Advanced non-linear 3D FEM modeling of masonry structures for the preservation of cultural heritage

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Abstract
The Italian building stock is largely made up of masonry structures of different ages, having high social and cultural value. Such constructions constitute a heritage to be preserved. Therefore, it is necessary to evaluate their safety level, making use of proper numerical models that should be sufficiently advanced, accurate and detailed, with a specific attention to the 3D modelling, but still computationally feasible. The development of such models presents several difficulties because of the peculiar behavior of these systems, which is very variable according to the observation scale. Therefore, the modelling and analysis of masonry structures is still an open question in the scientific world. In the literature, Concrete Damage Plasticity (CDP) is widely applied to the numerical modeling of this type of structures by means of FEM approaches. This is a material model originally developed for the analysis of concrete elements (reinforced and not). In this work, we report some applications of Concrete Damage Plasticity to problems on masonry structures at different scales.

Keywords
concrete damage plasticity, masonry, FEM, cultural heritage

1. Introduction

Masonry is a building technology which has been widely used in the past, and many Italian cities have historic masonry structures that play valuable social and urban roles.

Analyzing the structural mechanics of masonry buildings, it can be noticed that this kind of structures, if built according to a certain technological rigor, almost always develop only partial collapses, thus showing a better behavior than other structures towards collapse [1, 2]. On the other hand, buildings built according to other construction technologies often collapse completely if not designed with adequate redundancy and robustness [1].

However, this construction technology presents some disadvantages related to the mechanics of the materials used and the overall structural behavior: negligible tensile strength, low compressive strength, limited ductility [1].

A fundamental difficulty in the analysis of the behavior of existing masonry structures is the modelling of the structural element. The modelling of masonry structures is difficult both mechanically and geometrically [3, 4]. Masonry panels, in fact, are characterized by a particular behavior in several aspects, for example, the masonry has a strongly non-linear response with a continuous degradation of stiffness and strength, due to the progressive development of a consistent cracking framework (with inelastic deformations and not negligible hysteretic

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dissipation) [2]. Furthermore, the variability of the mechanical characteristics of the material can lead to the development of unpredictable collapse mechanisms.

With regard to geometrical issues, the complexity in the modelling of masonry structures lies in the variability of structural components used: walls, arches, vaults, domes, beams and columns [4]. Moreover, as D'Altri et al. point out [3], the constructive details, that play a key role in the behavior of the building and are the result of a continuous process of modification of the structure, are often difficult to know and model. To this list, it is necessary to add that many structures are characterized by multi-leaves masonries, with uncertain connection and characterization of each leaf [5].

For several years, the interest of scientific research has been oriented towards the development of mathematical models that simulate the structural response of masonry elements. A summary of existing strategies is reported in this paper. Among the available modeling strategies, in this paper, a FE-model for a test masonry panel has been used, adopting for the constitutive behavior the CDP model, which takes into account the material damage and its evolution during the load history. First, a micro-model of a tuff masonry shear wall has been implemented and has been considered as the benchmark model for the subsequent analysis. Then, the CDP parameters of a homogeneous panel have been calibrated through a sensitivity analysis. Finally, the obtained parameters have been adopted to study the seismic response of a masonry aggregate with a FE macro model.

2. Modelling strategies available in literature: between micro and macro

Different modelling strategies have been developed in the literature in order to deal with the many peculiarities of the masonry structures, introduced in the previous section. Different approaches have been developed for different scale of the analysis [3, 6-11]. A major distinction can be made between micro and macro-models.

The essential element that characterizes the micro-modeling strategy is the discretization of the masonry wall in mortar and blocks, reproducing the wall topology and considering in this way its influence on the elastic and post-elastic response.

Different types of approaches belong to the micro-model family, depending on the formulation adopted and the interaction between the different masonry components [3]. Some of these approaches are: interface element-based approaches [12], contact-based approaches [13], textured continuum-based approaches [14], block-based limit analysis approaches [15], and extended finite element approaches [16, 17].

All the micro-modelling strategies presents pros and cons. On one hand, it is easier to characterize masonry components than the homogenous masonry; on the other hand, the modelling and computational efforts are not negligible, especially when the analysis is conducted to investigate the behavior of a building or of an entire aggregate. In addition, at the building or aggregate scale, it is improbable that the exact masonry texture at any point is known [3].

Macro model approaches are based on the definition of a continuum with mechanical properties defined through a homogenization procedure in order to simulate the behavior of masonry both for the elastic and the post-elastic response. The parameters of the material model of the homogeneous continuum can be obtained by experimental results or multi-scale procedures, in which the homogeneous properties are defined in each step of the analysis considering an RVE (Representative Volume Element) micro-model [18-32].

3. Concrete Damage Plasticity

3.1. Introduction

The Concrete Damage Plasticity is a material model based on the theory of plasticity and on the theory of damage mechanics [33]. This model was created in 1989 by Lubliner et al. [34], with the aim of analyzing the post-elastic behavior of concrete considering: the mechanical non-linearities, the degradation of stiffness and the occurrence of cracks (also quantifying them) [34].

Further developments to the model were made subsequently (in 1998) by Lee and Fenves [35]. These authors propose a modification regarding the use of two independent scalar damage variables (one for compression and one for tensile) able to take into account both the variation of the effective stresses [36-38] and the degradation of stiffness [35].

The material model presented by Lee et al. [35] is now widely used in literature, not only for the analysis of concrete structures (reinforced or not) but also for the analysis of problems related to geomechanics [33] and for the analysis of masonry elements and materials with similar frictional and brittle behavior.

3.2. The Drucker-Prager Yield Criterion and the flow potential rule

In order to understand the properties of the CDP, it is necessary to analyze the Drucker-Prager yield criterion [39], developed by D. C. Drucker and W. Prager in 1952 and involved in the definition of this material model.

The Drucker-Prager Criterion was developed in 1952 by D. C. Drucker and W. Prager, as a generalization of the Mohr-Coulomb criterion [39]. Over time, the original criterion has been modified and nowadays several formulations are available in the literature [40]. The original criterion is known as "Linear Drucker-Prager", later modified to be a nonlinear function. The most frequently used formulations are the hyperbolic (the one implemented in CDP) and the exponential [40]. In the hyperbolic formulation the yielding function is [40]:

$$\sqrt{(c - p_t \cdot \tan\beta)^2 + q^2} = p \cdot tan\,\beta + c, \tag{1}$$

Where: c is the cohesion, p_t is the hydrostatic tensile strength, β is the friction angle, q is the Von Mises equivalent stress and p is the hydrostatic pressure.

Instead, the flow potential G for the hyperbolic Drucker-Prager is defined as [40]:

$$G = \sqrt{(e\overline{\sigma_0}\tan\psi)^2 + q^2} - p \cdot \tan\psi \tag{2}$$

Where q is the Von Mises equivalent stress, p is the hydrostatic pressure, e is the eccentricity, $\overline{\sigma_{t0}}$ is the initial effective yield stress and ψ is the dilation angle.

The CDP uses the exact same flow potential of the hyperbolic Drucker-Prager.

3.3. Damage parameters and yield function

For multiaxial stress states, the stress–strain relationship can be governed by the scalar damage elasticity equation [41]:

$$\boldsymbol{\sigma} = (1 - d)\boldsymbol{D}_{\boldsymbol{0}}^{\boldsymbol{el}}\colon \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\boldsymbol{pl}}\right), \tag{3}$$

where d is the scalar damage variable (function both of the damage scalar variable in tension and in compression), $\boldsymbol{D}_{\boldsymbol{0}}^{\boldsymbol{el}}$ is the elasticity tensor referred to the initial condition (undamaged); $\boldsymbol{\varepsilon}$ is the strain tensor and $\boldsymbol{\varepsilon}^{\boldsymbol{pl}}$ is the plastic part of the strain tensor [35].

The CDP yield function is a function of the effective stresses $\overline{\boldsymbol{\sigma}}$, which are always bigger than the stresses $\boldsymbol{\sigma}$ because the crack formation leads to a reduction in the load-carrying area [36-38]. Then the CDP yield function can be introduced [35]:

$$F(\bar{\sigma}, \tilde{\varepsilon}^{pl}) = \frac{1}{1-\alpha}\left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\varepsilon}^{pl})\langle\hat{\bar{\sigma}}_{max}\rangle - \gamma\langle-\hat{\bar{\sigma}}_{max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}), \tag{4}$$

where:

- $\bar{p}$ is the effective hydrostatic pressure,
- $\bar{q}$ is the Von Mises equivalent of effective stress,
- α is a function of the ratio between the initial equi-biaxial and uniaxial compressive yield stresses σ_{b0} and σ_{c0}:
- $\beta(\tilde{\varepsilon}^{pl})$ is a function of the effective cohesion stresses, $\bar{\sigma}_t(\tilde{\varepsilon}_t^{pl})$ and $\bar{\sigma}_c(\tilde{\varepsilon}_c^{pl})$ (for the respective levels of plastic deformations $\tilde{\varepsilon}_c^{pl}$) and of α:
- $\hat{\bar{\sigma}}_{\max}$ is the maximum eigenvalue of the effective stress tensor,
- γ is a direct function of the K_C, which is defined as the ratio between the Von Mises equivalent effective stress on the tensile meridian $\bar{q}_{TM}$ and on the compressive meridian $\bar{q}_{CM}$. The closer K_C is to 1, the closer the yield surface is to a circle on the deviatoric plane.

4. Micro-Modelling and Macro-Modelling of a Tuff Masonry Panel Subjected to a Control Displacement Shear Test

4.1. Models

In the past, tuff has been widely used in construction in the south of Italy, both for masonry structures and infill walls in reinforced concrete frame structures. Regardless of the role that tuff walls play in the structural organization, it is essential to numerically model the behavior of these masonry elements in existing buildings.

In this study, for the numerical analyses, two FE models have been developed in Abaqus [41] for the test panel, which is 1.00 x 1.00 m^2 large and 0.1 m deep.

The first is a micro-model with a discretization of blocks and mortar joints. Blocks' dimensions are 0.25 x 0.19 x 0.10 m^3 and mortar joints are 0.01 m thick (see Figure 1, left).

The second is a macro-model that is also implemented as a 3D panel, but it is not discretized or partitioned defining a homogeneous equivalent material (see Figure 1, center).

Figure 1 (right) shows the boundary conditions applied to the panel.

Each model has been subjected to two different analysis steps.

During the first step, a vertical and uniform pressure with a magnitude of 0.30 MPa on the top has been applied, instead the base of the panel is kept fixed. At the end of the first step, the panel's upper face is constrained not to move along the Y and Z directions.

During the second step, a horizontal increasing displacement up to 0.0031 m has been applied at the top of the panel along the X direction, keeping constant the constrains applied at the end of the first step.

Hence, the lower face is always kept fixed, while the upper face is bound to move parallel to the lower face in the second step.

For both numerical models, it is used a mesh with an approximate global size of 1.5 cm.

In the micro-model, different CDP parameters have been assigned to the blocks and to the mortar, in order to take in account their different behavior, as reported in Tables 1 and 2. In particular, compressive response is described by a tri-linear curve, whereas tensile one is described by a bilinear curve. The corresponding characteristic points are reported in Tables 1-2.

The values adopted for the mechanical parameters of the macro-model are reported in Table 3. They are obtained from an accurate sensitivity analysis and not from a numerical optimization procedure. The sensitivity analysis has been conducted as illustrated in [42].

For further details about CDP Model, the reader is addressed to ABAQUS User Manual [41].

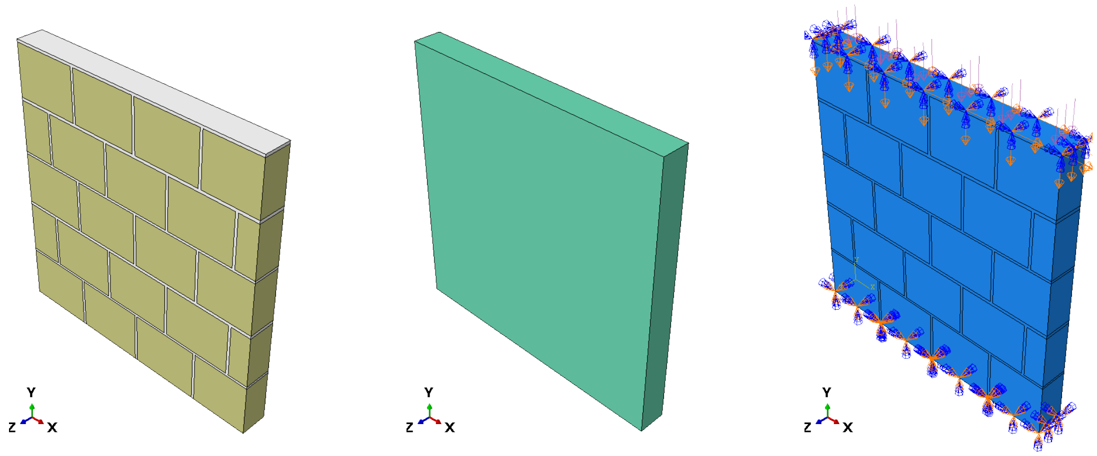

Figure 1: Tuff micro and macro-models implemented in Abaqus and boundary conditions applied.

Table 1
CDP parameters set 1 adopted for blocks in the tuff micro-model

Elasticiy parameters	E [Pa] 16,700,000,000			ν 0.15	
Plasticity parameters	ψ [°] 10	e 0.1	σ_{b0}/σ_{c0} 1.16	K_C 2/3	μ 0.0001
Compressive response	Yielding stress [Pa]	Inelastic strain [%]	Damage parameter	Inelastic strain [%]	
	4,412,700	0	0	0	
	4,903,000	0.005	0.1	0.005	
	0	0.015	0.2	0.015	
Tensile response	Yielding stress [Pa]	Displacement [m]	Damage parameter	Displacement [m]	
	326,867	0	0	0	
	0	0.00049	0.1	0.00015	

Table 2
CDP parameters set 2 adopted for mortar in the tuff micro-model

Elasticiy parameters	E [Pa] 800,000,000			ν 0.15	
Plasticity parameters	ψ [°] 15	e 0.1	σ_{b0}/σ_{c0} 1.16	K_C 2/3	μ 0.0001
Compressive response	Yielding stress [Pa]	Inelastic strain [%]	Damage parameter	Inelastic strain [%]	
	9,450,000	0	0	0	
	10,500,000	0.005	0.1	0.005	
	0	0.015	0.2	0.015	
Tensile response	Yielding stress [Pa]	Displacement [m]	Damage parameter	Displacement [m]	
	250,000	0	0	0	
	0	0.000144	0.1	0.000144	

Table 3
CDP parameters set 3adopted for the tuff macro-model

Elasticiy parameters	E [Pa] 6,000,000,000			ν 0.15	
Plasticity parameters	ψ [°] 36.9	e 0.1	σ_{b0}/σ_{c0} 1.16	K_C 2/3	μ 0.0001
Compressive response	Yielding stress [Pa]	Inelastic strain [%]	Damage parameter	Inelastic strain [%]	
	4,412,700	0	0	0	
	4,903,000	0.004	0.1	0.005	
	0	0.01	0.2	0.015	
Tensile response	Yielding stress [Pa]	Displacement [m]	Damage parameter	Displacement [m]	
	200,000	0	0	0	
	0	0.00018	0.1	0.00018	

4.2. Results

To analyze the behavior of the panel subjected to the shear displacement-controlled test, the results are reported in terms of the tensile damage maps (Figure 2, left) and of capacity curves which show the relationship between the horizontal reaction force measured at the bottom of the panel and the horizontal displacement measured at the top (Figure 2, right).

The damage map in Figure 2 (left) shows that the damage process involves both the mortar and the blocks in the micro-model. The tensile damages are developing mostly along the diagonal of the panel and this is an obvious result caused by the distribution of tensile and compressive stresses along the principal direction. The capacity curve of the micro-model shows that the panel has an elastic behavior initially, then it reaches a peak value of horizontal reaction force. Between the peak and the failure, there is a softening branch.

The macro-model (Figure 2, center) well reproduces the results obtained with the micro-model (Figure 2, left). However, the damage map obtained can't grasp the difference between blocks and mortar in terms of damage but only can grasp the global collapse mechanism (diagonal cracking). The capacity curve, instead, can reproduce the result obtained with the micro-model, with a lower computational effort.

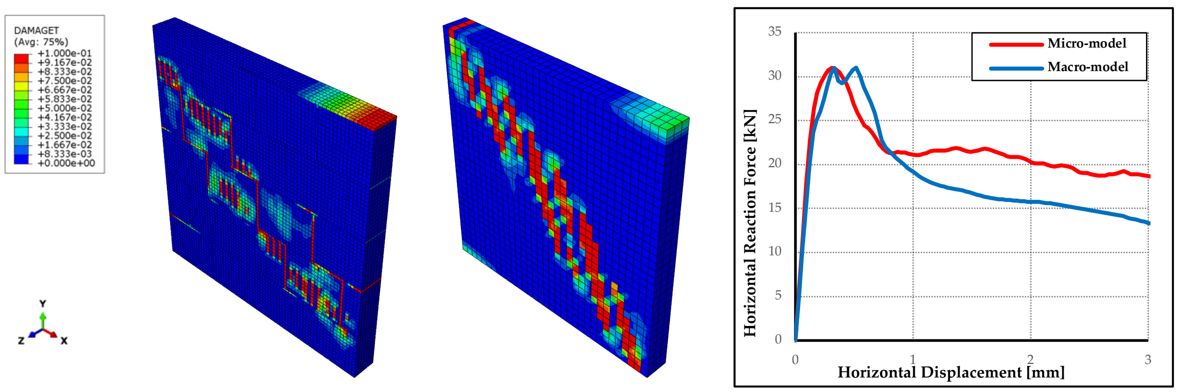

Figure 2: From the left to the right: map of the tensile damage developed on the tuff micro-model, map of the tensile damage developed on the tuff macro-model and capacity curves obtained from both the models.

5. Macro-modelling of an aggregate: a case study

The calibrated CDP parameters are used to study the seismic response of a structural aggregate. More in detail, the seismic response is evaluated through a non-linear explicit dynamic analysis conducted on a 3D FEM model of the aggregate.

The structural aggregate considered is located in the historical center of the Municipality of Foggia [43].

The implemented model is composed by shell elements with a mesh of approximate global size of 0.15 m. The model is simply supported and the seismic accelerations have been applied at the base acting with the same intensity in the two directions of the plane that contains the base of the model.

Since the objective of this work is to verify that the constitutive model used is capable of adequately reproducing the damage maps usually detected in post-seismic scenarios and not to evaluate the seismic vulnerability of the aggregate, only one natural accelerogram has been used for this first analysis. The one used is compatible with the requirements of the National Building Code for the area and limit state considered (life-safety). It was obtained through the REXEL [44] software, an application capable of providing accelerograms of real seismic events and available in the ESD (European Strong-Motion Database) [45] and the ITACA (Italian Accelerometric Archive) [46].

Figure 3 shows the 3D model implemented. Figure 4-6 show the damage maps obtained from the analysis.

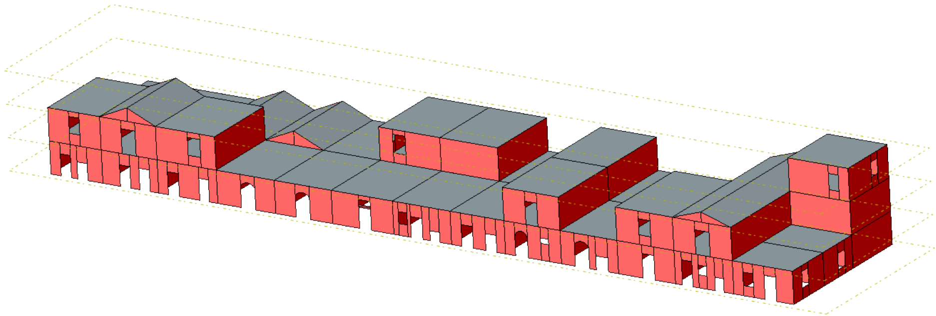

Figure 3: 3D model of the masonry aggregate.

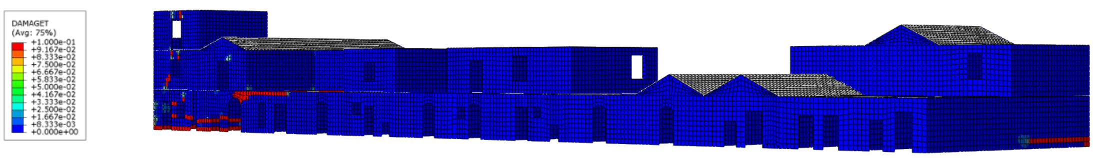

Figure 4: Axonometric view of the damage map obtained on the west and south façades.

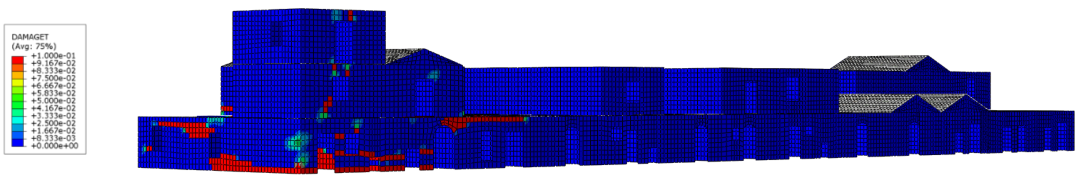

Figure 5: Axonometric view of the damage map obtained on the north and west façades.

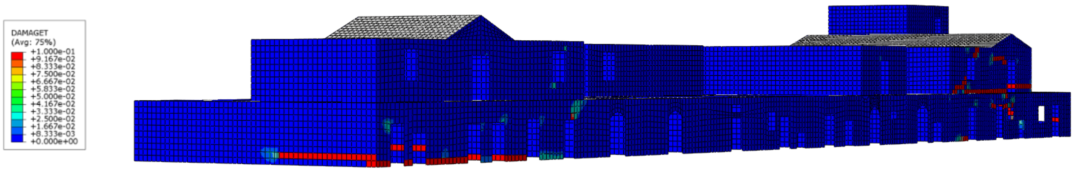

Figure 6: Axonometric view of the damage map obtained on the east and south façades.

The analysis of Figure 4-6 shows that the model implemented is able to grasp the possible damage of the material. In fact, the pattern produced by the analysis is typical of masonry buildings subject to seismic actions that cyclically reverse their sign. We can also note the similarity between the diagonal crack patterns obtained for the panel and the ones obtained for the building. Clearly, in the latter case, there is presence of other types of collapse mechanisms (such as sliding) that on the panel could only be reproduced by changing the boundary conditions.

6. Conclusions

In this work, after a brief introduction about the essential issues and the still open challenges in the scientific world about masonry modelling, we focused on the possibility of using the Concrete Damage Plasticity material model for tuff-masonry modeling.

The procedure here proposed is based on 3 main steps. First of all, the shear response of a single masonry panel was studied with both a micro and a macro model. Knowing from literature the parameters for blocks and mortar the CDP parameters for the macro-model have been calibrated. Subsequently the calibrated parameters have been adopted for the study of a masonry aggregate.

The application of the set of CDP parameters, calibrated on a micro-model with tuff elements, to the nonlinear dynamic analysis of a structural aggregate, has allowed to obtain damage maps with patterns that are coherent with the typical cracks detected in post-seismic scenarios on masonry buildings. In particular, the set of calibrated parameters allowed to reproduce the shear-sliding failure mechanisms, with horizontal, vertical or diagonal damage patterns.

In the future, more detailed studies at the scale of the aggregate could help to understand how the CDP parameters affect these collapse mechanisms.

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