Mathematical models and methods for decision coordination in critical infrastructure operations

Hryhorii Hnatiienko[1,†], Oleksii Hnatiienko[1,†], Tetiana Babenko[1,2,†] and Larysa Myrutenko[1,*,†]

[1] *Taras Shevchenko National University of Kyiv, 64/13 Volodymyrska str., 01601 Kyiv, Ukraine*
[2] *International Information Technology University, 34/1 Manas str., A15M0E6 Almaty, Kazakhstan*

Abstract

This paper considers the problems associated with ensuring the functioning of the critical infrastructure network. It is proposed to consider a poorly formalized system of ensuring the functioning of critical infrastructure facilities using expert information processing methods. The urgency of solving the problem under consideration is confirmed by the massive attacks on critical infrastructure by the Russian troops during Russia's large-scale aggression against Ukraine. The paper presents a mathematical model of the problem of maintenance of a network of critical infrastructure facilities developed by the authors. A scheme of sequential analysis of options for solving the problem of ensuring the operation of the critical infrastructure system is proposed. Methods for finding a valid solution to the problem, searching for a reference solution to the problem, and algorithms for improving the reference solution in various variations are described. The problem statement and the mathematical model of decision coordination in a three-level hierarchical system for ensuring the operation of a network of critical infrastructure facilities are also described. An algorithm for coordinating decisions in a three-level hierarchical system is presented.

Keywords

organizational system, functional stability, critical elements, weighting factors, layering method

1. Introduction

The possibilities of applying mathematical models and decision-making methods to study the problems of vulnerability, protection, and management of critical infrastructure systems are in the field of view of many researchers [1, 2]. This issue has been studied by scientists from different countries for many years [3–5] and its relevance is not decreasing [6–8]. To date, a large number of approaches and mathematical models have been developed that demonstrate the authors' attempts to ensure the effective functioning and protection of critical infrastructure [9–11]. At the same time, the problems of protecting critical infrastructure from terrorist attacks remain extremely relevant [12, 13]. This is explained, in particular, by the fact that the problem of critical infrastructure protection is poorly structured, and the systems that describe the network of critical infrastructure facilities are poorly formalized organizational systems [14, 15].

In many practical decision-making situations in poorly formalized organizational systems, the decision-maker is forced to act in poorly structured subject areas [16, 17]. To ensure the quality of decision support in poorly structured subject areas, expert knowledge is traditionally and effectively involved. In addition, building a preference structure in a formalized form is a difficult task for humans: in particular, it is difficult for subject matter experts to build metric relations on a set of objects [18, 19].

In particular, a person cannot set reliable weighting coefficients for the relative importance of parameters or criteria [20, 21], expert competence coefficients [22, 23], elements of metricated pairwise comparison matrices [24, 25], or build a reasonable reliable membership function using direct methods [26]. Meanwhile, such problems regularly arise in everyday life and require their solution.

2. Critical infrastructure facilities

Critical infrastructure facilities are those that are

- Particularly important for the state.
- Capable of significantly affecting other critical infrastructure facilities.
- Whose disruption causes a crisis of national importance.
- Vital at the regional level.
- Whose disruption or malfunction causes a crisis of regional, local, or local significance.

Critical infrastructure facilities include enterprises and institutions operating in the following industries:

- Energy

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* Corresponding author.
† These authors contributed equally.
g.gna5@ukr.net (H. Hnatiienko); oleksii.hnatiienko@knu.ua (O. Hnatiienko); babenkot@ua.fm (T. Babenko); myrutenko.lara@gmail.com (L. Myrutenko)
0000-0002-0465-5018 (H. Hnatiienko); 0000-0001-8546-5074 (O. Hnatiienko); 0000-0003-1184-9483 (T. Babenko); 0000-0003-1686-261X (L. Myrutenko)

- Chemical
- Food
- Transport
- Financial and banking
- Information technology and telecommunications
- Utilities: water, heat, and gas supply
- Healthcare, etc.

According to [27], critical infrastructure assets are systems that are essential for maintaining vital social functions, health, safety, security, and economic or social well-being of people.

Timely restoration of critical infrastructure facilities during military operations is particularly important. As a result of unprovoked aggression by Russia, Ukraine faced large-scale and targeted attacks on its critical infrastructure [28]. Attacks were carried out in more than a hundred cities in Ukraine [29].

3. The task of maintaining a network of critical infrastructure facilities

The maintenance task was considered and studied in [30] as the task of maintaining a network of communication elements. Subsequently, the problem statement, approaches to its solution, and the algorithm for sequential analysis of options were adapted to the extremely relevant problem of timely and efficient operation of the C&I system.

3.1. Setting the task of ensuring the operation of the OCI network

Let $R_b = \{r_i^b\}, i = 1, \dots, N_b, b = 1, \dots, B, N_b \leq D, R_b$ is the set of working intervals in a month, N_b is the number of intervals (windows) for the b is the team that ensures the operation of the OCI network; B is the number of teams, $(\cap r_i^b,\ i = 1, \dots, N_b) = \varnothing$ (in particular, the system of intervals may coincide for all teams); r_i^b is determined by the beginning η_i^b and duration $\delta_i^b, r_i^b = (\eta_i^b, \delta_i^b),\ i = 1, \dots, N_b$; D number of working days in a month; $\{\alpha_j\}, j = 1, \dots, n$, is the set of requests for activation of the work of the CMI NMS teams, the duration of which is equal to $\tau_j = \tau(\alpha_j), 1 \leq \tau_j \leq A$; n is number of requests; A is the maximum length of the request; $f_{ij} = f_i(\alpha_j), i = 1, \dots, D$, $j = 1, \dots, n$, is the efficiency of execution of the j is request if it starts to be executed on the i^{th} day of the month. Target function of the task:

$$F(\alpha) = \sum_{j=1}^{n} f_{k_j}(\alpha_j) \to \max_{k_j \in \{1, \dots, D\}}, \quad (1)$$

where k_j is the number of working days in the calendar month on which the j is request starts to be executed. Let $g_{i_j}^l = g_i^l(\alpha_j)$ is the resource of the l is the type required to satisfy the j is request if it starts executing on the i^{th} day, $l = 1, \dots, m, i = 1, \dots, D,\ \ j = 1, \dots, n$. Then the resource constraints are

$$\sum_{j=1}^{n} g_{k_j}^l(\alpha_j) \leq G_l,\ l = 1, \dots, m, \quad (2)$$

β_i here k_j is the number of days when the execution of the$j$ is request starts; m is the number of types of resources; $\beta_i, i = 1, \dots, D$ are restrictions on the readiness of the OCI's command center for the EWM brigades, which means that no more than i is requests can be serviced on a given day,

$$\beta_i \leq B,\ i = 1, \dots, D. \quad (3)$$

It should be noted that requests for activation of the next I&CS teams are received directly from CI facilities, I&CS subsystems (SS), or the governing bodies of the I&CS system.

Array of inconsistencies (incompatibilities) of applications

$$C = (C_{sh_s}),\ s = 1, \dots, S, h_s = 1,2, \dots, \quad (4)$$

where C_{sh_s} is the number of requests that cannot be executed simultaneously; S is the number of array lines; h_s is the indices of incompatible requests of the s^{th} line of the unstacked array.

3.2. Scheme for solving the problem of ensuring the operation of the QMS

The solution to the problem is sought in two stages: building a reference solution and building an optimal solution. To describe the algorithms, we present the necessary definitions.

Definition 1. A variant of the problem (1)–(4) is a vector $v = (v^1, \dots, v^n)$, whose elements are triples$v^i = (v_{i_1}^i, v_{i_2}^i, v_{i_3}^i)$, where i is the number of the request; $v_{i_1}^i$ is the duration of the i^{th} request; $v_{i_2}^i$ is the number of the team that executes i^{th} the request; $v_{i_3}^i$ is the day the request starts i^{th} the request.

Definition 2. An admissible (complete) variant (solution) of the problem (1)–(4) is the variant $v = (v^1, \dots, v^n)$, that satisfies constraints (2)–(4).

Definition 3. A partial solution to problem (1)–(4) is *a* vector, $v = (v^1, \dots, v^s), s < n$, that satisfies constraints (2)–(4).

Definition 4. A locally valid subvariant $v^l = (v_{i_1}^l, v_{i_2}^l, v_{i_3}^l), l \in \{1, \dots, n\}$ of problem (1)–(4) is the placement of a request in some working interval that satisfies conditions (2)–(4).

Definition 5. An admissible subvariant of problem (2)–(4) is a locally admissible subvariant that leads to an admissible variant.

Definition 6. A reference solution to problems (1)–(4) is a feasible solution (variant) that can, without being optimal, make the most of resources and at the same time deliver a local optimum to the objective function.

3.3. Method for finding a valid solution to the problem of ensuring the operation of the QMS

The method of finding a feasible solution consists of the sequential construction of a reference solution as a union of locally feasible subvariants. Therefore, it is reduced to the sequential application of the following procedure.

Procedure for finding a locally admissible subvariant PS. The basis of the method is the formation of a set of admissible subvariants $V = (v_{il}), i = 1, \dots, m + 2,\ \ l =$

$1, \dots, L$, from which a compromise subvariant is selected, L—the number of admissible subvariants. The condition for generating a sub-variant is, firstly, $\delta_i^1 \geq \alpha_j, i = 1, \dots, N_b$, $b = 1, \dots, B$, $j = 1, \dots, n$, i.e., the application α_j should not exceed the interval in which it is supposed to be placed, and secondly, the compatibility of the current application with the one already accepted in the partial solution. In parallel to the set of sub-options, a set of indices corresponding to them is formed, $T = (t_{h_l}), h = 1,2,3,\ l = 1, \dots, L,$ where $t_{1_l} \leq n$ is the number of the application that generates the sub-option, $t_{2_l} \leq B$ is the number of the team proposed to execute the application, $t_{3_l} \leq D$ is the day the execution of the application t_{1_l} by the team starts t_{2_l}.

Consider the procedure for forming a valid subvariant $v_l = (v_{1l}, \dots, v_{m+2,l})$. For each request $\alpha_j, j = t_{1_l}$, all possible combinations of its placement in the working intervals of the teams are selected. At the same time, $v_{1l} = \frac{(A-\tau_j)}{(A-1)}, \tau_j \leq A$ is the length of the request t_{1_l}; $v_{2l} = \omega_s, \omega_s = \left(\frac{(f^0-f_s)}{(f^0-f^H)}\right)^\mu$, where $s = t_{3_l}, F^0(F^H) = \max_{i=1,\dots D}\left(\min f_i\right)$; $\mu > 1$ is a multiplier that plays the role of a weighting factor for the relative importance of the request length for finding a compromise sub-option; $v_{i+2,l} = \left(\frac{g_s}{G_s'}\right)^\mu, i = 1, \dots, m$, where $G_i' = G_i - \Delta G_i^0, s = 1, \dots, n$, ΔG_i^0 is the amount of resource i of the type spent when including the next compromise sub-optionv^k in a partial solution to the problem:

$$G_i' = 0, i = 1, \dots, m\ .$$

Thus, the search for a partial solution to the original problem is reduced to a discrete multicriteria optimization model with a set of feasible solutions V and $(m+2)$ criteria to be minimized. If the set of admissible sub-options is not empty,$V \neq \varnothing$, and not trivial, i.e. $|V| > 1$, we will look for a compromise. To find a single solution to a multicriteria optimization problem, it is necessary to set the weighting coefficients of the criteria [31–33]. Let's fix the weighting factor for the length of the application as ρ_1—for the sake of certainty, let's assume$\rho_1 = 0.5$. Let's denote by ρ_2 the weighting factor of the objective function of the initial problem of the OCI MOC; the criteria that are "responsible" for resources are aggregated and denoted by the total weighting factor

$$\rho_3^A = \sum_{i=3}^{m+2} \rho_i, \sum_{i=1}^{m+2} \rho_i = 1.$$

We are looking for a compromise option as

$$v^k = arg \min_{l=1,\dots,L} \max_{i=1,\dots,m+2} \rho_i \cdot v_{il}. \quad (5)$$

In the case when the solution of (5) is not unique, a linear convolution is applied

$$v^k = arg \min_{v_{i_k} \in U} \sum_{i=1}^{m+2} \rho_i \cdot v_{i_k},$$

where U is the set of indices of sub-variants equivalent by criterion (5).

As a result of the search for the "best" sub-variant, we complement the partial solution. This modifies the original problem. The number of requests is reduced, i.e. $n = n - 1$, and the number and/or length of work intervals of the teams are changed.

There are four options for modifying the system of working intervals of brigades by changing the interval in which the compromise order is placed:

1. The length of the interval $r_k^{t2_k}, k \in \{1, \dots, N^b\}$ is equal to the length of the bid τ_{t1_k} and the interval is completely excluded from consideration. At the same time, $N^b = N^b - 1, b = t_{2_k}$, where k—is the index of the compromise bid.

2. $r_k^{t2_k} > \tau_{t1_k}$ and the compromise application is placed at the beginning of the interval $\eta_k^{t2_k} > t_{3k}$. In this case, $\eta_k^{t2_k} = \eta_k^{t2_k} + t_{3k}$, $\delta_k^{t2_k} = \delta_k^{t2_k} - t_{2k}$.

3. $r_k^{t2_k} > \tau_{t1_k}$ and the compromise bid is placed at the end of the interval. Then $\eta_k^{t2_k} = \eta_k^{t2_k}$, $\delta_k^{t2_k} = \delta_k^{t2_k} - t_{3k}$.

4. $r_k^{t2_k} \geq \tau_{t1_k}$ and the placement of the application does not correspond to any of the three cases. This generates an additional interval with the index, i.e. $d, d = N^b + 1, b = t_{2k}$, $\delta_d^{t2_k} = t_{3k} + \tau_{t1_k}$, $\delta_d^{t2_k} = \eta_k^{t2_k} + \delta_k^{t2_k} - t_{3k} - \tau_{t1_k} - 1$, and $\eta_k^{t2_k} = \eta_k^{t2_k}, \delta_k^{t2_k} = t_{3k} - \eta_k^{t2_k}$.

In addition, the availability conditions are checked by comparing the number of requests accepted for execution on each day of the month $z_i, i = 1, \dots, D$, with the availability limits $\beta_i, i = 1, \dots, D$. If they are equal $z_i = \beta_i$, $i = 1, \dots, D$, the interval system is adjusted on some days: working days of teams for which $z_i - \beta_i = 0$, become days off, which affects the structure of intervals.

As a result of applying the PS procedure to the original problem, a partial solution is constructed and the problem is modified. After n is the application of the described procedure, three cases are possible:

1. The solution to the problem is found and one of the resources is completely exhausted:

$$\exists i \colon \Delta G_i^n = G_i,$$

and hence, $G_i' = 0$. The method of finding the reference solution is completed.

2. The solution to the problem is found, but

$$\text{for } \forall i = 1, \dots, m, \Delta G_i^n < G_i;$$

in this case, it is necessary to reduce the total weighting of resources ρ_3^A by increasing the weight of the objective function ρ_2, taking into account the following condition:

$$\rho_2 + \rho_3^A = 1 - \rho_1; \quad (6)$$

3. The task is incompatible. This, in turn, is possible when:

3.1) one or more resources have been exhausted to obtain a complete solution to the problem. Therefore, the total weight of the resources ρ_3^A should be increased, taking into account condition (6). This reduces the weight ρ_1 of the objective function, which could also influence the "unfavourable" placement of the suboption.

3.2) there is no valid working interval for the next order, and therefore $V \neq \varnothing$. Such a situation is possible if the coefficient ρ_1, which is "responsible" for the length of the order, is not large enough. In this case, orders of short length were likely prioritized and "cut" the working intervals that could accommodate orders of longer length. Such a situation can be managed by reducing the μ indicator. In this case, $v_{1_l}, l = 1, \dots, L$, remain unchanged, and $v_{i_l}, i = 2, \dots, m+2$, $l = 1, \dots, L$, increase by reducing μ with unchanged weighting factors $\rho_i, i = 1, \dots, m+2$, and thus "move away" from the optima.

As a result of applying the described method, we obtain the reference solution $v^0 = (v^{01}, \dots, v^{0n})$ or make sure that the initial problem is incompatible. In this case, the conditions of incompatibility are constructively formulated.

If the initial problem is admissible, you can improve the solution. To describe this method, let's introduce a definition.

Definition 7. A P-admissible sub-variant of $\alpha_i \in \alpha, i = 1, \dots, s$, is one or more bids placed in the same working interval (s is number of admissible sub-variants, P—admissible placement option). Moreover, the P—valid sub-option must be valid.

Definition 8. The length P—of a valid sub-variant will be the distance from the start of the first order in a fixed working interval to the end of the last order in that interval.

Definition 9. P—Valid sub-options are comparable when the sum of the order lengths of one sub-option does not exceed the length of the working interval containing the second sub-option, and vice versa.

Definition 10. P is valid sub-option α will be *more promising* than P is valid sub-option y. If these sub-options are comparable for a fixed interval z and $f(\alpha) > f(y)$, or $f(\alpha) = f(y)$ and ($g(\alpha) = g(y)$ α dominates y in terms of resources).

$f(\alpha)$ denotes $\max_{\alpha \in X} f_z(\alpha)$, where X—is the set of options for placing requests on the interval z.

3.4. Algorithm for improving the reference solution by changing its P-valid variants

The initial data for this algorithm are the data described in the problem statement, as well as the reference solution $v^0 = (v^{01}, \dots, v^{0n})$, obtained as a result of the previous method and described in terms of P—admissible subvariants.

Step 0. Ordering by the quality P—of the admissible subvariants that make up the reference solution. If $f(\alpha) = f(y)$ and the vector $\left(g^1(\alpha), \dots, g^m(\alpha)\right)$ are incomparable with the vector $\left(g^1(y), \dots, g^m(y)\right)$, then the subvariant of shorter length is considered more promising.

Step 1. The master selects P—the best quality admissible sub-variant contained in the reference solution. An attempt is made to improve it by placing it in other working intervals or by permissible permutations in its working interval.

Step 2. If the leading sub-option cannot be made more promising, the next best sub-option is considered. If no P is valid sub-option has improved during the algorithm, the algorithm ends.

With improvement, such cases are possible:

a) the application α is "exchanged" by the working interval of placement with the application y.

b) the application α shall be placed in the time slot previously occupied by the application y, and the application y shall be placed in the previously free time slot.

c) cyclical replacement $\alpha \to y \to z \to \alpha$,

where z—is an additional application involved in the replacement chain to generate additional solution options.

It is easy to see that all the more complex cases are reduced to the cases a)—c) described above.

If the option remains valid, its P is valid sub-options are replaced (permuted) and the process proceeds to step 0. If the option is not valid, an attempt is made to make the permutation valid by making concessions on the sub-options found for the permutation.

At the same time, if $f\left(\alpha^{(k+1)}\right) > f\left(\alpha^{(k)}\right)$ or $f\left(\alpha^{(k+1)}\right) = f\left(\alpha^{(k)}\right)$ and $g\left(\alpha^{(k+1)}\right) > g\left(\alpha^{(k)}\right)$, are used, the option $\alpha^{(k+1)}$, where k—is the iteration number is accepted.

3.5. Algorithm for improving the reference solution by changing its P-admissible subvariants

Step 1. Search for the maximum possible length of the empty segment in the intervals that make up P—valid subvariants.

Step 2. Applications whose length does not exceed the value of the found segment are sorted in descending order.

Step 3. The applications of the found set are "tried on" to the empty segments in which they can be placed. If $f\left(\alpha^{(k+1)}\right) = f\left(\alpha^{(k)}\right)$, the request is moved. If not, other applications are considered. If there is no improvement as a result, the option is locally optimal.

The combinatorial formulation of the problem of QMS and the sequential algorithm for its solution is a convenient tool for research, structuring the subject area and "penetration" of the user into the problem and information content of the QMS problem. At the same time, the described heuristic algorithms are an effective apparatus for finding a locally optimal solution to the problem, since they allow generating an acceptable variant of request service with its subsequent improvement and identifying incompatibilities of the problem.

4. The task of coordinating decisions in a three-level hierarchical system for ensuring the operation of a network of critical infrastructure facilities

In group decision-making and determining the properties of an object, there is almost always a problem reconciling assessments [34, 35]. Experts' opinions often do not coincide and must be aggregated to obtain a single conclusion [36, 37]. In some practical tasks, the definition of an aggregated (integral, resultant, etc.) solution is carried out in the form of intervals or a membership function of a fuzzy set [38].

In [39], the authors considered and studied the problem of coordinating decisions in a two-level hierarchical model for choosing the mode of operation of a communication system. Due to the urgency of the problem of critical infrastructure protection at the regional and state levels, this task was adapted to the problems of ensuring the operation of a hierarchical system of critical infrastructure facilities. The technology for coordinating decisions in a hierarchical system was improved and refined to ensure that decisions are coordinated in a three-tier hierarchical system. The interpretation of the problem area of research was also naturally adapted to the issues related to the functioning and characteristics of the critical infrastructure network.

4.1. Formulation of the problem of decision coordination in a three-level hierarchical system of critical infrastructure

Let there be given a set of alternatives (objects, options, plans, projects, etc.) $a_i \in A$, $i \in I = \{1, \dots, n\}$, each of which is characterized by m features (attributes, characteristics, factors, etc.)

$$a_i = (a_i^1, \dots, a_i^m), \qquad i \in I.$$

To build a model of a specific task, it is often necessary to determine the relative weight of characteristics and their influence on decision-making—to increase certainty and increase the structure of the subject area. Since a person in most cases is not able to adequately assign relative weights, indirect methods are a promising direction for solving the problem of determining the weighting coefficients of characteristics [40, 41].

As a rule, there is a history of preferences between objects (alternatives, players, projects, units, etc.)—based on the results of measuring experts' preferences or any other natural information. This can be a series of tournaments or a ranking of objects, i.e. a complete preference relation.

We will assume that the topology of the set (network) of IEDs requiring regular maintenance and ensuring uninterrupted operation in the event of emergency outages or planned rolling outages is given: the number of IEDs and their geolocation coordinates on the plane. It is necessary to determine:

- A sufficient number of QMS centers.
- Their location (the QIS RM center can only be located in an element of the QIS network).
- Provision of the CMI centers with brigades (network equipment), i.e. the optimal number of brigades in each center (we will assume that all brigades are integrated, interchangeable, and of the same type).
- Distribution (division) of the network into service zones, i.e. finding the best option for clustering network elements with the identification of the element number in which the OCI MIS center is located and the number of elements served by each center.

At the same time, it is necessary to ensure the minimum cost of building the I&CMS system and operating the network in three modes of operation and the maximum probability of maintaining operability for each mode while ensuring restrictions on the average recovery time of each network element. We will interpret the task in terms of a three-level hierarchical model, considering that the modes of operation of the network of elements are subsystems (SS) of the lower level, and the capital costs for the creation and maintenance of the system, operation of the I&C and maintenance of crews are arguments for the quality criterion of the upper-level SS [42, 43].

Let us consider the problem of reconciling the decisions of a certain set of PS of a hierarchical organizational system with indices $I = \{0, \dots, M\}$, connected by a three-level hierarchical structure [44]. We will assume that the analytical or tabular dependencies of the values of $f_l(u)$, $l \in I$, of the quality criteria of the PS on the attributes (characteristics, attributes, etc.) are known. The relationships between the criteria (also given analytically or tabularly) are denoted by

$$M^l(D), l \in I,$$

where D is the set of permissible variants of the values of the features that affect the values of the quality criteria $f_l(u)$, $l \in I$, $u \in D$.

4.2. A mathematical model of the problem of decision coordination in a three-level hierarchical system

The three-tier hierarchical management system under consideration consists of:

- One top-level PS (denoted by the index 0).
- n_1 of medium-sized substations isolated from each other with a set of indices $I_1 = \{1, \dots, n_1\}$.
- n_2 subordinate systems of the set I_1 of isolated lower-level substations with a set of indices $I_2 = \{n_1 + 1, \dots, n_1 + n_2\}$.

As a rule, the model of the hierarchical system of the above structure is built as follows.

The top-level PS SSH^0 coordinates the operation of the middle-level PS SSM^i, $i = 1, \dots, n_1$, using control vectors $u^l, l \in I_1$, whose values are determined when solving the top-level optimization problem according to the top-level optimality criterion. The control influences of each medium-level AC $u_l, l \in I_2$, are determined when solving optimization problems according to the medium-level optimality criterion, taking into account the calculated control influences $u^l, l \in I_1$, received from the upper-level AC. In turn, each l—and $(l \in I_1)$ of the middle level coordinates the work of isolated lower level $SSL^i, i = n_1 + 1, \dots, n_2$, associated with it (the set of indices of which is denoted by $I^l, l \in I_1$, $\left(\sum|I^l| = n_2\right)$. Each l and lower level substation, taking into account the control influence of l is the middle-level substation u_l^H, finds its solution u_l according to its optimality criterion.

A diagram of the relationships between subsystems of different levels of the three-tier hierarchical system is shown in Fig. 1.

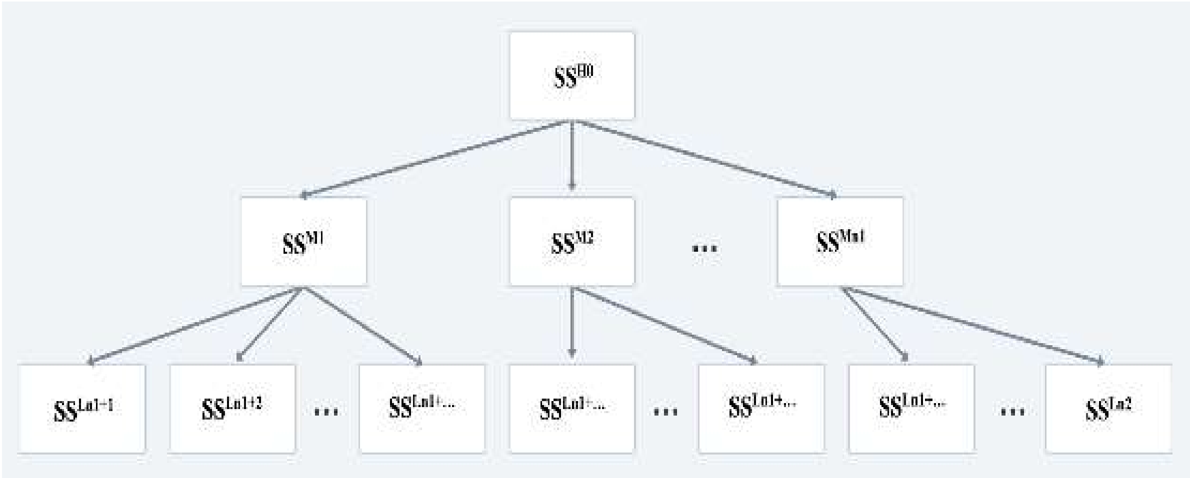

Figure 1: Schematic diagram of the three-level hierarchical structure for ensuring the sustainable operation of the critical infrastructure network

We will assume that the choice of control influences u_l^H in the l of the lower-level AC $SSL^i, i = n_1 + 1, \ldots, n_2$, is carried out when solving a discrete optimization problem of the form

$$f_l^H(u_l^H) \to min, \quad (7)$$

$$H_l^H(u_l^H) \le H^{H_*}, \quad (8)$$

$$g_l^H(u_l^H) \le u_l^C, \quad (9)$$

$$u_l^H \in U_l^H = \prod_{j=1}^{j_l} U_{l_j}^H, \quad (10)$$

where u_l^H is the solution vector, the dimension of which is determined by condition (10); $U_{l_j}^H$ is the finite sets of possible variants j and components of the vector u_l^H; f_l^H are scalar functions; H_l^H, g_l^H are vector functions of a discrete argument of the corresponding dimension, specified analytically or in the form of tables, such that for each component of $u_{l_j}^H$ vectors u_l^H the values of $arg \quad min\left(\max ()_l^H\left(H_l^H, g_l^H\right)\right)$ to u_l^H at fixed values of other variables do not depend on the value of this component (this condition is satisfied, for example, by monotonic or separable functions); H^{H_*}—vector of given constants.

Relationship (9) characterizes the own constraints of the PS, i.e. the relationship between the own parameters of the PS (H^{H_*}—its resource, so condition (9) sets the law of distribution of the own resources of the PS), and relations (8) determine the relationship with the medium-level PS (i.e. the relationship between the own parameters of the PS and external influences—interpreted as the “external” resource of the PS). This is carried out with the help of control influences u_l^C, that is determined by solving a similar problem for each l of the medium-level PS SSM^i, $i = 1, \ldots, n_1$,

$$f_l^C(u_l^C) \to min, \quad (11)$$

$$H_l^C(u_l^C) \le H^{C_*}, \quad (12)$$

$$g_l^C(u_l^C) \le u_l^B, \quad (13)$$

$$u_l^C \in U_l^C = \prod_{j=1}^{j_l} U_{l_j}^C, \quad (14)$$

where all notations correspond to those in problems (7)–(10), u_l^B is control influence transmitted to l that middle-level PS from the upper-level PS SSH^0.

The choice of control influences $u^l, l \in I_1$, in the upper-level control system is carried out when solving the discrete optimization problem of the form

$$f^0(u) \to min, \quad (15)$$

$$H(u) \le H^*, \quad (16)$$

$$u \in U = \prod_{j=1}^{n_1} U^j, \quad (17)$$

where $u = \{u^l, l \in I_1\}$; U^l are finite sets; f^0 is a scalar function; H is a vector function; H^* is a vector of constants (the requirements for f^0 are similar to those for problem (7)–(10)).

The variants of the values of characteristics (factors, features, attributes, etc.) that satisfy condition (8) for a particular model are set explicitly (taking into account information about the peculiarities of the functioning of l of the PS obtained from experts or by solving auxiliary tasks of finding the quality of functioning of l of the PS). Conditions (9) in a particular model are set by tables of correspondence of variants of values of external characteristics to internal ones.

A “partial” mathematical model of the general model discussed above is presented.

We will assume that the set of possible values of characteristics U is divided into the set of $U^з$ general characteristics, on which the quality of the system functioning as a whole depends, and $U_l, l \in I_1$, the set of own characteristics of the PS, i.e., characteristics that affect only the functioning of individual PS; U_{i_j} $u^l, l, i_j \in I, u^l, l \in$ $i \ne j$, characteristics that reflect the “vertical” links between PS of different levels.

Let us denote by $U^\Sigma = U^з \times \prod_{l \in I} U_l \times \prod_{i,j \in I} U_{i_j}$, the set of possible values of characteristics in the functioning of a hierarchical system.

Then, the partial mathematical model of the OCI CMM system is interpreted in terms of the general mathematical model as follows. The values of the criterion functions (7), (11), (15) are set as a function of the given points of the corresponding hyperparallelepipeds (10), (14), (17). The constraints (9) and (13) are set by the tables of correspondence between the binding and own control influences. The eigenconstraints of the PS (8), (10), and (12) are given by the table dependencies on their eigenparameters.

A consistent solution of a hierarchical system is calculated as a "compromise" solution:

$$u^k = arg \min_{u \in U^{\Sigma}} \max_{i \in I} \rho_i \omega_i(u), \quad (18)$$

where $\omega_i(u) = \omega_i(f_i(u)), i \in I$ relative deviations from the optimums of the quality criteria for the functioning of i^{th} PS at the values of parameters u, $0 \le \omega_i(u) \le 1$; ρ_i is weighting coefficients of the importance of PS for achieving the goals of the entire hierarchical system. Finding the weighting coefficients is an independent task. We only note that additional restrictions may be imposed on the weighting factors, for example,

$$\sum_{i \in I_1} \rho_i = \rho_0, i \in I_1,$$

where ρ_i—are the coefficients of the lower-level PS, ρ_0—are the "weights" of the upper-level PS.

U^k, $u^k \in U^k$. In the case when the solution (18) is not unique, an additional criterion of the form

$$\sum_{i \in I} p_i w_i(u^k) \to \min_{u^k \in U^k}.$$

The function of the quality of the upper-level PS functioning is calculated by the formula

$$S = E_{H^*} \sum_{j=1}^{N} S_{1j} + \sum_{j=1}^{N} \sum_{b \in B_j} S_{2j_b} + \sum_{j=1}^{N} \sum_{r \in R_j} S_{3j_r}, i \in I_1,$$

where E_H is the coefficient related to capital expenditures; N is the number of QMS centers (a factor whose value should be found); $S_{1j} = S_1 = const$ is the capital expenditures for the establishment and operation of the $S_{1j} = S_1 = const$ center (for the sake of simplicity, we assume $S_{1j} = S_1 = const$ for); $\forall j = 1, \dots, N$, $R_j, j = 1, \dots, N$ is the number of service teams in the $S_{2j_b} = S_2 = const$ center (we assume that all teams are of the same type, i.e. $S_{2j_b} = S_2 = const$ for $b \in B_j, \forall j = 1, \dots, N$) characteristics whose values are to be calculated; $r \in R_j, \forall j = 1, \dots, N$ is the composition of sets of indices of elements served by the S_{3j_r} center values of characteristics whose values are to be calculated; S_{3j_r} is operating costs to maintain the operability of the r—network element served by the $S_{3j_r} = S_3 = S_3(S)$, the center of the MIS OCI.

The value $S_{3j_r} = S_3 = S_3(S)$, where S is the distance between the center of the OCI RFM with the index j and the network element to be served with the index r. The value of the distance S is calculated by the formula

$$S = \left(\left(x_j^1 - x_r^1\right)^2 + \left(x_j^2 - x_r^2\right)^2 \right)^{1/2},$$

where x_j^1, x_j^2, are the geolocation coordinates of the j of the OCI network element on the plane (this network element may or may not contain an OCI center).

The meaning of S_3 is calculated as follows:

$$S_3 = S_{top} + S_{tp},$$

where S_{top} are the costs of CMM O&M and repairs, S_{tp} are transport costs.

$$S_{top} = T_E \cdot (C_B \cdot (T_{TO} - T_{БP}) + C_{TO} \cdot \tau_{TO})/T_{TO},$$

where C_B, C_{TO} are the average specific costs of restoring a network element and maintenance, respectively; τ_{TO}, T_{TO} are the given constants;$T_{БP}$ is the average failure-free life, which is determined by the formula

$$T_{БP} = T_{TO} \cdot \frac{\mu}{(\mu + \lambda)} + \lambda \cdot \frac{(1 - exp(-(\mu + \lambda) \cdot T_{TO}))}{(\mu + \lambda)^2},$$

in which λ is a constant: μ is the failure rate, $\mu = 1/T_B, T_B$ is the average recovery time $T_B = T_{BP} + \tau_s$; T_{BP} is a constant related to the restoration of operability; $\tau_s = S/v$ is the time spent on moving from the OCI's emergency response center to the network element,$v = const$ is the average speed of brigades' movement, S is the distance between points.

$$S_{TP} = T_E \cdot C_{TP} \cdot (1/T_{БP} + 1/T_{TO})$$

T_E and T_{TO} are the specified constants.

The values of the functionalities of the quality of operation of the lower-level substation are given in the form of a table in the form of a correspondence

$$\left(u_q, u_j^i\right) \Leftrightarrow f_{i_q j}, q \in Q, j \in I^i, i = 1,2,3,$$

where u_q, q is a string of values of "connecting" ("linking") characteristics, u_j^i, j is a string of eigenvalues of characteristics i of the lower-level PS, I^i is a set of indices of variants of values of characteristics of the lower-level PS.

4.3. Algorithm for reconciling decisions in a three-level hierarchical system

The algorithm for matching solutions of a three-level hierarchical system that models the solution of the OCI WRM problem can be described by the following sequence of steps.

Step 0. Reading the data required for the algorithm to function: variants of the values of general characteristics, variants of the values of the characteristics of the substations of all three levels. Calculate or explicitly enter the weighting coefficients of the relative importance of the PS for the functioning of the system as a whole. Calculation or specification of the optimal and worst values of the quality criteria for the functioning of all the PSs of the hierarchical system.

Step 1. Formation of sets of indices of variants of values of characteristics of those PSs that are connected by the same common characteristics. If a certain variant of the common values of characteristics is absent in a PS, then it cannot be included in the search for an agreed solution—it is concluded that it is inadmissible.

Step 2. Based on the coordinates of the network elements, distances between them are calculated to use their values in calculating the quality of the upper-level substation, which is set in an analytical form.

Step 3. Set the initial values of the trade-off $\omega^k = 1$, $\omega^s = 1$, where ω^k is the initial value of the parameter k_0, ω^s is the initial value of the linear function of the PS criteria with the weighting coefficients set in step 0.

Step 4. A complete search of the sets of indices of variants of the values of the characteristics of the PS, connected by the same common characteristics, is organized. If the search is completed, an agreed solution is displayed, i.e., a variant of the values of the characteristics of the PS that delivers a minimum of ω^k and ω^s. This completes the algorithm.

Step 5. The value of the upper-level PS quality criterion is calculated on the next variant of the parameter values, the values of the lower-level PS criteria are searched for in the correspondence tables and these values are converted to a dimensionless form. The next ω^{kH} and values of ω^{sH}, ω^k, and ω^s are also calculated, respectively. If $\omega^{kH} > \omega^k$ or to a dimensionless form. The next ω^{kH} and values of ω^{sH}, ω^k and ω^s, are also calculated, respectively. If $\omega^{kH} > \omega^k$ or $\omega^{kH} = \omega^k$, and $\omega^{sH} > \omega^s$, proceed to step 4. Otherwise, the values are reassigned to $\omega^k = \omega^{kH}$, $\omega^s = \omega^{sH}$. Go to step 4.

5. Prospects for further research

In further research, it is advisable to consider and improve the solution of the problem described in this paper for other classes of problems. In particular, it is promising to formalize the described problem in other classes of mathematical problems:

- Determination of the collective resultant ranking of all applications [45] based on individual applications received from the QIS MQM centers, i.e. formalization of the QIS system in ordinal scales.
- Determination of the relative importance of individual applications [46] from the QMS centers in the form of normalized weighting factors—fixed or interval.
- Formalization of the relative importance of individual applications from the QIS RMA centers in the form of membership functions of a fuzzy set [47].
- Clustering of applications received from the CMI centers and prioritization of the response of teams to the needs of the CMI centers [48].
- Building a functionally stable QMS system, i.e. ensuring redundancy in the QMS system and its reasonable use [49, 50].
- Introduction of the concept of criticality categories of CMI [51] and consideration of this indicator when making decisions on the reliable and uninterrupted operation of CMI's CMM [52].

6. Conclusions

Thus, the paper considers the problems of ensuring the functioning of the critical infrastructure system. The relevance of the study is due, in particular, to the massive attacks on Ukraine's critical infrastructure during Russia's large-scale aggression since February 2024. The paper presents a mathematical model of the problem of maintenance of a system of critical infrastructure facilities developed by the authors. The authors propose a scheme for solving the problem of ensuring the operation of a system of critical infrastructure facilities. The procedures for finding a valid solution to the problem, searching for a reference solution to the problem, and algorithms for improving the reference solution in various variations are described. The problem statement and the mathematical model of decision coordination in a three-level hierarchical system for ensuring the operation of a network of critical infrastructure facilities are also presented. An algorithm for coordinating decisions in a three-level hierarchical system is developed and described.

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