# Towards Temporal Many-valued Conditional Logics for Gradual Argumentation: a Preliminary Report

Mario Alviano[1,*], Laura Giordano[2,*] and Daniele Theseider Dupré[2,*]

[1] *DEMACS, University of Calabria, Via Bucci 30/B, 87036 Rende (CS), Italy*
[2] *DISIT, University of Piemonte Orientale, Viale Michel 11, 15121 Alessandria, Italy*

**Abstract**

In this paper we propose a many-valued temporal conditional logic, and exploit it in the verification of properties of an argumentation graph, in a gradual semantics. We start from a many-valued logic with typicality, and extend it with the temporal operators of the Linear Time Temporal Logic (LTL), thus providing a formalism which is able to capture the dynamics of a system, trough strict and defeasible temporal properties. We then consider an instantiation of the formalism for gradual argumentation.

**Keywords**

Preferential and Conditional reasoning, Many-valued logics, Temporal Reasoning, Argumentation

## 1. Introduction

Preferential approaches to commonsense reasoning [1, 2, 3, 4, 5, 6, 7, 8, 9] have their roots in conditional logics [10, 11], and have been used to provide axiomatic foundations of non-monotonic or defeasible reasoning. In recent work [12], we have proposed a many-valued multi-preferential conditional logic with typicality to define a preferential interpretation of an argumentation graph in gradual argumentation semantics [13, 14, 15, 16, 17, 18], provided some weak conditions on the domain of argument interpretation are satisfied by the semantics. The many-valued conditional logic with typicality is not only intended for reasoning about argumentation graphs. It can be used as the basis for a general formalism for reasoning about the dynamic of a system, as well as for reasoning about the dynamic of *weighted Knowledge Bases* (KBs). Actually, in the static case, a weighted knowledge base can be seen as a weighted argumentation graph, and vice-versa. The relationships between *weighted argumentation graphs*, under a specific gradual semantics (the $\varphi$-coherent semantics [19, 20]), and weighted conditional KBs in a Description Logic (DL) formalism [21, 22] has been studied in [23].

This paper deals with the propositional setting, and aims at extending the many-valued conditional logic with typicality developed in [12] by adding to the language the temporal operators of Linear Time Temporal Logic (LTL), thus defining a *propositional many-valued temporal logic with typicality*. The resulting temporal conditional formalism allows considering the temporal dimension of a weighted conditional KB and reasoning about the defeasible properties of a system for explanation. Capturing the dynamics of a weighted knowledge base can be exploited, for instance, to prove properties about the transient behavior of a (recurrent) neural network (which can indeed be characterized as a weighted KB [22]), or to reason about the dynamics of a weighted argumentation graph [23] under a gradual semantics. In particular, as we will point out in Section 5, the proposed formalism allows reasoning about the evolution of a weighted argumentation graph, when the weights of edges of the argumentation graph change in time, e.g. when the weights of the argumentation graph (or of the weighted KB) are

*8th Workshop on Advances in Argumentation in Artificial Intelligence ($AI^3$ 2024)*
*Corresponding author.
mario.alviano@unical.it (M. Alviano); laura.giordano@uniupo.it (L. Giordano); dtd@uniupo.it (D. Theseider Dupré)
https://alviano.net/ (M. Alviano); https://people.unipmn.it/laura.giordano/ (L. Giordano); https://people.unipmn.it/dtd/ (D. Theseider Dupré)
0000-0002-2052-2063 (M. Alviano); 0000-0001-9445-7770 (L. Giordano); 0000-0001-6798-4380 (D. Theseider Dupré)

learned[1]. As another example, the structure of an argumentation graph can be updated through the interaction of different agents in time, such as in the framework [27], via Argumentative Exchanges.

Preferential extensions of LTL with defeasible temporal operators have been recently studied [28, 29, 30] to enrich temporal formalisms with non-monotonic reasoning features, by considering defeasible versions of the LTL operators. Our approach, instead, consists in adding the standard LTL operators to a (many-valued) conditional logic with typicality, an approach similar to the temporal preferential extension considered for Description Logics (DLs) in [31], where the logic $LTL_{\mathcal{ALC}}$ [32], extending $\mathcal{ALC}$ with LTL operators, has been further extended with a *typicality operator*, to develop a (two-valued) temporal $\mathcal{ALC}$ with typicality, called $LTL^{\mathbf{T}}_{\mathcal{ALC}}$.

As in the Propositional Typicality Logic by Booth et al. [33] (and in the DLs with typicality [34]) the conditionals are formalized based on material implication (resp., concept inclusion) plus the *typicality operator* $\mathbf{T}$. The typicality operator allows for the definition of *conditional implications* $\mathbf{T}(\alpha) \rightarrow \beta$, meaning that "normally if $\alpha$ holds, $\beta$ holds". They correspond to conditional implications $\alpha \mathrel{|\!\sim} \beta$ in KLM logics [4, 6]. More precisely in this paper, as in [12], we consider a *many-valued semantics*, so that a formula is given a value in a *truth degree set* $\mathcal{D}$, and the two-valued case can be regarded as a specific case, obtained for $\mathcal{D} = \{0, 1\}$. As the logic is many-valued, we consider *graded conditionals* of the form $\mathbf{T}(\alpha) \rightarrow \beta \geq l$ (resp., $\mathbf{T}(\alpha) \rightarrow \beta \leq l$), meaning that "normally if $\alpha$ holds then $\beta$ holds with degree at least (resp., at most) $l$". For instance, the formalism allows for representing graded implications such as:

$$(living\_in\_Town \wedge Young \rightarrow \mathbf{T}(\Diamond Granted\_Loan)) \geq l,$$

meaning that living in town and being young implies that normally the loan is eventually granted with degree at least $l$, where the interpretation of some propositions (e.g., $Young$) may be non-crisp.

The preferential semantics of the logic exploits *multiple preference relations* $<_\alpha$ with respect to different formulas $\alpha$, following the approach developed for ranked and weighted KBs in description logics, based on a *multi-preferential semantics* [35, 36] and for the propositional calculus in [37], where preference are allowed with respect to different aspects.

The schedule of the paper is the following. Section 2 develops a *many-valued preferential logic with typicality*, which is inspired to [12]. Section 3 extends such logic with LTL modalities to develop a *temporal many-valued conditional logic*, by considering *temporal graded formulas*. In Section 4, we introduce *weighted temporal knowledge bases* and their semantics. In Section 5, an instantiation of the logic for gradual argumentation is considered, in the direction of providing a temporal conditional semantics for reasoning about the dynamics of gradual argumentation graphs. Section 6 concludes the paper.

## 2. A Many-valued Preferential Logics with Typicality

In this section, we recall a many-valued preferential logic with typicality developed in [12].

Let $\mathcal{L}$ be a propositional many-valued logic, whose formulas are built from a set $Prop$ of propositional variables using the logical connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$, as usual. We assume that $\bot$ (representing falsity) and $\top$ (representing truth) are formulas of $\mathcal{L}$. We consider a many-valued semantics for formulas, over a *truth degree set* $\mathcal{D}$, equipped with a preorder relation $\leq^{\mathcal{D}}$, a bottom element $0^{\mathcal{D}}$, and a top element $1^{\mathcal{D}}$. We denote by $<^{\mathcal{D}}$ and $\sim^{\mathcal{D}}$ the related strict preference relation and equivalence relation (often, we will omit explicitly referring to $\mathcal{D}$, and simply write $\leq$ $<$, $\sim$, 0 and 1).

Let $\otimes$, $\oplus$, $\ominus$ and $\rhd$ be the *truth degree functions* in $\mathcal{D}$ for the connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$ (respectively). When $\mathcal{D}$ is $[0, 1]$ or the finite truth space $\mathcal{C}_n = \{0, \frac{1}{n}, \ldots, \frac{n-1}{n}, \frac{n}{n}\}$, for an integer $n \geq 1$, as in our case of study [20], $\otimes$, $\oplus$, $\rhd$ and $\ominus$ can be chosen as a t-norm, an s-norm, an implication function, and a negation function in some system of many-valued logic [38]; for instance, in Gödel logic (that we will consider later): $a \otimes b = min\{a, b\}$, $a \oplus b = max\{a, b\}$, $a \rhd b = 1$ *if* $a \leq b$ *and* $b$ *otherwise*; and $\ominus a = 1$ *if* $a = 0$ *and* $0$ *otherwise*.

[1] Indeed, a multilayer neural network can be regarded as an argumentation graph [24, 25], or as a weighted knowledge base [26, 22], based on the strong relationships of the two formalisms [23].

We further extend the language of $\mathcal{L}$ by adding a typicality operator, as introduced by Booth et al. [33] for the propositional calculus, and by Giordano et al. for preferential description logics [39]. Intuitively, "a sentence of the form $\mathbf{T}(\alpha)$ is understood to refer to the *typical situations in which* $\alpha$ *holds*" [33]. The typicality operator allows the formulation of *conditional implications* (or *defeasible implications*) of the form $\mathbf{T}(\alpha) \rightarrow \beta$ whose meaning is that "normally, if $\alpha$ then $\beta$", or "in the typical situations when $\alpha$ holds, $\beta$ also holds". They correspond to conditional implications $\alpha \mid\sim \beta$ of KLM preferential logics [6]. As in PTL [33], the typicality operator cannot be nested. When $\alpha$ and $\beta$ do not contain occurrences of the typicality operator, an implication $\alpha \rightarrow \beta$ is called *strict*. We call $\mathcal{L}^{\mathbf{T}}$ the language obtained by extending $\mathcal{L}$ with a unary typicality operator $\mathbf{T}$. In the logic $\mathcal{L}^{\mathbf{T}}$, we allow *general implications* $\alpha \rightarrow \beta$, where $\alpha$ and $\beta$ may contain occurrences of the typicality operator.

The interpretation of a typicality formula $\mathbf{T}(\alpha)$ is defined with respect to a preferential interpretation. The KLM preferential semantics [4, 6, 3] exploits a set of worlds $\mathcal{W}$, with their valuation and a preference relation $<$ among worlds, to provide an interpretation of conditional formulas. A conditional $A \mid\sim B$ is satisfied in a preferential interpretation, if $B$ holds in all the most normal worlds satisfying $A$, i.e., in all $<$-minimal worlds satisfying $A$.

Here we consider a many-valued multi-preferential semantics. The propositions at each world $w \in \mathcal{W}$ have a value in $\mathcal{D}$ and multiple preference relations $<_A \subseteq W \times W$ are associated to formulas $A$ of $\mathcal{L}$.

Multi-preferential semantics have previously been proposed by Gill in [40], used in defining refinements of the rational closure construction [37], and for description logics, in ranked defeasible KBs [41] and, in the many-valued case, in weighted conditional KBs [21, 22]. The semantics below exploits a set of preference relations $<_{A_i}$ associated to formulas $A_i$ of $\mathcal{L}$ [2].

**Definition 1.** *A* (multi-)preferential interpretation *is a triple* $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ *where:*

- $\mathcal{W}$ *is a non-empty set of worlds;*
- *each* $<_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ *is an irreflexive and transitive relation on* $\mathcal{W}$*;*
- $v : \mathcal{W} \times Prop \longrightarrow \mathcal{D}$ *is a valuation function, assigning a truth value in* $\mathcal{D}$ *to any propositional variable in each world* $w \in \mathcal{W}$*.*

The valuation $v$ is inductively extended to all formulas in $\mathcal{L}^{\mathbf{T}}$ as follows:

$$v(w, \bot) = 0_{\mathcal{D}} \qquad v(w, \top) = 1_{\mathcal{D}}$$
$$v(w, A \wedge B) = v(w, A) \otimes v(w, B) \qquad v(w, A \vee B) = v(w, A) \oplus v(w, B)$$
$$v(w, A \rightarrow B) = v(w, A) \rhd v(w, B) \qquad v(w, \neg A) = \ominus v(w, A)$$

and the interpretation of a typicality formula $\mathbf{T}(A)$ in $\mathcal{M}$, at a world $w$, is defined as:

$$v(w, \mathbf{T}(A)) = \begin{cases} v(w, A) & \text{if } \nexists w' \in \mathcal{W} \text{ s.t. } w' <_A w \\ 0_{\mathcal{D}} & \text{otherwise} \end{cases}$$

When $v(w, \mathbf{T}(A)) \neq 0_{\mathcal{D}}$, $w$ is a typical/normal $A$-world in $\mathcal{M}$. Note that we do not assume well-foundedness of $<_A$.

A *ranked* interpretation is a (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ for which each preference relation $<_{A_i}$ is modular, i.e., for all $x, y, z$, if $x <_{A_i} y$ then $x <_{A_i} z$ or $z <_{A_i} y$.

In general, some conditions may be needed to enforce an *agreement* between the truth values of a formula $A_i$ at the different worlds in $\mathcal{M}$ and preference relation $<_{A_i}$ among them. The preferences $<_{A_i}$ might have been determined by some *closure construction*, such as those exploiting the ranks or weights of conditionals, as in [41, 21]. Similar conditions, called coherence, faithfulness and $\varphi$-coherence conditions, have for instance been introduced in the multi-preferential semantics for DLs with typicality in [21, 22].

[2] If we limit our consideration to finite KBs, we can restrict our attention to interpretations with a finite set of preference relations $<_{A_i}$, one for each formula $A_i$ such that $\mathbf{T}(A_i)$ occurs in the KB or in the query.

We call a (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ *coherent* if, for all $w, w' \in \mathcal{W}$, and preference relation $<_{A_i}$,

$$v(w, A_i) > v(w', A_i) \iff w <_{A_i} w'$$

that is, the ordering among $A_i$ valuations in $w$ and $w'$ is justified by the preference relation $<_{A_i}$; and vice-versa. A weaker condition is faithfulness. A (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ is *faithful* if, for all $w, w' \in \mathcal{W}$, and preference relation $<_{A_i}$,

$$v(w, A_i) > v(w', A_i) \Rightarrow w <_{A_i} w'$$

Clearly, a preferential interpretation $\mathcal{M}$ might be coherent with respect to a preference relation $<_{A_i}$, while being only faithful with respect to another one $<_{A_j}$.

Let us now define the satisfiability of a *graded implication* with form $A \rightarrow B \geq l$ (or $A \rightarrow B \leq u$) with respect to an interpretation $\mathcal{M}$, where $l$ and $u$ are constants corresponding to truth values in $\mathcal{D}$ and $A$ and $B$ are formulas of $\mathcal{L}^{\mathbf{T}}$.

We can first define the truth degree of an implication $A \rightarrow B$ in $\mathcal{M}$ as follows:

**Definition 2.** *Given a preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$, the* truth degree of an implication $A \rightarrow B$ wrt. $\mathcal{M}$ *is defined as:*
$(A \rightarrow B)^{\mathcal{M}} = inf_{w \in \mathcal{W}}(v(w, A) \rhd v(w, B))$.

The satisfiability of a *graded implication* in a preferential interpretation $\mathcal{M}$ is evaluated globally to the preferential interpretation $\mathcal{M}$.

**Definition 3.** *A preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$,* satisfies a graded implication $A \rightarrow B \geq l$ *(written $\mathcal{M} \models A \rightarrow B \geq l$) iff $(A \rightarrow B)^{\mathcal{M}} \geq l$. Similarly, $I$ satisfies a graded implication $A \rightarrow B \leq u$ (written $\mathcal{M} \models A \rightarrow B \leq u$) iff $(A \rightarrow B)^{\mathcal{M}} \leq u$.*

Let a *knowledge base* $K$ be a set of graded implications. A *model of* $K$ is an interpretation $\mathcal{M}$ which satisfies all the graded implications in $K$. Given a knowledge base $K$, we say that $K$ *entails* a graded implication $A \rightarrow B \geq l$ if $A \rightarrow B \geq l$ is satisfied in all the models of $K$ (and similarly for a graded implication $A \rightarrow B \leq l$). In the following, we will refer to the entailment of graded implications $A \rightarrow B \geq 1$ as 1-*entailment*.

It can be proven that the the usual KLM preferential semantics [4] can be regarded as a special case of the multi-preferential semantics above. The KLM properties of a *preferential consequence relation* can be reformulated in the many-valued setting, and they are satisfied by multi-preferential interpretations for some choice of combination functions, when the preference relations $<_{A_i}$ are well-founded.

## 3. A Temporal Preferential Logic with Typicality

In this section we extend the language of the logic $\mathcal{L}^{\mathbf{T}}$ with the temporal operators $\bigcirc$ (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of Linear Time Temporal Logic (LTL) [42].

We extend the language of graded implications, by allowing temporal and typicality operators to occur in a graded implication $A \rightarrow B \geq l$ (or $A \rightarrow B \geq l$) in $A$ and in $B$, with the only restriction that $\mathbf{T}$ should not be nested. For instance,

$$lives\_in\_town \land young \rightarrow \mathbf{T}(\Diamond granted\_loan) \geq 0.8$$
$$\Diamond \mathbf{T}(granted\_loan) \rightarrow lives\_in\_town \land young \geq 0.8.$$

are graded implications. We define the semantics of the logic in agreement with the fuzzy LTL semantics by Frigeri et al. [43].

**Definition 4.** *A* temporal (multi-)preferential interpretation *is a triple $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ where:*

- *$\mathcal{W}$ is a non-empty set of worlds;*

- *each $<^n_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ is an irreflexive and transitive relation on $\mathcal{W}$;*
- *$v : \mathbb{N} \times \mathcal{W} \times Prop \longrightarrow \mathcal{D}$ is a valuation function assigning, at each time point $n$, a truth value to any propositional variable in each world $w \in \mathcal{W}$.*

When there is no $w' \in \mathcal{W}$ s.t. $w' <^n_A w$, we say that $w$ is a normal situation for $A$ at timepoint $n$.

In a preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$, the valuation $v(n, w, A)$ of a formula $A$, in world $w$, at time point $n \in \mathbb{N}$, can be defined inductively as follows:

$$v(n,w,\bot) = 0_{\mathcal{D}} \qquad v(n,w,\top) = 1_{\mathcal{D}} \qquad v(n,w,\neg A) = \ominus v(n,w,A)$$
$$v(n,w,A \wedge B) = v(n,w,A) \otimes v(n,w,B)$$
$$v(n,w,A \vee B) = v(n,w,A) \oplus v(n,w,B)$$
$$v(n,w,\mathbf{T}(A)) = \begin{cases} v(n,w,A) & \text{if } \nexists w' \in \mathcal{W} \text{ s.t. } w' <^n_A w \\ 0_{\mathcal{D}} & \text{otherwise} \end{cases}$$
$$v(n,w,\bigcirc A) = v(n+1,w,A)$$
$$v(n,w,\Diamond A) = \bigoplus_{m \geq n} v(m,w,A) \qquad v(n,w,\Box A) = \bigotimes_{m \geq n} v(m,w,A)$$
$$v(n,w,A\mathcal{U}B) = \bigoplus_{m \geq n}(v(m,w,B) \otimes \ \bigotimes_{k=n}^{m-1} v(k,w,A))$$

The semantics of $\Diamond$, $\Box$ and $\mathcal{U}$ requires a passage to the limit. Following [43], we introduce a bounded version for $\Diamond$, $\Box$ and $\mathcal{U}$, by adding new temporal operators $\Diamond_t$ (eventually in the next $t$ time points), $\Box_t$ (always within $t$ time points) and $\mathcal{U}_t$, with the interpretation:

$$v(n,w,\Diamond_t A) = \bigoplus_{m=n}^{n+t} v(m,w,A) \qquad v(n,w,\Box_t A) = \bigotimes_{m=n}^{n+t} v(m,w,A)$$
$$v(n,w,A\mathcal{U}_t B) = \bigoplus_{m=n}^{n+t}(v(m,w,B) \otimes \ \bigotimes_{k=n}^{m-1} v(k,w,A))$$

so that

$$v(n,w,\Diamond A) = lim_{t \to +\infty}\ v(n,w,\Diamond_t A)$$
$$v(n,w,\Box A) = lim_{t \to +\infty}\ v(n,w,\Box_t A)$$
$$v(n,w,A\mathcal{U}B) = lim_{t \to +\infty} v(n,w,A\mathcal{U}_t B)$$

The existence of the limits is ensured by the fact that $v(n, w, \Diamond_t A)$ and $v(n, w, A\mathcal{U}_t B)$ are increasing in $t$, while $v(n, w, \Box_t A)$ is decreasing in $t$ (assuming the monotonicity properties of t-norms and t-conorms for $\otimes$ and $\oplus$) [43].

The notions of coherence and faithfulness can be extended to temporal multi-preferential interpretation. E.g., a multi-preferential interpretation is *coherent* if, for all $w, w' \in \mathcal{W}$, $n \in \mathbb{N}$ and preference relation $<_{A_i}$, $v(n, w, A_i) > v(n, w', A_i) \iff w <^n_{A_i} w'$ (and similarly for faithfulness).

Note that, here, we have not considered the additional temporal operators ("soon", "almost always", etc.) introduced by Frigeri et al. [43] for representing vagueness in the temporal dimension (which can be considered for future work). As a consequence, for the case $\mathcal{D} = [0, 1]$, without the typicality operator, the semantics corresponds to the semantics of FLTL (Fuzzy Linear-time Temporal Logic) by Lamine and Kabanza [44].

**Proposition 1.** *For any formulas $A$ and $B$, time point $n$ and world $w$, the following holds:*
$v(n,w,\Diamond A) = v(n,w,A) \oplus v(n+1,w,\Diamond A)$
$v(n,w,\Box A) = v(n,w,A) \otimes v(n+1,w,\Box A)$
$v(n,w,A\mathcal{U}B) = v(n,w,B) \oplus\ (v(n,w,A) \otimes v(n+1,w,A\mathcal{U}B))$

It can be seen that a temporal many-valued interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ can be regarded as a sequence of (non-temporal) preferential interpretations $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots$ over the same set of worlds $\mathcal{W}$, where each $\mathcal{M}_n$ is defined as follows: $\mathcal{M}_n = \langle \mathcal{W}, \{<^n_{A_i}\}, v^n \rangle$, where $w <^n_{A_i} w'$ holds in $\mathcal{M}_n$ iff $w <^n_{A_i} w'$ holds in $\mathcal{I}$, for all $w, w' \in \mathcal{W}$; and $v^n(w, A) = v(n, w, A)$, for all $w \in \mathcal{W}$ and propositional variable $A$.

**Definition 5.** *Given a temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *the* truth degree of an implication $A \rightarrow B$ in $\mathcal{I}$ at time point $n$ *is defined as:*

$$(A \rightarrow B)^{\mathcal{I},n} = inf_{w\in\mathcal{W}}(v(n, w, A) \rhd v(n, w, B)).$$

Let us now define the *satisfiability of a graded implication* in a preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$. Rather than regarding graded implications as global constraints, that have to hold at all time points, we can allow for boolean combination of graded implications (as in [12]) and also for temporal operators to occur in front of the graded implications and of their boolean combinations. We call such formulas temporal graded formulas.

### 3.1. Temporal graded Formulas

A *temporal graded formula* is defined as follows:

$$\alpha ::= A \rightarrow B \geq l \mid A \rightarrow B \geq l \mid \alpha \wedge \beta \mid \neg\alpha \mid \bigcirc\alpha \mid \Diamond\alpha \mid \Box\alpha \mid \alpha\mathcal{U}\beta,$$

where $\alpha$ and $\beta$ stand for temporal graded formulas. Note that temporal operators may occur both within graded implications ($A \rightarrow B \geq l$) and in front of them, and of their boolean combinations.

An example of temporal graded formula is the following conjunction:

$$\Box(\mathbf{T}(\mathit{professor}) \rightarrow \mathit{teaches}\,\mathcal{U}\ \mathit{retired} \geq 0.7) \wedge$$
$$(\mathit{lives\_in\_town} \wedge \mathit{young} \rightarrow \mathbf{T}(\Diamond \mathit{granted\_loan}) \geq 0.8)$$

where the graded implication in the first conjunct is prefixed by a $\Box$ operator, while the second one is not.

We will evaluate the satisfiability of a temporal graded formula at the initial time point 0 of a temporal preferential interpretation $\mathcal{I}$. Let us first define the interpretation of temporal graded formulas at a time point $n$ of a temporal interpretation $\mathcal{I}$ as follows:

$\mathcal{I}, n \models A \rightarrow B \geq l$ iff $(A \rightarrow B)^{\mathcal{I},n} \geq l$
$\mathcal{I}, n \models A \rightarrow B \leq l$ iff $(A \rightarrow B)^{\mathcal{I},n} \leq l$
$\mathcal{I}, n \models \alpha \wedge \beta$ iff $\mathcal{I}, n \models \alpha$ and $\mathcal{I}, n \models \beta$
$\mathcal{I}, n \models \neg\alpha$ iff $\mathcal{I}, n \not\models \alpha$
$\mathcal{I}, n \models \bigcirc\alpha$ iff $\mathcal{I}, n + 1 \models \alpha$
$\mathcal{I}, n \models \Diamond\alpha$ iff exists $m \geq n$ such that $\mathcal{I}, m \models \alpha$
$\mathcal{I}, n \models \Box\alpha$ iff for all $m \geq n$, $\mathcal{I}, m \models \alpha$
$\mathcal{I}, n \models \alpha\mathcal{U}\beta$ iff exists $m \geq n$ such that $\mathcal{I}, m \models \beta$ and, for all $n \leq k < m$, $\mathcal{I}, k \models \alpha$

Let us define the notions of satisfiability and entailment.

**Definition 6** (Satisfiability and entailment)**.** *A temporal graded formula* $\alpha$ *is* satisfied *in a temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *if* $\mathcal{I}, 0 \models \alpha$.

*A preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *is a* model *of a temporal conditional knowledge base* $K$*, if* $\mathcal{I}$ *satisfies all the temporal graded formulas in* $K$.

*A temporal conditional knowledge base* $K$ entails *a temporal graded formula* $\alpha$ *if* $\alpha$ *is satisfied in all the models* $\mathcal{I}$ *of* $K$.

Observe that any graded implication $A \rightarrow B \geq l$ is either satisfied or not at a time point $n$ of a temporal interpretation $\mathcal{I}$, i.e., either $\mathcal{I}, n \models A \rightarrow B \geq l$ or $\mathcal{I}, n \not\models A \rightarrow B \geq l$ (and similarly for the graded implications $A \rightarrow B \leq l$). Hence, the interpretation above of temporal graded formulas in $\mathcal{I}$ at a time point $n$ is two-valued (although it builds over the degree of an implication $A \rightarrow B$ in $\mathcal{I}$ at time point $n$, which has a truth value $(A \rightarrow B)^{\mathcal{I},n}$ in $\mathcal{D}$, see Definition 5).

Note that, in the temporal graded formula given above, the graded implication in the first conjunct ($\mathbf{T}(\mathit{professor}) \rightarrow \mathit{teaches}\,\mathcal{U}\ \mathit{retired} \geq 0.7$) is required to hold at all the time points of the interpretation $\mathcal{I}$ (as it is prefixed by $\Box$), while the second conjunct ($\mathit{lives\_in\_town} \wedge \mathit{young} \rightarrow \mathbf{T}(\Diamond \mathit{granted\_loan}) \geq 0.8$) has to hold only at time point 0.

Decidability and complexity of the different decision problems (the satisfiability, the model checking and entailment problems) have to be studied for this temporal many-valued conditional logic, for different choices of $\mathcal{D}$ and of combination functions. Satisfiability is decidable in the two-valued case, when we restrict to preference relations $<_{A_i}$ with respect to a finite number of formulas (for instance, by restricting to the formulas occurring in a finite KB, and to the respective preferences). Under such conditions, and assuming that all the temporal graded formulas in a KB are prefixed by the $\Box$ operator, the propositional temporal logic with typicality introduced above can be regarded as a special case of $LTL_{\mathcal{ALC}}$ with typicality, which has been shown to be decidable in the two-valued case and for a finite number of preference relations [31].

## 4. Weighted temporal knowledge bases

As in the two-valued non-temporal case, the notion of preferential entailment considered in the previous section is rather weak. For the KLM logics, different closure constructions have been proposed to strengthen entailment by restricting to a subset of the preferential models of a conditional knowledge base $K$. Let us just mention, the rational closure [6] (or system Z [3]) and the lexicographic closure [45], but also other constructions, such as the MP-closure [37], which exploits a similar idea, only using a different kind of lexicographic ordering to define the preference relation.

In the following we consider a construction that has been proposed for weighted knowledge bases in defeasible description logics, where defeasible implications have a weight. We reformulate the semantics of weighted KBs in [21, 22] in the propositional context, for the temporal case, by assuming that $\mathcal{D}$ is the unit interval $[0, 1]$ or a subset of it (e.g., the finite set $\mathcal{D} = \mathcal{C}_n$, for some $n \geq 1$). The two-valued case $\mathcal{D} = \{0, 1\}$ is also a special case.

A *weighted KB* is a set of *weighted typicality implication* of the form $(\mathbf{T}(A_i) \rightarrow B_j, w_{ij})$, where $A_i$ and $B_j$ are propositions, and the weight $w_{ij}$ is a real number, representing the plausibility or implausibility of the conditional implication. For instance, for a proposition $student$, we may have a set of weighted defeasible implications:

$(\mathbf{T}(student) \rightarrow has\_Classes$, +50) $\qquad$ $(\mathbf{T}(student) \rightarrow \Diamond holds\_Degree$,+30)
$(\mathbf{T}(student) \rightarrow has\_Boss$, -40)

that represent *prototypical properties* of students, i.e., that a student normally has classes and will eventually reach the degree, and she usually does not have a boss (negative weight). Accordingly, a student having classes, but not a boss, is more typical than a student having classes and a boss. Similarly, one may introduce a set of weighted conditionals for other formulas, e.g., for $employee$.

Based on the set of weighted conditionals for a formula $A_i$, one can constrain the preferences between worlds according to $<_{A_i}$. For instance, consider an interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ in which a world $w$ describes a student ($v(0, w, student) = 1$) that in the initial state has classes ($v(0, w, has\_Classes) = 1$) but not a boss ($v(0, w, has\_Boss) = 0$), and that at time point 8 will reach the degree ($v(8, w, hold\_Degree) = 1$) ; while world $w'$ describes a student ($v(0, w', student) = 1$) that in the initial state has classes ($v(0, w', has\_Classes) = 1$) and has a boss ($v(0, w', has\_Boss) = 1$), and will reach the degree at time point 7 ($v(7, w', hold\_Degree) = 1$.

The idea is that the preference relation $<^0_{student}$ in $\mathcal{I}$ should consider the situation described at $w$ at time point 0, more normal than the situation described by $w'$ (i.e., $w <^0_{student} w'$), as the sum of the weights of the defeasible implications satisfied by world $w$ at time point 0 ($50 + 30 = 80$) is greater than the sum of the weights of the defeasible implications satisfied by world $w'$ ($50 + 30 - 40 = 40$) at time point 0.

We have to further consider that the propositions may be non-crisp, e.g., $v(0, w, has\_Classes) = 0.7$, and this has some impact on the degree to which a conditional implication (e.g., $\mathbf{T}(student) \rightarrow has\_Classes$), is satisfied.

Given a weighted knowledge base $K$, we call *distinguished propositions* those propositions $A_i$ such that at least a weighted defeasible implications of the form $(\mathbf{T}(A_i) \rightarrow B_j, w_{ij})$ occurs in $K$.

Let $K$ be a temporal weighted KB. Given a many-valued temporal interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}$

$\}_{n\in\mathbb{N}}, v\rangle$, the *weight of a world* $x \in \mathcal{W}$ *with respect to a distinguished proposition* $A_i$ *at time point* $n$ is given by

$$W^{\mathcal{I}}_{A_i,n}(x) = \sum_{(\mathbf{T}(A_i)\rightarrow B_j, w_{ij})\in K} \;\; w_{ij} \cdot v(n, x, B_j).$$

Intuitively, the higher the value of $W^{\mathcal{I}}_{i,n}(x)$, the more normal is the state of affairs $x$, at time point $n$, concerning the properties of $A$ in $K$. This constrains the preference relation $<_{A_i}$ in $\mathcal{I}$. We extend the coherent and faithful multi-preferential semantics for weighted knowledge bases to the temporal case:

**Definition 7.** *A many-valued temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ satisfies a weighted KB $K$ *if, for all distinguished formulas* $A_i$ *and time points* $n$*, it holds that:*

$$x <^n_{A_i} y \qquad \Longleftrightarrow \qquad W^{\mathcal{I}}_{i,n}(x) > W^{\mathcal{I}}_{i,n}(y)$$

The condition in Definition 7, together with the coherence (faithfulness) condition introduced in Section 2, guarantees that the many-valued interpretation $\mathcal{I}$ agrees with the weighted inclusions in $K$, at each time point $n$.

A weighted (defeasible) knowledge base $K_D$ can coexist with a strict knowledge base $K_S$ (i.e., a set of temporal graded formulas). This is in agreement with the usual approach in defeasible DLs, which distinguishes between a strict TBox and a defeasible TBox.

## 5. Towards a temporal conditional logic for gradual argumentation

In previous sections, we have developed a many-valued, temporal logic with typicality, extending with $LTL$ operators the many-valued conditional logic with typicality proposed in [12]. In this section we aim at instantiating the proposed temporal logic to the gradual argumentation setting, to make it suitable for capturing the dynamics of an argumentation graph (e.g., the changes of weights of edges in time).

The idea in [12] was to provide a general approach for developing a preferential interpretation from an argumentation graph $G$ under a gradual semantics $S$, provided some weak conditions on the domain of argument interpretation are satisfied by $S$ and, specifically, that the *domain of argument interpretation* $\mathcal{D}$ is equipped with a *preorder relation* $\leq$ (which is a widely agreed requirement [16, 17]). As it may be expected, the domain of argument interpretation $\mathcal{D}$ plays the role of the truth degree set of our many-valued semantics introduced above.

For the definition of an argumentation graph, let us adapt the notion of *edge-weighted QBAF* by Potyka [24] to a generic domain $\mathcal{D}$. A *(weighted) argumentation graph* is a quadruple $G = \langle \mathcal{A}, \mathcal{R}, \sigma_0, \pi\rangle$, where $\mathcal{A}$ is a set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ a set of edges, $\sigma_0 : \mathcal{A} \rightarrow \mathcal{D}$ assigns a *base score* of arguments, and $\pi : \mathcal{R} \rightarrow \mathbb{R}$ is a weight function assigning a positive or negative weight to edges (when the graph is not weighted, we assume function $\pi$ only distinguishes between attacks and supports).

A *many-valued labelling* $\sigma$ *of* $G$ *over* $\mathcal{D}$ is a function $\sigma : \mathcal{A} \rightarrow \mathcal{D}$, which assigns to each argument an *acceptability degree* (or a *strength*) in the domain of argument valuation $\mathcal{D}$ (for $A_i \in \mathcal{A}$, $\sigma(A_i)$ is the acceptability degree of argument $A_i$ in $\sigma$). Whatever gradual semantics $S$ is considered for the argumentation graph $G$ (see, e.g., [16, 17, 24]) we assume that $S$ identifies a *set* $\Sigma^S$ *of labellings* of the graph $G$ over a domain of argument valuation $\mathcal{D}$. A *semantics* $S$ *of* $G$ can then be regarded, abstractly, as a pair $(\mathcal{D}, \Sigma^S)$: a domain of argument valuation $\mathcal{D}$ and a set of many-valued labellings $\Sigma^S$ over the domain.

If we consider all arguments $A_i \in \mathcal{A}$ as propositional variables ($Prop = \mathcal{A}$), each many-valued labelling $\sigma$ can be regarded as a world $w_\sigma \in \mathcal{W}$ in a many-valued preferential interpretation $\mathcal{M}^G = \langle \mathcal{W}, \{<_{A_i}\}, v\rangle$, such that $v(w_\sigma, A_i) = \sigma(A_i)$.

More precisely, in [12] a gradual semantics $(\mathcal{D}, \Sigma^S)$ of the argumentation graph $G$ is mapped into a preferential interpretation $\mathcal{M}^G = \langle \mathcal{W}, \{<_{A_i}\}, v\rangle$, defined as in Section 2, by letting:

- $\mathcal{W} = \{w_\sigma \mid \sigma \in \Sigma^S\}$
- $v(w_\sigma, A_i) = \sigma(A_i)$, for all the arguments $A_i \in Prop$

- $w_\sigma <_{A_i} w_{\sigma'}$ iff $\sigma(A_i) > \sigma'(A_i)$

Such a preferential interpretation can then be used in the verification of strict and conditional graded implications. For finitely-valued $\varphi$-coherent argumentation semantics, in the finitely-valued case, an ASP approach has been presented for conditional reasoning over an argumentation graph [20, 23], by mapping weighted argumentation graphs into weighted knowledge bases.

The approach can be extended to the temporal case, based on the temporal many-valued logic with typicality developed in Section 3. It can allow to reason about the dynamics of an argumentation graph, for instance, when the weights of edges might change in time, e.g. when learning the weights. Indeed, a multilayer neural network can be regarded as a weighted knowledge base [26, 22], but also as a weighted argumentation graph [24, 25], based on the strong relationships of the two formalisms [23]. As another example, the structure of an argumentation graph can be updated through the interaction of different agents in time, such as in [27] via Argumentative Exchanges, giving rise to a sequence of argumentation graphs $G^n$ (over the same set of arguments $\mathcal{A}$).

When the argumentation graph changes dynamically, a temporal many-valued interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ can be seen as a sequence of (non-temporal) preferential interpretations $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots$ (as in Section 3), where each $\mathcal{M}_n = \langle \mathcal{W}, \{<^n_{A_i}\}, v^n \rangle$ is constructed from the labellings $\Sigma^S_n$ of the argumentation graph $G_n$ at time point $n$, as for $\mathcal{M}^G$ above. Temporal graded formulas over arguments, e.g., $\Box(\mathbf{T}(A_1) \rightarrow A_2 \mathcal{U} A_3 \vee A_3) \geq 0.7$, can then be verified over $\mathcal{I}$.

As mentioned above, this verification approach has been studied, for the non-temporal case, in the verification of properties of argumentation graphs under the $\varphi$-coherent gradual semantics [20], and an ASP approach has been developed for the verification of graded conditional implications over arguments and over boolean combination of arguments. Extending the ASP approach to deal with the temporal case, for specific fragments of the language, is a direction for future work.

## 6. Conclusions

The paper proposes a framework in which different (many-valued) preferential logics with typicality can be captured, together with their temporal extensions, with LTL operators. The interpretation of the typicality operator is based on a multi-preferential semantics, and an extension of weighted conditional knowledge bases to the temporal (many-valued) case is proposed.

The approach is parametric with respect to the choice of a specific many-valued logic (and their combination functions), but also with respect to the definition of the preference relations $<_{A_i}$, which might exploit different closure constructions, among the many studied in the literature, in the spirit of Lehmann's lexicographic closure [45]. The two-valued case, with a single preference relation can as well be regarded as a special cases of this preferential temporal formalism.

On a different route, a preferential logics with defeasible LTL operators has been studied in [29, 46]. The decidability of different fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [28, 46]. Our approach does not consider defeasible temporal operators nor preferences over time points, but combines standard LTL operators with the typicality operator in a many-valued temporal logic. In our approach, preferences between worlds change over time.

Much work has been recently devoted to the combination of neural networks and symbolic reasoning [47, 48, 49]. While conditional weighted KBs have been shown to capture (in the many-valued case) the stationary states of a neural network (or its finite approximation) [21, 22, 23], and allow for combining empirical knowledge with elicited knowledge for reasoning and for post-hoc verification, adding a temporal dimension opens to the possibility of verifying properties concerning the dynamic behavior of a network, based on a model checking approach or an entailment based approach.

Extending the above mentioned ASP encodings to deal with temporal preferential interpretations is a direction of future work. Future work also includes studying the decidability for fragments of the logic and exploiting the formalism for explainability, and for reasoning about the dynamics of gradual argumentation graphs in gradual semantics.

## Acknowledgments

We thank the anonymous referees for their helpful comments. This research was partially supported by INDAM-GNCS. Mario Alviano was partially supported by Italian Ministry of University and Research (MUR) under PRIN project PRODE "Probabilistic declarative process mining", CUP H53D23003420006, under PNRR project FAIR "Future AI Research", CUP H23C22000860006, under PNRR project Tech4You "Technologies for climate change adaptation and quality of life improvement", CUP H23C22000370006, and under PNRR project SERICS "SEcurity and RIghts in the CyberSpace", CUP H73C22000880001; by Italian Ministry of Health (MSAL) under POS projects CAL.HUB.RIA (CUP H53C22000800006) and RADIOAMICA (CUP H53C22000650006); by Italian Ministry of Enterprises and Made in Italy under project STROKE 5.0 (CUP B29J23000430005); and by the LAIA lab (part of the SILA labs).

## References

[1] J. Delgrande, A first-order conditional logic for prototypical properties, Artificial Intelligence 33 (1987) 105–130.
[2] D. Makinson, General theory of cumulative inference, in: Non-Monotonic Reasoning, 2nd International Workshop, Grassau, FRG, June 13-15, 1988, Proceedings, 1988, pp. 1–18.
[3] J. Pearl, Probabilistic Reasoning in Intelligent Systems Networks of Plausible Inference, Morgan Kaufmann, 1988.
[4] S. Kraus, D. Lehmann, M. Magidor, Nonmonotonic reasoning, preferential models and cumulative logics, Artificial Intelligence 44 (1990) 167–207.
[5] J. Pearl, System Z: A natural ordering of defaults with tractable applications to nonmonotonic reasoning, in: TARK'90, Pacific Grove, CA, USA, 1990, pp. 121–135.
[6] D. Lehmann, M. Magidor, What does a conditional knowledge base entail?, Artificial Intelligence 55 (1992) 1–60.
[7] S. Benferhat, C. Cayrol, D. Dubois, J. Lang, H. Prade, Inconsistency management and prioritized syntax-based entailment, in: Proc. IJCAI'93, Chambéry,, 1993, pp. 640–647.
[8] R. Booth, J. B. Paris, A note on the rational closure of knowledge bases with both positive and negative knowledge, Journal of Logic, Language and Information 7 (1998) 165–190.
[9] G. Kern-Isberner, Conditionals in Nonmonotonic Reasoning and Belief Revision - Considering Conditionals as Agents, volume 2087 of *LNCS*, Springer, 2001.
[10] D. Lewis, Counterfactuals, Basil Blackwell Ltd, 1973.
[11] D. Nute, Topics in conditional logic, Reidel, Dordrecht (1980).
[12] M. Alviano, L. Giordano, D. Theseider Dupré, Typicality, conditionals and a probabilistic semantics for gradual argumentation, in: Proc. 21st International Workshop on Non-Monotonic Reasoning (NMR 2023), Rhodes, Greece, September 2-4, 2023, volume 3464 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2023, pp. 4–13.
[13] C. Cayrol, M. Lagasquie-Schiex, Graduality in argumentation, J. Artif. Intell. Res. 23 (2005) 245–297.
[14] P. E. Dunne, A. Hunter, P. McBurney, S. Parsons, M. J. Wooldridge, Weighted argument systems: Basic definitions, algorithms, and complexity results, Artif. Intell. 175 (2011) 457–486.
[15] L. Amgoud, J. Ben-Naim, D. Doder, S. Vesic, Acceptability semantics for weighted argumentation frameworks, in: IJCAI 2017, Melbourne, Australia, 2017, pp. 56–62.
[16] P. Baroni, A. Rago, F. Toni, How many properties do we need for gradual argumentation?, in: Proc. AAAI 2018, New Orleans, Louisiana, USA, February 2-7, 2018, pp. 1736–1743.
[17] P. Baroni, A. Rago, F. Toni, From fine-grained properties to broad principles for gradual argumentation: A principled spectrum, Int. J. Approx. Reason. 105 (2019) 252–286.
[18] L. Amgoud, D. Doder, Gradual semantics accounting for varied-strength attacks, in: Proceedings AAMAS '19, Montreal, QC, Canada, May 13-17, 2019, 2019, pp. 1270–1278.
[19] L. Giordano, From weighted conditionals of multilayer perceptrons to a gradual argumentation

semantics, in: 5th Workshop on Advances in Argumentation in Artif. Intell., 2021, Milan, Italy, Nov. 29, volume 3086 of *CEUR Workshop Proc.*, 2021.
[20] M. Alviano, L. Giordano, D. Theseider Dupré, Preferential reasoning with typicality in ASP over weighted argumentation graphs in a gradual semantics, in: ASPOCP 2023, Proc. ICLP Workshops 2023, CEUR Workshop Proceedings 3437, 2023.
[21] L. Giordano, D. Theseider Dupré, Weighted defeasible knowledge bases and a multipreference semantics for a deep neural network model, in: Proc. JELIA 2021, May 17-20, volume 12678 of *LNCS*, Springer, 2021, pp. 225–242.
[22] M. Alviano, F. Bartoli, M. Botta, R. Esposito, L. Giordano, D. Theseider Dupré, A preferential interpretation of multilayer perceptrons in a conditional logic with typicality, Int. Journal of Approximate Reasoning 164 (2024).
[23] M. Alviano, L. Giordano, D. Theseider Dupré, Weighted knowledge bases with typicality and defeasible reasoning in a gradual argumentation semantics, Intelligenza Artificiale 18 (2024).
[24] N. Potyka, Interpreting neural networks as quantitative argumentation frameworks, in: Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021, February 2-9, 2021, AAAI Press, 2021, pp. 6463–6470.
[25] A. S. d'Avila Garcez, K. Broda, D. M. Gabbay, Symbolic knowledge extraction from trained neural networks: A sound approach, Artif. Intell. 125 (2001) 155–207.
[26] L. Giordano, From weighted conditionals of multilayer perceptrons to a gradual argumentation semantics, in: 5th Workshop on Advances in Argumentation in Artif. Intell., volume 3086 of *CEUR Workshop Proceedings*, 2021.
[27] A. Rago, H. Li, F. Toni, Interactive explanations by conflict resolution via argumentative exchanges, in: Proc. 20th Int. Conf. on Principles of Knowledge Representation and Reasoning, 2023, pp. 582–592.
[28] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, A one-pass tree-shaped tableau for defeasible LTL, in: TIME 2021, September 27-29, 2021, Klagenfurt, Austria, volume 206 of *LIPIcs*, 2021.
[29] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, On the decidability of a fragment of preferential LTL, in: TIME 2020, September 23-25, 2020, Bozen-Bolzano, Italy, volume 178 of *LIPIcs*, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2020.
[30] A. Chafik, Defeasible temporal logics for the specification and verification of exception-tolerant systems, PhD Thesis, Artois University, 2022.
[31] M. Alviano, L. Giordano, D. Theseider Dupré, Preferential temporal description logics with typicality and weighted knowledge bases, in: Proc. 38th Italian Conf. on Computational Logic, Udine, Italy, June 21-23, 2023, volume 3428 of *CEUR Workshop Proceedings*, 2023.
[32] C. Lutz, F. Wolter, M. Zakharyaschev, Temporal description logics: A survey, in: TIME, 2008, pp. 3–14.
[33] R. Booth, G. Casini, T. Meyer, I. Varzinczak, On rational entailment for propositional typicality logic, Artif. Intell. 277 (2019).
[34] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, ALC+T: a preferential extension of Description Logics, Fundamenta Informaticae 96 (2009) 1–32.
[35] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning in a concept-aware multipreferential lightweight DL, TPLP 10(5) (2020) 751–766.
[36] M. Alviano, L. Giordano, D. Theseider Dupré, Complexity and scalability of defeasible reasoning in many-valued weighted knowledge bases, in: Logics in Artificial Intelligence - 18th European Conf., JELIA 2023, volume 14281 of *LNCS*, Springer, 2023, pp. 481–497.
[37] L. Giordano, V. Gliozzi, A reconstruction of multipreference closure, Artif. Intell. 290 (2021).
[38] S. Gottwald, A Treatise on Many-valued Logics, Research Studies Press, 2001.
[39] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Preferential Description Logics, in: LPAR 2007, volume 4790 of *LNAI*, Springer, Yerevan, Armenia, 2007, pp. 257–272.
[40] O. F. Gil, On the Non-Monotonic Description Logic ALC+$T_{min}$, nonmonotonic reasoning workshop 2014, CoRR abs/1404.6566 (2014).
[41] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning in a concept-aware multi-

preferential lightweight DL, Theory Pract. Log. Program. 20 (2020) 751–766.
[42] E. M. Clarke, O. Grumberg, D. A. Peled, Model checking, MIT Press, 1999.
[43] A. Frigeri, L. Pasquale, P. Spoletini, Fuzzy time in linear temporal logic, ACM Trans. Comput. Log. 15 (2014) 30:1–30:22.
[44] K. Lamine, F. Kabanza, History checking of temporal fuzzy logic formulas for monitoring behavior-based mobile robots, in: 12th IEEE Int. Conf. on Tools with Artificial Intelligence (ICTAI 2000), 13-15 November 2000, Vancouver, BC, Canada, 2000, pp. 312–319.
[45] D. J. Lehmann, Another perspective on default reasoning, Ann. Math. Artif. Intell. 15 (1995) 61–82.
[46] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, Defeasible linear temporal logic, J. Appl. Non Class. Logics 33 (2023) 1–51.
[47] L. Serafini, A. S. d'Avila Garcez, Learning and reasoning with logic tensor networks, in: XVth Int. Conf. of the Italian Association for Artificial Intelligence, AI*IA 2016, Genova, Italy, Nov 29 - Dec 1, volume 10037 of *LNCS*, Springer, 2016, pp. 334–348.
[48] L. C. Lamb, A. S. d'Avila Garcez, M. Gori, M. O. R. Prates, P. H. C. Avelar, M. Y. Vardi, Graph neural networks meet neural-symbolic computing: A survey and perspective, in: C. Bessiere (Ed.), Proc. IJCAI 2020, ijcai.org, 2020, pp. 4877–4884.
[49] M. Setzu, R. Guidotti, A. Monreale, F. Turini, D. Pedreschi, F. Giannotti, GlocalX - from local to global explanations of black box AI models, Artif. Intell. 294 (2021) 103457.