# A Bin-Packing Formulation for Radiotherapy Treatment Scheduling

Chiara Camilla Rambaldi Migliore[1,2,*], Giovanni Iacca[1,†] and Marco Roveri[1,†]

[1]University of Trento, Italy
[2]University of Pisa, Italy

**Abstract**

The scheduling of radiation therapy is a complex problem that significantly impacts patient outcomes and the use of healthcare resources. This paper proposes a novel formalization of the radiotherapy scheduling problem (RTSP) as a modified one-dimensional bin-packing problem (BPP). This formalization offers several advantages, including leveraging state-of-the-art solvers for the one-dimensional BPP and extending the formulation to various BPP variants that align with the complexities of the RTSP. Preliminary results on a synthetic instance demonstrate the feasibility of the proposed approach.

**Keywords**

Radiotherapy Scheduling, Bin-Packing Problem, Operational Research

## 1. Introduction

Radiotherapy is a significant component of cancer treatment. It involves delivering precise radiation doses to the tumor sites while minimizing damage to surrounding healthy tissues. It is critical for a patient to start treatment as soon as possible for a successful cure. Thus, effective scheduling of radiation therapy treatments is a complex problem that directly impacts patient outcomes and good utilization of healthcare resources. Existing approaches often rely on novel formalizations of the problem, often solved with exact methods from Operational Research or metaheuristics such as Genetic Algorithms (GAs). In this paper, we propose a formalization of the problem starting from a well-known combinatorial problem, the one-dimensional Bin-Packing Problem (BPP). The one-dimensional BPP is a well-studied combinatorial optimization problem that involves packing items of varying sizes into a minimum number of bins of equal capacity. By mapping radiotherapy treatments to items and the linear accelerator machine (LINAC) to a bin, we establish a clear correspondence between these two problems. This new formalization offers several advantages:

- **Leveraging State-of-the-Art Solvers**: The one-dimensional BPP has been extensively researched, leading to the development of efficient, effective, and scalable algorithms. By mapping the Radiotherapy Scheduling Problem (RTSP) as a one-dimensional BPP, we can use these existing solvers to improve the quality of treatment schedules.

- **Extending to other BPP versions**: The versatility of the one-dimensional BPP allows its extension to various variants that align with the complexities of the RTSP. For instance, *online* BPP can address scenarios in which the number and characteristics of patients are not known in advance, while *extensible* BPP (EBP) can incorporate constraints related to potential working overtime of the medical

*AI4CC-IPS-RCRA-SPIRIT 2024: International Workshop on Artificial Intelligence for Climate Change, Italian Workshop on Planning and Scheduling, RCRA Workshop on Experimental evaluation of algorithms for solving problems with combinatorial explosion, and SPIRIT Workshop on Strategies, Prediction, Interaction, and Reasoning in Italy. November 25-28th, 2024, Bolzano, Italy [1].*
*Corresponding author.
†These authors contributed equally.
cc.rambaldimigliore@unitn.it (C. C. Rambaldi Migliore); giovanni.iacca@unitn.it (G. Iacca); marco.roveri@unitn.it (M. Roveri)
https://sites.google.com/site/giovanniiacca (G. Iacca); https://sites.google.com/view/marco-roveri (M. Roveri)
0009-0000-1581-6086 (C. C. Rambaldi Migliore); 0000-0001-9723-1830 (G. Iacca); 0000-0001-9483-3940 (M. Roveri)

staff. Stochastic EBP can further capture the innate variability in treatment times arising from pre- and post-treatment procedures.

The remainder of the article is organized as follows. Section 2 presents the background concepts on the RTSP and the main formalization of the BPP. Section 3 describes our proposed formalization for the RTSP, which is the main contribution of this paper. Section 4 summarizes the relevant related works on the scheduling of healthcare operations and, more deeply, on radiotherapy scheduling. Section 5 shows the preliminary results of our new formalization, starting from a synthetic instance solved with an Integer Linear Programming (ILP) solver. Finally, Section 6 ends this article.

## 2. Background

Hereafter we first briefly summarize the main concepts of radiotherapy and its workflow, then we briefly present the BPP.

### 2.1. Radiotherapy

Radiotherapy is a cancer treatment that involves the use of radiation to attack and kill cancer cells. Since radiation has limited precision, although research in the field has advanced considerably, it is the responsibility of doctors and medical physicists to test and calibrate both the radiation doses and the position of the machine in relation to the patient's body as effectively as they can. Optimizing the workflow that goes from the time the tumor is diagnosed and the choice of radiotherapy as a treatment to the actual moment the patient undergoes it is therefore of paramount importance. Figure 1 visually shows the macro-stages of a typical radiotherapy workflow, which are briefly described below:

- **Consultation**: After the patient is diagnosed with the tumor and radiation therapy is recommended as treatment, a preliminary consultation is conducted between the patient and the radiotherapist.
- **Scanning**: As the consultation ends with positive feedback from both the patient and the radiotherapist, the pre-treatment flow starts. Firstly, the patient undergoes a series of scans to delineate the tumor's position.
- **Image Post-Processing**: The resulting image of the scan is post-processed to become as usable and understandable as possible.
- **Contouring**: At this point, the contouring phase starts: the scanned image, after being post-processed, is then used to contour the tumor. This phase is actually very delicate, as the contouring continuously undergoes a review to make sure that it is perfectly executed.
- **Treatment Planning**: The final phase before starting the treatments is used to define the treatment planning. In this context, the dose distribution and the plan for the radiotherapy sessions are decided.
- **Treatment**: After all the pre-treatment phases are completed, the actual treatment can start. This last phase is crucial, as the schedule of the treatment sessions can significantly impact the patient's health and well-being.

The above-mentioned phases underline the importance of developing a formalization for the RTSP that allows the optimization of the entire radiotherapy workflow. Sub-optimal solutions to this problem can in fact lead to significant disruptions for patients, including long waiting times, underutilized equipment, and disruptions due to unforeseen events.

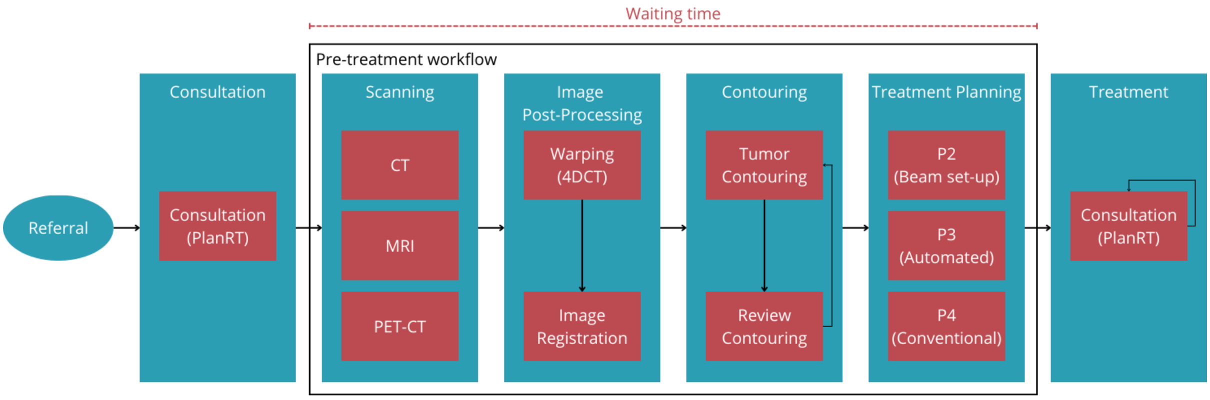


**Figure 1:** Typical workflow of a radiotherapy treatment.

## 2.2. One-dimensional Bin-Packing Problem

The one-dimensional BPP is one of the most known and studied combinatorial problems, known to belong to the class of NP-hard problems. In the BPP, objects of different sizes must be packed into a finite number of bins or containers of fixed capacity, in order to minimize the number of bins used. It is a combinatorial problem with several areas of application, e.g., in logistics, cloud computing, and resource allocation. It can be formalized as a linear programming problem [2]:

**Parameters**

| | | |
|---|---|---|
| $s_j$ | $(j = 1, \dots, J)$ | size of item $j$ |
| $c_i$ | $(i = 1, \dots, I)$ | capacity of bin $i$ |

**Variables**

| | | |
|---|---|---|
| $y_i$ | $(i = 1, \dots, I)$ | 1 if bin $i$ is used, 0 otherwise |
| $x_{ij}$ | $(i = 1, \dots, I; j = 1, \dots, J)$ | 1 if item $j$ is packed in bin $i$, 0 otherwise |

**Objective**

$$\min \quad \sum_{i=1}^{I} y_i \tag{1}$$

**Constraints**

$$\sum_{i=1}^{I} x_{ij} = 1 \quad \forall j \in [1, \dots, J] \tag{2}$$

$$\sum_{j=1}^{J} s_j x_{ij} \leq c_i y_i \quad \forall i \in [1, \dots, I] \tag{3}$$

The objective presented in Equation (1) describes the goal of the BPP, namely minimizing the number of used bins. The constraint described in Equation (2) states that each item must be packed in exactly one container, while the constraint in Equation (3) states that the amount packed in each container cannot exceed its capacity. A visual example of the BPP is shown in Figure 2. It can be seen that the different colors of the items correspond to different sizes of them. The goal of the problem is to minimize the number of bins used, assuming that one knows in advance the quantity and size of the items to be packed.

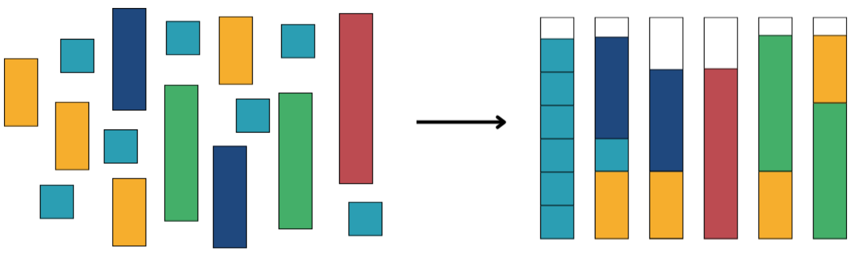

**Figure 2:** A visual example of the one-dimensional BPP.

## 3. Proposed modified one-dimensional Bin-Packing Problem

We propose a new formulation of the one-dimensional BPP which is used to formalize the RTSP. In this new formulation, each item is part of a group, and all items within the same group must be packed together in consecutive groups of bins. The bins are also grouped, with each group containing a fixed number of bins with a set capacity. When packing items from a group $k$, each item $j$ must be placed in any bin $i$ within a group $d$ that has enough space. In a further development of the problem, some constraints on which bin $i$ of the group $d$ can be used for item $j$ may arise. This problem can be expressed as a linear programming model:

**Parameters**

| | | |
|---|---|---|
| $s_{j,k}$ | $(k = 1, \dots, K; j = 1, \dots, J_k)$ | size of item $j$ of group $k$ |
| $c_{i,d}$ | $(d = 1, \dots, D; i = 1, \dots, I_d)$ | capacity of bin $i$ of group $d$ |
| $M_d = I_d$ | $(d = 1, \dots, D)$ | maximum bins' number for each group $d$ |
| $I_k$ | $(k = 1, \dots, K)$ | sets of bins allowed for each group $k$ |

**Variables**

| | | |
|---|---|---|
| $y_{i,d}$ | $(d = 1, \dots, D; i = 1, \dots, I_d)$ | 1 if bin $i$ of group $d$ is used, 0 otherwise |
| $x_{j,k,i,d}$ | $(k = 1, \dots, K; j = 1, \dots, J_k; d = 1, \dots, D; i = 1, \dots, I_d)$ | 1 if item $j$ of group $k$ is packed in bin $i$ of group $d$, 0 otherwise |
| $z_d$ | $(d = 1, \dots, D)$ | 1 if a bin of group $d$ is used, 0 otherwise, i.e. 1 if $\sum_{i=1}^{I_d} y_{i,d} \geq 1$, 0 otherwise |

**Objective**

$$\min \quad \sum_{d=1}^{D} z_d d \tag{4}$$

**Constraints**

$$\sum_{d=1}^{D} \sum_{i=1}^{I_d} x_{j,k,i,d} = 1 \qquad \forall k \in [1, \dots, K], \forall j \in [1, \dots, J_k] \tag{5}$$

$$\sum_{k=1}^{K} \sum_{j=1}^{J_k} s_{j,k} x_{j,k,i,d} \leq y_{i,d} c_{i,d} \qquad \forall d \in [1, \dots, D], \forall i \in [1, \dots, I_d] \tag{6}$$

$$\sum_{i} x_{j,k,i,d} = \sum_{i} x_{j+1,k,i,d+1} \qquad \forall k \in [1, \dots, K], \forall j \in [1, \dots, J_k - 1], \forall d \in [1, \dots, D-1] \tag{7}$$

$$\sum_{i}^{I_d} y_{i,d} \leq M z_d \qquad \forall d \in [1, \dots, D] \tag{8}$$

$$1 - \sum_{i}^{I_d} y_{i,d} \leq M(1 - z_d) \qquad \forall d \in [1, \dots, D] \tag{9}$$

$$\sum_{i}^{I_d \setminus I_k} x_{j,k,i,d} = 0 \qquad \forall d \in [1, \dots, D], \forall k \in [1, \dots, K], \forall j \in [1, \dots, J_k] \tag{10}$$

The objective presented in Equation (4) describes the goal of the modified BPP, namely minimizing the number of used groups of bins. The constraint described in Equation (5) states that each item $j$ of each group $k$ must be packed in exactly one bin $i$ of group $d$, while constraint in Equation (6) states that the amount packed in each bin cannot exceed its capacity. In Equation (7), another constraint assures that all items must be packed consecutively in bins' groups: if item $j$ is in group $d$, then item $j + 1$ must be in group $d + 1$. Equation (8) and Equation (9) force each variable $z_d$ to take value 1 if $\sum_{i}^{I_d} y_{i,d} \geq 1$, and 0 otherwise. Finally, Equation (10) states that no item of group $k$ can be packed in a bin that is not allowed by the group.

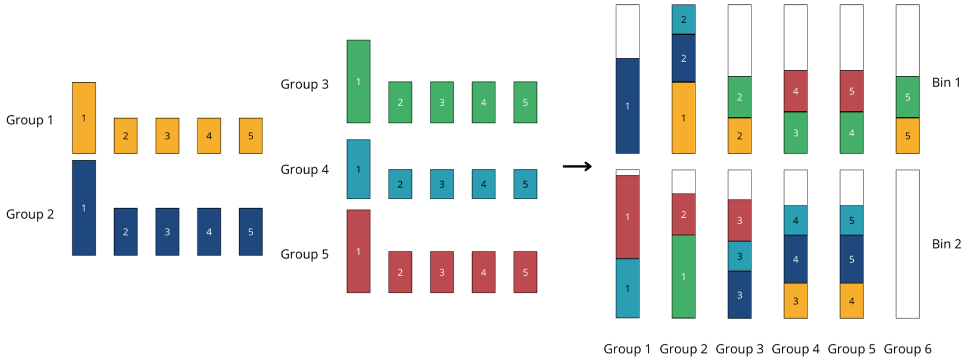


**Figure 3:** A visual example of the proposed modified one-dimensional BPP.

A visual example of the proposed modified BPP is shown in Figure 3. Different colors indicate different groups of items. Each item in a group has a number identifying it. It can be seen how each item in a group is packed consecutively, as stated in Equation (7). In this case, the minimum number of groups of bins to be used is 6.

The RTSP can be straightforwardly mapped to this modified one-dimensional BPP formulation. In fact, each $k$ is a patient and each fraction is an item $j$. Each group of bins$d$ corresponds to a day and each bin $j$ in a group $d$ is a specific machine for the day $d$. For what concerns the availability of machines in terms of maintenance and in terms of medical personnel that can use the machine, it can be modulated by varying the capacity of the bin that correspond to that machine in that specific day. In a broader view, all the workflow of the patient radiotherapy can be formulated as our modified BPP, by adding some more constraints.

## 4. Related works

In this section, we first introduce articles that tackle the wider topic of healthcare scheduling, and then we deepen the literature for the RTSP. We categorize those works into (1) workflow scheduling, (2) appointment scheduling, (3) online appointment scheduling, and (4) appointment scheduling formalizations.

**Healthcare scheduling.** In the last few years, several research efforts have been carried out to develop healthcare resource allocation decision support tools. For example, Kokangul [3], Oddoye et al. [4], Zhang et al [5], Ma and Demeulemeester [6], and Holm et al. [7] developed tools for solving the bed capacity problem. Instead, Wang et al. [8] conducted a study related to operating room capacity. Ordu et al. [9] followed a forecasting-simulation-optimization approach for optimizing the level of resources of a National Health Service. Concerning the problem of patient appointments, Squires et al. [10] developed a novel genetic algorithm for the scheduling of repetitive Transcranial Magnetic Stimulation (rTMS) appointments.

**Workflow scheduling.** For the specific case of planning and scheduling of patients' treatments in radiotherapy, there are several studies that have explored the field. Some authors explored the scheduling of the entire radiotherapy process. For instance, Petrovic et al. [11] developed three patient-priority GAs to schedule the entire radiotherapy process–from consultation to treatment–aiming to (i) minimize patient waiting times and (ii) minimize breaches of waiting time targets. These objectives were normalized and weighted using a scalarization approach. Their model considers real-life constraints like doctors' schedules, machine availability, patient categories, and waiting time targets. Vieira et al. focused on specific stages of radiotherapy. Firstly, they introduced a stochastic Mixed-Integer Linear Programming (MILP) model to optimize the allocation of radiation therapist technologists (RTTs) during pre-treatment, considering stochastic patient inflows [12]. Later, they developed a MILP model to develop weekly treatment schedules, taking patient time window preferences into account [13]. Eventually, this model was tested on real data from two radiotherapy centers, where it demonstrated improved performance by reducing LINAC switches and better aligning with patient preferences compared to manual scheduling methods [14]. More recently, Hoffmans-Holtzer et al. combined a multi-objective Genetic Algorithm (NSGA-II) with MILP to optimize pre-treatment preparation scheduling [15]. Their model balances average patient preparation time with the risk of overtime, offering tactical decision-making insights based on factors like patient volume, staff composition, and clinic hours.

**Appointment scheduling.** Several works have focused solely on scheduling radiotherapy treatment appointments, bypassing the pre-treatment phases. The majority of these works focused on using Operational Research approaches. Early work by Conforti et al. [16] formulated the scheduling problem as an ILP, focusing on maximizing the number of patients to be scheduled under four conditions: (i) patient priority, (ii) number of treatment sessions, (iii) consecutive treatment days, and (iv) treatment duration in weeks. Later, they added minimizing patient waiting times as an objective [17], and then further extended the initial model to incorporate patient availability [18]. Sauré et al. formulated the RTSP as a discounted infinite-horizon Markov Decision Process (MDP) model [19]. They expanded the work by Partick et al. [20] by introducing multiple appointment requests, different session durations, and allowing overtime. They transformed the MDP into a Linear Programming (LP) model and solved its dual problem using the Column Generation (CG) algorithm. More recently, Pham et al. proposed a two-phase approach: first, assigning sessions to specific LINACs and days using ILP, and then deciding the sequence of patients on each day/LINAC and the specific appointment times with MILP and Constraint Programming models [21]. This approach was tested on data from CHUM, a large cancer center in Montréal. Finally, Frimodig et al. compared three optimization methods: (i) Integer Programming (IP), (ii) Column Generation IP (CG-IP), and (iii) Constraint Programming, focusing on minimizing waiting times and maximizing patient preference fulfilment [22]. They also modeled urgent patient arrivals and machine interruptions. A later study by the same authors focused specifically on the Column Generation algorithm, addressing additional constraints such as machine compatibility, individualized protocols, and multiple hospital sites [23].

**Online appointment scheduling.** Another challenge in radiotherapy scheduling is the continuous and uncertain arrival of patients. Legrain et al. addressed this by integrating patient arrival uncertainty into a method that combines stochastic and online optimization [24]. Their model utilizes future patient arrival information to better predict resource utilization, adapting an online stochastic algorithm developed by Legrain and Jaillet [25] for real-world radiotherapy scheduling, to determine the initial treatment day and time slot on a LINAC. Additionally, Braune et al. considered the uncertainty of patient preparation and exit times during treatment scheduling [26]. They developed a model for planning appointment times under uncertain duration, employing a combination of GAs and Monte Carlo simulations to heuristically solve the problem.

**Appointment scheduling formalizations.** In the context of radiotherapy, Vogl et al. modeled treatment appointments at an ion beam facility as a modified job shop scheduling problem (JSP) [27]. They established custom constraints and aimed to minimize the operation time of the bottleneck resource, the particle beam, while also reducing violations of time window constraints. To solve this problem, they employed three metaheuristic methods: (i) a GA, (ii) Iterated Local Search, and (iii) a combination of the two.

While previous research has explored the JSP paradigm for ion beam therapy, our work focuses on traditional RTSP. Unlike the first, where treatments require precise start times, the latter primarily involves assigning patients to suitable treatment days and then finding the best start times.

| Patient | Fractions Duration | Machines allowed |
|---|---|---|
| p1 | 1: 12, 2: 6, 3: 6, 4: 6, 5: 6 | m1, m2 |
| p2 | 1: 16, 2: 8, 3: 8, 4: 8, 5: 8 | m2 |
| p3 | 1: 14, 2: 7, 3: 7, 4: 7, 5: 7 | m1 |
| p4 | 1: 10, 2: 5, 3: 5, 4: 5, 5: 5 | m2 |
| p5 | 1: 14, 2: 7, 3: 7, 4: 7, 5: 7 | m1 |

**Table 1**
Patients data.

| Machine | Day [capacity in minutes] |
|---|---|
| m1 | 1 [25], 2 [20], 3 [20], 4 [25], 5 [20], 6 [25], 7 [25], 8 [20], ... |
| m2 | 1 [25], 2 [15], 3 [25], 4 [20], 5 [25], 6 [25], 7 [25], 8 [25], ... |

**Table 2**
Machines capacities.

## 5. Preliminary Results

This section introduces and explains the synthetic instance created to generate preliminary results to test the proposed formalization.

**Synthetic instance** We created a synthetic instance, as shown in Table 1, composed of 5 patients with 5 fractions each, which can be allocated on two machines whose capacities are reported in Table 2. Each patient has a different duration for the fractions, and the duration of the first fraction is always twice as long as that of the other fractions, since the first time the patient undergoes radiotherapy treatment, they need more time for positioning. Each patient also has a set of allowed machines, which are in total two with different capacities each day, as shown in the "Previous Occupation" column in Table 3. When completely unloaded, without previous scheduled appointments, the machines have a maximum capacity of 25 minutes.

**Numerical results** To validate the proposed formulation, we implemented a prototype using Google OR-Tools[1] with the SCIP solver [28] in Python 3.12. The prototype has been run on a MacBook Air with an Apple M1 chip, 8 cores, and 16 GB of RAM. The solver was configured with the constraints and objective described in Section 3. Table 3 presents the results of solving the problem instance described in Section 5. The schedule optimizes patient assignments to machines while adhering to machine restrictions and minimizing treatment days.

| | m1 [(Frac n°, Patient)] | Prev. Occ. | Plan Occ. | m2 [(Frac n°, Patient)] | Prev. Occ. | Plan Occ. |
|---|---|---|---|---|---|---|
| day 1 | [] | 0 min | 0 min | [(1, 'p4')] | 0 min | 10 min |
| day 2 | [(1, 'p1')] | 5 min | 12 min | [(2, 'p4')] | 10 min | 5 min |
| day 3 | [(2, 'p1'), (1, 'p3')] | 5 min | 20 min | [(1, 'p2'), (3, 'p4')] | 0 min | 21 min |
| day 4 | [(2, 'p3'), (1, 'p5')] | 0 min | 21 min | [(3, 'p1'), (2, 'p2'), (4, 'p4')] | 5 min | 19 min |
| day 5 | [(4, 'p1'), (3, 'p3'), (2, 'p5')] | 5 min | 20 min | [(3, 'p2'), (5, 'p4')] | 0 min | 13 min |
| day 6 | [(5, 'p1'), (4, 'p3'), (3, 'p5')] | 0 min | 20 min | [(4, 'p2')] | 0 min | 8 min |
| day 7 | [(5, 'p3'), (4, 'p5')] | 0 min | 14 min | [(5, 'p2')] | 0 min | 8 min |
| day 8 | [(5, 'p5')] | 5 min | 7 min | [] | 0 min | 0 min |

**Table 3**
Results of the proposed modified BPP on the synthetic instance. "Prev. Occ." refers to the minutes the machine was already booked for, while "Plan Occ." refers to the minutes scheduled on the machine by the solver.

## 6. Conclusion

This paper introduced a novel approach to formalizing the RTSP as a *modified one-dimensional Bin-Packing Problem.* The proposed formalization is designed to exploit the research and algorithms developed for the one-dimensional BPP to address the challenges of the RTSP. By mapping radiotherapy

[1]https://developers.google.com/optimization

treatments to items and the LINAC to a bin, we established a clear correlation between these two problems. The proposed formulation offers several advantages, including the ability to leverage state-of-the-art solvers used for the plain one-dimensional BPP and the possibility of extending the formulation to other extensively studied versions of the BPP to tackle the complexities of the RTSP.

Future work includes evaluating the performance of the proposed formulation on real-world RTSP instances, incorporating patient priorities and uncertainties in treatment times, and tailoring existing one-dimensional BPP algorithms to the specific requirements of the new formulation. A practical enhancement could involve integrating physicians' expertise to refine the model's objectives and constraints, leading to more clinically feasible and optimal treatment plans. In conclusion, this paper provides a promising foundation for addressing the complex problem of radiotherapy scheduling. By leveraging the power of the one-dimensional BPP, we can potentially improve the efficiency and effectiveness of cancer treatment.

## Acknowledgments

This work has been partially supported by the PNRR project FAIR - Future AI Research (PE00000013), the NRRP MUR program funded by the NextGenerationEU, and by the project MUR PRIN 2020 - RIPER - Resilient AI-Based Self-Programming and Strategic Reasoning - CUP E63C22000400001.

## References

[1] D. Aineto, R. De Benedictis, M. Maratea, M. Mittelmann, G. Monaco, E. Scala, L. Serafini, I. Serina, F. Spegni, E. Tosello, A. Umbrico, M. Vallati (Eds.), Proceedings of the International Workshop on Artificial Intelligence for Climate Change, the Italian workshop on Planning and Scheduling, the RCRA Workshop on Experimental evaluation of algorithms for solving problems with combinatorial explosion, and the Workshop on Strategies, Prediction, Interaction, and Reasoning in Italy (AI4CC-IPS-RCRA-SPIRIT 2024), co-located with 23rd International Conference of the Italian Association for Artificial Intelligence (AIxIA 2024), CEUR Workshop Proceedings, CEUR-WS.org, 2024.

[2] S. Martello, P. Toth, Knapsack problems: algorithms and computer implementations, John Wiley & Sons, Inc., 1990.

[3] A. Kokangul, A combination of deterministic and stochastic approaches to optimize bed capacity in a hospital unit, Computer methods and programs in biomedicine 90 (2008) 56–65.

[4] J. P. Oddoye, D. F. Jones, M. Tamiz, P. Schmidt, Combining simulation and goal programming for healthcare planning in a medical assessment unit, European Journal of Operational Research 193 (2009) 250–261.

[5] Y. Zhang, M. L. Puterman, M. Nelson, D. Atkins, A simulation optimization approach to long-term care capacity planning, Operations research 60 (2012) 249–261.

[6] G. Ma, E. Demeulemeester, A multilevel integrative approach to hospital case mix and capacity planning, Computers & Operations Research 40 (2013) 2198–2207.

[7] L. B. Holm, H. Lurås, F. A. Dahl, Improving hospital bed utilisation through simulation and optimisation: with application to a 40% increase in patient volume in a norwegian general hospital, International journal of medical informatics 82 (2013) 80–89.

[8] T. Wang, A. Guinet, B. Besombes, A sizing tool for allocation planning of hospital bed resources, Intelligent Patient Management (2009) 113–125.

[9] M. Ordu, E. Demir, C. Tofallis, M. M. Gunal, A novel healthcare resource allocation decision support tool: A forecasting-simulation-optimization approach, Journal of the operational research society 72 (2021) 485–500.

[10] M. Squires, X. Tao, S. Elangovan, R. Gururajan, X. Zhou, U. R. Acharya, A novel genetic algorithm based system for the scheduling of medical treatments, Expert Systems with Applications 195 (2022) 116464.

[11] D. Petrovic, M. Morshed, S. Petrovic, Multi-objective genetic algorithms for scheduling of radiotherapy treatments for categorised cancer patients, Expert Systems with Applications 38 (2011) 6994–7002.
[12] B. Vieira, D. Demirtas, J. B. van de Kamer, E. W. Hans, W. van Harten, A mathematical programming model for optimizing the staff allocation in radiotherapy under uncertain demand, European journal of operational research 270 (2018) 709–722.
[13] B. Vieira, D. Demirtas, J. B. van de Kamer, E. W. Hans, L.-M. Rousseau, N. Lahrichi, W. H. van Harten, Radiotherapy treatment scheduling considering time window preferences, Health care management science 23 (2020) 520–534.
[14] B. Vieira, D. Demirtas, J. B. van de Kamer, E. W. Hans, W. Jongste, W. van Harten, Radiotherapy treatment scheduling: Implementing operations research into clinical practice, Plos one 16 (2021) e0247428.
[15] N. Hoffmans-Holtzer, L. Smolenaers, M. Olofsen-van Acht, M. Hoogeman, M. Balvert, S. Petit, Robust optimization of a radiotherapy pretreatment preparation workflow, Physics in Medicine & Biology 69 (2024) 025022.
[16] D. Conforti, F. Guerriero, R. Guido, Optimization models for radiotherapy patient scheduling, 4or 6 (2008) 263–278.
[17] D. Conforti, F. Guerriero, R. Guido, Non-block scheduling with priority for radiotherapy treatments, European Journal of Operational Research 201 (2010) 289–296.
[18] D. Conforti, F. Guerriero, R. Guido, M. Veltri, An optimal decision-making approach for the management of radiotherapy patients, OR Spectrum 33 (2011) 123–148.
[19] A. Saure, J. Patrick, S. Tyldesley, M. L. Puterman, Dynamic multi-appointment patient scheduling for radiation therapy, European Journal of Operational Research 223 (2012) 573–584.
[20] J. Patrick, M. L. Puterman, M. Queyranne, Dynamic multipriority patient scheduling for a diagnostic resource, Operations research 56 (2008) 1507–1525.
[21] T.-S. Pham, L.-M. Rousseau, P. De Causmaecker, A two-phase approach for the radiotherapy scheduling problem, Health Care Management Science (2022) 1–17.
[22] S. Frimodig, P. Enqvist, M. Carlsson, C. Mercier, Comparing optimization methods for radiation therapy patient scheduling using different objectives, in: Operations Research Forum, volume 4, Springer, 2023, p. 83.
[23] S. Frimodig, P. Enqvist, J. Kronqvist, A column generation approach for radiation therapy patient scheduling with planned machine unavailability and uncertain future arrivals, arXiv preprint arXiv:2303.10985 (2023).
[24] A. Legrain, M.-A. Fortin, N. Lahrichi, L.-M. Rousseau, Online stochastic optimization of radiotherapy patient scheduling, Health care management science 18 (2015) 110–123.
[25] A. Legrain, P. Jaillet, Stochastic online bipartite resource allocation problems, CIRRELT, 2013.
[26] R. Braune, W. J. Gutjahr, P. Vogl, Stochastic radiotherapy appointment scheduling, Central European Journal of Operations Research (2022) 1–39.
[27] P. Vogl, R. Braune, K. F. Doerner, Scheduling recurring radiotherapy appointments in an ion beam facility: Considering optional activities and time window constraints, Journal of Scheduling 22 (2019) 137–154.
[28] S. Bolusani, M. Besançon, K. Bestuzheva, A. Chmiela, J. Dionísio, T. Donkiewicz, J. van Doornmalen, L. Eifler, M. Ghannam, A. Gleixner, C. Graczyk, K. Halbig, I. Hedtke, A. Hoen, C. Hojny, R. van der Hulst, D. Kamp, T. Koch, K. Kofler, J. Lentz, J. Manns, G. Mexi, E. Mühmer, M. E. Pfetsch, F. Schlösser, F. Serrano, Y. Shinano, M. Turner, S. Vigerske, D. Weninger, L. Xu, The SCIP Optimization Suite 9.0, Technical Report, Optimization Online, 2024. URL: https://optimization-online.org/2024/02/the-scip-optimization-suite-9-0/.