On Balancing Energy Consumption in Multi-Interface Networks (short paper)*

Alessandro Aloisio[1,*,†]

[1]University of International Studies of Rome, Via Cristoforo Colombo, 200 – 00147, Roma, Italy

Abstract

In heterogeneous networks, devices can communicate using multiple interfaces. By selectively activating interfaces on each device, various connections can be established. A connection is formed when the devices at its endpoints share at least one active interface. Each interface activation incurs a cost, representing the percentage of energy consumed. This paper focuses on scenarios where each device can activate at most a fixed number, q, of its interfaces. Specifically, we address the *Coverage* problem in a network $G = (V, E)$, where nodes in V represent devices and edges in E represent potential connections. The goal is to activate up to q interfaces per node to establish all connections in E while minimizing the total cost. The parameter q ensures balanced energy consumption across devices, preventing any single device from being overburdened. Additionally, we consider a model where each interface has both a cost and a profit associated with it, with the establishment of a connection yielding a profit. This paper presents two negative results and several positive findings related to these scenarios.

Keywords

Multi-Interface Network, Coverage, Optimal algorithms

1. Introduction

As technology advances, powerful devices have become increasingly accessible, enabling seamless communication among heterogeneous devices through various protocols and interfaces. This paper explores networks composed of diverse devices that utilize different communication interfaces to establish connections. While many devices are equipped with multiple interfaces—such as Bluetooth, Wi-Fi, 4G, and 5G—their full potential is often underutilized. Optimal interface selection depends on factors like availability, communication bandwidth, energy consumption, and the device's environment. For example, an experimental study in [1] investigates the choice between Bluetooth and Wi-Fi based on energy consumption for data transmission among smartphones. Given the portable nature of these devices, managing energy consumption is crucial for extending network lifespan and preventing device failures due to battery depletion.

This optimization problem is particularly relevant in networks composed of diverse devices, where each device possesses multiple interfaces, and connections rely on shared active interfaces

ICTCS'24: Italian Conference on Theoretical Computer Science, September 11–13, 2024, Torino, Italy

*Corresponding author.
†These authors contributed equally.
alessandro.aloisio@unint.eu (A. Aloisio)
0000-0003-3911-4008 (A. Aloisio)

between links. Activating an interface consumes energy but enables communication with neighboring devices that also have that interface active.

Formally, a network of devices is represented by a graph $G = (V, E)$, where V is the set of devices and E represents potential connections based on device proximity and shared interfaces. Each device $u \in V$ has a set of available interfaces $W(u)$. The total set of interfaces in the network is denoted by $\bigcup_{u \in V} W(u)$, represented as $\{1, \ldots, k\}$. A connection is established when the endpoints of an edge share at least one active interface. Activating an interface i at node u incurs an energy cost $c(i)$, enabling communication with all neighbors that also have interface i active.

This paper focuses on two well-known versions of the *Coverage* problem. The first version aims to establish all connections in the graph G at the lowest cost, ensuring a common active interface at the endpoints of each edge while minimizing the overall activation cost in the network [2, 3, 4, 5, 6, 7]. This model also includes a constraint on the number of active interfaces per device, limited to q, to manage energy consumption and optimize costs under specific limitations [2]. We refer to this model as *CMI(q)*.

The second version, denoted as *CMI(q,b)*, combines the Coverage problem with constraints on both the overall cost budget b and the maximum number q of interfaces that each device can activate. Additionally, each interface is associated with a profit, introducing an incentive for interface activation. This models an environment where users are motivated to activate interfaces not only to access services but also to earn profits from establishing connections. Specifically, if a connection is established, another profit function reflects the gains from this connection [8, 9, 10, 11].

In recent years, significant research has been conducted on *multi-interface networks*, emphasizing the benefits of using multiple interfaces per device across various applications. This research addresses core challenges in network optimization, particularly in routing and network connectivity (e.g., [12, 13, 14]). The exploration of combinatorial problems in multi-interface wireless networks dates back to early studies, such as those in [15]. A version of the multi-interface problem related to the maximum subgraph edge-colorable problem [16, 17, 18], but with distance differences, was studied in [19]. Subsequent investigations have explored variants of the Coverage problem, as evidenced by studies in papers like [6, 20, 21]. Additionally, the cheapest path problem, a reinterpretation of the classical shortest path problem within multi-interface networks, was analyzed in [22]. These studies have potential applications in various fields, including tactical military operations [23], where they can be used to implement emergency networks.

2. Preliminaries and models definitions

For a graph G, let V denote its node set and E its edge set. Unless specified otherwise, the graph $G = (V, E)$ representing the network is assumed to be undirected, connected, and free of multiple edges and loops. For any positive integer k, we define $[k] = \{1, \ldots, k\}$. When discussing the network graph G, we refer to the number of nodes as n and the number of edges as m. The global assignment of interfaces to the nodes in V is specified by an appropriate interface function W, as detailed below.

Definition 1. A function $W: V \to 2^{[k]}$ is said to *cover* graph G if $W(u) \cap W(v) \neq \emptyset$, for each $\{u, v\} \in E$.

The cost of activating an interface for a node is assumed to be identical for all nodes and given by a cost function $c: [k] \to \mathbb{R}_{>0}$, i.e., the cost of interface i is denoted as $c(i)$. The considered *CMI(q)* optimization problem is formulated as follows.

CMI(q): Coverage in Multi-Interface Networks

Input: A graph $G = (V, E)$, an allocation of available interfaces $W: V \to 2^{[k]}$ covering graph G, an interface cost function $c: [k] \to \mathbb{R}_{>0}$, and an integer $q \geq 1$.

Solution: An allocation of active interfaces $W_A: V \to 2^{[k]}$ covering G such that for all $u \in V$, $W_A(u) \subseteq W(u)$ and $|W_A(u)| \leq q$.

Goal: Minimize the total cost of the active interfaces, $c(W_A) = \sum_{u \in V} \sum_{i \in W_A(u)} c(i)$.

It is worth noting that we can explore two variations of the aforementioned problem: the cost function c may range over $\mathbb{R}_{>0}$, or $c(i) = 1$, for every $i \in [k]$ (*unit cost case*). In both instances, we presume $k \geq 2$, as the case $k = 1$ has a straightforward and unique solution (all nodes must activate their sole interface).

In [2], we demonstrated that *CMI(q)* is a specific instance of the broader *CMI(∞)* problem (refer to [14, 24]), where each node is limited to activating at most q interfaces. Notably, the basic variant with $q = 2$ proves to be generally more challenging than *CMI(∞)*. However, certain graph classes exhibit more manageable characteristics. For instance, in trees and complete graphs, *CMI(∞)* has been established as *APX*-hard and not approximable within $O(\log n)$, respectively, while *CMI(2)* is solvable in polynomial time.

The considered *CMI(q,b)* optimization problem is formulated as follows.

CMI(q,b): Coverage in Multi-Interface Networks

Input: A graph $G = (V, E)$, an allocation of available interfaces $W: V \to 2^{[k]}$ covering graph G, an interface cost function $c: [k] \to \mathbb{N}_{>0}$, two integers $q, b \geq 1$, two profit functions $p: [k] \to \mathbb{N}_{\geq 0}$ and $p: [k]^2 \to \mathbb{N}_{\geq 0}$.

Solution: An allocation of active interfaces $W_A: V \to 2^{[k]}$ covering G such that for all $u \in V$, $W_A(u) \subseteq W(u)$ and $|W_A(u)| \leq q$; with $c(W_A) = \sum_{u \in V} \sum_{i \in W_A(u)} c(i) \leq b$.

Goal: Maximize the total profit $p(W_A) = \sum_{u \in V} \sum_{i \in W_A(u)} p(i) + \sum_{\{u,v\} \in E} \sum_{i \in (W_A(u) \cap W_A(v))} p(i, i)$.

As for *CMI(q)*, there are two variations to consider: firstly, the cost function c may vary over $\mathbb{N}_{>0}$, or alternatively, $c(i)$ equals 1 for each i in the set $[k]$, (unitary cost case). In both scenarios, we assume $k \geq 2$, as the case $k = 1$ is straightforward. This version is difficult even when feasibility is guaranteed, as we showed it to be *NP*-hard, even when the input instance admits a feasible solution and $q = 2$ [10, 9].

3. Our results

In this section, we will report the main results that we achieved for *CMI(2)* and *CMI(2,b)*.

Table 1
Main results for *CMI(2)*

Graph class	Costs	Complexity of *CMI(2)*	Reference
Graphs with $\Delta \geq 4$	Unitary	*NP*-complete (feasibility)	[2, 4]
Series–Parallel graphs	Arbitrary	Optimally solvable in $O(k^6 \cdot n)$	[3, 4]
Complete Bipartite graphs	Arbitrary	Optimally solvable in $O(k^4 \cdot n)$	[2, 4]
Complete graphs	Arbitrary	Optimally solvable in $O(k^3 \cdot n)$	[2, 4]
Rings	Arbitrary	Optimally solvable in $O(k^3 \cdot n)$	[2, 4]
Paths	Arbitrary	Optimally solvable in $O(k \cdot n)$	[2, 4]
Trees	Arbitrary	Optimally solvable in $O(\Delta \cdot k^2 \cdot n)$	[2, 4]
Carvingwidth h	Arbitrary	Optimally solvable in $O(k^{4h} \cdot n)$	[5]
Pathwidth h	Arbitrary	Optimally solvable in $O(k^{2(h+1)} \cdot n)$	[5]
Branchwidth h	Arbitrary	Optimally solvable in $O\left(\left(\frac{k^2}{2}\cdot\right)^{2h} \cdot h \cdot m\right)$	[6]
Treewidth h	Arbitrary	Optimally solvable in $O\left(\left(\frac{k^2}{2}\cdot\right)^{4(h+1)} \cdot h \cdot m\right)$	[6]

3.1. *CMI(2)*

Table 1 contains the results we obtained for the first coverage problem, *CMI(2)*. We provided the following negative result by a polynomial time reduction of the well-known 3-SAT problem with bounded occurrences.

Theorem 1 ([2, 4]). *Finding a feasible solution for CMI(2) is NP-complete for graphs with* $\Delta \geq 4$*, even for the unitary cost case and bipartite graphs.*

Then we analyzed the complexity of *CMI(2)* for several classes of graphs, describing ad-hoc deterministic algorithms. In particular, we tackled *series-parallel* graphs, *complete bipartite* graphs, *complete* graphs, *paths*, *rings*, *trees*, and graphs with bounded *carvingwidth*, *pathwidth*, *branchwidth*, and *treewidth*. Please note that the results given in the table can also be seen as fixed-parameter tractability (FPT) [25] results by choosing appropriate parameters.

3.2. *CMI(2,b)*

First, we proved the following theorem.

Theorem 2 ([9, 10]). *CMI(q,b) is NP-hard, even when the input admits a feasible solution and* $q = 2$.

We then presented seven positive results for three specific classes of graphs: series-parallel graphs, graphs with bounded carvingwidth, and graphs with bounded pathwidth. For series-parallel graphs, we developed two deterministic algorithms: one focusing on the maximum total cost b, and the other on an upper bound μ for the maximum profit. Additionally, we introduced a fully polynomial-time approximation scheme (FPTAS) by scaling the profits down sufficiently so that all the objects' profits become polynomially bounded in n. Furthermore, we proposed two optimal algorithms for graphs with bounded carvingwidth and pathwidth, addressing both

Table 2
Main results for *CMI(2,b)*

Graph class	Costs	Complexity of *CMI(2)*	Reference
Graphs that allow for a feasible solution.	Unitary	*NP*-hard	[10, 9]
Series–Parallel graphs	Arbitrary	Optimally solvable in $O(b^2 \cdot k^6 \cdot n)$	[10, 9]
Series–Parallel graphs	Arbitrary	Optimally solvable in $O(\mu^2 \cdot k^6 \cdot n)$	[10, 9]
Series–Parallel graphs	Arbitrary	in FPTAS	[10, 9]
Carvingwidth h	Arbitrary	Optimally solvable in $O\left(b^2 \cdot k^{4h} \cdot n\right)$	[8, 10]
Carvingwidth h	Arbitrary	Optimally solvable in $O\left(\mu^2 \cdot k^{4h} \cdot n\right)$	[8, 10]
Pathwidth h	Arbitrary	Optimally solvable in $O\left(b \cdot h \cdot \left(\frac{k^2}{2}\right)^{h+1} \cdot n\right)$	[11, 26]
Pathwidth h	Arbitrary	Optimally solvable in $O\left(\mu \cdot h \cdot \left(\frac{k^2}{2}\right)^{h+1} \cdot n\right)$	[11, 26]

b and μ. These results are summarized in Table 2. As mentioned in Section 3.1, the results in Table 2 can be interpreted as FPT results with appropriate parameters [25].

4. Conclusion and future works

In this study, we explored two variants of the well-known Coverage problem within the context of multi-interface networks.

The first variant, *CMI(q)*, focuses on identifying the most cost-effective way to establish all connections defined by an input graph. This is achieved by activating appropriate subsets of interfaces at the network nodes. Unlike the original Coverage model, this variant introduces an additional constraint, limiting each node to activating no more than q interfaces, with particular attention to the case where $q = 2$. Although this problem is *NP*-hard, we developed several optimal algorithms for specific classes of graphs.

The second variant, *CMI(q,b)*, aims to find the most profitable way to establish all connections in the graph while keeping the overall cost within a given budget.

Future research could explore *CMI(q)* and *CMI(q,b)* in relation to other parameters, such as local treewidth and cliquewidth, to derive both positive and negative results. Additionally, while *NP*-hardness has been established for general graphs with a maximum degree of 4, and the problem is solvable in polynomial time for graphs with a maximum degree of 2, the case of graphs with a maximum degree of 3 remains unexplored.

Another promising research direction involves analyzing the multi-interface coverage problem through the lens of game theory. In this approach, the problem becomes decentralized, with each device functioning as an agent aiming to maximize its utility (e.g., connections) while managing overall energy consumption. This perspective could model the problem as a type of classic polymatrix game [27, 28] or more recent general and specific versions [29, 30, 31, 32, 33, 34].

Acknowledgments

This work is partially supported by the project 'Soluzioni innovative per il problema della copertura nelle multi-interfacce e relative varianti', UNINT, and by the Italian National Group for Scientific Computation (GNCS-INdAM).

References

[1] R. Friedman, A. Kogan, Y. Krivolapov, On power and throughput tradeoffs of wifi and bluetooth in smartphones, in: Proc. 30th Int'l Conf. on Computer Communications (INFOCOM), IEEE, 2011, pp. 900–908.

[2] A. Aloisio, A. Navarra, Balancing energy consumption for the establishment of multi-interface networks, in: Proc. 41st Int'l Conf. on Current Trends in Theory and Practice of Computer Science, (SOFSEM-2015), volume 8939, 2015, pp. 102–114.

[3] A. Aloisio, A. Navarra, L. Mostarda, Distributing energy consumption in multi-interface series-parallel networks, in: Proc. 33rd Int.'l Conf. on Advanced Information Networking and Applications, (AINA Workshops), volume 927 of *Advances in Intelligent Systems and Computing*, Springer, 2019, pp. 734–744.

[4] A. Aloisio, A. Navarra, L. Mostarda, Energy consumption balancing in multi-interface networks, J. Ambient Intell. Humaniz. Comput. 11 (2020) 3209–3219.

[5] A. Aloisio, A. Navarra, Constrained connectivity in bounded X-width multi-interface networks, Algorithms 13 (2020) 31.

[6] A. Aloisio, Algorithmic aspects of distributing energy consumption in multi-interface networks, in: Conf. Advanced Information Networking and Applications (AINA), volume 204, Springer, 2024, pp. 114–123.

[7] A. Aloisio, D. Cacciagrano, Distributing energy consumption in multi-interface networks: dimension of the cycle space, in: Proc. of the 19th International Conference on P2P, Parallel, Grid, Cloud and Internet Computing (3PGCIC-2024), San Benedetto, Italy, 13-15 November 2024, volume To appear, Springer, 2024.

[8] A. Aloisio, Coverage subject to a budget on multi-interface networks with bounded carving-width, in: Web, Artificial Intelligence and Network Applications - Proceedings of the Workshops of the 34th International Conference on Advanced Information Networking and Applications, AINA Workshops 2020, Caserta, Italy, 15-17 April, volume 1150 of *Advances in Intelligent Systems and Computing*, Springer, 2020, pp. 937–946.

[9] A. Aloisio, A. Navarra, Budgeted constrained coverage on series-parallel multi-interface networks, in: WAINA, Advances in Intelligent Systems and Computing, volume 1151, Springer, 2020, pp. 458–469.

[10] A. Aloisio, A. Navarra, Budgeted constrained coverage on bounded carving-width and series-parallel multi-interface networks, Internet of Things. 11 (2020) 100259.

[11] A. Aloisio, A. Navarra, On coverage in multi-interface networks with bounded path-width, in: Conf. Advanced Information Networking and Applications (AINA), volume 204, Springer, 2024, pp. 96–105.

[12] G. D'Angelo, G. Di Stefano, A. Navarra, Minimizing the Maximum Duty for Connectivity in

Multi-Interface Networks, in: Proc. 4th Annual Int'l Conf. on Combinatorial Optimization and Applications (COCOA), volume 6509 Part II of *LNCS*, Springer, 2010, pp. 254–267.
[13] G. D'Angelo, G. D. Stefano, A. Navarra, Minimize the maximum duty in multi-interface networks, Algorithmica 63 (2012) 274–295.
[14] G. D'Angelo, G. Di Stefano, A. Navarra, Multi-interface wireless networks: Complexity and algorithms, in: S. R. Ibrahiem M. M. El Emary (Ed.), Wireless Sensor Networks: From Theory to Applications, CRC Press, Taylor & Francis Group, 2013, pp. 119–155.
[15] M. Caporuscio, D. Charlet, V. Issarny, A. Navarra, Energetic Performance of Service-oriented Multi-radio Networks: Issues and Perspectives., in: Proc. 6th Int'l Workshop on Software and Performance (WOSP), ACM, 2007, pp. 42–45.
[16] A. Aloisio, V. Mkrtchyan, Algorithmic aspects of the maximum 2-edge-colorable subgraph problem, in: Conf. Advanced Information Networking and Applications (AINA), volume 227, Springer, 2021, pp. 232–241.
[17] A. Aloisio, Fixed-parameter tractability for branchwidth of the maximum-weight edge-colored subgraph problem, in: Conf. Advanced Information Networking and Applications (AINA), volume 204, Springer, 2024, pp. 86–95.
[18] V. Mkrtchyan, The maximum 2-edge-colorable subgraph problem and its fixed-parameter tractability, J. Graph Algorithms Appl. 28 (2024) 129–147.
[19] A. Kosowski, A. Navarra, D. Pajak, C. Pinotti, Maximum matching in multi-interface networks, Theoretical Computer Science 507 (2013) 52–60.
[20] A. Aloisio, Min-max coverage in multi-interface networks: pathwidth, in: Proc. of the 19th International Conference on P2P, Parallel, Grid, Cloud and Internet Computing (3PGCIC-2024), San Benedetto, Italy, 13-15 November 2024, volume To appear, Springer, 2024.
[21] A. Aloisio, F. Piselli, Min-max coverage in multi-interface networks: series-parallel graphs, in: Proc. of the 19th International Conference on Broad-Band and Wireless Computing, Communication and Applications (BWCCA-2024), San Benedetto, Italy, 13-15 November 2024, volume In press., Springer, 2024.
[22] A. Kosowski, A. Navarra, M. Pinotti, Exploiting Multi-Interface Networks: Connectivity and Cheapest Paths, Wireless Networks 16 (2010) 1063–1073.
[23] A. Perucci, M. Autili, M. Tivoli, A. Aloisio, P. Inverardi, Distributed composition of highly-collaborative services and sensors in tactical domains, in: Proc. of 6th Int. Conference in Software Engineering for Defence Applications (SEDA), volume 925 of *Advances in Intelligent Systems and Computing*, Springer, 2019, pp. 232–244.
[24] R. Klasing, A. Kosowski, A. Navarra, Cost minimization in wireless networks with a bounded and unbounded number of interfaces, Networks 53 (2009) 266–275.
[25] J. Flum, M. Grohe, Parameterized Complexity Theory, Springer, 2006.
[26] A. Aloisio, A. Navarra, Parameterized complexity of coverage in multi-interface iot networks: Pathwidth, Internet of Things 28 (2024) 101353.
[27] J. Howson, Equilibria of polymatrix games, Management Sci. 18 (1972) 312–318.
[28] B. C. Eaves, Polymatrix games with joint constraints, SIAM Journal on Applied Mathematics 24 (1973) 418–423.
[29] A. Aloisio, Distance hypergraph polymatrix coordination games, in: Proc. 22nd Conf. Autonomous Agents and Multi-Agent Systems (AAMAS), 2023, pp. 2679–2681.

[30] A. Aloisio, M. Flammini, B. Kodric, C. Vinci, Distance polymatrix coordination games, in: Proc. 30th Intl. Joint Conf. Artif. Intell. (IJCAI), 2021, pp. 3–9.
[31] A. Aloisio, M. Flammini, B. Kodric, C. Vinci, Distance polymatrix coordination games (short paper), in: SPIRIT co-located with 22nd International Conf. AIxIA 2023, November 7-9th, 2023, Rome, Italy, volume 3585, 2023.
[32] A. Aloisio, M. Flammini, C. Vinci, The Impact of Selfishness in Hypergraph Hedonic Games, in: Proc. 34th Conf. Artificial Intelligence (AAAI), 2020, pp. 1766–1773.
[33] A. Aloisio, M. Flammini, C. Vinci, Generalized distance polymatrix games, in: Proc. 49th Intl. Conf. Current Trends in Theory & Practice of Comput. Sci. (SOFSEM), Springer, 2024, pp. 25–39.
[34] A. Aloisio, M. Flammini, C. Vinci, Generalized distance polymatrix games (short paper), in: Proc. of the 26th Italian Conference on Theoretical Computer Science, Torino, Italy, September 11-13, 2024, volume In press., Springer, 2024.