Optimization geometric design in intelligent systems for ensuring safety

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Abstract
Packing optimization problems have a wide spectrum of real-word applications. In today's technologically advanced world, the need for safety systems is ubiquitous and paramount. From industrial processes to everyday applications, safety systems play a crucial role in ensuring the smooth and secure operation of various systems. One such critical application is the safe storage of spent nuclear fuel (SNF), a significant scientific problem in the present day. The solution of the problem can be reduced to the solution of the problem of finding the optimal placement of a given set of congruent circles into a multiconnected domain taking into account technological restrictions. A mathematical model of the problem is constructed and its peculiarities are considered. Our approach is based on the mathematical modelling of relations between geometric objects by means of phi-function technique. That allowed us to reduce the problem solving to nonlinear programming.

Keywords
Intelligent systems for ensuring safety, optimization packing problem, mathematical modeling, phi-function, non-linear programing

1. Introduction

In the technologically advanced world of today, safety systems are ubiquitous and paramount. They play a crucial role in ensuring the smooth and secure operation of various systems across a wide range of applications. These safety systems are designed to protect both people and infrastructure from potential harm, and their importance cannot be overstated.

Safety systems are particularly critical in industries where the consequences of system failure can be catastrophic, such as in nuclear power plants, chemical manufacturing, and aviation [1]. In these sectors, safety systems are designed to prevent accidents and mitigate the effects of any that do occur.

In addition to protecting against potential threats, safety systems also play a key role in the detection and response to incidents. Through continuous monitoring and advanced analytics, modern safety systems can quickly identify potential issues and initiate appropriate response measures.

One of the areas where these safety systems are particularly crucial is in the storage of hazardous materials, such as flammable liquids, gases, and chemically active substances. The safe storage of these materials is not just about containing them in a secure manner, but also about ensuring that they are stored in a way that minimizes the risk of accidents and facilitates quick response in case of incidents.

This is where the specific problem of safe storage of flammable liquids, gases, and chemically active substances in containers comes into play. The challenge lies not only in the physical storage of these substances but also in the strategic placement of the containers. This involves considering factors such as the type of substance, its quantity, the type of container, and the distance between containers.

[1]ICST-2024: Information Control Systems & Technologies, September , 23 – 25, 2024, Odesa, Ukraine

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CEUR-WS.org/Vol-3790/paper45.pdf
CEUR Workshop Proceedings ceur-ws.org ISSN 1613-0073

The importance of maintaining appropriate separation distances between containers storing hazardous materials is well-documented in the literature. For instance, a study [2] discusses how safety distance determination is a key design issue in the process industry. The paper presents a risk-based methodology for determining safety distances, which encompasses all major hazard scenarios including jet fires, flash fires, explosions, boiling liquid expanding vapour explosion, and toxic releases.

Article [3] addresses the critical issue of safety distance determination in the process industry. The authors emphasize that this determination is often made too late in the project lifecycle, potentially leading to safety concerns or inefficient use of space.

The paper [4] presents state-of-the-art research on determining safety distances globally. The specific safety distances can vary depending on the type of substance, the quantity, and the specific regulations and guidelines in place.

The problem of finding the optimal placement of containers, considering given technological restrictions, can be formulated as an optimization problem of geometric design [5-7]. The theory of optimization geometric design is one of the tools for studying and optimizing complex technical systems to achieve their optimal functioning state. It is designed to solve a number of applied optimization problems of placing geometric objects. These problems are associated with the creation of energy- and resource-saving technologies in priority sectors of the national economy (energy, machine-, ship-, aircraft-building, construction, chemical industry, as well as in scientific research in the field of nanotechnologies, in modern tasks of biology, mineralogy, medicine, materials science, in robotics, tasks of coding information, systems of image recognition, control systems of spacecraft) during the automation and modeling of the processes of placing various objects.

Various heuristic approaches for solving the problem of placing equal circles in a circular container have been proposed [8,9]. Currently, many articles consider the task of packing a fixed set of objects into a given object, with the aim of minimizing its dimensions. Several articles solve this problem using nonlinear models and optimization methods [10]. A large variety of nonlinear programming models for the task of minimizing object sizes in 2D packing tasks (square, rectangle, triangle, and circle) is also considered.

The aim of the study is to build an adequate mathematical model of the problem of finding the optimal placement of containers and to develop effective methods of finding a solution. This paper focuses on creating an intelligent system that determines the optimal arrangement of containers on a storage area. By formulating the problem as an optimization problem involving the placement of congruent circles within a multiconnected domain, while adhering to stringent technological and safety constraints, we address a critical issue in nuclear, thermal and chemical safety. The system leverages advanced mathematical modeling techniques, particularly the phi-function method, to represent the geometric relationships between objects. This transformation simplifies the problem into a nonlinear programming one.

The proposed intelligent system integrates these mathematical models and algorithms to provide a robust solution for the safe and efficient storage. Key features and benefits of the system are highlighted, showcasing its potential to enhance operational safety and storage efficiency various facilities.

2. Problem formulation

The overarching objective of geometric design optimization can be articulated as follows: to ascertain a spatial configuration of a specific set of geometric entities within a designated container that adheres to all rules, requirements, and technological constraints, such that the optimization criterion achieves an extreme value.

In order to frame the problem of container placement with safety system in the context of geometric design optimization, it is imperative to analytically define some elements of the system.

- The spatial form of the placement area
- The spatial form of the entities to be placed within the placement area

- The technological conditions for positioning a specified set of entities within the placement zone
- The optimization criterion

Let's delve into each of these components.

Spatial form of the placement region. The external contour of the geometric shape can be determined in alignment with the topographic conditions of the selected locality.

The topological form of the placement area can be in general multiply connected due to the presence of areas where container placement is not feasible (prohibition areas) (1):

$$A = A_0 \setminus \operatorname{int} \bigcup_{l=1}^{\sigma} P_l \, . \tag{1}$$

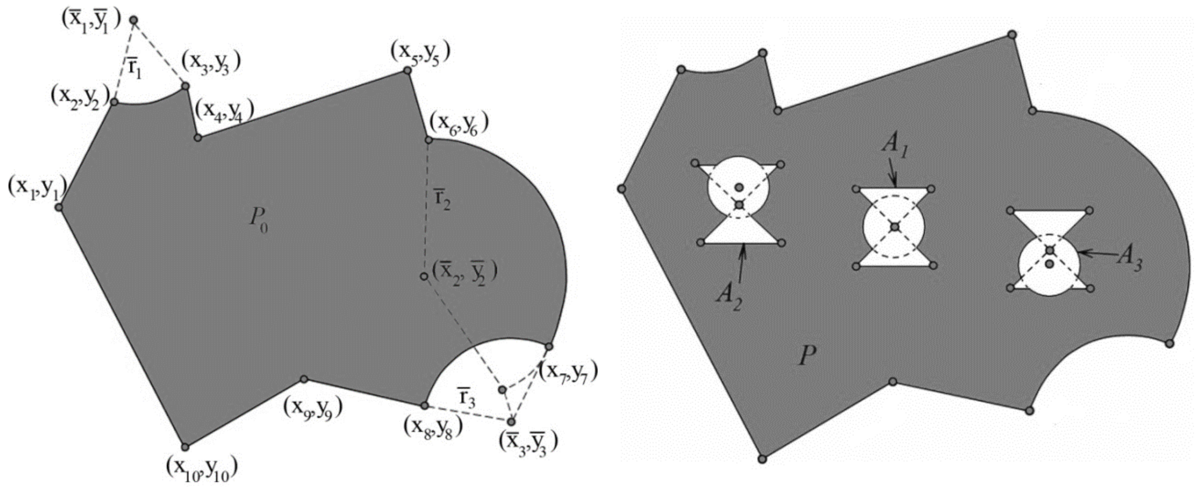
For mathematical modeling of the problem, the external boundary of the area is approximated with a collection of line segments and circle arcs. So we assume that A_0 is a connected set, the frontier of which is formed with line segments $[x_i, y_i, x_{i+1}, y_{i+1}]$ and circle arcs $[x_i, y_i, x_{i+1}, y_{i+1}, \bar{x}_j, \bar{y}_j, \bar{r}_j]$.

Here (x_i, y_i) and (x_{i+1}, y_{i+1}) are the coordinates of the start and end of the segment or arc, respectively; $(\bar{x}_j, \bar{y}_j)$ and $\bar{r}_j$ are the center coordinates and radius of the circle, respectively; A_l is a prohibited area which can be represented as (2)

$$P_l = \bigcup_{g=1}^{\psi} C_{lg} \bigcup_{q=1}^{\vartheta} M_{lq} \, , l \in I_\sigma = \{1, 2, ..., \sigma\} \, ; \tag{2}$$

$$C_{lg} = \{(x, y) \in R^2 : (x - x_{lg}^0)^2 + (y - y_{lg}^0)^2 - (\rho_{lg}^0)^2 \le 0\}, g \in I_\psi = \{1, 2, ..., \psi\},$$

where M_{lq} is a convex polygon, specified by m_{lq} vertices, $q \in I_\vartheta = \{1, 2, ..., \vartheta\}$, i.e., each prohibited area is a non-convex set that can be represented as a finite union of different circles and convex polygons (Figure 1).



(a) (b)

Figure 1: An example of the area: (a) simply connected domain; (b) multiply connected domain

The methodology delineated for demarcating the area *P* facilitates an approximation of any given form with a reasonable degree of accuracy.

The spatial form of the objects to be placed in the placement area. Let's assume a set of *N* cylindrical containers $K_i, i \in I_N$, with nuclear power plants and a site for their storage are given. An important feature of the problem is all containers being congruent (Figure 2). The containers should be located only on one level (placement of containers one above the other is not allowed).

Considering the specified features, the set of containers can be represented as a set of congruent circles K_i, $i \in I_N$, can be represented as a set of congruent circles C_i with radii r, $i \in I_N$. Let's denote

$u_i = (x_i, y_i)$ as the coordinates of the center of circle C_i. Then, the location of all C_i, $i \in I_N$, in $\mathbf{R}^2$ can be determined using the vector $u = (u_1, u_2, ..., u_N) \in R^{2N}$. Hereafter, the circle C_i translated by the vector $u_i = (x_i, y_i)$ is denoted as $C_i(u_i)$.

The technological restrictions for placing a given set of objects within the placement area can be divided into two types of constraints.

Figure 2: The spatial form of the containers

The first type of constraints is dictated by the need to ensure necessary safety parameters, namely, not exceeding the overall level of ionizing radiation and adhering to the thermal storage regimes of spent nuclear fuel.

To describe the first type of container placement conditions, we formalize the impact of each container on the overall thermal regime and the level of ionizing radiation of the site. Each container being characterized by its physical properties (temperature and level of ionizing radiation), which need to be taken into account to comply with safe storage conditions, we associate an integral indicator of thermal and radiation properties of the container with spent nuclear fuel to each circle C_i.

This parameter will determine the integral level of influence of each container on the temperature of the containers located nearby and, at the same time, on the overall thermal level and the level of ionizing radiation of the area *P.*

We assume that the values of the integral coefficients k_i are determined by expert judgment k_i Then, according to the values of k_i, $i \in I_N$, we distribute the circles C_i, $i \in I_N$, into groups G_j, $j \in I_g$. Let each group consist of q_i, $i \in I_g$, circles, where g is the number of groups obtained.

In order to minimize the mutual influence of ionizing radiation and the temperature regime of the containers, we set the minimum permissible distances $d_{ij}^g, i, j \in I_g$, between the circles of each group G_i, $i \in I_g$, and between the circles within one group.

Thus, taking into account the first type of technological constraints will ensure an increase in fuel temperatures and the level of ionizing radiation from containers with spent nuclear fuel, and ensure a uniform distribution of ionizing radiation within the site when storing spent fuel on it.

The second type of constraints is conditioned by ensuring the conditions for servicing the containers.

It is necessary to provide the possibility of approaching each container with special service equipment in order to rotate the container or move the container within the site.

To ensure this condition, it is necessary to consider the placement on the site of the so-called "service network" for moving equipment. Let's assume that for moving equipment, it is necessary to ensure the presence of lanes with a width of d.

Then, when placing objects, it is necessary to ensure the condition of touching C_i, $i \in I_N$, with a lane of a given width d, which will ensure the approach of special service equipment to the container.

The criterion for optimizing the placement of objects. As a criterion for optimization, we will choose to find the maximum filling of the selected site with circles C_i from the set I_N.

Thus, after formalizing all the conditions of the optimization problem of geometric design in an analytical form, we will formulate the problem statement as follows.

Problem. Determine the vector $u = (u_1, u_2, ..., u_n) \in \mathbf{R}^{2n}$, that ensures the placement of the maximum number of circles from the set C_i, $i \in I_N$, in the given area P while ensuring the fulfillment of the specified technological constraints (Figure 3).

Figure 3: Formulation of the problem of placing containers

Based on the problem statement, we can construct a mathematical model of the problem and conduct a study of its properties.

3. Mathematical model

One of the most crucial and complex tasks in computer and mathematical modeling of this class of problems is analytically describing the interaction between circles and the area. In this study, we will use the method of phi-functions, as presented in works such as [11, 12]. This method is currently regarded as the most effective for solving similar problems.

Formalization of the conditions for packing circles C_i in the area P is based on construction of the following set $G = cl(\mathbf{R}^2 \setminus P_0)$. This set can always be represented as a finite union of basic objects Q_{ij}, (3) i.e.

$$G = \bigcup_{j=1}^{\delta} Q_{1j} \bigcup_{j=1}^{\gamma} Q_{2j} \bigcup_{j=1}^{\xi} Q_{3j} \bigcup_{j=1}^{\zeta} Q_{4j}, \tag{3}$$

where $Q_{1j} = \{X \in \mathbf{R}^2 : \chi_{1j}(X) \geq 0\}$, $j \in J_\delta = \{1, 2, ..., \delta\}$, $\chi_{1j}(X) = a_{1j}x + b_{1j}y + c_{1j}$, $Q_{2j} = \{X \in \mathbf{R}^2 : \chi_{2jl}(X) \geq 0,\ l = 1, 2\}$, $j \in J_\gamma = \{1, 2, ..., \gamma\}$, $\chi_{2jl}(X) = a_{2jl}x + b_{2jl}y + c_{2jl}$, $Q_{3j} = \{X \in \mathbf{R}^2 : \widehat{\omega}_{3j}(X) \geq 0\}$, $j \in J_\xi = \{1, 2, ..., \xi\}$, $\widehat{\omega}_{3j}(x, y) = \widehat{r}_j^2 - (x - \widehat{x}_j)^2 - (y - \widehat{y}_j)^2$, $Q_{4j} = \{X \in \mathbf{R}^2 : \chi_{4jl}(X) \geq 0,\ \breve{\omega}_{4j}(X) \geq 0,\ l = 1, 2\}$, $j \in J_\zeta = \{1, 2, ..., \zeta\}$, $\chi_{4jl}(X) = a_{4jl}x + b_{4jl}y + c_{4jl}$, $\breve{\omega}_{4j}(X) = (x - \breve{x}_j)^2 + (y - \breve{y}_j)^2 - \breve{r}_j^2$, $\rho(A, B) \geq 2r$, $\rho(A, B)$ is the distance between points A and B, i.e. Q_{1j} is a half-plane, Q_{2j} is a convex cone; Q_{3j} is a circle and Q_{4j} is the intersection of the half-plane and the complement of the circle to $\mathbf{R}^2$.

To solve the problem, we propose an approach that allows us to reduce the solution of the problem to solving a sequence of problems with linear objective functions. For this purpose r_i for C_i, $i \in I_\tau$, are supposed to be variable. The vector of radii is $v^\tau = (r_1, r_2, ..., r_\tau) \in \mathbf{R}^\tau$.

The values of the phi-function are certain measures of both the intersection of two geometric objects and the shortest distance between them, depending on their mutual arrangement in space. Using phi-functions for primary objects and complex 2D objects, the mathematical model of the sequence of problems can be represented as follows (4), (5):

$$F_n(X^{n*}) = \max F_n(X^n) = \max \sum_{i=1}^{n} r_i \tag{4}$$

where

$$X^n = (u^n, v^n) \in W_n, \quad n = 1, 2, ..., \tau + 1, \tag{5}$$

$\Phi_i(X^n)$ is phi-function for C_i and $G_1 = cl(\mathbf{R}^2 \setminus P)$,

I_C is a set of clusters that include circles C_i. Function $\Phi_i(u_i, r_i)$ describes the belonging of circle C_i to the area P.

The area of feasible solutions in mathematics is described by two types of constraints. The first type of constraints sets the conditions for the placement of objects within a given area of feasible solutions, considering the specified permissible distances. The second type of constraints describes the conditions for the placement of objects at specified technological distances. Both types of constraints are described using phi-functions from systems (6). The objective function maximizes the sum of the radii of the placed circles subject to restrictions on their maximum sizes.

A phi-function for C_i and G_1 can be represented as follows (6):

$$\Phi_i = \min\left\{\Phi_{il}^{CA},\ \Phi_i^{CG},\ l \in I_\sigma\right\}, \tag{6}$$

where

$$\Phi_{il}^{CA} = \min\left\{\Phi_{ilg}^{CC}, \Phi_{ilq}^{CM}, g \in I_\psi, q \in I_\vartheta\right\}, \tag{7}$$

Φ_{il}^{CA} is a phi-function for C_i and A_l;

$$\Phi_{ilg}^{CC} = (x_i - x_{lg}^0)^2 + (y_i - y_{lg}^0)^2 - (r_i + \rho_{lg}^0)^2, \tag{8}$$

Φ_{ilg}^{CC} is a phi-function for C_i and C_{lg};

$$\Phi_{ilq}^{CM} = \max_{k=1,2,...,m_{lq}} \left\{\max\left\{\min\left\{\psi_{ilqk}, \omega_{ilqk}\right\}, \chi_{ilqk}^*\right\}\right\}, \tag{9}$$

Φ_{ilq}^{CM} is a phi-function for C_i and M_{lq};

$$\Phi_i^{CG} = \min\left\{\Phi_{ij}^{CQ_1}, j \in I_\delta, \Phi_{ij}^{CQ_2}, j \in I_\gamma, \Phi_{ij}^{CQ_3}, j \in I_\xi, \Phi_{ij}^{CQ_4}, j \in J_\zeta\right\}, \tag{11}$$

Φ_i^{CG} is a phi-function for C_i and G;

$$\Phi_{ij}^{CQ_1} = \chi_{i1j}^* = -\chi_{1j} - r_i, \tag{12}$$

$$\Phi_{ij}^{CQ_2} = \max\left\{\min\{\psi_{ij}, \omega_{ij}\}, \chi_{i2j1}^*, \chi_{i2j2}^*\right\}, \tag{13}$$

$$\Phi_{ij}^{CQ_3} = (x_i - \hat{x}_j)^2 + (y_i - \hat{y}_j)^2 - (r_i + \hat{r}_j)^2, \tag{14}$$

$$\Phi_{ij}^{CQ_4} = \max\left\{\varphi_{ij1}, \varphi_{ij2}, \varphi_{ij3}, \omega_{ij3i}, \chi_{i4j1}^*, \chi_{i4j2}^*\right\}, \tag{15}$$

$\Phi_{ij}^{CQ_1}$, $\Phi_{ij}^{CQ_2}$, $\Phi_{ij}^{CQ_2}$, $\Phi_{ij}^{CQ_4}$ are phi-functions for C_i and basic objects.

The components of the function $\Phi_{ij}^{CQ_4}$ are as follows (16),(17):

$$\varphi_{ij1} = \min\left\{\omega_{ij1}, \omega_{ij2}, \psi_{ij1}, \psi_{ij2}, \psi_{ij3}\right\}, \tag{16}$$

$$\varphi_{ij2} = \min\{\omega_{ij1}, \psi_{ij4}\},\ \varphi_{ij3} = \min\left\{\omega_{ij2}, \psi_{ij5}\right\}, \tag{17}$$

$$\omega_{ij1} = (x_i - x_{j1})^2 + (y_i - y_{j1})^2 - r_i^2,\ \omega_{ij2}(X^n) = (x_i - x_{j2})^2 + (y_i - y_{j2})^2 - r_i^2,$$

$$\omega_{ij3} = (\breve{r}_j - r_i)^2 - (x_i - \breve{x}_j)^2 - (y_i - \breve{y}_j)^2$$

$$\psi_{ij1} = a_{j1}x_i + b_{j1}y_i + c_{j1},\ a_{1j} = y_{j4} - y_{j3},\ b_{j1} = -(x_{j4} - x_{j3}),\ c_{1j} = -(a_{1j}x_{j3} + b_{1j}y_{j3}),$$

$$\psi_{ij2} = a_{j2}x_i + b_{j2}y_i + c_{j2},\ a_{j2} = y_{j6} - y_{j5},\ b_{j2} = -(x_{j6} - x_{j5}),\ c_{j2} = -(a_{j2}x_{j5} + b_{j2}y_{j5}),$$

$$\psi_{ij3} = a_{j3}x_i + b_{j3}y_i + c_{j3},\ a_{j3} = y_{j5} - y_{j4},\ b_{j3} = -(x_{j5} - x_{j4}),\ c_{j3} = -(a_{j3}x_5 + b_{j3}y_5),$$

$$\psi_{ij4} = a_{j4}x_i + b_{j4}y_i + c_{j4},\ a_{j4} = y_{j3} - y_{j7},\ b_{j4} = -(x_{j3} - x_{j7}),\ c_{j4} = -(a_{j4}x_{j3} + b_{j4}y_{j3}),$$

$$\psi_{ij5} = a_{j5}x_i + b_{j5}y_i + c_{j5},\ a_{j5} = y_{j8} - y_{j6},\ b_{j5} = -(x_{j8} - x_{j6}),\ c_{j5} = -(a_{j5}x_{j6} + b_{j5}y_{j6}),$$

$$\chi^*_{i4j1} = -\chi_{4j1} - r_i,\ \chi^*_{i4j2} = -\chi_{4j2} - r_i.$$

If the boundary of the set Q_{4j} is formed by one straight line, then $\Phi_{ij}^{CQ_4}$ can be written more simply (18)

$$\Phi_{ij}^{CQ_4} = \max\left\{\varphi_{ij1}, \omega_{ij3}, \chi^*_{i4j1}\right\}. \tag{18}$$

Thus, $\Phi_i \geq 0$ holds if at least one of the systems of inequalities of the form

$$\Gamma_i^s = \begin{cases} \Phi_{ij}^{CQ_1} \geq 0, j \in J_\delta, \\ \Phi_{ij}^{CQ_2} \geq 0, j \in J_\gamma, \\ \Phi_{ij}^{CQ_3} \geq 0, j \in J_\xi, \\ \Phi_{ij}^{CQ_4} \geq 0, j \in J_\zeta, \\ \Phi_{ilg}^{CC} \geq 0, l \in I_\sigma, g \in I_\psi, \\ \Phi_{ilq}^{CM} \geq 0, l \in I_\sigma, q \in I_\vartheta \end{cases} \tag{19}$$

is satisfied where $\Phi_{ij}^{CQ_2} \geq 0$ is either one of the inequalities $\chi^*_{i2j1} \geq 0$, $\chi^*_{i2j2} \geq 0$ or the inequality system

$$\begin{cases} \omega_{ij} \geq 0, \\ \psi_{ij} \geq 0; \end{cases} \tag{20}$$

$\Phi_{ij}^{CQ_4} \geq 0$ is either one of the inequalities $\omega_{ij3} \geq 0$, $\chi^*_{i4j1} \geq 0$, $\chi^*_{i4j2} \geq 0$ or one of the inequality systems (21)

$$\begin{cases} \omega_{ij1} \geq 0, \\ \omega_{ij2} \geq 0, \\ \psi_{ij1} \geq 0, \\ \psi_{ij2} \geq 0, \\ \psi_{ij3} \geq 0, \end{cases} \quad \begin{cases} \omega_{ij1} \geq 0, \\ \psi_{ij4} \geq 0, \end{cases} \quad \begin{cases} \omega_{ij2} \geq 0, \\ \psi_{ij5} \geq 0; \end{cases} \tag{21}$$

$\Phi_{ilq}^{CM} \geq 0$ or the inequality $\chi^*_{ilqk} \geq 0$ or the inequality system

$$\begin{cases} \omega_{ilqk} \geq 0 \\ \psi_{ilqk} \geq 0 \end{cases}.$$

4. General solution approach

To solve the problem, a multi-stage methodology is proposed for packing container considering specified technological constraints. At each stage, methods of nonlinear optimization and modern NLP solvers are applied.

The proposed methodology is based on a multi-stage solution search. For optimal filling of an area considering given constraints, the first stage involves solving an optimization problem to place the maximum number of containers in a complex area with restricted zones. To ensure safety from increased thermal and ionizing levels and to achieve uniform distribution of ionizing radiation within the area when storing spent nuclear fuel, constraints on the minimum allowable distances between groups of containers are considered. At this stage, a modification of the feasible direction method with an active set strategy is developed for local optimization. For global optimization, a sequential statistical optimization method is designed.

In the second stage, to ensure servicing conditions for containers on the area, the problem of placing clusters of various geometric shapes is addressed. These clusters must maintain specific distances to allow the passage of service equipment. A nonlinear optimization method based on the interior point method with a special decomposition algorithm is developed to solve this problem.

In the third stage, the total ionizing field of the area is calculated. If its value does not meet the established criteria, the problems from the first two stages are revisited iteratively until the desired field parameters are achieved.

To solve the problem in the first stage, a strategy has been developed based on the following sequence of methods:

- for constructing starting points, the regular placements method and the block coordinate descent method are utilized
- for local extrema search, a modified method of feasible directions combined with an active set strategy on subdomains is employed
- for approaching the global extrema, a modified method of narrowing neighborhoods is applied.

The main strategy focuses on optimizing the objective function defined over a set of permutations. To ensure effective global extremum search, the objective function should be quasi-separable and have multiple extrema. The number of local extrema should be such that their distribution law can be asserted, considering each as a realization of a random variable.

To construct starting points within the feasible domain, methods are employed that use a sequence of placing three-dimensional geometric objects. This involves the block coordinate descent method or the regular placements method, which focuses on arranging congruent three-dimensional geometric objects.

Due to the possibility of establishing correspondence between permutations of three-dimensional geometric objects and local extrema in the problem under consideration, a strategy is employed to approximate the global extremum using a modified method of narrowing neighborhoods.

The method employs a randomized search aimed at optimizing the objective function defined over a set of permutations. The narrowing neighborhoods method is based on the probabilistic distribution properties of local extrema of the objective function. It allows organizing the exploration of sequences of three-dimensional geometric objects to be placed in a way that, within a relatively short time, a solution close to the global extremum of the problem can be obtained.

To implement this method, a specific metric is introduced in the space of permutations. The search for optimal values of the objective function takes place within neighborhoods defined over the permutation set. At each step of the method, centers and radii of new neighborhoods are selected based on accumulated statistical information. If the objective function value does not improve during the transition to the next search stage, the neighborhood radius is reduced. This process results in a convergent sequence.

As is well-known, nonlinear optimization methods require a feasible starting point. Among the techniques commonly used for constructing initial points in problems related to the placement of three-dimensional geometric objects, various modifications of ‘greedy’ algorithms are prevalent. However, due to the NP-hard nature of geometric packing problems, the application of ‘greedy’ algorithms significantly limits the exploration of a vast number of local extrema (whose count exceeds n!).

The use of the phi-function method for constructing a mathematical model enables the application of modern nonlinear optimization techniques at all stages of problem-solving, including initial point generation, local extremum search, and exploration of local extrema.

In this regard, a specialized approach is proposed for constructing feasible points. The main idea involves increasing the dimensionality of the problem by introducing metric variables related to geometric object characteristics and their homothetic transformations.

In the context of geometric objects allowing homothetic transformations, we introduce variable coefficients for the homothety of these objects. To determine a feasible starting point, we randomly generate coordinates for the placed geometric objects within a container. Subsequently, we solve a nonlinear programming problem maximizing the sum of homothety coefficients for all geometric objects. If this optimization yields a point corresponding to a local maximum where all homothety coefficients are equal to one, we accept it as the starting point for searching the local extremum of the main problem. Notably, unlike 'greedy' algorithms for constructing starting points, which may produce good but repetitive points, the developed method allows for diverse initial points through random coordinate generation for the centers of geometric objects.

Given the large number of inequalities defining the feasible region, direct application of nonlinear optimization methods for finding local extrema would result in significant computational costs. Therefore, a specialized decomposition method has been developed to search for local extrema in formulated optimization problems. This approach substantially reduces computational expenses by significantly decreasing the number of inequalities during the search for local extrema. By leveraging the fact that the feasible solution space can be represented as a union of subdomains, we can notably reduce the time required for finding local minima by solving a sequence of subproblems, each defined with a much smaller set of inequalities

The key idea of the method allows selecting a subregion within the feasible region at each stage and generating subregions of the chosen subregion iteratively. Based on an analysis of the starting point, an additional system of constraints is introduced for the placement parameters of each object, enabling movement within individual containers. Next, inequalities are removed for all pairs of three-dimensional geometric objects whose individual containers do not intersect. By doing so, we reduce the number of constraints and, in the case of quasi-phi functions, the number of additional variables. Subsequently, a search for a local minimum point is conducted for the constructed subproblem. The obtained local extremum of the subproblem serves as the starting point for the next iteration.

5. Local optimization in the problem of packing congruent objects considering prohibited areas

To find local maxima in the considered problem, which is a nonlinear programming problem, an optimization method based on the feasible direction strategy is applied.

Based on the properties of the constructed mathematical model, to find the local minimum corresponding to the obtained initial point $X^i \in W$, a subregion is highlighted $W_{k_1} \subset W$, that contains X^i and the local minimum is calculated X^{1*} in this subregion. If there are other subregions $W_{k_j} \subset W$, for which $X^{1*} \in W_{k_j}$, $j = 1, 2, ..., \vartheta$, and X^{1*} is not a local minimum, then the problem of finding a local minimum on one of these subregions is solved again. The process is repeated until a local minimum of the basic problem is found. Thus, the calculation of the local minimum of the problem can be reduced to the solution of a sequence of problems of nonlinear programming of the form (22)

$$F(X^{j*}) = \min_{X \in W_{k_j}} F(X), \quad j = 1,2,...,m << \eta_0 \; . \tag{22}$$

To solve the problem (22), a modification of the Zoutendijk's method of feasible directions together with the strategy of ε-active inequalities are used. A standard iterative process is used to find the local minimum for each of the problems $X^{(k+1)} = X^k + tZ^k, \; k = 1,2,...,\zeta$, where $Z^k \in R^{3n+1}$ – solving the following problem of linear programming (23),(24)

$$\max_{(\alpha^k, Z^k) \in G^k} \alpha^k , \tag{23}$$

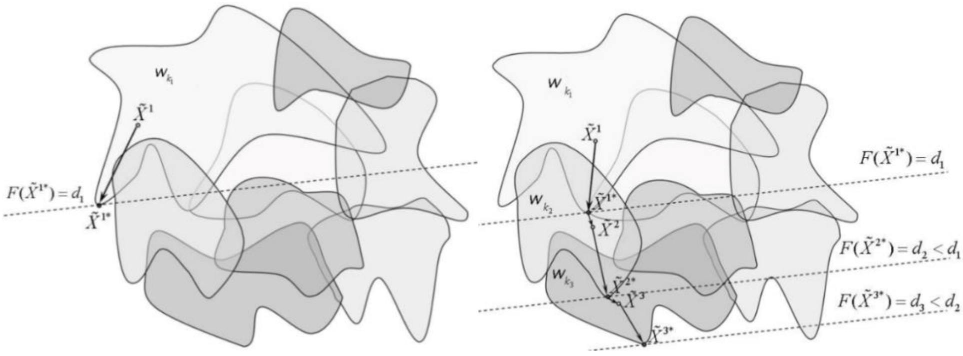
$$G^k = \Big\{ (\alpha^k, Z^k) \in R^{3n+1} : \left(-\nabla F(X^k), Z^k \right) \ge \alpha^k,$$
$$\left(\nabla \Psi_{k_j}(X^k), Z^k \right) \ge \alpha^k,$$
$$j = 1,2,...,\varsigma_k(\varepsilon_k), -1 \le z_i^k \le 1, i = 1,2,...,3n+1 \Big\}, \tag{24}$$

where $\Psi_{k_j}(X^k)$ is left parts of ε-active inequalities from the system separated from the system at a point X^k.

The transition from one task to another is carried out in the following way. Let $X^i = \bar{X}^1 \in W$ is the starting point. Then from the system specifying the subregion W_{k_1}, the defining system is chosen $W_{k_1} \subset W$, such that $\bar{X}^1 \in W_{i_1}$. Using a point $\bar{X}^1$ as a starting point, the problem is solved

$$F\,(X^{1*}) = \min_{X \in W_{k_1}} F(X) \, . \tag{25}$$

Point received X^{1*} can be either a local minimum over the entire area W (Figure 4,a) , or geometric objects relative to the subregion W_{i_1} (Figure 4,b).



(a) (b)

Figure 4: The scheme of finding a local minimum on subregions of the feasible domain

In order to determine whether X^{1*} a local minimum relative to W, it is necessary to investigate subregions W_{k_j} with $X^{1*} \in W_{k_j}$, $j \in \{1,2,...,\eta_0\}$. For this purpose, all are chosen from the system ε-active inequalities in X^{1*} and problem is solved.

If $\alpha > 0$, then X^{1*} is not a local minimum of the basic problem of the form (1) – (2). This makes it possible to calculate a new point $\bar{X}^2 = (X^{1*} + tZ) \in W$, in which $F(X^{1*}) > F(\bar{X}^2)$. After that, a new system of inequalities is formed that defines the subdomain $W_{k_2} \subset W$, such that $\overline{X}^2 \in W_{k_2}$. Using $\overline{X}^2$ as a starting point, the problem is solved

$$F(X^{2*}) = \min_{X \in W_{k_2}} F(X). \tag{26}$$

The described process continues until a local minimum of the basic problem is obtained.

It should be noted that the search for local extrema on subdomains from the domain of admissible solutions made it possible to significantly reduce the computational complexity of the method and time spent on the search for a local extremum.

6. Computational results

To test the effectiveness of the constructed mathematical model and the proposed approach for solving the problem, we addressed the problem of packing 80 cylindrical containers containing hazardous waste on an area with a complex geometry (as depicted in Figure 5,a). The geometric contours of the area are defined by a sequence of line segments and circular arcs. Within the placement area, a zone prohibits container placement. Additionally, according to the problem statement, there are technological constraints on container placement. The results of solving this problem are shown in Figure 5,b.

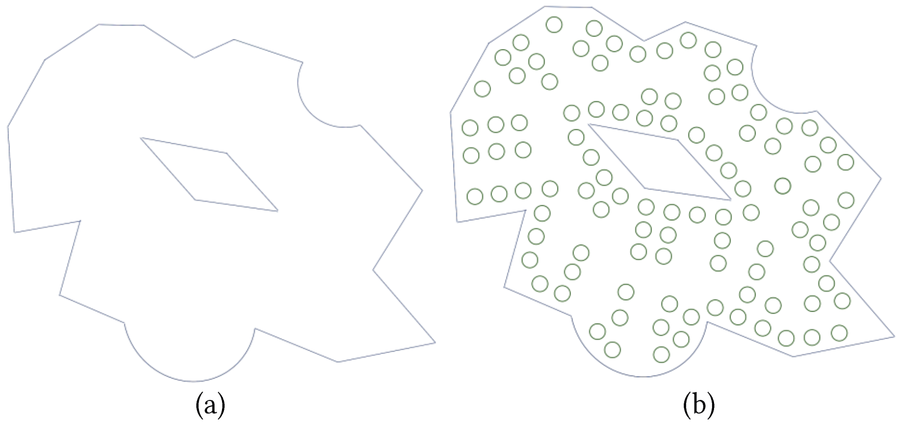


Figure 5: A numerical example

The computational time required to solve the problem illustrated in the figure was 45 minutes and 40 seconds. Our solution utilized a computer equipped with an Intel(R) Core i5-10400F 2.90GHz processor and 16 GB of RAM, running software developed in C#.

Recent advancements in optimization methodologies, particularly in nonlinear optimization, have revolutionized approaches to solving optimization problems. These advancements greatly enhance the reliability, speed, and accuracy of locating both local and global solutions.

They are applicable across diverse domains, leveraging user-developed external procedures for computing objective functions, residual constraints, and Jacobi and Hessian matrices.

In this study, we leveraged the IPOPT library [13] to enhance the efficiency of searching for local extrema within subregions. IPOPT excels particularly in large-scale packing optimization tasks due to its ability to efficiently handle high-dimensional problems. Such problems often involve numerous variables and constraints, making them challenging for traditional optimization methods to resolve within reasonable timeframes as problem size increases. However, IPOPT's advanced algorithms, based on interior-point methods, navigate high-dimensional solution spaces swiftly and accurately.

IPOPT uses powerful strategies to exploit the inherent structure of optimization problems, ensuring efficient search for optimal solutions while adhering to all constraints. By leveraging sparse linear algebra and problem-specific properties, IPOPT scales effectively to manage problems with up to millions of variables. This capability makes IPOPT indispensable for large-scale packing optimization tasks. Moreover, IPOPT provides ample customization capabilities and interfaces, allowing users to adjust optimization procedures to suit particular requirements and smoothly integrate them into current operational workflows.

7. Conclusion

In this work, the problem of optimally placing containers with nuclear waste, considering given technological constraints, is formulated as a geometric design optimization problem. All the conditions of the geometric design problem are described in detail. A mathematical model for the problem of packing congruent circles into a multiply connected region, whose boundary consists of arcs of circles and line segments, has been constructed. This model is presented as a nonlinear optimization problem.

The method of phi-functions is utilized to construct a mathematical model where the feasible solution region can be represented as a union of subregions. Each subregion is described by systems of inequalities with continuous functions on the left-hand sides. This representation allows for the application of modern nonlinear optimization methods to solve the problem effectively.

The results of this study hold substantial practical significance, particularly in the context of optimizing safety-critical systems. The developed intelligent system for the optimal placement of containers is directly applicable to the storage of spent nuclear fuel and other hazardous materials. By leveraging advanced mathematical modeling techniques this approach addresses complex real-world constraints, such as spatial limitations, safety regulations, and technological restrictions.

The effectiveness of the proposed model is demonstrated through its application to a practical problem: the optimal packing of 80 cylindrical containers in a complex, multiconnected storage area with prohibited zones. This problem, reflective of real-world challenges, underscores the system's capability to enhance the safety and efficiency of storage operations in nuclear, thermal, and chemical industries.

The practical implications extend beyond theoretical advancements, offering tangible benefits in operational settings. The system's ability to handle nonlinear programming challenges with high precision and reliability ensures that it can be effectively integrated into existing safety protocols. This integration can lead to improved storage density, reduced risk of accidents, and optimized use of available space—all of which are critical in environments where safety is paramount.

Moreover, the flexibility of the approach allows for its adaptation to a wide range of applications, from industrial storage facilities to transportation of hazardous materials. As safety regulations continue to evolve and become more stringent, the need for such intelligent systems will only grow, making the results of this study not only relevant but essential for future developments in safety optimization.

In summary, the practical significance of the obtained results lies in their ability to provide a robust, efficient, and scalable solution to the pressing problem of safe storage of hazardous materials, with potential applications across multiple domains where safety and optimization are critical.

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