Statistical Syllogistic Tableaux

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Abstract
After presenting Sommers and Englebretsen's Term Functor Logic, and Thompson's statistical syllogistic, we produce some tableaux for a fragment of Thompson's syllogistic.

Keywords
Tableaux, syllogistic, Term Functor Logic

1. Introduction

Term logics are interesting logics. They are Aristotelian in principle, rather than Fregean, and maybe because of that, they have been disparaged in various ways, particularly since the late 19th and the early 20th; however, nowadays, far from being superseded (*contra* [1, 2, 3]), they are in a path of revision and revival (*v.gr.* [4, 5, 6, 7, 8, 9, 10]). In this contribution we follow this path and so we offer a tableaux proof method for statistical reasoning by using a particular term logic. More specifically, after presenting Sommers and Englebretsen's Term Functor Logic, and Thompson's statistical syllogistic, we produce some tableaux for a fragment of Thompson's syllogistic.

2. Preliminaries

2.1. Term Functor Logic

Term Functor Logic [4, 11, 12, 6, 13] is a plus-minus algebra that employs terms and functors, in Aristotelian fashion, rather than Fregean, first order language elements such as individual variables or quantifiers. According to this algebra, the four categorical statements of syllogistic, $\mathcal{SYLL}$, can be represented by the following syntax [6]:

$$\text{All S is P} := -\mathsf{S} + \mathsf{P}$$
$$\text{All S is not P} := -\mathsf{S} - \mathsf{P}$$
$$\text{Some S is P} := +\mathsf{S} + \mathsf{P}$$
$$\text{Some S is not P} := +\mathsf{S} - \mathsf{P}$$

Given this representation, Term Functor Logic, $\mathcal{TFL}$, provides a simple rule for syllogistic inference: a conclusion follows validly from a set of premises if and only if *i)* the sum of the premises is algebraically equal to the conclusion and *ii)* the number of conclusions with particular quantity (*viz.*, zero or one) is the same as the number of premises with particular quantity [6, p.167]. Thus, for instance, if we consider a valid syllogism, we can see how the application of this rule produces the right conclusion (Table 1).

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Table 1
A valid syllogism

Statement	$\mathcal{TFL}$
1. All computer scientists are animals.	$-\mathsf{C}+\mathsf{A}$
2. All logicians are computer scientists.	$-\mathsf{L}+\mathsf{C}$
$\vdash$ All logicians are animals.	$-\mathsf{L}+\mathsf{A}$

In this example, we can clearly see how the rule works: *i)* if we add up the premises we obtain the algebraic expression $(-\mathsf{C}+\mathsf{A})+(-\mathsf{L}+\mathsf{C}) = -\mathsf{C}+\mathsf{A}-\mathsf{L}+\mathsf{C}= -\mathsf{L}+\mathsf{A}$, so that the sum of the premises is algebraically equal to the conclusion and the conclusion is of the form $-\mathsf{L}+\mathsf{A}$, rather than $+\mathsf{A}-\mathsf{L}$, because *ii)* the number of conclusions with particular quantity (zero in this case) is the same as the number of premises with particular quantity (zero in this case).[1] In contrast, just for the sake of comparison, consider an invalid syllogism that does not add up (Table 2).

Table 2
An invalid syllogism

Statement	$\mathcal{TFL}$
1. All computer scientists are animals.	$-\mathsf{C}+\mathsf{A}$
2. All computer scientists are logicians.	$-\mathsf{C}+\mathsf{L}$
$\nvdash$ All logicians are animals.	$-\mathsf{L}+\mathsf{A}$

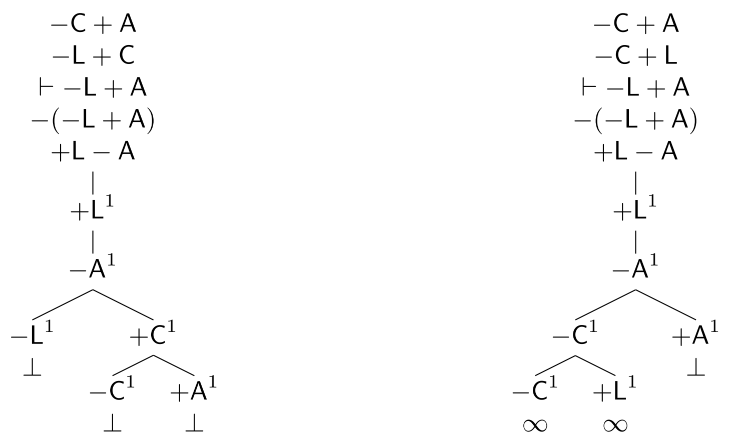
Now, as exposed elsewhere [14, 15], we can develop a tableaux proof method for $\mathcal{TFL}$. So, let us say a *tableau* for $\mathcal{TFL}$ is an acyclic connected graph determined by nodes and vertices. The node at the top is called *root*. The nodes at the bottom are called *tips*. Any path from the root down a series of vertices is a *branch*. To test an inference for validity we construct a tableau which begins with a single branch at whose nodes occur the premises and the rejection of the conclusion: this is the *initial list*. We then apply the expansion rules that allow us to extend the initial list (Figure 1).

Figure 1: $\mathcal{TFL}$ tableaux expansion rules

Figure 3a depicts the rule for universal statements, while Figure 3b shows the rule for particular statements. After applying a rule we introduce some index $i \in \{1, 2, 3, \ldots\}$. For universal statements the index may be any natural number; for particular statements the index has to be a new natural number if they do not already have an index. Also, following $\mathcal{TFL}$ tenets, we assume the following rules of rejection: $-(\pm\mathsf{T}) = \mp\mathsf{T}$, $-(\pm\mathsf{T}\pm\mathsf{T}) = \mp\mathsf{T}\mp\mathsf{T}$, and $-(--\mathsf{T}--\mathsf{T}) = +(-\mathsf{T})+(-\mathsf{T})$.

A tableau is *complete* if and only if every rule that can be applied has been applied. A branch is *closed* if and only if there are terms of the form $\pm\mathsf{A}^i$ and $\mp\mathsf{A}^i$ on two of its nodes; otherwise it is *open*. A closed branch is indicated by writing a $\perp$ at the end of it; an open branch is indicated by writing ∞. A tableau is *closed* if and only if every branch is closed; otherwise it is *open*. So, as usual, $\pm\mathsf{T}$ is a logical consequence of the set of terms Γ (*i.e.* $\Gamma \vdash \pm\mathsf{T}$) if and only if there is a complete closed tableau whose initial list includes the terms of Γ and the rejection of $\pm\mathsf{T}$ (*i.e.* $\Gamma \cup \{\mp\mathsf{T}\} \vdash \perp$). As an example, consider Figure 2, which shows the inferences exposed in Tables 1 and 2.

[1] Although we are exemplifying this logic with syllogistic inferences, this system is capable of representing relational, singular, and compound inferences with ease and clarity. Furthermore, $\mathcal{TFL}$ is arguably more expressive than classical first order logic [12, p.172].



(a) A valid syllogism

(b) An invalid syllogism

Figure 2: A pair of examples

2.2. Statistical syllogistic

Peterson [16] and Thompson [17] developed an extension of $\mathcal{SYLL}$ by adding three intermediate quantifiers: "few" (for predominant statements), "many" (for majority statements), and "most" (for common statements). The result was an intermediate syllogistic, $\mathcal{SYLL}^+$, with which we can model inference between universal, particular, predominant, majority, and common statements. Thompson's Statistical Syllogistic, $\mathcal{SYLL}^{stat}$, is an extension of $\mathcal{SYLL}^+$ that models inference between statements using statistical quantifiers [18]. To observe the differences among these logics, consider Table 3.

Table 3
Statements in $\mathcal{SYLL}$, $\mathcal{SYLL}^+$, and $\mathcal{SYLL}^{stat}$

Statement	$\mathcal{SYLL}$	$\mathcal{SYLL}^+$	$\mathcal{SYLL}^{stat}$
Universal	All S are (not) P	All S are (not) P	100% of S are (not) P
Predominant		Few S are (not) P	Almost 100% of S are (not) P
Majority		Most S are (not) P	More than 100% of S are (not) P
Common		Many S are (not) P	Much more than 0% of S are (not) P
Particular	Some S are (not) P	Some S are (not) P	More than 0% of S are (not) P

According to Thompson, in order to specify these statements in $\mathcal{SYLL}^{stat}$ we need to consider a distribution index defined by two components:

- A limit $n \in \Re$ such that $0 \leq n \leq 100$, for all the quantifiers that receive a minimal interpretation: n is the percentual quantifier.[2]
- A modifier written as a subindex of the limit that measures the vagueness of a quantifier in a given context. This modifier is expressed by way of two variables, σ and ι:
  - σ is a significance level. Given some context, σ is the value such that "much more than $n\%$ of S are P" is true when the actual percentage of S that are S is $(n + \sigma)$ or more. By the way in which "much more than $n\%$" is defined, σ is also the value such that "almost $n\%$ of S are P" is false when the actual percentage of S that are P is $(n - \sigma)$ or less. σ is thus arbitrarily defined, but if it works with its usual meaning, it cannot be less than or equal to 0 or greater than 100, and, like the significance level of statistical tests, it is rarely greater than 5.
  - ι denotes an infinitesimal positive magnitude with two properties:

[2] As explained in [17], a quantifier receives a minimal interpretation when it means at least a certain amount or more; a quantifier receives a maximal interpretation when it means no more than a certain amount or less. Thus, for example, "25% of S is P" is true if the percentage of S that are P is exactly 25%, 50%, or even 100%.

* $(n + \iota) > n$, and
* if $m < n$, then $m < n - (x \times \iota)$, where ι is a positive infinitesimal and m, n, and x are real numbers.

Being greater than 0, ι is a value such that "more than $n\%$ of S are P" is true when the actual percentage of S that are P is $(n + \iota)$ or more. Consequently, ι is also a value such that nearly $n\%$ of S are P is true when the percentage of S that are P is greater than or equal to $(\sigma - \iota) = n + (\iota - \sigma)$.

With these assumptions, the next rules of distribution allow us to associate a distribution index to each term in a given statement:

1. Distribution by quality.
   a) For positive statements, the predicate term has a distribution index of 0_ι.
   b) For negative statements, the predicate term has a distribution index of 100_0.
2. Distribution by quantity.
   a) For statements with a quantifier of the form "$n\%$" the subject term has a distribution index of n_0.
   b) For statements with a quantifier of the form "almost $n\%$" the subject term has a distribution index of $n_{(\iota-\sigma)}$.
   c) For statements with a quantifier of the form "more than $n\%$" the subject term has a distribution index of n_ι.
   d) For statements with a quantifier of the form "Many more than $n\%$" the subject term has a distribution index of n_σ.
   e) For statements with a quantifier of the form "Less than $n\%$" the subject term has a distribution index of $(100 - n)_\iota$.

Given these preliminary considerations, Thompson offers the following rules of validity, where $M1$ and Pp are the distribution indices of the terms of the major premise (the middle term and the major term, respectively); $M2$ and Sp are the distribution indices of the minor premise (the middle term and the minor term, respectively); Sc and Pc are the distribution indices of the terms of the conclusion (the minor term and the major term, respectively); and finally, PM is the distribution index of the predicate of the major premise and Pm is the distribution index of the predicate of the minor premise. The maximum distribution value that an occurrence of a term can receive is 100_0, such that a term with a distribution index of 100_ι is maximally distributed. Thus, a syllogism is valid in $\mathcal{SYLL}^{stat}$ if and only if:

1. The middle term is more that maximally distributed in the premises, *i.e.*, $M1 + M2 > 100_0$.
2. The minor term in the premises is distributed at least to the same degree as in the conclusion, *i.e.*, $Sp \geq Sc$.
3. The major term in the premises is distributed at least to the same degree as in the conclusion, *i.e.*, $Pp \geq Pc$.
4. The number of negative premises is equal to the number of negative conclusions, *i.e.*, $PM + Pm = Pc + 0_\iota$.

Thus, for example, the syllogisms in Tables 4, 5 are valid in $\mathcal{SYLL}^{stat}$, while the syllogism in Table 6 is invalid. The syllogism in Table 4 is valid because it follows all the rules. It satisfies rule 1, because $(M1 + M2) = (100_0 + 0_\iota) = (100 + 0)_\iota = 100_\iota$, and $100_\iota > 100_0$, since $(100 - 100) = 0 > -\iota = 0 - \iota$. It also satisfies rule 2, since $37, 2_0 \geq 37, 2_0$; and rule 3, because $0_\iota \geq 0_\iota$. Also, vacuously, it satisfies rule 4. The example shown in Table 5 also satisfies rule 1 insofar as $(M1 + M2) = (27_{\iota-\sigma} + 73_\sigma) = (27 + 73)_{((\iota-\sigma)+\sigma)} = 100_\iota$. Clearly, the other rules are also satisfied. The example in Table 6 is invalid because the middle term is not more than maximally distributed (*i.e.* $5_\iota + 0_\iota < 100_0$) and the major term in the premises is not distributed to at least the same degree as the major term in the conclusion (*i.e.* $Pp < Pc$).

Table 4
A valid syllogism in $\mathcal{SYLL}^{stat}$

Statement	$\mathcal{SYLL}^{stat}$
1. All Greeks are human.	$M1 = 100_0, Pp = 0_\iota$
2. 37,2% of philosophers are Greek.	$Sp = 37,2_0, M2 = 0_\iota$
⊢ 37,2% of philosophers are human.	$Sc = 37,2_0, Pc = 0_\iota$

Table 5
Another valid syllogism in $\mathcal{SYLL}^{stat}$

Statement	$\mathcal{SYLL}^{stat}$
1. Almost 27% of philosophers are not friendly.	$M1 = 27_{\iota-\sigma}, Pp = 0_\iota$
2. Much more than 73% of philosophers are strage.	$M2 = 73_\sigma, Sp = 0_\iota$
⊢ Some strange people are not friendly.	$Sc = 0_\iota, Pc = 0_\iota$

Table 6
An invalid syllogism in $\mathcal{SYLL}^{stat}$

Statement	$\mathcal{SYLL}^{stat}$
1. More than 5% of philosophers are vegan.	$M1 = 5_\iota, Pp = 0_\iota$
2. Less than 100% of philosophers are not smart.	$M2 = 0_\iota, Sp = 100_0$
⊬ Almost 95% of smart people are vegan.	$Sc = 95_{\iota-\sigma}, Pc = 100_0$

3. $\mathcal{TFL}^{stat}$

As can be seen up to this point, $\mathcal{SYLL}^{stat}$ offers an interesting approach to model statistical syllogistic; however, it does not offer a more general algebraic model. Given this state of affairs, in this section we propose the logic $\mathcal{TFL}^{stat}$ in order to unify the virtues of $\mathcal{SYLL}^{stat}$ with those of $\mathcal{TFL}$. To achieve this goal we follow two steps: first, we propose an adaptation of the $\mathcal{TFL}$ syntax to include the statistical quantifiers of $\mathcal{SYLL}^{stat}$, and then we modify the $\mathcal{TFL}$ rules.

3.1. Syntax

In order to accommodate the statements of $\mathcal{SYLL}^{stat}$ within the signature of $\mathcal{TFL}$, consider Table 7.

Table 7
Syntax of $\mathcal{TFL}^{stat}$

Positive statements		Negative statements	
n% of S is P	$-\mathsf{S}^{n_0}+\mathsf{P}^{0_\iota}$	n% of S is not P	$-\mathsf{S}^{n_0}-\mathsf{P}^{100_0}$
Almost n% of S is not P	$-\mathsf{S}^{n_{\iota-\sigma}}+\mathsf{P}^{0_\iota}$	Almost n% of S is P	$-\mathsf{S}^{n_{\iota-\sigma}}-\mathsf{P}^{100_0}$
More than n% of S is P	$+\mathsf{S}^{n_\iota}+\mathsf{P}^{0_\iota}$	More than n% of S is not P	$+\mathsf{S}^{n_\iota}-\mathsf{P}^{100_0}$
Much more than n% of S is P	$+\mathsf{S}^{n_\sigma}+\mathsf{P}^{0_\iota}$	Much more than n% of S is not P	$+\mathsf{S}^{n_\sigma}-\mathsf{P}^{100_0}$
Less than n% of S is not P	$+\mathsf{S}^{(100-n)_\iota}+\mathsf{P}^{0_\iota}$	Less than n% of S is P	$+\mathsf{S}^{(100-n)_\iota}-\mathsf{P}^{100_0}$

3.2. Rules

Now, we say that a syllogism is valid in $\mathcal{TFL}^{stat}$ if and only if *i)* the sum of the premises is algebraically equal to the conclusion, *ii)* the number of conclusions with particular quantity (*i.e.*, zero or one) is equal to the number of premises with particular quantity; *iii)* the sum of the distribution indices of the middle terms is greater than 100_0; and *iv)* the distribution indices of the conclusion do not exceed the distribution indices of the premises. To illustrate this definition, let us reconsider the previous examples (Tables 8, 9 and 10).

Table 8
A valid syllogism in $\mathcal{TFL}^{stat}$

Statement	$\mathcal{TFL}^{stat}$
1. All Greeks are human.	$-\mathsf{G}^{100_0}+\mathsf{H}^{0_\iota}$
2. 37,2% of philosophers are Greek.	$-\mathsf{P}^{37,2_0}+\mathsf{G}^{0_\iota}$
⊢ 37,2% of philosophers are human.	$-\mathsf{P}^{37,2_0}+\mathsf{H}^{0_\iota}$

Table 9
Another valid syllogism in $\mathcal{TFL}^{stat}$

Statement	$\mathcal{TFL}^{stat}$
1. Almost 27% of philosophers are not friendly.	$-\mathsf{P}^{27_{\iota-\sigma}}+\mathsf{F}^{0_\iota}$
2. Much more than 73% of philosophers are strage.	$+\mathsf{P}^{73_\sigma}+\mathsf{S}^{0_\iota}$
⊢ Some strange people are friendly.	$+\mathsf{S}^{0_\iota}+\mathsf{F}^{0_\iota}$

Table 10
An invalid syllogism in $\mathcal{TFL}^{stat}$

Statement	$\mathcal{TFL}^{stat}$
1. More than 5% of philosophers are vegan.	$+\mathsf{P}^{5_\iota}+\mathsf{V}^{0_\iota}$
2. Less than 100% of philosophers are not smart.	$+\mathsf{P}^{0_\iota}-\mathsf{A}^{100_0}$
⊬ Almost 95% of smart people are vegan.	$-\mathsf{A}^{95_{\iota-\sigma}}-\mathsf{V}^{100_0}$

3.3. Tableaux

Given these ideas, we would like to offer some tableaux for $\mathcal{TFL}^{stat}$. So, consider the following expansion rules (Figure 3):

Figure 3: $\mathcal{TFL}^{stat}$ tableaux expansion rules

These rules work as expected. After applying a rule we introduce some subindex $i \in \{1, 2, 3, \ldots\}$ as in $\mathcal{TFL}$, but also, we use a superindex d that represents the distribution index of a given term according to $\mathcal{SYLL}^{stat}$. With these assumptions, we say tableau is *complete* if and only if every rule that can be applied has been applied. A branch is *closed* if and only if *i)* there are terms of the form $\pm\mathsf{A}_i^d$ and $\mp\mathsf{A}_i^d$ on two of its nodes or *ii)* there are terms of the form $\pm\mathsf{A}_i^d$ and $\mp\mathsf{A}_i^d$ and the sum of the distribution indexes is greater than 100_0; otherwise it is *open*. A closed branch is indicated by writing a $\perp$ at the end of it; an open branch is indicated by writing ∞. A tableau is *closed* if and only if every branch is closed; otherwise it is *open*. Thus, again as usual, $\pm\mathsf{T}$ is a logical consequence of the set of terms Γ if and only if there is a complete closed tableau whose initial list includes the terms of Γ and the rejection of $\pm\mathsf{T}$. As an example, consider Figure 4, which shows the inferences exposed in Tables 8, 9, and 10.

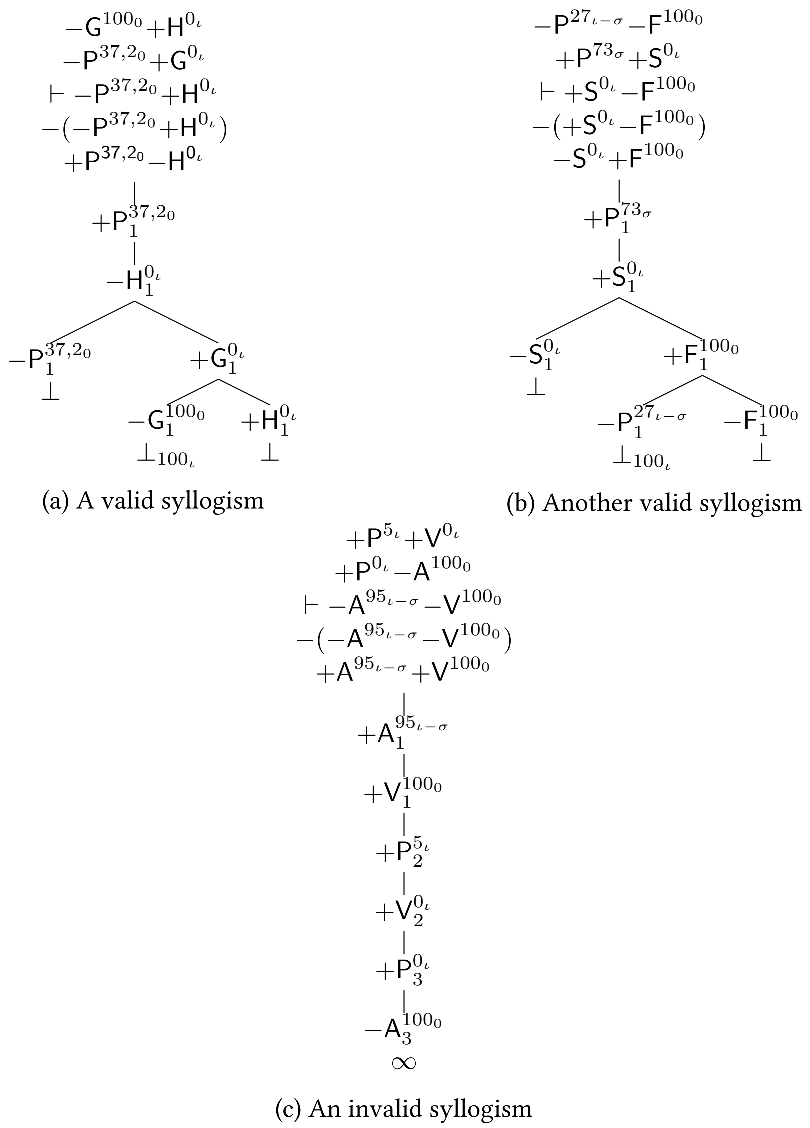


Figure 4: More examples

Now, before we continue with some formal results, let us consider a couple of features of this proposal. First, we have to point out that our proposal differs from Thompson's insofar as $\mathcal{SYLL}^{stat}$ allows universal statements to entail particular statements, but since our proposal follows the tenets of $\mathcal{TFL}$, we have to add another rule to the $\mathcal{TFL}^{stat}$ framework: if the premises have a subject term with the functor "$-$", then the conclusion cannot have a subject term with the functor "$+$". This consideration causes inferences such as those in Table 11 to be conditionally or enthymematically correct, as in Figure 5.

Table 11
A conditionally valid inference in $\mathcal{TFL}^{stat}$

Statement	$\mathcal{TFL}^{stat}$
0. There are more than 63% of philosophers.	$+P^{63_\iota}+P^{0_\iota}$
1. Every Greek is human.	$-G^{100_0}+H^{0_\iota}$
2. 37% of philosophers are Greek.	$-P^{37_0}+G^{0_\iota}$
⊢ More than 0% of philosophers are human.	$+P^{0_\iota}+H^{0_\iota}$

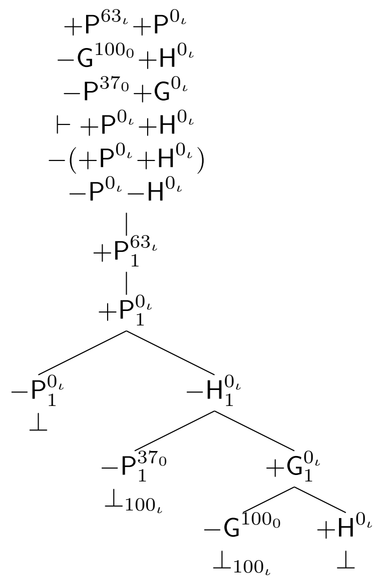


Figure 5: A conditionally valid inference

Second, it seems that the expressive power of $\mathcal{TFL}$ for dealing with relations can be used to produce statistical syllogisms with relations, for example, as in Table 12 and Figure 6.

Table 12
A relational valid inference in $\mathcal{TFL}^{stat}$

Statement	$\mathcal{TFL}^{stat}$
1. 37% of philosophers hate some logicians.	$-P^{37_0}+(+H^{0_\iota}+L^{0_\iota})$
2. More than 89% of philosophers are cynical.	$+P^{89_\iota}+C^{0_\iota}$
⊢ Some cynical hates some logician.	$+C^{0_\iota}+(+H^{0_\iota}+L^{0_\iota})$

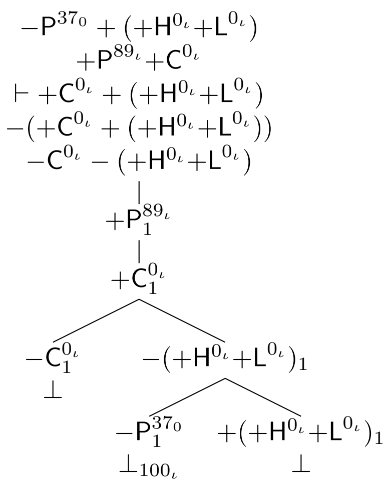


Figure 6: A relational valid inference

Finally, before we close this contribution, we would like to offer some formal results:

Proposition 1 (Soundness)**.** *If a syllogism is valid in $\mathcal{SYLL}^{stat}$, then its tableau is closed complete.*

Proof. Notice that when the statements have indices 100_0 and 0_ι only, the proof is trivial: the syllogisms have closed complete tableaux because in that case $\mathcal{SYLL}^{stat}$ collapses with $\mathcal{TFL}$. For the rest of syllogisms let us suppose, for *reductio*, that $\mathfrak{s}$ is an arbitrary syllogism that is valid in $\mathcal{SYLL}^{stat}$ but its tableau is not closed complete. In other words, $\mathfrak{s}$ satisfies the rules of $\mathcal{SYLL}^{stat}$ but its corresponding tableau is open.

By following the rules of $\mathcal{SYLL}^{stat}$, we can build an exhaustive and exclusive array of arbitrary valid syllogisms for whatever terms S, P, and M where $+\mathsf{P}^x = +\mathsf{P}^{0_\iota}$, $-\mathsf{P}^x = -\mathsf{P}^{100_0}$, $+\mathsf{S}^y = +\mathsf{S}^{\{k_\iota \leq n_\iota, k_\sigma \leq n_\sigma\}}$, and $-\mathsf{S}^y = -\mathsf{S}^{\{k_0 \leq n_0, k_{(\iota-\sigma)} \leq n_{(\iota-\sigma)}\}}$, as in Table 13.

Table 13
An array of valid syllogisms in $\mathcal{TFL}^{stat}$

	I		II		III		IV
1.	$-\mathsf{M}^{100_0} \pm \mathsf{P}^{\mathsf{x}}$	1.	$-\mathsf{P}^{100_0} - \mathsf{M}^{100_0}$	1.	$-\mathsf{P}^{100_0} + \mathsf{M}^{0_\iota}$	1.	$-\mathsf{M}^{n_0} \pm \mathsf{P}^x$
2.	$\pm\mathsf{S}^y + \mathsf{M}^{0_\iota}$	2.	$\pm\mathsf{S}^y + \mathsf{M}^{0_\iota}$	2.	$\pm\mathsf{S}^y - \mathsf{M}^{100_0}$	2.	$+\mathsf{M}^{(100-n)_\iota} + \mathsf{S}^{0_\iota}$
$\vdash$	$\pm\mathsf{S}^y \pm \mathsf{P}^x$	$\vdash$	$\pm\mathsf{S}^y - \mathsf{P}^{100_0}$	$\vdash$	$\pm\mathsf{S}^y - \mathsf{P}^{100_0}$	$\vdash$	$+\mathsf{S}^{0_\iota} \pm \mathsf{P}^x$
	V		VI		VII		VIII
1.	$+\mathsf{M}^{(100-n)_\iota} \pm \mathsf{P}^x$	1.	$-\mathsf{M}^{n_{(\iota-\sigma)}} \pm \mathsf{P}^x$	1.	$+\mathsf{M}^{(100-n)_\sigma} \pm \mathsf{P}^x$	1.	$-\mathsf{P}^{100_0} - \mathsf{M}^{100_0}$
2.	$-\mathsf{M}^{n_0} + \mathsf{S}^{0_\iota}$	2.	$+\mathsf{M}^{(100-n)_\sigma} + \mathsf{S}^{0_\iota}$	2.	$-\mathsf{M}^{n_{(\iota-\sigma)}} + \mathsf{S}^{0_\iota}$	2.	$+\mathsf{M}^{n_\iota} + \mathsf{S}^{0_\iota}$
$\vdash$	$+\mathsf{S}^{0_\iota} \pm \mathsf{P}^x$	$\vdash$	$+\mathsf{S}^{0_\iota} \pm \mathsf{P}^x$	$\vdash$	$+\mathsf{S}^{0_\iota} \pm \mathsf{P}^x$	$\vdash$	$+\mathsf{S}^{0_\iota} - \mathsf{P}^{100_0}$

If we develop tableaux for each of these syllogisms, we will see that all them are closed complete, but since $\mathfrak{s}$ is a syllogism built after the rules of $\mathcal{SYLL}^{stat}$, it must be included in Table 13, and hence its tableau must be closed complete, which contradicts our assumption. □

Previously in [19], we have shown that:

Proposition 2. *If a syllogism is valid in $\mathcal{TFL}^{stat}$, then it is also valid in $\mathcal{SYLL}^{stat}$.*

So, from these results it follows that:

Corollary 1. *If a syllogism is valid in $\mathcal{TFL}^{stat}$, then its tableau is closed complete.*

4. Final Remarks

After presenting Sommers and Englebretsen's Term Functor Logic, and Thompson's statistical syllogistic, we have offered some tableaux for a fragment of Thompson's syllogistic. This result, albeit humble, updates the research on term logics with the purpose of dealing with non-deductive inference, namely, inductive and abductive inference, in a terministic, Aristotelian fashion. Our future work consists in studying the formal properties of this proposal and developing fine-tuned implementations.

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