R.C \mid \nabla\!\!\!\!\nabla^t\big(w_1 : C_1, \ldots, w_m : C_m\big)$$

where $\nabla\!\!\!\!\nabla^t\big(w_1 : C_1, \ldots, w_m : C_m\big)$ forms a new concept which, in a fixed interpretation I, contains exactly those individuals d who satisfy ‘enough’ of the concepts C_i such that their accumulative weight reaches the threshold value t, formally:

$$d \in \big(\nabla\!\!\!\!\nabla^t\big(w_1 : C_1, \ldots, w_m : C_m\big)\big)^I \iff \sum_{i \in \{1,\ldots,m\}} \{w_i \mid d \in C_i^I\} \geq t$$

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This basic shift of formalisation opens up a surprisingly rich landscape of theoretical and practical developments, and new considerations come into play at the friction point of modelling common sense, defeasible reasoning, and aspects of computational logics such as DLs.

The resulting perceptron logic family, often called tooth logic, has numerous applications, including in concept learning, modelling exceptions, modelling psychological phenomena in the use of concepts, and modelling some of the mechanics of conceptual blending / combination, as well as in concept abstraction. We briefly summarise the main contributions. The first detailed discussion of threshold operators, their formal definition and basic algebraic and logical properties, can be found in [5]. In [6], a distinction is made between knowledge-dependent and knowledge-independent tooth expressions, where the core idea of knowledge-dependence is that the features whose weights may be accumulated must provably hold for an individual according to a knowledge base, rather than accidentally in a model. In [7], summarised also in [8], the application of tooth expressions to modelling issues with concepts are studied, particularly those stemming from psychological studies, such as the effects of over- or under-extension studied by James Hampton [9, 10]. For example, an over-extension of a conjunctive concept $A \sqcap B$ means that we consider more things to enter the conjunctive concept than would be expected when taking a set-theoretic intersection of the extensions of the concepts. For instance, something might be considered ‘a sport that is a game’ without being considered to be a ‘game’ proper. This work is further refined in [11], where the combination problem is studied in more detail algorithmically by employing a distinction between logically impossible and necessary features, a topic further explored in [12]. The paper [13] contains two important contributions. It illustrates on the one hand the applicability of the tooth logic approach to the problem of concept learning from data, and on the other hand it shows that reasoning can be done with off-the-shelf tools by providing an encoding into standard description logics without increasing the computational complexity.

The interpretability of threshold expressions was studied in [14], providing some evidence that tooth operators are easier to understand by people without formal logic education than equivalent disjunctive normal forms. Regarding further application areas, the core idea of using an accumulation of weighted features was explored in a novel logical approach to exceptions in [15], where it is exploited to provide a knowledge-dependent definition for the idea that a certain individual d is more an A than it is a B, i.e. providing a notion of conceptual distance that depends on a given knowledge base. The philosophical and conceptual foundations of this approach are further discussed in [16].

Finally, in the most recent extension of the basic tooth logic, [17] studies the complexity of reasoning with tooth expressions that include counting perceptrons and illustrates this extension with some modelling examples. Each individual instance of a role successor in DL is considered, and their weights are accumulated. For instance, whilst the original tooth logic might give a certain weight to the concept of ‘having a child’, it does not properly extend the expressivity of $\mathcal{ALC}$. The counting tooth, on the other hand, can express the concept of ‘having as many daughters as sons’, which indeed extends the expressivity of standard DLs.

Future work is foreseen along the dimensions sketched above. However, we see particular potential in applications of tooth logics to the concept learning problem, and in providing a formal and conceptual bridge between the statistical world of learning approaches and the world of computational logics.

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