A General Dialogue Framework for Logic-based Argumentation

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Abstract

There is an extensive body of work in logic-based argumentation, which links logic and argumentation, which is a potential solution to address inconsistencies or conflicting information in knowledge bases (KBs) by offering dialogue games as proof procedures to determine and explain the acceptance of propositions. Most existing work, though, focuses on specific logics (such as description logics, existential rules, defeasible and propositional logics), has limitations of representational aspects, for selected semantics and binary conflicts. In this paper, we generalise this work by introducing G-SAF, which generalises the notions of arguments, dialogues and dialogue trees for more general logical reasoning with inconsistencies, including the most common semantics and to facilitate reasoning with non-binary conflicts using argumentation with collective attacks (SAFs).

Keywords

Argumentation, Inconsistency-tolerant reasoning, Explanation

1. Introduction

Argumentation is a potential solution for inconsistencies or conflicting information in knowledge bases; it offers dialogue games as proof procedures to determine and explain the acceptance of propositions. To benefit from argumentation in explaining and querying answers in (possibly inconsistent) KBs, *Logic-based argumentation frameworks* (LAFs) have emerged, linking logic with abstract argumentation frameworks (AFs), providing argument game-based proofs.

Several different approaches to logical argumentation have been introduced in the literature. Most previous LAFs frameworks link Dung's AFs [1] to specific logics. For instance, these works present an instantiation of AFs for classical logic [2], Description Logic [3] and for existential rules (Datalog$^{\pm}$) [4, 5]. In [6, 7], LAFs with collective attacks are proposed to capture non-binary conflicts in Datalog$^{\pm}$, i.e., assuming that every conflict has more than two formulas. In [8, 9], ASPIC/ASPIC+ is introduced for defeasible logic. Following [10], the logical formalism in ASPIC+ is ill-defined, i.e., the contrariness relation is not general enough to consider n-ary constraints. This issue is stated in [6] for Datalog$^{\pm}$.

[11, 12] proposes *sequence-based argumentation* for (propositional) logic and provides non-monotonic extensions for Gentzen-style proof systems in terms of argumentation-based. Moreover, the authors conclude [12] by wishing future work to include "the study of more expressive formalisms, like those that are based on first-order logics". In [13, 14] DeLP/DeLP with collective

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attacks are introduced for defeasible logic programming. However, [6] claims they cannot instantiate DeLP for Datalog$^{\pm}$, since it only considers ground rules. In [15, 16] assumption-based argumentation (ABA)/ ABA with collective attacks for logic programming encodes a single conflict/multi-conflicts for each assumption, while in [17], ABA is linked to Answer Set Programming. In [15, 16], the focus is on *flat* ABA frameworks, i.e., it ignores the cases of the inferred assumption being conflicting, which is allowed in the existential rules or other logics. From the observation, earlier works focus on specific logics or have limitations of representational aspects. This paper, using abstract logic with no requirement on the language $\mathcal{L}$ and CN (a function from $2^{\mathcal{L}}$ to $2^{\mathcal{L}}$), provides a very flexible environment for logical argumentation. Like our approach, the work of [18] proposed "non-flat" ABA frameworks with collective attacks. While the work of [18] mainly focuses on representation considerations, our work further studies proof procedures used in AFs with collective attacks.

To compute and explain answers w.r.t the semantics of logical reasoning, argumentation provides dialogue models as proof procedures, but, there is no unifying approach for the most common inconsistency-tolerant semantics. In the Datalog$^{\pm}$ context, the framework of [19] focuses on ICR semantics (i.e., the query being entailed under the intersection of closure of repairs).The dialogue model of [20, 4] considers queries entailed under IAR semantics (the intersection of all repairs) or Brave semantics (some repairs [1]), while AR semantics have not yet been considered. This paper gives possible translations for the most common types of semantics (as defined in Definition 2).

Previous proposed dialogue models are used in LAFs with binary attacks, which are not generic enough to deal with collective attacks. In [21, 20, 4] the authors propose a dialogue model as an abstract dialectic proof procedure and only involving arguments and binary attacks. The models in [22, 23], applied to ABA, are limited to express the contrariness relation for existential rules with non-binary conflicts. The models in [24, 25] related domain expert and a system, but are only applied to a specific domain (agronomy).

Main contributions. The existing approaches are mostly restricted to: (1) specific logics or limitations of representational aspects; (2) specific inconsistency-tolerant semantics; (3) (dialectical) proof procedures used in AFs with binary attacks. This paper closes this gap: (1) we propose a unifying framework, G-LAF, that allows a translation of KBs in a broad family of logics into argumentation frameworks; (2) our unified approach includes inconsistency-tolerant semantics considered in previous literature (e.g. IAR and Brave in the case of Datalog$^{\pm}$) as well as new ones (AR in the case of Datalog$^{\pm}$); (3) we propose an extended version of the dialogue model that applies to AFs/LAFs with collective attacks, generalizing the dialogue model used in AFs/LAFs with binary attacks.

We only sketch the proofs for our main results in this paper, more details can be found in the appendix.

2. Preliminaries

Most of our discussion applies to arbitrary logics (monotonic and non-monotonic) consisting of a set $\mathcal{L}$ (of *sentences*) and a function CN from $2^{\mathcal{L}}$ to $2^{\mathcal{L}}$ returning the logical consequences of a

[1]The notion of *repair* corresponds to "*maximal consistent subsets*" in our work

set of sentences (the *consequence operator*), that satisfies the axiom: **Expansion** $X \subseteq \mathrm{CN}(X)$, **Idempotence** $\mathrm{CN}(\mathrm{CN}(X)) = \mathrm{CN}(X)$. Fix a logic $(\mathcal{L}, \mathrm{CN})$ and a set of sentences $X \subseteq \mathcal{L}$. We say that:

- X is *consistent* wrt $(\mathcal{L}, \mathrm{CN})$ iff $\mathrm{CN}(X) \neq \mathcal{L}$. It is *inconsistent* otherwise;
- A *knowledge base* (KB) is any subset $\mathcal{K}$ of $\mathcal{L}$. A knowledge base may be inconsistent.

Reasoning in inconsistent KBs $\mathcal{K} \subseteq \mathcal{L}$ amounts to:

1. Constructing *maximal consistent subsets*,
2. Applying *classical entailment mechanism* on a choice of the maximal consistent subsets.

Motivated by this idea, we give the following definition.

Definition 1. *Let $\mathcal{K}$ be a KB and $X \subseteq \mathcal{K}$ be a set of sentences. X is a* maximal (for set-inclusion) consistent subset *of $\mathcal{K}$ iff X is consistent, there is no X' such that $X \subset X'$ and X' is consistent. We denote the set of all maximal consistent subsets by* $\mathtt{MCS}(\mathcal{K})$.

Inconsistency-tolerant reasoning in KB has semantics to determine different types of answers.

Definition 2. *Let $\mathcal{K}$ be a KB and* $\vdash: 2^{\mathtt{MCS}(\mathcal{K})} \mapsto \mathtt{MCS}(\mathcal{K})$ *be an entailment mechanism. A sentence $f \in \mathcal{L}$ is entailed in*

- all maximal consistent subsets, *written* $\mathcal{K} \vdash_{\bigcup \mathtt{MCS}(\mathcal{K})} f$, *iff for all* $\Delta \in \mathtt{MCS}(\mathcal{K})$, $f \in \mathrm{CN}(\Delta)$;
- some maximal consistent subset, *written* $\mathcal{K} \vdash_{\mathtt{MCS}(\mathcal{K})} f$, *iff for some* $\Delta \in \mathtt{MCS}(\mathcal{K})$, $f \in \mathrm{CN}(\Delta)$;
- *the* intersection of all maximal consistent subsets, *written* $\mathcal{K} \vdash_{\bigcap \mathtt{MCS}(\mathcal{K})} f$, *iff for* $\Psi = \bigcap\{\Delta \mid \Delta \in \mathtt{MCS}(\mathcal{K})\}$, $f \in \mathrm{CN}(\Psi)$.

3. A General Framework of Explanatory Dialogues

In this section, we introduce: (1) A **general logic-based argumentation with collective attacks** (G-SAF) translated from a KB. (2) **General formal model of dialogue** (for G-SAF) considered as a *dialectical proof procedure* to determine and explain reasoning outcomes in KBs. This dialogue model is inspired by previous works [26, 20, 21].

3.1. General Logic-based Argumentation

Arguments are built from a KB. They represent *proofs* for conclusions. Thus, an argument has two parts: a support (also called a set of premisses) and a conclusion.

Definition 3 (G-SAF arguments). *An* argument *induced from a KB $\mathcal{K}$ is the pair $A = (\Gamma, f)$ such that $\Gamma \subseteq \mathcal{K}$ and $f \in \mathrm{CN}(\Gamma)$. For such argument, we denote by* $\mathtt{Sup}(A) = \Gamma$ *the* support set *of A and by* $\mathtt{Con}(A) = f$ *the* consequence *of A. We use* $\mathtt{Arg}_{\mathcal{K}}$ *to denote the* set of arguments *constructed from $\mathcal{K}$. Fix* $\mathcal{S} \subseteq \mathtt{Arg}_{\mathcal{K}}$, $\mathtt{Cons}(\mathcal{S}) = \bigcup_{A \in \mathcal{S}} \{\mathtt{Con}(A)\}$ *are the conclusions in $\mathcal{S}$.*

Some proposals for logic-based argumentation stipulate additionally that the argument's support is consistent and/or that none of its subsets entails the argument's conclusion (see [27]). However, such restrictions are not be substantial (although required for some specific logics). To keep our framework as general as possible, we do not consider the extra restrictions for our definition of arguments. We refer to [27, 11] for further justifications of this choice.

Different attack relations have been considered in the literature for logic-based argumentation. However, some definitions of attack are not suitable to capture non-binary conflicts [8, 13, 15, 3, 17]. The attack definitions in [4, 28, 11] can capture non-binary conflicts, and these frameworks can generate a large number (See [7, 6] for justifications in the case of Datalog$^{\pm}$). To overcome this, we use the notion of *collective attacks.*

Definition 4 (Collective Attacks). *Let $A = (\Gamma, \alpha)$ be an argument and $\mathcal{X} \subseteq \texttt{Arg}_{\mathcal{K}}$ be a set of arguments.*

- $\mathcal{X}$ undercut-attacks A *iff there is* $\Gamma' \subseteq \Gamma$ *s.t* $\bigcup_{X \in \mathcal{X}} \{\texttt{Con}(X)\} \cup \Gamma'$ *is inconsistent.*
- $\mathcal{X}$ rebuttal-attacks A *iff* $\bigcup_{X \in \mathcal{X}} \{\texttt{Con}(X)\} \cup \{\alpha\}$ *is inconsistent.*

We can say that $\mathcal{X}$ attacks A *for short. We use* $\texttt{Att}_{\mathcal{K}} \subseteq 2^{\texttt{Arg}_{\mathcal{K}}} \times \texttt{Arg}_{\mathcal{K}}$ *to denote* the set of attacks *induced from* $\mathcal{K}$.

For KBs with only binary conflicts (e.g. core DL-Lite dialects), we have that $|\mathcal{X}| = 1$. We refer this case to (Dung) AFs where they consider an attack between two arguments (i.e. a binary attack).

Definition 5. *Let $\mathcal{K}$ be a KB, the corresponding* general logic-based argumentation (G-SAF) $\mathcal{AF}_{\mathcal{K}}$ *is the pair* $(\texttt{Arg}_{\mathcal{K}}, \texttt{Att}_{\mathcal{K}})$, *where* $\texttt{Arg}_{\mathcal{K}}$ *is the set of arguments induced from* $\mathcal{K}$ *and* $\texttt{Att}_{\mathcal{K}}$ *is the set of attacks.*

Its semantics are now defined as in the definition of argumentation semantics for SAFs [29].

Given a G-SAF $\mathcal{AF}_{\mathcal{K}} = (\texttt{Arg}_{\mathcal{K}}, \texttt{Att}_{\mathcal{K}})$ and $\mathcal{S} \subseteq \texttt{Arg}_{\mathcal{K}}$. $\mathcal{S}$ *attacks* $\mathcal{X}$ iff $\exists A \in \mathcal{X}$ s.t. $\mathcal{S}$ attacks A. $\mathcal{S}$ *defends* A if for each $\mathcal{X} \subseteq \texttt{Arg}_{\mathcal{K}}$ s.t. $\mathcal{X}$ attacks A, some $\mathcal{S}' \subseteq \mathcal{S}$ attacks $\mathcal{X}$. An *extension* $\mathcal{S}$ is called

- *conflict-free* if it does not attack itself.
- *admissible* $(\texttt{ad})$ if it is conflict-free and defends itself.
- *complete* $(\texttt{cm})$ if it is an admissible extension containing all arguments that it defends.
- *preferred* $(\texttt{pr})$ if it is an inclusion-maximal admissible extension.
- *stable* $(\texttt{st})$ if it is conflict-free and attacks every argument which is not in it.
- *grounded* $(\texttt{gr})$ if it is an inclusion-minimal complete extension.

Let $\texttt{Ext}_{\texttt{s}}(\mathcal{AF}_{\mathcal{K}})$ denote *the set of all extensions* of $\mathcal{AF}_{\mathcal{K}}$ under the semantics $\texttt{s} \in \{\texttt{ad}, \texttt{cm}, \texttt{st}, \texttt{pr}, \texttt{gr}\}$. Let us define *acceptability* in our G-SAF:

Definition 6. *Let $\mathcal{AF}_{\mathcal{K}}$ be the corresponding G-SAF of a KB $\mathcal{K}$ and $\texttt{s} \in \{\texttt{ad}, \texttt{st}, \texttt{pr}\}$. A sentence $f \in \mathcal{L}$ is*

- credulously accepted *under* $\mathtt{s}$ *iff for some* $\mathcal{E} \in \mathtt{Ext}_{\mathtt{s}}(\mathcal{AF}_{\mathcal{K}})$, $f \in \mathtt{Cons}(\mathcal{E})$.
- sceptically accepted *under* $\mathtt{s}$ *iff for all* $\mathcal{E} \in \mathtt{Ext}_{\mathtt{s}}(\mathcal{AF}_{\mathcal{K}})$, $f \in \mathtt{Cons}(\mathcal{E})$.
- groundedly accepted *under* $\mathtt{gr}$ *iff for some* $\mathcal{E} \in \mathtt{Ext}_{\mathtt{gr}}(\mathcal{AF}_{\mathcal{K}})$, $f \in \mathtt{Cons}(\mathcal{E})$.

Proposition 1 shows a relation between extensions of G-SAFs and maximal consistent subsets of KBs.

Proposition 1. *Let $\mathcal{AF}_{\mathcal{K}}$ be the corresponding G-SAF of a KB $\mathcal{K}$. Then,*

- *the maximal consistent subset of $\mathcal{K}$ coincides with the stable/ preferred extension of $\mathcal{AF}_{\mathcal{K}}$;*
- *the intersection of the maximal consistent subsets of $\mathcal{K}$ coincides with the grounded extension of $\mathcal{AF}_{\mathcal{K}}$.*

Proof 1. *The idea of the proof is to show that every preferred extension is the set of arguments generated from a MCS, that every such set of arguments is a stable extension, and that every stable extension is a preferred extension.*

The following is the main result of this section, which generalises related results of previous works.

Theorem 1. *Let $\mathcal{AF}_{\mathcal{K}}$ be the corresponding G-SAF of a KB $\mathcal{K}$, $f \in \mathcal{L}$ a sentence and $\mathtt{s} \in \{\mathtt{st}, \mathtt{ad}, \mathtt{pr}\}$. Then,*

- $\mathcal{K} \vdash_{\mathtt{MCS}(\mathcal{K})} f$ *iff f is* credulously accepted *under* $\mathtt{s}$.
- $\mathcal{K} \vdash_{\bigcup \mathtt{MCS}(\mathcal{K})} f$ *iff f is* sceptically accepted *under* $\mathtt{s}$.
- $\mathcal{K} \vdash_{\bigcap \mathtt{MCS}(\mathcal{K})} f$ *iff f is* groundedly accepted *under* $\mathtt{gr}$.

The proof of Theorem 1 follows Proposition 1.

To argue the quality of G-SAF, it can be shown that it satisfies the rationality postulates introduced in [2, 30].

Definition 7. *Let $\mathcal{AF}_{\mathcal{K}}$ be the corresponding G-SAF of a KB $\mathcal{K}$. Wrt.* $\mathtt{s} \in \{\mathtt{ad}, \mathtt{st}, \mathtt{pr}, \mathtt{gr}\}$, *$\mathcal{AF}_{\mathcal{K}}$ is*

1. closed under CN *iff for all* $\mathcal{E} \in \mathtt{Ext}_{\mathtt{s}}(\mathcal{AF}_{\mathcal{K}})$, $\mathtt{Cons}(\mathcal{E}) = \mathrm{CN}(\mathtt{Cons}(\mathcal{E}))$*;*
2. consistent *iff for all* $\mathcal{E} \in \mathtt{Ext}_{\mathtt{s}}(\mathcal{AF}_{\mathcal{K}})$, $\mathtt{Cons}(\mathcal{E})$ *is consistent.*

Proposition 2. *Wrt. to any semantics in $\{\mathtt{ad}, \mathtt{st}, \mathtt{pr}, \mathtt{gr}\}$, $\mathcal{AF}_{\mathcal{K}}$ satisfies consistency, closure.*

To further illustrate the generality of our framework for monotonic and nonmonotonic logics, let us consider the following example.

Example 1. *Let A be a set of propositional atoms. Any atoms $a \in A$ is a well-formed formula wrt. A. If ϕ and α are well-formed formulas wrt. A then $\neg\phi$, $\phi \wedge \alpha$, $\phi \vee \alpha$ are well-formulas wrt. A (we also assume that the usual abbreviations $\rightarrow$, $\leftrightarrow$ are defined accordingly). Then $\mathcal{L}_p$ is the set of well-formed formulas wrt. A. Let $\models$ be the entailment relation, i.e., $\phi \models \alpha$ if all models of ϕ are* *models of* α *in the propositional semantics. A consequence operator* $\text{CN}_p : 2^{\mathcal{L}_p} \to 2^{\mathcal{L}_p}$ *is defined for each* $X \subseteq \mathcal{L}_p$ *by* $\text{CN}_p(X) = \{\alpha \in \mathcal{L}_p \mid X \models \alpha\}$*. The propositional logic can be defined as* $(\mathcal{L}_p, \text{CN}_p)$*.*

Consider the propositional atoms $A_1 = \{x, y\}$ *and the knowledge base* $\mathcal{K}' = \{x, y, x \to \neg y\} \subseteq \mathcal{L}_p$*. The following set of arguments:*
$C_1 = (\{x\}, x)$*;* $C_2 = (\{y\}, y)$*;* $C_3 = (\{x \to \neg y\}, x \to \neg y)$*;*
$C_4 = (\{x, x \to \neg y\}, \neg y)$*;* $C_5 = (\{y, x \to \neg y\}, \neg x)$*;* $C_6 = (\{x, y\}, x \wedge y)$*.*
The attack relations:$\{(C_1, C_5), (C_5, C_1), (C_2, C_4), (C_4, C_2), (C_3, C_6), (C_6, C_3)\}$*.*
The preferred extensions: $\{C_1, C_2, C3\}$, $\{C_4, C_5, C_6\}$*.*

Example 2. *Let* $(\mathcal{L}_d, \text{CN}_d)$ *be a defeasible logic such as used in defeasible logic programming [13], assumption-based argumentation (ABA) [15], ASPIC systems [8]. The language for defeasible logic* $\mathcal{L}_d$ *includes a set of (strict and defeasible) rules and a set of literals. The rules is the form of* $x_1, \dots x_i \to_s x_{i+1}$ *(*$x_1, \dots x_i \to_d x_{i+1}$*) where* $x_1, \dots x_i, x_{i+1}$ *are literals and* $\to_s$ *(denote strict rules) and* $\to_d$ *(denotes defeasible rules) are implication symbols.*

The consequence operator CN_d *is a function from* $2^{\mathcal{L}_d}$ *to* $2^{\mathcal{L}_d}$ *such that for all* $X \subseteq \mathcal{L}_d$*,* $x \in \text{CN}_d(X)$ *iff there exists a* sequence $x_1, \dots, x_n$ *such that*

1. x *is* x_n*, and*
2. *for each* $x_i \in \{x_1, \dots, x_n\}$*,*
    - *there is* $y_1, \dots, y_j \to_s x_i \in X$*, or* $y_1, \dots, y_j \to_d x_i \in X$*, such that* $\{y_1, \dots, y_j\} \subseteq \{x_1, \dots, x_{i-1}\}$*,*
    - *or* $x_i \in X$

Consider $\mathcal{K}'' = \{a, \neg b, a \to_s \neg c,\ \neg b \wedge \neg c \to_d s,\ s \to_s t,\ a \wedge t \to_d u\}$*, the following is an argument in defeasible logic* $B = (\{a, \neg b, a \to_s \neg c,\ \neg b \wedge \neg c \to_d s\}, s)$*.*

Example 3. *Let* $(\mathcal{L}_{da}, \text{CN}_{da})$ *be Datalog*$^{\pm}$ *[31].*

Let $\mathsf{N_t}$ *be a set of* terms *that contain variables, constants and function terms. An atom is of the form* $P(\vec{t})$*, with* P *a predicate name and* $\vec{t}$ *a vector of terms, which is* ground *if it contains no variables. A* database *is a finite instance. A* tuple-generating dependency *(TGD)* σ *is a first-order formula of the form* $\forall\vec{x}\forall\vec{y}\phi(\vec{x}, \vec{y}) \to \exists\vec{z}\psi(\vec{x}, \vec{z})$*, where* $\phi(\vec{x}, \vec{y})$ *and* $\psi(\vec{x}, \vec{z})$ *are non-empty conjunctions of atoms.* [2] *A* negative constraint *(NC)* δ *is a rule of the form* $\forall\vec{x}\ \phi(\vec{x}) \to \bot$ *where* $\phi(\vec{x})$ *is a conjunction of atoms. The language* $\mathcal{L}_{da}$ *contain a set of facts, TGDs, and NCs.*

Assume that $\models_{\mathcal{L}_{da}}$ *is an entailment of first-order formulas, i.e.,* $\Delta \models_{\mathcal{L}_{da}} \alpha$ *holds iff every model of all elements in* Δ *is also a model of* α*. The consequence operator* CN_{da} *is defined for* $X \subseteq \mathcal{L}_{da}$ *by* $\text{CN}_{da}(X) = \{\alpha \in \mathcal{L}_{da} \mid X \models_{\mathcal{L}_{da}} \alpha\}$ *in the first-order semantics.*

A knowledge base $\mathcal{K}$ *is a tuple* $(\mathcal{F}, \mathcal{R}, \mathcal{C})$ *where a database* $\mathcal{F}$*, a set* $\mathcal{R}$ *of TGDs and a set* $\mathcal{C}$ *of NCs.*

Consider KB $\mathcal{K} = \{\mathcal{R}, \mathcal{C}, \mathcal{F}\} \subseteq \mathcal{L}$*, where:*

$$\mathcal{R} = \{r_1 : \mathtt{Le}(x) \to \mathtt{Em}(x),\ r_2 : \mathtt{Re}(x) \to \mathtt{Em}(x),$$

[2] We usually leave out the universal quantification, and refer to $\phi(\vec{x}, \vec{y})$ and $\psi(\vec{x}, \vec{z})$ as the *body* ad *head* of σ.

Table 1
Supports and conclusions of arguments

Argument	$\mathtt{Sup}(A_i)$	$\mathtt{Con}(A_i)$
A_0	$\{\mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{t}(\mathrm{v}, \mathrm{KR})$
A_1	$\{\mathrm{GC}(\mathrm{KR})\}$	$\mathrm{GC}(\mathrm{KR})$
A_2	$\{\mathrm{GC}(\mathrm{KR}), \mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{FP}(\mathrm{v})$
A_3	$\{\mathrm{GC}(\mathrm{KR}), \mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{Re}(\mathrm{v})$
A_4	$\{\mathtt{t}(\mathrm{v}, \mathrm{KD})\}$	$\mathtt{t}(\mathrm{v}, \mathrm{KD})$
A_5	$\{\mathtt{ta}(\mathrm{v}, \mathrm{KD})\}$	$\mathtt{ta}(\mathrm{v}, \mathrm{KD})$
A_6	$\{\mathrm{UC}(\mathrm{KD})\}$	$\mathrm{UC}(\mathrm{KD})$
A_7	$\{\mathtt{ta}(\mathrm{v}, \mathrm{KD}),\ \mathrm{UC}(\mathrm{KD})\}$	$\mathtt{TA}(\mathrm{v})$
A_9	$\{\mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{Le}(\mathrm{v})$
A_{10}	$\{\mathtt{t}(\mathrm{v}, \mathrm{KD})\}$	$\mathtt{Le}(\mathrm{v})$
A_{11}	$\{\mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{Em}(\mathrm{v})$
A_{12}	$\{\mathtt{t}(\mathrm{v}, \mathrm{KD})\}$	$\mathtt{Em}(\mathrm{v})$
A_{13}	$\{\mathrm{GC}(\mathrm{KR}), \mathtt{t}(\mathrm{v}, \mathrm{KR})\}$	$\mathtt{Em}(\mathrm{v})$

$$
\begin{aligned}
& r_3 : \mathtt{FP}(x) \rightarrow \mathtt{Re}(x),\ r_5 : \mathtt{t}(x,y) \rightarrow \mathtt{Le}(x),\\
& r_4 : \mathtt{ta}(x,y) \wedge \mathrm{UC}(y) \rightarrow \mathtt{TA}(x),\\
& r_6 : \mathtt{t}(x,y) \wedge \mathrm{GC}(y) \rightarrow \mathtt{FP}(x)\}\\
\mathcal{C} =& \{c_1 : \mathtt{TA}(x), \mathtt{Re}(x) \rightarrow \bot,\ c_2 : \mathtt{Le}(x), \mathtt{TA}(x) \rightarrow \bot\}\\
\mathcal{F} =& \{\mathtt{ta}(\mathrm{v}, \mathrm{KD}),\ \mathrm{UC}(\mathrm{KD}),\ \mathtt{t}(\mathrm{v}, \mathrm{KR}),\ \mathrm{GC}(\mathrm{KR}), \mathtt{t}(\mathrm{v}, \mathrm{KD})\}
\end{aligned}
$$

Table 1 shows the supports and conclusions of arguments induced from $\mathcal{K}$.

The KB admits six MCSs (called repairs*) $\mathcal{B}_1, \dots, \mathcal{B}_6$, e.g., $\mathcal{B}_1 = \{\mathtt{ta}(\mathrm{v}, \mathrm{KD}), \mathrm{UC}(\mathrm{KD})\}$. The corresponding G-SAF admits extensions: $\mathtt{Ext}_{\mathtt{st/pr}}(\mathcal{AF}) = \{\mathcal{E}_1, \dots, \mathcal{E}_6\}$, where $\mathcal{E}_1 = \mathtt{Args}(\{\mathtt{ta}(\mathrm{v}, \mathrm{KD}), \mathrm{UC}(\mathrm{KD})\})$ [3] $= \{A_5, A_6, A_7\}$, and $\mathcal{E}_2, \dots, \mathcal{E}_6$ are obtained in an analogous way. By Theorem 1, the extensions correspond to the repairs of the KBs, for example, $\mathcal{E}_1$ corresponds to $\mathcal{B}_1$.*

Consider $q_1 = \mathtt{Re}(\mathrm{v})$. We have that "$\mathtt{Re}(\mathrm{v})$ is a possible answer" since q_1 is entailed in $\mathcal{B}_2$, $\mathcal{B}_3$, $\mathcal{B}_5$. In other words, q_1 is credulously accepted under st *(*pr*) extensions.*

3.2. Explanatory Dialogue Model for G-SAF

We have presented the translation of KBs into G-SAFs as an instantiation of SAFs. Subsequently, we shift the problem of determining the *entailment of a sentence f* to computing and explaining the *acceptance of arguments* for f. This section shows how to determine the acceptance of an argument, in various argumentation semantics from an *explanatory dialogue* (or "*dialogue*" for short).

Dialogues can be viewed as dialectical proof procedures of moving arguments as a 2-person dialogue game. Informally, a dialectical proof procedure is formalised by a dialogue between two players a proponent (P) and an opponent (O). The dialogue begins with P moving an initial

[3] Fix $\mathcal{F}' \subseteq \mathcal{F}$, $\mathtt{Args}(\mathcal{F}') = \{A \in \mathtt{Arg}_{\mathcal{K}} \mid \mathtt{Sup}(A) \subseteq \mathcal{F}'\}$

argument A that it wants to put to the test. O and P take turns in moving arguments that attack their counterpart's last move. We adapt the notion of dialogue model[4] from [21] for G-SAF:

- A *topic language* $\mathcal{L}_t$, which is argument-based.
- A *communication language* $\mathcal{L}_c$.
- An *utterance* is a pair $u = (\texttt{PL}, \texttt{C})$, where PL is the *player* P (or O) and $\texttt{C} \in \mathcal{L}_c$ is the *speech act* performed in the utterance. C is one of the following forms:
  - $\texttt{claim}(A)$: The move advances an argument $A \in \texttt{Arg}_{\mathcal{K}}$ that supports a query q of a KB. We set $\texttt{arg}(u) = A$.
  - $\texttt{contrary}(A)$ ($\texttt{contrary}(\mathcal{S})$ resp.): The move advances an argument A (a set of arguments $\mathcal{S}$ resp.) in $\texttt{Arg}_{\mathcal{K}}$ that attack the previously advanced arguments. We set $\texttt{arg}(u) = A$ ($\texttt{arg}(u) = \mathcal{S}$ resp.).
  - $\texttt{retract}(A, i)$: When O (P resp.) cannot advance any argument attacking (defending) the previously advanced arguments, it should retrace back and choose another argument A to start a new line of attack (defend resp.) at the i-th position. We set $\texttt{arg}(u) = A$.

  For such utterance u, we set $\texttt{pl}(u) = \texttt{PL}$. Let $\mathcal{U}$ denote the set of utterances.
- A *dialogue* D_{G} is a finite sequence $u_1, \ldots, u_k$ of utterances such that the first utterance u_1 is played by P, i.e. $\texttt{pl}(u_1) = \texttt{P}$. We denote by $\mathcal{D}^{<\infty}$ the set of all dialogues.
- A *protocol* Θ on $\mathcal{U}$ specifying the *legal moves* at each stage of a dialogue. Formally, a *protocol* on $\mathcal{U}$ is a function $\Theta : \mathcal{D} \mapsto 2^{\mathcal{U}}$ such that $\mathcal{D} \subseteq \mathcal{D}^{<\infty}$.
  The elements of $\mathcal{D}$ are called the *legal finite dialogues*. For $D_{\text{G}} \in \mathcal{D}$, the elements of $\Theta(D_{\text{G}})$ are called the moves allowed after D_{G}. If D_{G} is a legal dialogue and $\Theta(D_{\text{G}}) = \emptyset$, then D_{G} is said to be a *terminated* dialogue. Θ must satisfy the following condition: for all finite dialogues D_{G} and moves u, $D_{\text{G}} \in \mathcal{D}$ and $u \in \Theta(D_{\text{G}})$ iff $D_{\text{G}}, u \in \mathcal{D}$.

The dialogue represents a compact representation of a tree where nodes are arguments played by both parties. To display all disclosed information from the dialogues, we introduce the notion of *dialogue trees* in terms of a meta-level, which includes arguments and counter-arguments.

Definition 8. *Given a dialogue $D_{\text{G}} = u_1, \ldots, u_k$ (for an argument $A \in \texttt{Arg}_{\mathcal{K}}$). The* general dialogue tree *drawn from D_{G} is a labelled tree $\mathcal{T}_{\text{G}} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of hyperedges that are directed from nodes to their parent node. $\mathcal{T}_{\text{G}}$ is defined as follows:*

- *The set of all nodes is defined as $\mathcal{V} = \{\arg(u_k) \mid u_k \in D_{\text{G}}\}$ with $\arg(u_1)$ as the root node of the tree s.t. $\arg(u_1) = A$.*
- *$\mathcal{E}$ is recursively defined as:*
  - *$\mathcal{E}_1 = \emptyset$ if the player P utters u_1,*
  - *$\mathcal{E}_{k-1} \cup \{(\arg(u_{k-1}), \arg(u_k))\}$ if the player $\texttt{PL} \in \{\texttt{P}, \texttt{O}\}$ utters u_k.*

[4] Following [20, 21], this model uses Dung's very abstract notion of argument, and they do not need utterances to reflect particular procedures or forms of argument. Thus, the model has a rather small set of utterances.

A branch in a dialogue tree may be finite or infinite. A finite branch represents a *winning debate* (i.e., P wins) that ends with an argument by P against which O cannot attack. An infinite branch represents a winning debate in which P counterattacks every attack of O, ad infinitum. The requirement that "the proponent must counterattack every attack" does not guarantee that the proponent does not attack itself. This further requirement is incorporated in the definition of admissible dialogue tree: A dialogue tree is said to be *admissible* iff P wins and no argument labels both a proponent and an opponent node.

Notice that, in admissible dialogue trees, it is not required that the opponent and proponent nodes have no arguments in common. This is because the opponent can use the proponent's arguments against the proponent. If the opponent can attack the proponent using only the proponent's arguments, then the proponent loses. To win, the proponent must identify and counter-attack each opponent's attack with some culprit not part of their own defence.

To ensure credulous soundness, all possible O nodes must be considered. But if such a parent node is already in the dialogue tree, then deploying it will not help O win the dialogues. For this reason we call an admissible tree *non-redundant* [32] if there is no sequence of arguments $A_1, \ldots, A_m$ with A_{i+1} children of A_i and A_1 children of A_m.

The following definition defines successful dialogues.

Definition 9. *Let $\mathcal{T}_\mathrm{G}$ be the dialogue tree drawn from a dialogue D_G for an argument $A \in \mathtt{Arg}_\mathcal{K}$. D_G is called*

- admissible-successful *iff $\mathcal{T}_\mathrm{G}$ is admissible;*
- preferred-successful *iff it is admissible-successful;*
- grounded-successful *iff $\mathcal{T}_\mathrm{G}$ is admissible and finite;*
- sceptical-successful *iff $\mathcal{T}_\mathrm{G}$ is admissible and for no opponent node* O *in it there exists an admissible dialogue tree for the argument labelling* O*.*

We now determine the acceptance of an argument from its dialogues and dialogue trees.

Theorem 2. *Let D_G be a dialogue for an argument $A \in \mathtt{Arg}_\mathrm{G}$. Then A is*

- *credulously accepted in some admissible extension of $\mathcal{AF}_\mathrm{G}$ if D_G is admissible-succesful;*
- *credulously accepted in some preferred extension of $\mathcal{AF}_\mathrm{G}$ if D_G is preferred-successful;*
- *groundedly accepted in $\mathcal{AF}_\mathrm{G}$ if D_G grounded-successful;*
- *sceptical accepted in all preferred extensions of $\mathcal{AF}_\mathrm{G}$ if D_G is sceptical-successful.*

Proof 2. *Theorem 2 generalizes known results about the abstract dispute trees of [32, 33] to the dialogue trees of Definition 8.*

The following is a direct corollary of Theorem 1, 2 and Definition 9, which shows how to determine the entailment of a sentence wrt dialogues. The acceptance of a sentence f corresponds to the acceptance of a set of arguments $\mathcal{A}$ for f.

Corollary 1. *Let $\mathcal{K}$ be a KB and f be a sentence in $\mathcal{L}$. Let $\mathcal{AF}_\mathcal{K}$ be the corresponding G-SAF and $\mathcal{A} \subseteq \mathtt{Arg}_\mathcal{K}$ be the set of arguments for f. Then, f is*

$\mathtt{P}: \mathtt{claim}(A_3)$

$\mathtt{O}: \mathtt{contrary}(A_7)$

$\mathtt{P}: \mathtt{contrary}(A_9)$

$\mathtt{P}: \mathtt{contrary}(A_{10})$

$$A_3 : \langle\{\mathtt{t}(\mathrm{v}, \mathrm{KR}),\ \mathtt{GC}(\mathrm{KR})\},\ \mathtt{Re}(\mathrm{v})\rangle$$

$$\uparrow$$

$$A_7 : \langle\{\mathtt{t}(\mathrm{v}, \mathrm{KD}),\ \mathtt{UC}(\mathrm{KR})\},\ \mathtt{ta}(\mathrm{v})\rangle$$

$$\nearrow \qquad \nwarrow$$

$$A_9 : \langle\{\mathtt{t}(\mathrm{v}, \mathrm{KR})\},\ \mathtt{Le}(\mathrm{v})\rangle \qquad A_{10} : \langle\{\mathtt{t}(\mathrm{v}, \mathrm{KD})\},\ \mathtt{Le}(\mathrm{v})\rangle$$

Figure 1: Left: a dialectical proof procedure for abstract level. Right: the corresponding dialogue tree; the line indicates children conflict with their parents.

- *credulously accepted, and* $\mathcal{K} \vdash_{\mathtt{MCS}(\mathcal{K})} f$ *if there is a dialogue* D_{G} *for some* $A \in \mathcal{A}$ *s.t.* D_{G} *is preferred-successful;*
- *groundedly accepted, and* $\mathcal{K} \vdash_{\bigcap \mathtt{MCS}(\mathcal{K})} f$ *if there is a dialogue* D_{G} *for all* $A \in \mathcal{A}$ *s.t.* D_{G} *is grounded-successful;*
- *sceptically accepted, and* $\mathcal{K} \vdash_{\bigcup \mathtt{MCS}(\mathcal{K})} f$ *if*
  1. *there is a dialogue* D_{G} *for some* $A \in \mathcal{A}$ *such that* D_{G} *is sceptical-successful;*
  2. *or there is a dialogue* D_{G} *for all* $A \in \mathcal{A}$ *s.t.* D_{G} *is preferred-successful and* $\mathcal{A}$ *is contained in all preferred extensions of* $\mathcal{AF}_{\mathcal{K}}$.

Example 4 (Continue Example 3). *Assume that the user wants to understand why "*v *is a possible researcher". The deliver system considers a persuasion dialogue:* $\mathtt{P}$ *is persuading* $\mathtt{O}$ *to agree that* v *is a researcher. The agreement can be reached through a dialogue* $D(A_3)$. *Figure 1 shows the dialogue tree* $\mathcal{T}$ *drawn from the dialogue* $D(A_3)$.

4. Conclusions

In this paper, we generalized the translation of arbitrary logic into SAFs. We introduced an argumentation dialogue framework and investigated the relation between its outcome and inconsistency-tolerant semantics in KBs. Our dialogue framework functions as proof procedures to compute and explain the acceptability of propositions. It can be applied to formalisms with collective attacks, thereby we show that this framework can be a generalization of the dialogue model used in AFs/LAFs with binary attacks. However, our framework has limitations: (1) Our dialogue model is still defined abstractly, only considering arguments and attacks and ignoring the internal structure of arguments; (2) Space complexity wrt the large input size, since the procedure considers all arguments translated from KBs as input to compute attacks. In future, we would address the limitations.

Acknowledgments

This work is partially supported by the Hybrid Intelligence programme (https://www.hybrid-intelligence-centre.nl/), funded by a 10 year Zwaartekracht grant from the Dutch Ministry of Education, Culture and Science.

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A. Proof for Proposition 1

Notation 1. *Let $\mathcal{K}$ be a KB, $\mathcal{X} \subseteq \mathcal{K}$ be a set of formulas and $\mathcal{S} \subseteq \texttt{Arg}_{\mathcal{K}}$ be a set of arguments. Then,*

- $\texttt{Args}(\mathcal{X}) = \{A \in \texttt{Arg} \mid \texttt{Sup}(A) \subseteq \mathcal{X}\}$ *are the set of* arguments generated by $\mathcal{X}$,
- $\texttt{Base}(\mathcal{S}) = \bigcup_{A \in \mathcal{S}} \texttt{Sup}(A)$ *are the set of* supports *of arguments in* $\mathcal{S}$,
- *An argument B is a* subargument *of argument A iff* $\texttt{Sup}(B) \subseteq \texttt{Sup}(A)$. *We denote the set of subarguments of A as* $\texttt{Subs}(A)$.

- *X is a minimal conflict of $\mathcal{K}$ if $X' \subsetneq X$ implies X' is consistent. We denote the set of minimal conflicts of $\mathcal{K}$ by $\texttt{Conflicts}(\mathcal{K})$.*

We prove Proposition 1 through Lemma 1 and Lemma 2.

Lemma 1. *Let $\mathcal{K}$ be a KB, the corresponding G-LAF $\mathcal{AF}_{\mathcal{K}}$. Then, the maximal consistent subset of $\mathcal{K}$ coincides with the stable/ preferred extension of $\mathcal{AF}_{\mathcal{K}}$;*

Proof 3. *To prove the lemma, we first prove it with the undercut-attacks. For the rebuttal-attacks, the proof is similar to that of the undercut-attacks. We separate the proof of the lemma into two parts:*

1. *We prove $\{\texttt{Args}(\mathcal{A}) \mid \mathcal{A} \in \texttt{MCS}(\mathcal{K})\} \subseteq \texttt{Ext}_{\texttt{st}}(\mathcal{AF}_{\mathcal{K}})$*
2. *We prove $\texttt{Ext}_{\texttt{pr}}(\mathcal{AF}_{\mathcal{K}}) \subseteq \{\texttt{Args}(\mathcal{A}) \mid \mathcal{A} \in \texttt{MCS}(\mathcal{K})\}$*

1. We prove $\{\texttt{Args}(\mathcal{A}) \mid \mathcal{A} \in \texttt{MCS}(\mathcal{K})\} \subseteq \texttt{Ext}_{\texttt{st}}(\mathcal{AF}_{\mathcal{K}})$. Suppose that $\mathcal{A} \in \texttt{MCS}(\mathcal{K})$ (i.e., $\mathcal{A}$ is consistent) and $\mathcal{E} = \texttt{Args}(\mathcal{A})$, we show that $\mathcal{E}$ is a stable extension of $\mathcal{AF}_{\mathcal{K}}$, i.e., $\mathcal{E} \in \texttt{Ext}_{\texttt{st}}(\mathcal{AF}_{\mathcal{K}})$. Which means that, by definition, we prove that $\mathcal{E}$ is conflict-free and attacks all arguments not belonging to $\mathcal{E}$.

First, we prove that $\mathcal{E}$ is conflict-free. Suppose for a contradiction that $\mathcal{E}$ is not conflict-free. There exists $\mathcal{E}' \subseteq \mathcal{E}$ and an argument $A \in \mathcal{E}$ such that $\mathcal{E}'$ attacks A. By Definition 4, there exists $\beta \in \texttt{Sup}(A)$ such that $\bigcup_{E \in \mathcal{E}'}\{\texttt{Con}(E)\} \cup \{\beta\}$ is inconsistent. Thus $\bigcup_{E \in \mathcal{E}'} \texttt{Sup}(E) \cup \{\beta\}$ is inconsistent. Since $\mathcal{E}' \subseteq \mathcal{E}$, it follows that $\bigcup_{E \in \mathcal{E}} \texttt{Sup}(E) \cup \{\beta\}$ is inconsistent. Thus $\mathcal{A}$ is inconsistent which is a contradiction to $\mathcal{A}$ is consistent. Thus $\mathcal{E}$ must be conflict-free.

Second, we prove that $\mathcal{E}$ attacks all arguments not belonging to $\mathcal{E}$. Let $C \in \texttt{Arg}_{\mathcal{K}} \setminus \mathcal{E}$ and $\alpha \in \texttt{Sup}(C) \setminus \mathcal{A}$. Since $\alpha \notin \mathcal{A}$ and $\mathcal{A}$ is a maximal consistent subset of $\mathcal{K}$, then for every $\mathcal{S} \subseteq \mathcal{E} = \texttt{Args}(\mathcal{A})$ such that $\bigcup_{S \in \mathcal{S}} \texttt{Sup}(S) \cup \{\alpha\}$ is inconsistent, there follows that $\bigcup_{S \in \mathcal{S}}\{\texttt{Con}(S)\} \cup \{\alpha\}$ is inconsistent. By Definition 4, $\mathcal{S}$ attacks C. Since $\mathcal{E} = \texttt{Args}(\mathcal{A})$, this proves that $\mathcal{E}$ attacks all arguments not belonging to $\mathcal{E}$. Since $\mathcal{E}$ is conflict-free and attacks all arguments not belonging to $\mathcal{E}$, it follows that $\mathcal{E}$ is a stable extension.

2. We next prove $\texttt{Ext}_{\texttt{pr}}(\mathcal{AF}_{\mathcal{K}}) \subseteq \{\texttt{Args}(\mathcal{A}) \mid \mathcal{A} \in \texttt{MCS}(\mathcal{K})\}$. Suppose that $\mathcal{E} \in \texttt{Ext}_{\texttt{pr}}(\mathcal{AF}_{\mathcal{K}})$, we prove that there exists a maximal consistent subset $\mathcal{A}$ such that $\mathcal{E} = \texttt{Args}(\mathcal{A})$. Which means that, by definition, we prove that $\mathcal{A}$ is maximal consistent.

First, suppose that $\mathcal{A} = \texttt{Base}(\mathcal{E})$ and we prove $\mathcal{A}$ is consistent. Assume for a contradiction that $\mathcal{A}$ is inconsistent. Let $\{\beta_1, \ldots, \beta_m\} = \mathcal{A}' \subseteq \mathcal{A}$ be an inconsistent set and every proper subset of it is consistent. Let $C \in \mathcal{E}$ be an argument such that $\beta_m \in \texttt{Sup}(C)$ and $\mathcal{S}$ be the set of arguments generated from $\mathcal{A}' \backslash \{\beta_m\}$ (i.e., $\mathcal{S} = \{S \in \texttt{Arg} \mid \texttt{Sup}(S) \subseteq \mathcal{A}' \backslash \{\beta_m\}\}$) such that $\bigcup_{S \in \mathcal{S}} \texttt{Sup}(S) \cup \{\beta_m\}$ is inconsistent. It follows that $\bigcup_{S \in \mathcal{S}}\{\texttt{Con}(S)\} \cup \{\beta_m\}$ is inconsistent, then $\mathcal{S}$ attacks C (by Definition 4). Since $\texttt{Sup}(C) \subseteq \mathcal{A}'$, $\mathcal{S}$ attacks C. Since $\mathcal{E}$ is conflict-free, we have $\mathcal{S} \nsubseteq \mathcal{E}$; and since $\mathcal{E}$ is also an admissible set, there is a set of arguments $\mathcal{D} \subseteq \mathcal{E}$ such that $\mathcal{D}$ attacks $\mathcal{S}$. Then, there exist some arguments $S_i \in \mathcal{S}$ and $\beta_i \in \texttt{Sup}(S_i)$, $i \in \{1, \ldots, m-1\}$ such that $\bigcup_{D \in \mathcal{D}}\{\texttt{Con}(D)\} \cup \{\beta_i\}$ is inconsistent. Next, we will show $\mathcal{D}$ attacks an argument in $\mathcal{E}$. Since $\beta_i \in \texttt{Base}(\mathcal{E})$, there exists an argument $F \in \mathcal{E}$ s.t. $\beta_i \in \texttt{Sup}(F)$, which implies $\mathcal{D}$ attacks F. Clearly, C is attacked by $\mathcal{S}$ but cannot be defended, which is in contradiction with the fact that $\mathcal{E}$ is admissible. Hence $\mathcal{A}$ is consistent.

Second, we show that $\mathcal{A}$ is maximal consistent, i.e., there is no maximal consistent subset $\mathcal{A}' \in \texttt{MCS}(\mathcal{K})$ such that $\mathcal{A}' \subseteq \mathcal{A}$ and $\mathcal{A}'$ is inconsistent. Assume the contrary. Since $\mathcal{E}$ is a preferred extension, it follows that $\texttt{Args}(\mathcal{A})\backslash\texttt{Args}(\mathcal{A}') \neq \emptyset$, and let $\Omega \subseteq \mathcal{A}\backslash\mathcal{A}'$ and $\mathcal{M}$ be the set of arguments generated by Ω, i.e., $\mathcal{M} = \{M \in \texttt{Arg} \mid \texttt{Sup}(M) \subseteq \Omega\}$. Since $\Omega \subseteq \mathcal{A}\backslash\mathcal{A}'$, it follows that $\mathcal{M} \subseteq \texttt{Args}(\mathcal{A})\backslash\texttt{Args}(\mathcal{A}')$. By the first part of the proof, $\texttt{Args}(\mathcal{A}')$ is a stable extension of $\mathcal{AF}_{\mathcal{K}}$, there must be $\Phi \subseteq \mathcal{A}'$ such that the set of all arguments generated by Φ, that is $\mathcal{X} = \{X \in \texttt{Args}(\mathcal{A}') \mid \texttt{Sup}(X) \subseteq \Phi\}$, attacks $\mathcal{M}$. Since $\mathcal{E}$ is the preferred extension, there must be a set of arguments $\mathcal{H} \subseteq \mathcal{E}$ and $\mathcal{H}$ attacks $\mathcal{X}$. Since $\mathcal{H} \subseteq \mathcal{E}$ and $\mathcal{A} = \texttt{Base}(\mathcal{E})$, it follows that $\texttt{Base}(\mathcal{H}) \subseteq \mathcal{A}$. By assumption, i.e., $\mathcal{A}' \subseteq \mathcal{A}$, it follows that $\mathcal{A}'$ is inconsistent, which contradicts the assumption. Hence $\mathcal{A}$ is maximal consistent. We conclude that $\mathcal{A} \in \texttt{MCS}(\mathcal{K})$.

Since every stable extension is a preferred one [29], we can proceed as follows. From the first item, we proved that every maximal consistent subset is a stable extension, thus the proposition holds for preferred semantics. From the second item, we have that every preferred extension is a maximal consistent subset, thus the proposition holds for stable semantics.

Lemma 2. *Let $\mathcal{K}$ be a KB, the corresponding G-LAF $\mathcal{AF}_{\mathcal{K}}$. Then, the intersection of the maximal consistent subsets of $\mathcal{K}$ coincides with the grounded extension of $\mathcal{AF}_{\mathcal{K}}$.*

Proof 4. *Let $\mathcal{G}$ be the grounded extension of $\mathcal{AF}_{\mathcal{K}}$ and $\mathcal{A}$ be the intersection of the maximal consistent subsets of $\mathcal{K}$. Let $\mathcal{A}' \in \texttt{MCS}(\mathcal{K})$ and $\mathcal{A} = \bigcap_{\mathcal{A}' \in \texttt{MCS}(\mathcal{K})} \mathcal{A}'$. By Lemma 1, $\mathcal{A}'$ is a stable extension of $\mathcal{AF}_{\mathcal{K}}$. Since every stable extension is also a complete extension, and $\mathcal{G}$ is the minimal (w.r.t. set inclusion) complete extension of $\mathcal{AF}_{\mathcal{K}}$, it follows that $\mathcal{G} \subseteq \texttt{Args}(\mathcal{A}')$. Thus $\mathcal{G} \subseteq \texttt{Args}(\mathcal{A})$.*

In the other direction, let $\Delta \subsetneq \mathcal{A}$. Then there is no $\mathcal{C} \in \texttt{Conflicts}(\mathcal{K})$ such that $\Delta \subsetneq \mathcal{C}$ (If $\Delta \subsetneq \mathcal{C}$, $\mathcal{C} \setminus \Delta$ would be consistent and could be extended to a maximal consistent subset of $\mathcal{K}$ that would not contain Δ). Hence, there are no arguments that attack the argument $D \in \texttt{Args}(\mathcal{A})$ where $\texttt{Sup}(D) = \Delta$. Since D has no attackers, then any extension trivially defends it, so D must be in $\mathcal{G}$. It follows that $\texttt{Args}(\mathcal{A}) \subseteq \mathcal{G}$.

B. Proof of Proposition 2

We prove each postulate:

1. *Closure:* Let us show that for every $\mathcal{E} \in \texttt{Ext}_{\texttt{s}}(\mathcal{AF}_{\mathcal{K}})$, $\texttt{Cons}(\mathcal{E}) = \texttt{CN}(\texttt{Cons}(\mathcal{E}))$. From Expansion axiom, it follows that $\texttt{Cons}(\mathcal{E}) \subseteq \texttt{CN}(\texttt{Cons}(\mathcal{E}))$.

In the other direction, we prove $\texttt{CN}(\texttt{Cons}(\mathcal{E})) \subseteq \texttt{Cons}(\mathcal{E})$. Let $\alpha \in \texttt{CN}(\texttt{Cons}(\mathcal{E}))$. Since $\mathcal{E}$ is a preferred or stable extension, there is a set of sentences $\mathcal{A} \subseteq \mathcal{K}$ such that $\mathcal{E} = \texttt{Args}(\mathcal{A})$ (by Proposition 1). Thus $\mathcal{E} = \texttt{Args}(\texttt{Base}(\mathcal{E}))$. Since the supports of the arguments of $\mathcal{E}$ include the sentences in $\mathcal{A}$, it follows that $\alpha \in \texttt{CN}(\mathcal{A})$. Hence, there is an argument $A \in \mathcal{E}$ such that $\texttt{Con}(A) = \alpha$.

3. *Consistency:*

We prove that for $\mathcal{E} \in \texttt{Ext}_{\texttt{s}}(\mathcal{AF}_{\mathcal{K}})$, $\texttt{Cons}(\mathcal{E})$ is consistent. First, we consider the case of stable or preferred extensions. Let $\mathcal{E}$ be a stable or preferred extension of $\mathcal{AF}_{\mathcal{K}}$. By Proposition 1, there is a maximal consistent subset $\mathcal{A} \in \texttt{MCS}(\mathcal{K})$ such that $\mathcal{E} = \texttt{Args}(\mathcal{A})$. Since $\mathcal{A}$ is a preferred maximal consistent subset, we see that $\mathtt{CN}(\mathcal{A})$ is consistent. Since $\mathtt{Cons}(\mathcal{E}) = \mathtt{CN}(\mathcal{A})$ and $\mathtt{CN}(\mathcal{A})$ is consistent, it follows that $\mathtt{Cons}(\mathcal{E})$ is consistent.

We consider the case of grounded semantics. We denote $\mathcal{G}$ as the grounded extension of $\mathcal{AF}_{\mathcal{K}}$. We prove that for every $\mathcal{E} \in \mathtt{Ext}_{\mathtt{gr}}(\mathcal{AF}_{\mathcal{K}})$, $\mathtt{Cons}(\mathcal{E})$ is consistent. Since the grounded extension is a subset of the intersection of the preferred extensions, and since there is at least one preferred extension $\mathcal{E}_1$, then $\mathcal{G} \subseteq \mathcal{E}_1$. From Proposition 1, there is a maximal consistent subset $\mathcal{A} \in \mathtt{MCS}(\mathcal{K})$ s.t. $\mathcal{E}_1 = \mathtt{Args}(\mathcal{A})$. By the first part of the proof, $\mathtt{Cons}(\mathcal{E}_1)$ is consistent. It follows that $\mathtt{Cons}(\mathcal{G})$ is consistent.

C. Proof of Theorem 1

- $\mathcal{K} \vdash_{\mathtt{MCS}(\mathcal{K})} f$ if f is *credulously accepted* under $\mathtt{s}$ semantics.
- $\mathcal{K} \vdash_{\bigcup \mathtt{MCS}(\mathcal{K})} f$ if f is *sceptically accepted* under $\mathtt{s}$ semantics.
- $\mathcal{K} \vdash_{\bigcap \mathtt{MCS}(\mathcal{K})} f$ if f is *groundedly accepted* under $\mathtt{gr}$ semantics.

By Proposition 1, there exists a link between the set of maximal consistent subsets on $\mathcal{K}$ and the set of extensions on $\mathcal{AF}_{\mathcal{K}}$. Obviously, for every sentence $f \in \mathcal{L}$ and maximal consistent $\mathcal{A} \in \mathtt{MCS}(\mathcal{K})$, it is the case that $f \in \mathtt{CN}(\mathcal{A})$ iff $f \in \mathtt{Cons}(\mathtt{Args}(\mathcal{A}))$. The theorem is now proven as follows:

1. $\mathcal{K} \vdash_{\bigcup \mathtt{MCS}(\mathcal{K})} f$ iff for every $\mathcal{A} \in \mathtt{MCS}(\mathcal{K})$, $f \in \mathtt{CN}(\mathcal{A})$ iff for every $\mathtt{Args}(\mathcal{A}) \in \mathtt{Ext}(\mathcal{AF}_{\mathcal{K}})$, $f \in \mathtt{Cons}(\mathtt{Args}(\mathcal{A}))$ iff f is sceptically accepted.
2. $\mathcal{K} \vdash_{\mathtt{MCS}(\mathcal{K})} f$ iff for some $\mathcal{A} \in \mathtt{MCS}(\mathcal{K})$, $f \in \mathtt{CN}(\mathcal{A})$ iff for some $\mathtt{Args}(\mathcal{A}) \in \mathtt{Ext}(\mathcal{AF})$, $f \in \mathtt{Cons}(\mathtt{Args}(\mathcal{A}))$ iff f is credulously accepted.
3. $\mathcal{K} \vdash_{\bigcap \mathtt{MCS}(\mathcal{K})} f$ iff for every $\mathcal{A}_i \in \mathtt{MCS}(\mathcal{K})$, $f \in \mathtt{CN}(\bigcap \mathcal{A}_i)$ iff for every $\mathtt{Args}(\mathcal{A}_i) \in \mathtt{Ext}(\mathcal{AF}_{\mathcal{K}})$, $f \in \mathtt{Cons}(\bigcap \mathtt{Args}(\mathcal{A}_i))$ iff f is accepted under grounded semantics.

This ends the proof of Theorem 1.

D. Proof for Theorem 2

Proof 5. *It is clear that A is accepted in some admissible extension of $\mathcal{AF}_{\mathrm{G}}$ if D_{G} is admissible. Let $\mathcal{T}_{\mathrm{G}}$ be the dialogue tree drawn from D_{G}. The argument given in Definition 2 and Lemma 1 of [32] for binary attacks generalizes to collective attacks, implying that A is accepted in some admissible extension if $\mathcal{T}_{\mathrm{G}}$ is admissible which in turn holds iff $\mathtt{P}$ wins and no argument labels both a proponent and an opponent node. By Definition 9, D_{G} is admissible-successful iff $\mathcal{T}_{\mathrm{G}}$ is admissible. Thus, the statement is proved.*

D_{G} is preferred-successful if D_{G} is admissible-successful. This result directly follows from the results of [1] that states that an extension is preferred if it is admissible. Thus, A is credulously accepted in some preferred extension if D_{G} is preferred-successful.

The other statement follows in a similar way as a straightforward generalization of Theorem 1 of [32] for the "grounded-successful" semantic; Definition 3.3 and Theorem 3.4 of [33] for the "sceptical-successful" semantic.