Using STACK to support adaptive mathematics learning in LMS Moodle

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Abstract

The article describes some practices of using Systems for Teaching and Assessment using Computer Algebra Kernel (STACK) in LMS Moodle to organize and support adaptive mathematics learning at the university. It highlights the potential opportunities of this technology in the educational process to ensure effective teaching and learning in mixed or distance learning environments. Shifting the educational focus from providing static content, the same for all students, to a didactically motivated adaptive design of content and activities is an urgent task and goal of modern education. STACK is one of the leading technologies that allows the creation of dynamic, variable tests with automatic evaluation of student answers and individual feedback. However, there is no widespread practice of its use in Ukraine. Therefore, the author's examples focus on the advantages of interactive math tests using the potential response tree, particularly during formative assessment, and step-by-step tasks with separate prompts for self-study. Examples of STACK integration with LMS Moodle functions, such as Quiz and Grouping users, demonstrate the technology's capa-bilities to create personalized learning trajectories during the study of mathe-matical disciplines. Below are some feedback examples from students about the positive aspects of using STACK.

Keywords

adaptive mathematics learning, STACK, LMS Moodle

1. Introduction

Based on [1, 2, 3, 4, 5], adaptive learning means educational technology that ensures the personalization of the educational process, enables the construction of a flexible individual learning trajectory for each student that meets his/her needs, abilities, and pace of learning the material. Adaptive learning tools react in real time to the actions (answers) of the student, providing him/her with individual support.

Adaptive learning is based on the following key principles:

- personalization, which means providing each student with pedagogical support on the way to knowledge, taking into account his/her educational needs and individual characteristics (level of previous educational achievements, cognitive characteristics, temperament, etc.);
- real-time feedback and dynamic correction, which allow the student to quickly receive information about his/her achievements and mistakes, on the basis of which the educational trajectory is corrected;
- analysis of data on student progress, which allows the teacher and the administration of the educational institution to identify weak points, draw conclusions about the effectiveness of the used learning strategy, make corrections in the educational process, predict future results.

It is obvious that the implementation of these principles in the educational process of any educational institution without modern digital technologies, "by hand", so to speak, is an unattainable task. That

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is why adaptive learning is digital and is most effective in blended or remote learning. According to G. Borich, it is the use of digital tools that allows, “apply different instructional strategies to different groups of learners so the natural diversity prevailing in the classroom does not prevent any learner from achieving success” [1, p. 37].

In recent years, new and new adaptive educational web systems have been created, research into the possibilities and problems of their integration into the educational process has been intensified [6, 7, 8, 9, 10, 11, 12, 13]. One of the most common practices is the implementation of adaptive technology in LMS Moodle [14, 15, 16], in particular, in the teaching of mathematics [17, 18]. Among the main LMS Moodle tools used to support adaptive learning is the Quiz module. It provides a palette of flexible test settings, including an adaptive mode, when the student can see the correct answers and comments before completing the attempt. This mode makes it possible not so much to control the level of knowledge as to help in the assimilation of knowledge or the development of skills. However, the preparation of many options for test tasks, all comments, on incorrect answers, falls on the teacher, which, in the case of mathematical disciplines, is too time-consuming a task. This problem is well solved by interactive tests based on STACK. STACK can generate feedback automatically and adaptively, based on so-called “potential response tree”. In addition, it completely solves the problem of the number of test items of the same type (which is important for teaching mathematics), since the questions can be formulated with randomized variable parameters.

But, unfortunately, the use of STACK has not yet become part of the widespread practice of Ukrainian mathematics teachers. In March 2024, we interviewed 27 teachers of mathematics disciplines from 7 universities in different regions of Ukraine. To the question “Do you have experience using the STACK system for educational purposes?” $\frac{2}{3}$ of respondents answered that they do not know about such a system, the rest ($\frac{1}{3}$) – have heard about it, but do not use it.

The purpose of our article is to highlight some of the possibilities and advantages of the STACK system for supporting adaptive learning of mathematics in LMS Moodle, which, in particular, can increase the interest of university lecturers in its use.

2. Methodology

This study employed a mixed-methods approach to investigate the effectiveness of using STACK (System for Teaching and Assessment using Computer Algebra Kernel) in supporting adaptive mathematics learning within the LMS Moodle environment. The research was conducted at Borys Grinchenko Kyiv Metropolitan University as part of the scientific topic “Mathematical methods and digital technologies in education, science, technology” (state registration number: 0121U111924).

2.1. Research design

The study utilized a combination of qualitative and quantitative methods to provide a comprehensive understanding of the implementation and impact of STACK in adaptive mathematics learning.

1. An extensive review of existing literature on adaptive learning, mathematics education, and the use of digital tools in education was conducted to establish the theoretical framework for the study.
2. In March 2024, we conducted a survey of 27 mathematics teachers from 7 universities across different regions of Ukraine to assess their familiarity and experience with the STACK system.
3. The researchers developed a series of adaptive mathematics tasks using STACK, integrated within the LMS Moodle environment. These tasks covered various topics in mathematics and were designed to support different levels of learning.
4. Students majoring in mathematics at the bachelor’s level (1st and 2nd year) were engaged in completing the STACK-based tasks as part of their e-learning courses. Their interactions with the system were observed, and feedback was collected through open-ended questionnaires.

2.2. Data collection

Data was collected through multiple sources:

1. Quantitative data from the survey of mathematics teachers regarding their knowledge and use of STACK.
2. Data on student interactions with the STACK-based tasks, including attempts, scores, and time spent on tasks.
3. Qualitative data from student responses to open-ended questions about their experiences with the STACK-based tasks.
4. Qualitative data from teachers' observations of student engagement and performance with the STACK-based tasks.

2.3. Data analysis

The collected data was analyzed using both quantitative and qualitative methods:

1. Descriptive statistics was used to analyze the survey responses from mathematics teachers and quantitative data from STACK system logs.
2. Thematic analysis has been applied to the qualitative data from student feedback and teacher observations to identify key themes and patterns in the experiences and perceptions of using STACK for adaptive mathematics learning.
3. Detailed examination of specific examples of STACK implementation to provide in-depth insights into the adaptive learning process.

3. Research results

3.1. Features of adaptive mathematics learning and STACK

The specificity of mathematics as a science and as an educational discipline (hierarchical structure of knowledge, close relationship and interdependence of different sections and even different mathematical disciplines, complex logical structure of many mathematical concepts, high level of abstraction, interdisciplinary connections, universality of methods) not only dictates the need for adaptive learning of mathematics, but also causes certain features and requirements for such learning. This in particular:

- the need to identify gaps in knowledge and understanding and take timely measures to fill them in order to avoid obstacles to further learning, which should be focused on understanding;
- the need to use visualizations, interactive simulations and other multimedia resources not only to explain complex abstract concepts, but also to organize students' independent research and experimentation, which lead the student to his/her own discoveries of new knowledge for him/her;
- the need for constant feedback, which allows the student to quickly understand his/her mistakes and correct them; providing, if necessary, step-by-step instructions and explanations for completing the task;
- the exceptional importance of formative assessment, i.e. constant monitoring and assessment in the learning process and for learning;
- the need to solve many practical tasks with a gradual increase in the level of their complexity to develop stable procedural skills.

To meet the specified didactic requirements, we use Systems for Teaching and Assessment using Computer Algebra Kernel (STACK) in the process of teaching and learning mathematics [19]. STACK is an open-source computer-based automatic assessment system, inter alia compatible with the LMS Moodle. Mostly used in the study of mathematics with an emphasis on formative assessment. The system can automatically classify correct, half-correct and incorrect responses using a potential response tree (PRT) and provide appropriate feedback for each response. PRT is an acyclic directed graph, formed based on a series of true/false tests of student answers.

It should be noted that creating high-quality, useful interactive questions that would perform an adaptive function, help students manage their learning and develop metacognitive skills is not an easy task for a teacher. Creating questions requires skills in various areas, in particular:

- mathematics to design tasks, appropriately randomize and differentiate them;
- teaching mathematics to predict common errors and formulate adequate comments;
- programming using the Maxima computer algebra system to create problems, program a tree of potential answers, and etc.;
- LATEX for creating mathematical texts;
- LMS Moodle for storing questions, creating a test and organizing the learning process in general.

3.2. Examples of using STACK to implement an adaptive approach to teaching mathematics in LMS Moodle

Here are some examples of the use of interactive test tasks in STACK to support adaptive mathematics learning.

Example 1. The complex task on the topic "Derivative" (figure 1) requires the student to understand the concept of derivative and its geometric meaning, to have certain procedural skills.

One of the advantages of STACK is the ability to automatically create a huge number of tasks of the same type. In our example, these are tasks related to the derivative of the quadratic function $f(x) = ax^2 + bx + c$ with different randomly selected numerical values of parameters a, b, c. In the settings, the teacher sets **Question variables:**

```
a:rand_with_prohib(-3,3,[0]);
b:-3+rand(6);
c:-3+rand(6);
exp:a*x^2+b*x+c;
pt:rand(5);
ta1:diff(exp,x);
ta2:subst(x=pt,ta1);
ta3:remainder(exp,(x-pt)^2);
```

Separately, the teacher writes the text of the question in the Question text field and embeds the Geogebra Applet in the test task using the appropriate code (item 4 of the code):

```
1. Differenctiate {@exp@} with respect to \(x\).[[input:ans1]] [[validation:
ans1]] [[feedback:prt1]]
2. Evaluate your derivative at \(x={@pt@}\).[[input:ans2]] [[validation:ans2]]
[[feedback:prt2]]
3. Hence, find the equation of the tangent line.  \(y=\) [[input:ans3]]
[[validation:ans3]] [[feedback:prt3]]
4. Check your answer in point 3 using geogebra  [[geogebra set="a,b,c"
watch="ans1"]] params["material_id"]="wzmqgshr"; var params = {"appName":
"graphing", "width": 800,"height": 600, "showToolBar": true, "showAlgebraInput":
true, "showMenuBar": true };var applet = new GGBApplet(params, true);
window.addEventListener("load", function() { applet.inject('ggb-element');});
[[/geogebra]]
```

Another useful function of the system is automated checking of the student's answer and error diagnosis using PRT, pre-configured by the teacher (figure 2). In this way, the teacher does not need to "guess" after each answer of the student why he/she gave such an answer. STACK does it for him/her. For example, if a student incorrectly found the form of the derivative at a variable point x but based on

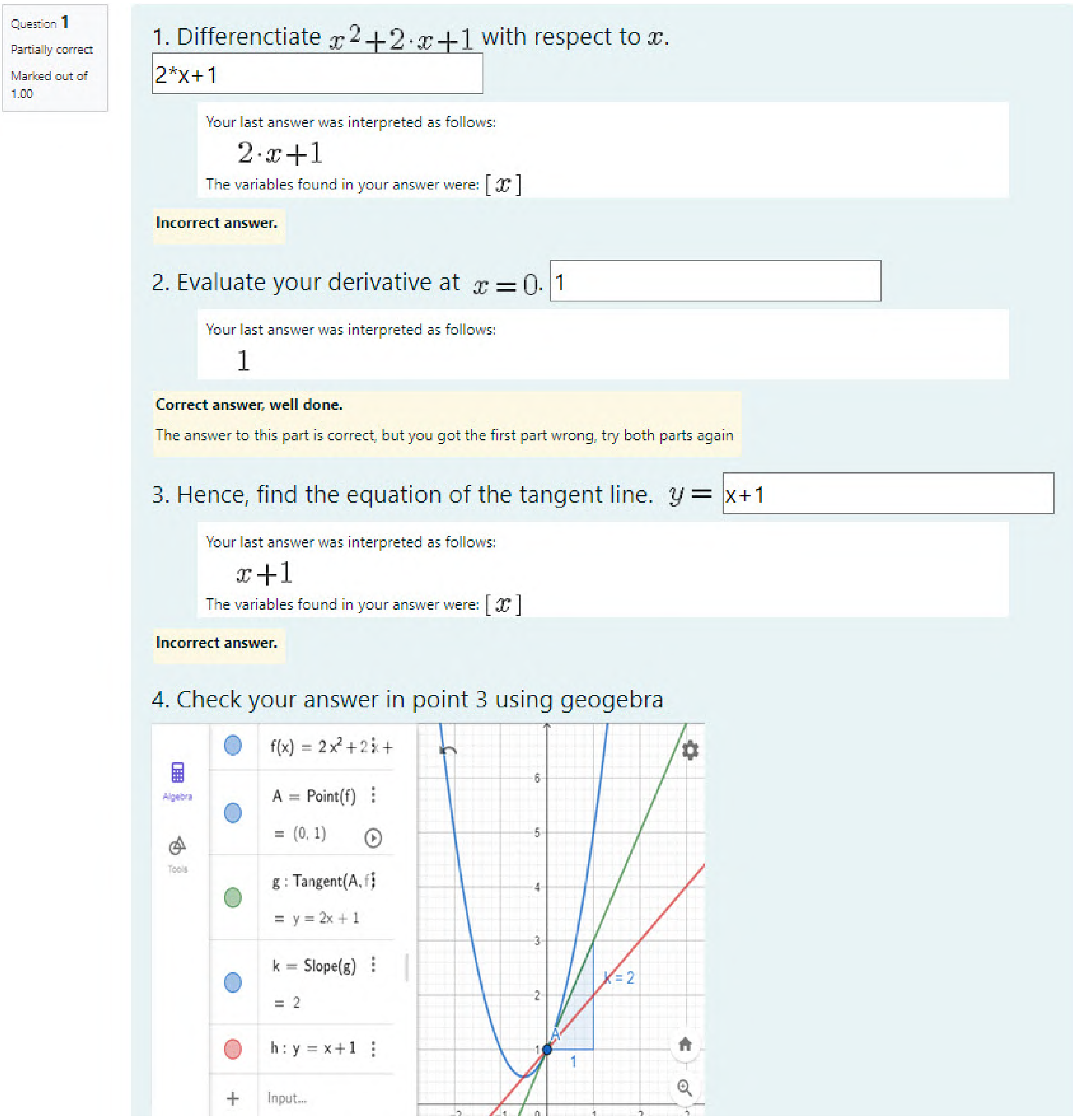

Figure 1: Screenshot of the task with the student’s answers.

“its” form of the derivative correctly calculated its value at a specific point, then STACK automatically recognizes this error and generates a corresponding comment (see comment to question 2 in figure 1). The GeoGebra window prompts the student to check whether he/she has correctly written the tangent equation (item 3 figure 1) and draw a conclusion (item 4 figure 1).

Example 2. The task on the topic “Irrational equations” is an example of a task differentiated by levels of complexity: I – reproductive level (the ability to apply the theory in standard situations to solve typical problems); II – the level of establishing connections (the ability to perform additional transformations, solve problems that require knowledge of various sections of mathematics and problems with variable conditions); III – search and research level (ability to conduct the necessary research, reveal a creative approach).

Using the Grouping users in Moodle function, based on previous knowledge, the teacher divides students into groups by level. Different groups are offered tasks with a corresponding level of cognitive requirements. The groups are dynamic and the student, according to his/her progress (or vice versa), migrates from one group to another, trying to achieve the best possible results.

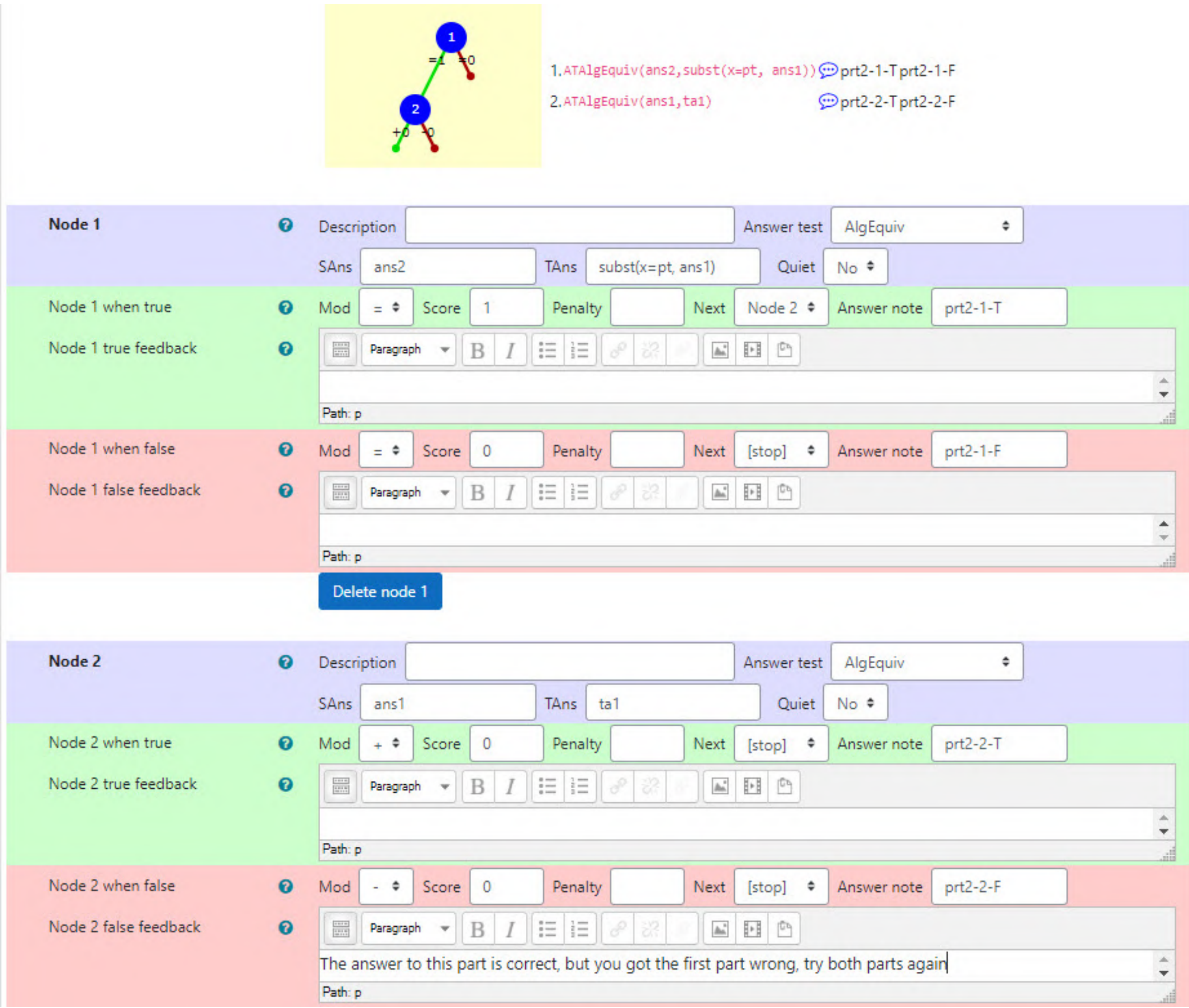

Figure 2: PRT setup page.

In our example, the I level group is asked to solve an equation of the type: $\sqrt{ax+b} = cx + d$ (by randomizing the values of parameters a, b, c and d, STACK generates a set of specific equations); for the II level group – to solve the type equation $m\sqrt{ax+b} + n\sqrt{cx+d} = p$; for group III level – an equation with a parameter.

Figure 3 presents a screenshot of the student's answer to the II level task with a comment on his/her incorrect answer. Please note that the comment does not directly indicate what the error is (of course, the author of the comment understands the reason for the error, he/she programmed this error), but encourages the student to figure it out him/herself. That is, we see the so-called delayed (timed) feedback, which is very useful in teaching mathematics.

Figure 4 shows a screenshot of the answer to the III level task. Here the student is asked to complete two tasks. The second of them involves an own experimental study, which the student performed with the help of GeoGebra. The student writes down his/her thoughts in the window provided for this, and the teacher checks them, which once again emphasizes digital technologies are only a good assistant for the teacher, but they cannot (and should not!) completely replace him/her.

Example 3. The task of finding the convergence set of a series (figure 5) is intended for self-learning, as it offers a step-by-step implementation of it: find the radius of convergence, the interval of convergence, check the convergence of the series at the ends of the interval and, finally, write down the convergence set of the series.

The task page also contains help in the form of questions, the answers to which you need to know to solve the task, and a hyperlink to a separate Help page, which contains the answers to these questions

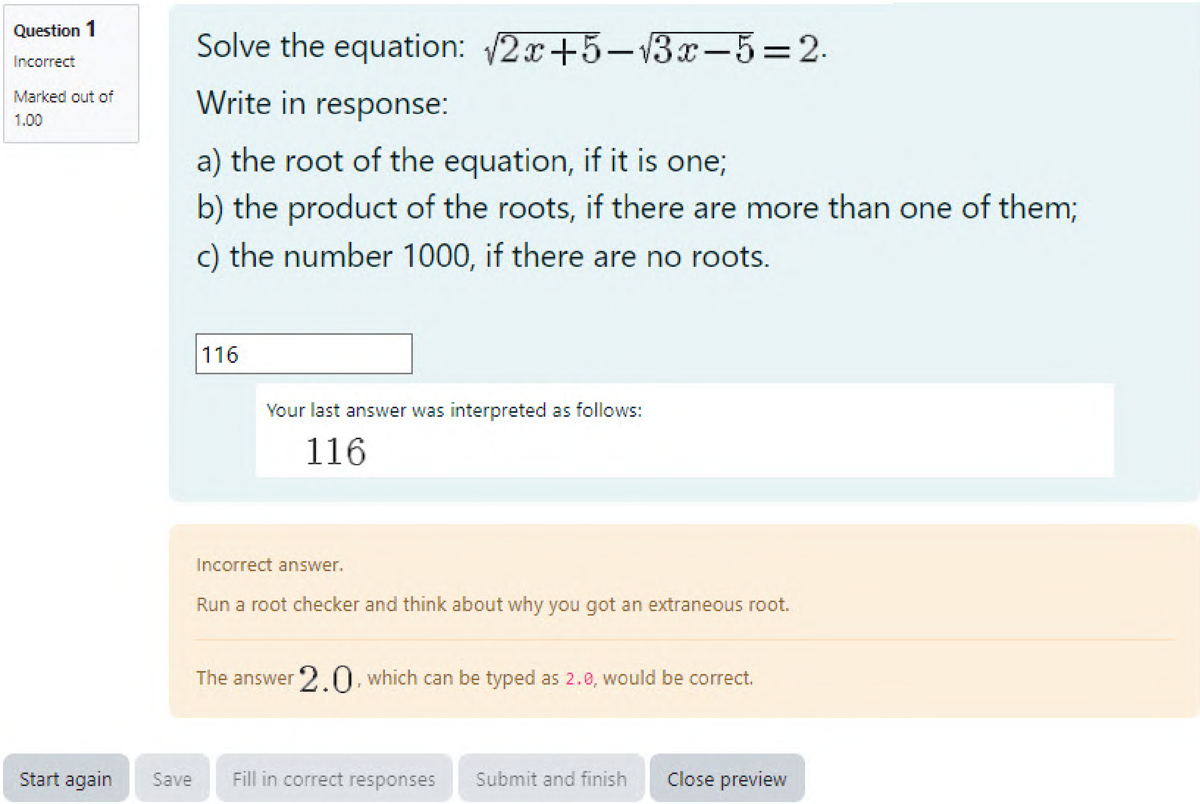

Figure 3: A page with the student's answer to the II level task.

in the form of hidden text (figure 6). The student can, as needed, use this help, not use it at all, or check him/herself after answering the questions him/herself.

Since the proposed task is step-by-step, the student's answer is checked at each step and the student receives a corresponding comment for each wrong answer. Because the task is intended for self-study, comments on incorrect answers encourage the student to find the correct answer on his/her own (figure 7).

3.3. Student feedback

We asked students majoring in mathematics, who practice performing test tasks in the STACK system in an e-learning course on the Moodle platform, to share their impressions of testing. Here are some fragments of these reviews.

Student 1 *(bachelor's level, 1st year of study)*: "Due to certain circumstances, I missed many classes on mathematical analysis, in fact the entire topic "Limits". But with the help of tests in the STACK system, I learned to calculate the limits of functions. Automatic prompts and instant feedback after each answer were especially valuable. It helped me quickly correct mistakes and understand where I made a wrong step. And the good thing is that I could train as much as needed to achieve the desired result, because STACK generated new and new exercises for each type of limits."

Student 2 *(bachelor's level, 1st year of study)*: "I enrolled in the first year of the mathematics major after almost a year of studying at another university, majoring in physics, which I left because I realized that this major was "not for me". That is why I am studying the course of mathematical analysis for the second time. I used to perform test tasks in Moodle, but here I met STACK-based tests for the first time. And I want to demonstrate their advantages on the example of test tasks on finding indefinite integrals (antiderivative). I will point out only two of them. Since the primitive integral is not a number, but a set of functions, most often in the test tasks that I performed before, it was suggested to choose the correct answer from several given ones or to establish correspondences between integrals and their

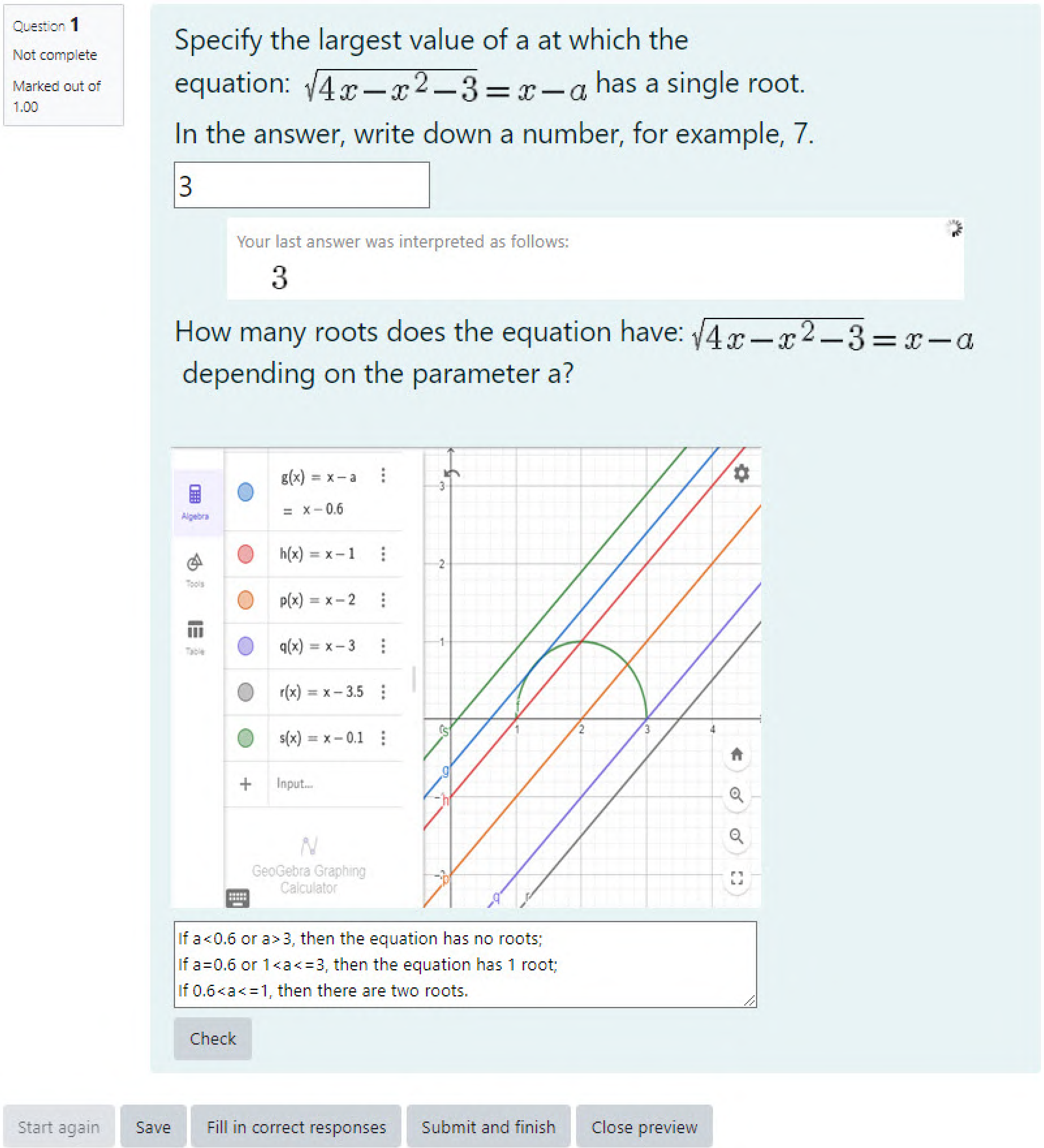

Figure 4: The page with the student's answer to the III level task.

corresponding functions. Such a task did not put me in front of the need to find the integral, because you can simply check by differentiation whether the indicated function is primitive. A similar test task in STACK is open-ended, it requires writing down the answer, and not choosing (guessing) it. That is, it is necessary to carry out the indefinite integration process, which is already much more difficult. In addition, as is known, the form of an antiderivative is not unambiguous, it all depends on which method of integration is used. We have repeatedly encountered the fact that the integration result we received differs from the printed (paper) collection of problems specified in the answer. STACK, on the other hand, "accepts" the answer in any form and checks whether it is correct or not."

Student 3 *(bachelor's level, 2nd year of study)*: "What I liked about the math tests on the STACK platform was the variety of tasks. Here you can find both simple tasks to consolidate basic skills, and more complex ones that really make you think. The presence of hints, as well as the ability to familiarize

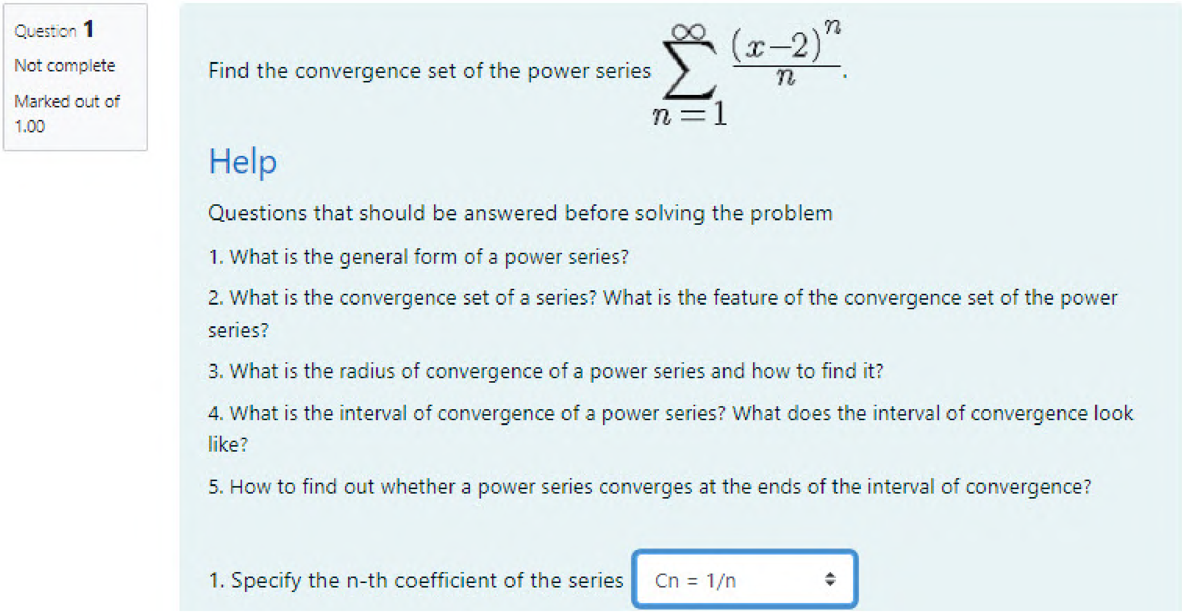

Figure 5: Tasks and questions that should help in solving. The Help hyperlink is a link to a separate page (figure 6) with answers to these questions.

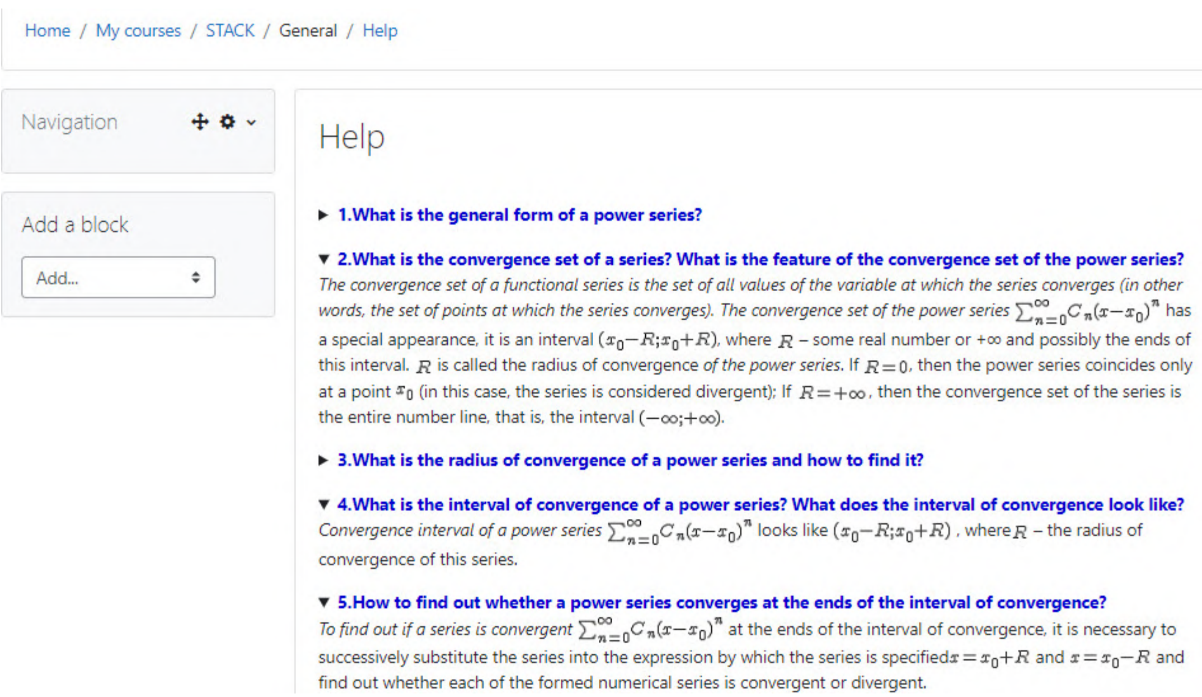

Figure 6: Hints page with hidden text created in Moodle in HTML code editing mode.

yourself with detailed solutions, are especially useful, because they allow you to better understand the material and learn from your own mistakes."

***Student 4** (bachelor's level, 2nd year of study)*: "Working with mathematical tests on the STACK platform was a real discovery for me. Tasks with different levels of difficulty helped me gradually improve my skills. Explanations of errors, their causes, and advice on what I should do to eliminate gaps in knowledge were very helpful."

These reviews confirm the effectiveness of tests in STACK for improving the results of learning mathematics, developing a dynamic educational environment, supporting students' efforts on the way to knowledge.

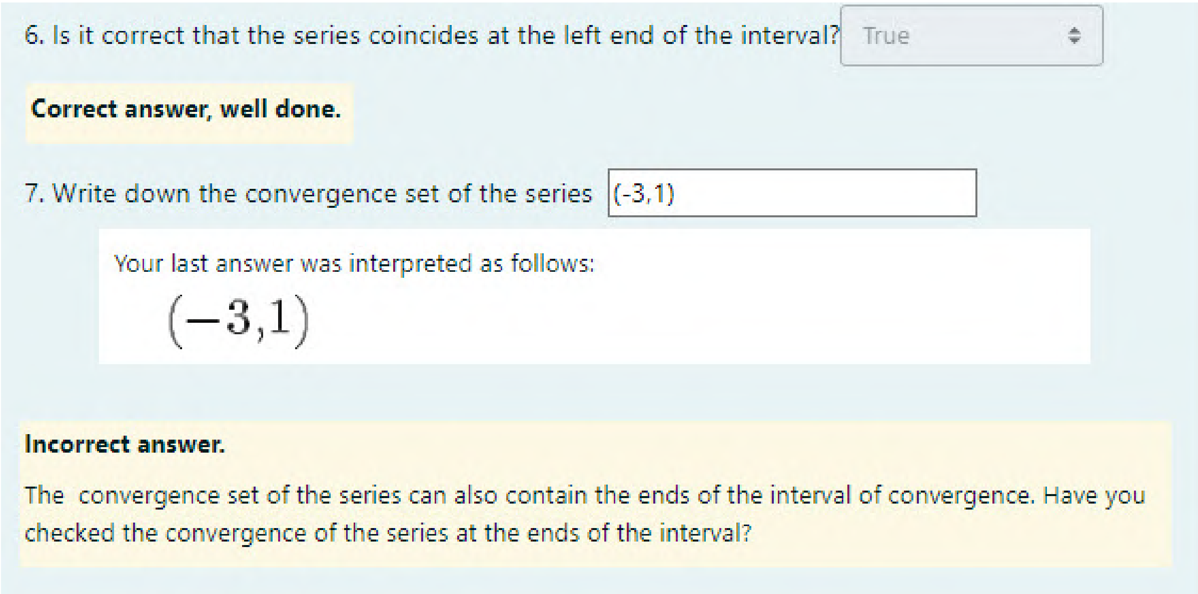

Figure 7: A fragment of the student's answer, which contains a comment on the wrong answer.

4. Conclusion

1. Based on the understanding of the specificity of mathematics as a science and educational discipline, special requirements for adaptive teaching of mathematics and adaptive tools are formulated to support self-regulated acquisition of knowledge by students and their achievement of optimal educational results.
2. Specific examples illustrate the expediency of using STACK in adaptive teaching of mathematics for creating and conducting interactive tests in the LMS Moodle environment. Using computer algebra, STACK automatically checks student answers, providing detailed feedback. This allows you to create interactive tests that not only check the correctness of the answer, but also consider different ways of solving the problem, which is important for an adequate assessment of knowledge in mathematics.
3. Student reviews show that the use of tests in STACK improves understanding of the material, helps to gradually increase the level of knowledge and the formation of skills, allows you to learn at your own pace. They also note the convenience and usefulness of instant and meaningful feedback.
4. As part of this study, a full-fledged experiment is planned, which will include statistical or other mathematical processing of the results to evaluate the effectiveness of using STACK tests to support adaptive mathematics learning.

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