A Cloud Full of Paths: Conceptual Blending as Betweenness Relation

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Abstract

Image schemas are vital for embodied cognition. In addition to image schemas considering one object or the interplay between two objects, there are also some having a ternary nature and allowing for the definition of some form of betweenness, such as for instance the introduction of a goal in between a start- and endpoint of a path. Betweenness is a central notion both in cognitive and formal aspects of modeling concepts. For example, in conceptual spaces, an object in between two other objects belonging to some concept again belongs to that same concept, according to the convexity assumption. This kind of betweenness understood as a path-searching procedure allows for increasing the comprehensibility of the resulting in-between objects because of a better understanding of the creation and origin of those objects. This view can also be applied to the task of finding new, creative objects based on given ones, thus finding paths that are not the straightforward solution, that is, not within a given convex conceptual space. This offers a direct connection to the framework of conceptual blending which allows for a structured creation of a blend of two input objects. We show that blending can be interpreted as a betweenness relation and that this path-based view is helpful to allow for a different viewpoint on conceptual blending. We argue that conceptual blending defined in this way as betweenness fulfills basic betweenness axioms and is thus a viable betweenness relation. As a proof of concept, these considerations are applied to visual blending.

Keywords

conceptual blending, image schemas, betweenness, generative models

1. Introduction

Image schemas [1, 2] can be seen as semantic building blocks of embodied cognition and early generalizations of experiences and therefore offer a direct link between embodied experiences and cognitive representations. An example is the image schema Support, denoting that an object is supported by another object, e.g., a cup on a table. Although many image schemas consider one object or the interplay between two objects, there are also image schemas such as Link and Source_Path_Goal that are ternary in nature, describing two (abstract or concrete) linked objects resp. a path from a source to a goal. Next to the two connected objects, there is a third object, namely the link resp. the path *in between* those. Considering the link resp. path as part of the image schema allows for achieving a deeper understanding of the scenery: it can be determined whether an intermediate goal is on the path, thus in between source and goal of the path, leading to the image schema Source_Path_via_Goal. Understanding Link means that children learn that when pulling on some object connected through a physical link with another object (a special case of Link), then, in addition to these two objects, also all objects in between them will move. Thus, *betweenness*, though not typically characterized as an image schema, is of vital importance for embodied cognition. However, betweenness is not restricted to this spatio-temporal view. It can also be used to decide on an abstract level whether some object is in between another two objects, e.g., in form of interpolation between rules: if it is known that a flat is cheap and a villa is expensive and it is known that a townhouse is in between a flat and a villa, then it can be concluded based on an assumed betweenness that a townhouse has a moderate price [3]. This principle of convexity of cognition has, e.g., been proposed by Gärdenfors in his theory of *conceptual spaces* [4]. He states that

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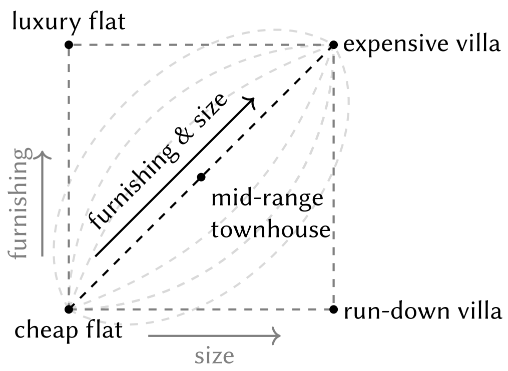

Figure 1: For the two given inputs "cheap flat" and "expensive villa", a non-creative solution would be the conclusion that in between there must be a mid-range townhouse. Creativity, however, allows for considering different paths not justified by the background knowledge, e.g., a luxury flat.

concepts are cognitively represented as convex (thus, betweenness-closed) sets and that especially learnability is increased by considering convex representations [5]. Schockaert and Prade [3] state that betweenness also allows for finding new in-between representations in conceptual spaces, e.g., for completing an incomplete rule base. Also this abstract notion of betweenness can be considered from an image schematic viewpoint, namely as a special type of the image schema Source_Path_via_Goal: If for two given objects an in-between representation is searched for, this process can be considered as a path-finding problem where a goal needs to be determined that is in-between the two objects, thus on an (abstract) path connecting them. In the example mentioned above, visualized in Figure 1, this would be the straight black line denoting a betweenness based on size and furnishing.

This straight connection denoting a straightforward and expected betweenness is not the only possibility. Beside the moderately priced houses (which are justified by the knowledge base) one could think of more creative and less justified results, such as a cheap villa or an expensive apartment. Therefore, since the novel combination of familiar ideas can be seen as a fundamental aspect of human creativity [6], choosing different betweenness relations enables a creative combination of given objects. Thus, the path of thought may take some detours to obtain creative results. This process is creative, as it allows for a wider variety of in-between objects and is not restricted to plausible results grounded in given background information as, e.g., the rule-base completion approach [3]. Analyzing the chosen path helps in understanding the creatively invented new object, as it allows for considering the iterative changes starting from start or end. However, the resulting path should be creative but not arbitrary, as, e.g., a creative combination of a house and a boat being a pink apple would not be considered as creative but as implausible and out of context. One type of betweenness suitable for this task is the intersection betweenness [7]: Roughly, it is defined such that some object is in between two other objects if the in-between object has all attributes the other two objects have in common. Thus, the creativity would be grounded on common attributes and thus would be more plausible. The problem is, however, that the intersection betweenness needs an attribute representation of both inputs and considers the intersection of these representations. Such a representation is not always available and the intersection is a too strong restriction, as it, e.g., does not consider generalizations of attributes which then may coincide. Following the basic principles of intersection betweenness while dismissing the representation of concepts as sets of attributes can be directly considered as a version of the framework of conceptual blending, as discussed by Fauconnier and Turner [8]. The basic idea is that two input concepts (e.g., "house" and "boat") are combined to form a new concept (typically not unique), e.g., "boathouse" or "houseboat". The first step is to find an abstraction of the commonalities of the two input concepts, the so-called *generic space*. Then, a *blend* is created, based on the information of the generic space and enriched with some features of the input spaces. Conceptual blending is a highly researched area, both in theory and in practice (see, e.g., [9]). When comparing the process of conceptual

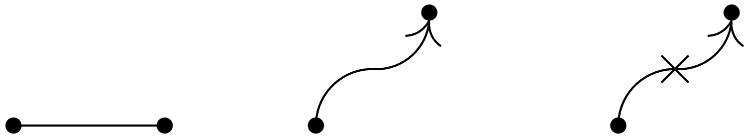

Figure 2: From left to right: the Link, the Source_Path_Goal, and the Source_Path_via_Goal image schema

blending with the betweenness relation, especially with the intersection betweenness discussed above, the similarity becomes evident: the generic space serves a function analogous to the intersection of attributes in the intersection betweenness (however, is not unique and allows for generalizations) and a blend can be considered as an object in between. This leads to the main statement of this paper: conceptual blending can be considered as a betweenness relation and therefore, in an image schematic view, the blending process can be seen as a search for a suitable path between two inputs, passing by the blend. As betweenness is an important notion, formally introduced by Huntington and Kline [10] and used in many different use cases, an interpretation of conceptual blending as betweenness allows to exploit this literature. It especially allows for a combination of the classical symbolic view of conceptual blending with a subsymbolic view of the betweenness relations and thus enables to incorporate the rich semantic structure of the subsymbolic space by meanwhile keeping the advantages of conceptual blending regarding conceptualization and reliance on a generic space. Additionally, the path-based view allows for understanding the creation of the blend by "walking the path", thus iteratively comprehending the changes necessary between start and end to reach the goal. As a proof of concept, an interpretation of visual blending as betweenness relation is given which shows that the process can profit from considering betweenness relations and their properties.

The rest of the paper is structured as follows: After a short introduction to image schemas, the importance of betweenness in an image schematic and general cognitive viewpoint is pointed out. After that, a short overview on conceptual blending is given, its connection to betweenness is discussed and necessary properties of the resulting betweenness relation are given. The last section considers an application to visual blending. The paper ends with a conclusion and an outlook.

2. Image Schemas and Betweenness

Image schemas offer a direct link between embodied experiences and representations in the mind and are therefore a basic semantic building block of embodied cognition. They have been independently introduced by Lakoff [1] and Johnson [2]. Image schemas are said to be learned in early infancy and are thus preverbal [11]. Therefore, image schemas can be seen as early generalizations of experiences. Examples for image schemas are Support, expressing the fact that an object is supported by another one, and Containment, thus the notion that an object can be placed inside another object without falling out. Next to the spatio-temporal use of image schemas, they also found their way into language in form of metaphors: "to be in love" is an example for an abstract use of the image schema Containment.

Image schemas such as Support and Contact are included in the two-object family as introduced and discussed in [12]. Thus, they are based on the interaction between two objects . Link is a member of this family and represents an "enforced connection between objects or regions, where transitivity ensures that the linked object reacts to the stimuli of the other object" [12]. It could be a concrete link like, e.g., a rope, or an abstract link like an emotional binding between persons or any form of causal relationship. A member of the Path-family is Source_Path_Goal, relating to movement from a source to a goal, including spatial primitives such as a path between them [12].

Although, these image schemas are on first sight based on two objects (the two linked objects, resp. source and goal), they actually incorporate a more complex relationship. Both include a third important element, namely the connection of the two objects, either the (possibly abstract) link or the path. Thus, Link and Source_Path_Goal are examples of image schemas that are ternary in nature. Therefore, though image schemas comprise basic notions easily perceptable, some elements of ternarity are needed.

This becomes even more obvious for the image schema SOURCE_PATH_VIA_GOAL where there is a goal in between the start and end of the path (see Figure 2 for an illustration).

Those three image schemas have an implicit notion of *betweenness* incorporated. Betweenness is not an image schema by itself and already a quite complex notion, however, SOURCE_PATH_GOAL and LINK require at least basic knowledge of special cases of betweenness. A child may consider the points in between source and goal on a path and may recognize beforehand that they will pass these points. They may even be able to consider different paths, thus different betweenness relations, and choose the best one. They are also able to recognize (at least for physical links) that an interaction with one linked object not only influences the other linked object but possibly also the objects in between. Therefore, it can be concluded that betweenness is at least implicitly incorporated in the catalog of image schemas. SOURCE_PATH_GOAL can also be interpreted metaphorically, such as in "life is a journey". There, as in the concrete case, the main point of this expression is not the source and the goal (thus, birth and death) but the path in between, all those experiences, one following another, on this journey. Whether betweenness should be considered an image schema by itself, or whether it is a too complex or abstract notion in its full generality, is an interesting question for future research.

2.1. A Formal Notion of Betweenness

A betweenness relation could be an arbitrary ternary relation. An object b is said to be *in between* a and c (written $Btw(a, b, c)$). It has been formally introduced by Huntington and Kline [10] and is based on an axiomatization of geometries by Hilbert [13]. Betweenness can be used to enrich the underlying structure of a space with an order, e.g., to determine an order of sets [7] or as a basis for ordered geometry. Another benefit of betweenness is that it allows for defining convex sets (in the sense of betweenness-closure) and thus allows, e.g., for convex optimization. One well known betweenness relation is the *metric betweenness*, e.g., in $\mathbb{R}^n$

$$Btw_m(a, b, c) \text{ iff } d(a, b) + d(b, c) = d(a, c) \text{ for } a, b, c \in \mathbb{R}^n \text{ with } a \neq b \neq c,$$

based on a distance $d(\cdot, \cdot)$, e.g., the Euclidean distance. With this betweenness, it is, in fact, possible to develop, together with the notion of equidistance, the whole elementary geometry axiomatically and, thus, it can be considered as a basic notion of elementary geometry [14]. Next to the metric betweenness, betweenness can also be used, e.g., for checking properties of lattices [15] or for knowledge graph embeddings with incorporation of expressive background logics [16] and in several other areas.

Though betweenness relations can be arbitrary ternary relations, to strengthen their expressivity, basic betweenness axioms have been considered to which an expressive betweenness relation should adhere to. They have been introduced by Huntington and Kline [10]. In the following, some basic betweenness axioms are presented.

Definition 1. *The following are basic betweenness axioms, based on a set X and $a, b, c \in X$*

(B0) *If $Btw(a, b, c)$, then a, b, c are pairwise distinct.* *(distinctness)*

(B1) *If $Btw(a, b, c)$, then $Btw(c, b, a)$.* *(symmetry in the end points)*

(B2) *If $Btw(a, b, c)$ and a, b, c are pairwise distinct, then not $Btw(b, a, c)$.* *(non-exchangeability)*

There are many further axioms considered, useful for determining the exact properties of a specific betweenness relation. One additional axiom concerns the density of the space, thus whether there is, for any two distinct elements of the space, an element in between.

(C1) For any two distinct points a and c in X, there is some point b such that $B(a, b, c)$. (density)

Another axiom, particularly interesting, as it is fulfilled by the Euclidean betweenness, is the following, stating that there is only one "path" in between two elements a and c, thus that two objects b and b' both in between a and c must be in a betweenness relation by themselves.

(C2) If for distinct b and b', $Btw(a, b, c)$ and $Btw(a, b', c)$, then either $Btw(a, b', b)$ or $Btw(a, b, b')$.

Though this is fulfilled by the Euclidean betweenness, it is a strong axiom not valid for every use case.

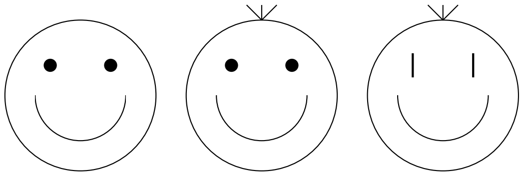

Figure 3: Face b being in between faces a and c based on intersection betweenness.

2.2. Betweenness in Cognition

Betweenness is implicitly incorporated into image schemas such as Source_Path_Goal and Link in a spatio-temporal manner. Next to this spatio-temporal view, image schemas allow for a metaphorical view directly resembling the spatio-temporal view in a metaphorical setting. However, these image schemas can be viewed in an even more abstract way: Source_Path_via_Goal can also be considered as a path between two objects trespassing several goals in form of other objects. When considering the example in Figure 1, to figure out the price of a townhouse, it is possible to imagine a path between the cheap flat and the expensive villa, changing furnishing and size successively, passing, e.g., a less cheap apartment, a near too expensive small villa and also the mid-range townhouse which is the answer to the question. This image-schematic view on betweenness allows for an understandable and explainable result since the way the in-between object has been approached is comprehensible.

As this imagined path does not need to have a direction, also the image schema Link is suitable in this regard: a physical link between two objects can be defined based on the objects in between (e.g., the position and other features of the parts of the link, e.g., of a chain). In the same way, two objects can be considered as being abstractly linked when there are objects in between. These in-between objects define whether the link is a straightforward or creative one. The view of applying betweenness to objects to obtain in between objects is justified by the consideration of *conceptual spaces*. They have been introduced by Gärdenfors [4] and are based on the idea that cognition is geometrical and that concepts are cognitively represented as convex sets (resp. combinations of convex sets). As convexity can be defined as betweenness-closure, this directly leads to the consideration of objects as in between other objects. This is motivated by cognitive economy and by the fact that inferring knowledge may work as follows: If one knows that two objects have a specific property, then it is natural to assume that an object in between the other two objects also has this property; this has been done for an incomplete rule-base by Schockaert and Prade [3]. One betweenness relation particularly useful for cognitive considerations is the *intersection betweenness*, a betweenness defined over sets of elements. Next to considering sets in general, this definition of betweenness supports also the declaring of a betweenness relation based on the attributes of objects. This is based on the idea that objects can be compared based on sets of their features [7]. An object b is said to lie in between a and c if it shares the features common to a and c (but may have ones that neither a nor c have) [17]. An example can be seen in Figure 3. Face b is in between faces a and c, as it shares the properties both have in common (the shape of the face and the mouth) and additionally has some distinctive properties, namely the hair of c and the eyes of a. This intersection betweenness is perfectly in line with the interpretation of betweenness as a path, as it is possible to iteratively change the first input to gain the in-between which then can iteratively be changed to obtain the second input. Intersection betweenness on the concept level has been discussed by Ibáñez-García et al. [18]. They argue in a description logic setting and state that a concept is in between two other concepts, if it is in between based on intersection betweenness on the feature level.

Even when considering intersection betweenness, there is not only the one possible path that is justified by the background knowledge. Next to this, there are other paths, though not justified by the data, however, acting as starting points to be creative: what would a cheap villa or an expensive flat look like? As creativity based on a new combination of two known concepts is a common way of creativity [6, 19], it would be interesting not only to consider the justified object in between but also other, more creative and less justified paths. However, these paths are not allowed to be arbitrary, in fact, the creativity needs to be controlled to lead to reasonable results. An area where this is considered is the area of conceptual blending. Therefore, in the following, it is discussed that conceptual blending can be interpreted as such a path-finding betweenness problem and can profit from such a view.

3. Conceptual Blending and Betweenness

In the following, it is argued that conceptual blending can be considered as a special type of betweenness relation. We begin with a quick reminder about conceptual blending.

3.1. Conceptual Blending

Conceptual blending was introduced by Fauconnier and Turner [20] and is a technique to achieve new concepts by creatively combining existing ones. The basic idea is to consider two input spaces, containing a representation of the input objects, e.g., “house” and “boat” such as that they have inhabitants, being on top of water resp. land etc. With these inputs, a *generic space* is defined, containing some general facts or abstractions that are common to both inputs. This space can be obtained through a generalization of both input spaces and is thus a more powerful device than an intersection of attribute representations as in intersection betweenness. The generic space is not unique, as different generalization strategies could lead to different results. With the help of the generic space, the blend can then be created by incorporating facts from both input spaces without contradictions. The last step is to “run the blend”, thus to enlarge the blend with information not given in the input spaces. For the house-boat example, the blend could be a houseboat and the running would incorporate questions such as how big a houseboat would be or whether it could be used offshore. The blend is again not unique, however, there are better and worse blends. To determine the quality of a blend, so called *optimality principles* may be used [20]. An example is the *unpacking principle* which states that both input spaces need to be reconstructable out of the blend, thus, that a sufficient amount of information of both input spaces should be incorporated. Therefore, a blend is of low quality if it is based solely on one input space and incorporates only a slight adjustment towards the other one. For a comprehensive discussion of optimality principles, see [20].

The formalization of conceptual blending is a challenging task, particularly the construction of the generic space. An example is an approach proposed by Eppe et al. [9] based on answer set programming. Although it was shown to work out for several examples, it is difficult to use in a general setting, as it suffers from complexity issues. As this is a general problem of symbolic approaches for conceptual blending, its interpretation with the help of betweenness could help to incorporate subsymbolic aspects into the process to better handle the complexity of the problem.

3.2. Conceptual Blending as Betweenness Relation

As discussed above, betweenness can be used to be creative and to construct new objects based on given ones. As betweenness relations have a wide variety of properties, such betweenness-based creativity could also lead to many different more or less reasonable results. Therefore, it is necessary to restrict the possible outcomes. This is the reason for considering conceptual blending: it creates out of two input spaces not only a blend but also a generic space restricting the blend, prevents contradictory results and allows for judging the quality of the blend with the help of optimality principles. However, the advantages of the betweenness-based and especially the path-based view should be kept, therefore, conceptual blending is interpreted as a special type of betweenness relation, related to intersection betweenness: the blend can be considered as being in between the input spaces, as similar to the intersection betweenness, there is an underlying space (the generic space) the blend must rely on and additionally, the blend consists of properties of the two inputs. Thus, conceptual blending can be considered as a generalization of intersection betweenness. However, there are vital differences: intersection betweenness is made for determining whether given objects have a betweenness relation. Using it for creativity has several issues: First, it is based on attribute representations and thus does not allow for the definition of an expressive generic space. Second, it is, even if the intersection of both

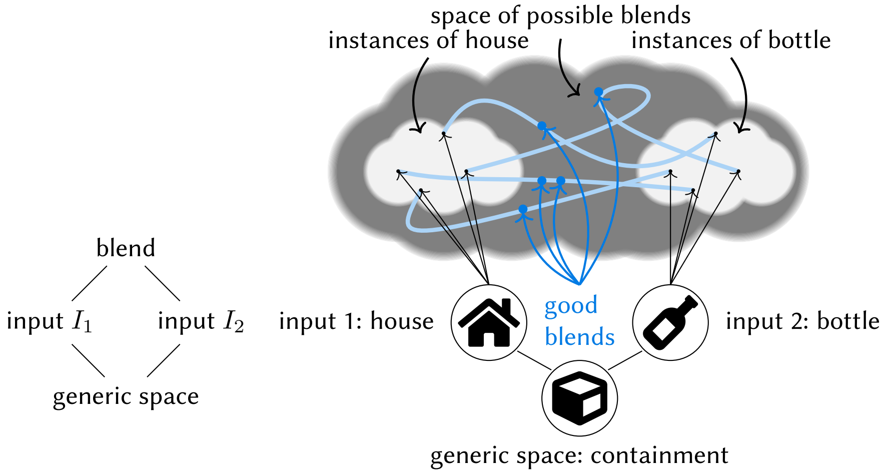

Figure 4: Left: The framework of conceptual blending [20]; right: the path-based view of conceptual blending: starting with two input concepts, first a couple of instantiations of these concepts are considered. In between these, paths (blue) are looked at, restricted to the generic space. They make up the cloud of possible blends (gray). Based on optimality principles, good blends are found (dark blue).

inputs is considered to be reasonable, not given how to define a reasonable new object in between which is expressive, creative and non-contradictory. Conceptual blending allows for all this and gives with the optimality principles also hints on how to judge the quality of the blends. Therefore, conceptual blending under this betweenness-based view can be interpreted as a path search from one input space to the other whereas this path is not allowed to be arbitrary, as it is restricted by the underlying generic space to which each object on the path needs to adhere to and additionally does not allow for contradictory results. The basic idea of considering betweenness instead of conceptual blending directly is to enable creating a blend in a more neuro-symbolic way. Whereas many conceptual blending approaches are based on symbol manipulation, betweenness allows for many different viewpoints, e.g., based on numeric or other abstract representations of the objects, thus allows for using the advantages of subsymbolic reasoning. Of course, betweenness alone does not lead to such viewpoints, it is additionally necessary to have data in an adequate representation to be used as basis for a betweenness relation. It would also be possible to interpret existing frameworks in a betweenness based setting. There are representations, e.g., in latent spaces that could be good starting points for a search for a suitable betweenness relation combining subsymbolic and symbolic information. This is discussed in Section 4.

Next to this view, it is also possible to consider conceptual blending as combination of different betweenness relations. This becomes evident when looking at the diagram of the framework of conceptual blending (see Figure 4 on the left). There, a graph-based betweenness can be applied, stating that a node is in between two other nodes if it lies on the shortest path between them on a graph. This view leads to four different betweenness relations $Btw(I_1, B, I_2)$, $Btw(I_1, GS, I_2)$, $Btw(GS, I_1, B)$ and $Btw(GS, I_2, B)$ (and their symmetric counterparts) where GS denotes the generic space and B the blend. In the following, the first two betweenness relations are looked at, as they match the view of having the input spaces given and determining generic space and blend. For future work, a consideration of the other two betweenness relations could also be interesting, e.g., for inverse blending by determining possible input spaces for a given blend. These definitions are not only justifiable by this abstract graph-based argument but also based on the basic principles of conceptual blending.

Considering solely the betweenness relation $Btw(I_1, B, I_2)$ pictures classical strategies of *visual blending* or *image morphing* where the basic idea is to combine two images on a visual level. Although, it is also based on a smooth transition between the two input images, it has not the same expressivity as conceptual blending, as the possible paths are not restricted based on a generic space and the focus does not lie on conceptual knowledge. An example of this betweenness is discussed in detail in Section 4.

The second betweenness, thus "the generic space is in between the two input spaces" is more abstract and not as easily justifiable: The basic idea is again to find a plausible path from one input to the other, in contrast to the betweenness of the blend with a different focus: instead of exchanging attributes of one input space with attributes of the other, here the attributes of both input spaces are generalized until they reach a point where both generalizations meet. This can be considered as a search for a Link. One way of stating that two objects are linked is to find their commonalities, thus a generalization in between them. This could also be interpreted as a basic case of intersection betweenness where only the intersection of properties (in this case the generalization of input spaces) is considered without adding further attributes from one or both of the inputs. In special cases this could be the outcome of a conceptual blending process, e.g., when a compromise of two contrastive opinions is searched for. Then, the generic space is the only possible blend, as all extensions would contradict the compromise. To use these two betweenness relations for conceptual blending, it is necessary to find a combination of them.

Betweenness Axioms For underlining the adequacy of considering conceptual blending as betweenness relation it is shown that betweenness based on the framework of conceptual blending fulfills the basic betweenness axioms. An interesting application would be to interpret existing conceptual blending methods as betweenness relations and to discuss their betweenness axioms in contrast to the betweenness axioms of the basic framework of conceptual blending. This is, however, left for future work. As there are many different betweenness axioms for different use cases, in the following, the basic axioms (B0)–(B2) and (C1) are discussed for all three cases of betweenness mentioned above (if not otherwise indicated, the explanation is applicable to all three). Additionally, to give a hint on the fact that some classical betweenness relations, such as the Euclidean betweenness, could be unsuitable for representing blending (even if a suitable geometric representation of the input spaces is found), the axiom (C2) is considered which depicts a basic property of Euclidean betweenness which is, however, in general too restrictive for conceptual blending. It turns out that all three basic axioms and the axiom of density (C1) are fulfilled in a conceptual blending setting. This underlines the validity of interpreting blending as a special type of betweenness relation.

First, the axiom (B0), thus the distinctness of the elements of a betweenness relation is considered. Clearly, the blend needs to be different from the two inputs, as otherwise no creativity is needed. However, it is not as obvious whether a blend can be created based on only one input space: an example is the blend of a dog leading to a dog with two heads. Even though the creation of the generic space is not unique and there are several options of generalizations even for identical input spaces, the most natural and thus also best generic space based on optimality principles would be a generic space identical to the input. In this case, no creativity is possible. Therefore, it can be argued that a dog with two heads could not be considered to be a blend of two dogs (at least not in the framework of conceptual blending).

In the conceptual blending framework the order of inputs is not specified, therefore symmetry (B1) is trivially valid. In a path-based perspective, this means that it does not matter whether the path starts at the one input or the other. This underlines the image schematic viewpoint that instead of a path also an (undirected) Link can be considered. It could be the case that the influence of one input is stronger than the influence of the other, this is, however, not because of the position of the input as first or second one but based on the properties of the input spaces.

The non-exchangeability principle (B2) can be seen to follow assuming the unpacking principle together with certain (contingent) features of blends, as follows: Assume, by contradiction, that both relations of (B2) are valid, thus $Btw(I_1, B, I_2)$ and $Btw(B, I_1, I_2)$. Due to the unpacking principle, the blend needs to be influenced by both inputs, thus it has properties one of the inputs has and the other does not. Thus, given $Btw(I_1, B, I_2)$, especially B and I_2 are sharing some properties that I_1 does not share, say property P is enjoyed by I_2 but not by I_1 (this also follows from the common assumption in blending that there is some conceptual clash between I_1 and I_2). Now when considering $Btw(B, I_1, I_2)$ and the generic space created by B and I_2, a natural condition of *maximality of the base* implies that these shared properties are incorporated into this base space. Thus, P belongs to the base. However, if that is the case, then since I_1 inherits properties from the generic space (assuming *complete inheritance from base*), it follows that I_1 needs to have the property P, too. This is a contradiction derived solely from the unpacking principle and certain maximality principles concerning the role of the base, outlining the conditions for validity of the non-exchangeability principle. Studying in full detail such dependencies between rules in blending construction and algebraic conditions on betweenness relations will be the subject of future work.

The above consideration shows that conceptual blending interpreted as betweenness relation fulfills basic betweenness axioms which again underlines that such an interpretation is suitable, as it opens up a wide choice of betweenness relations which could be useful for considering blendings.

Besides the classical betweenness axioms, it is possible to consider other, more specific ones. The *axiom of density* (C1) needs to be viewed from two perspectives: on the one hand, the two input spaces could be highly similar. Then, the blending process does not result in an overly creative blend, however, even for similar (but not equal) input spaces, a blend is possible. On the other hand, two input spaces could be highly dissimilar. Then, the generic space is a coarse generalization depicting highly general cases. This, however, is no restriction of the blending process, in fact, a coarse generic space allows for creative blends. Therefore, all two objects allow for a blend and thus the density axiom is valid. The quality of the blend, of course, depends on the chosen input spaces.

Axiom (C2) depicts that there is only one path in between two inputs. This is not generally valid. Considering the house-boat blend, there are (at least) two viable blends, “houseboat” and “boathouse”. When considering, e.g., the pictures on the path between house and boat in Figure 5, it becomes clear that it is not possible to place the boathouse reasonably on this path. It is thinkable of other betweenness relations leading to the houseboat, however, as houseboat and boathouse are both based on totally different generic spaces, it seems implausible that one of these could be considered as being in between the other one and “house” or “boat” (e.g. not $Btw(boathouse, houseboat, boat)$). Next to this example, it also seems in general to be natural to assume, even based on one generic space, different paths from one input to the other, respecting totally different facts of the input spaces.

A visualization of the interpretation of conceptual blending as betweenness relation can be seen in Figure 4 on the right. Two inputs, in this case “house” and “bottle” are considered and the generic space is either explicitly (as done here) or implicitly determined. Then, instances of both house and bottle are considered and between these instances, possible betweenness relations in form of paths are created. These paths could be arbitrary shaped as long as they remain in the space of possible blends, thus the region is restricted by the generic space and the consistency of the elements on the path. Based on the optimality principles, it is then possible to determine good blends on the paths. Possible blends are, e.g., genie in a bottle or a house in a bottle in style of a ship in a bottle. Thereby it is possible that a path does not contain any or contains several good blends.

It is possible to apply the above considerations to given conceptual blending approaches, analyze whether they are in line with these considerations and possibly proposing enhancements based on an appropriate betweenness relation. This is done in the next section for visual blending which is, though only a simplified version of conceptual blending, well interpretable with the help of a betweenness relation and is a step towards conceptual blending with the help of betweenness.

4. An Application to Visual Blending

An area that lends itself to use in context with betweenness is when a geometric representation of the objects is given, as those betweenness relations are widely studied and nicely applicable to examine the underlying structure of such representation spaces. Such representations are given, e.g., in the area of generative models. They are based on the idea that an input is mapped to a high-dimensional latent space representation which is then mapped to an output. Examples are text-to-image models such as *stable diffusion* [21]. Without considering the technical details here, the basic idea is to give a short text as input (e.g., “house”). This text is mapped to a vector representation in a high-dimensional *latent space* out of which an image is generated. For a comprehensive overview of such techniques, see, e.g., [21].

Such a latent space representation allows for considering betweenness of different representations, especially, as the aim of such models is to represent not only the training data but to be able to create

Figure 5: Visual blend of house and boat, created with attention interpolation[26]

new images based on the given ones which needs some sort of smoothness of the space.

How could this space be used for doing some sort of blending? A straightforward approach to achieve a visual blend is to combine the two inputs directly at text level, thus prompting, e.g., directly the word “boathouse”. This could lead to reasonable results, especially for cases where the combined word actually has the meaning the combination of words implies, such as “boathouse”. This has been done by Ge and Parikh [22]. However, this does not necessarily lead to the desired results and especially is no indicator of creativity. Consider, e.g., the blend of “butter” and “fly”. Whereas the combined word “butterfly” has an assigned and well-known meaning independent of butter and fly, a visual blend between butter and fly would be creative, e.g., a fly made out of butter.

An approach based on text-to-image models that considers betweenness was presented by Melzi et al. [23]. They implicitly considered betweenness by stating that something is a visual blend if it is in between the representation of the two input texts in the latent space. First, for both inputs, the latent space representation is created. Then, with the help of Euclidean betweenness, a point in between the two is found and for this representation, an image is created.

To improve this approach, it is possible to analyze the betweenness relation. As stated before, Euclidean betweenness fulfills the betweenness axiom (C2) which is as discussed in Section 3.2 not suitable for blending. Additionally, it is necessary to consider the underlying structure of the space to find a betweenness that allows for a smooth interpolation between the two inputs and thus for a path in between them. It has been observed, e.g., by Arvanitidis et al. [24] that the latent space is non-linear, because of its non-linear generation. Alternative betweenness relations are, e.g., spherical linear interpolation [25] or interpolation based on attention [26] for which an example can be seen in Figure 5. This is, due to its importance to visual blending, a highly researched area with many proposed interpolation functions from which the most can be interpreted as betweenness relations.

This underlines the advantage of considering betweenness for blending, especially, but not limited to geometric representations, as it allows for considering an order of elements that is not limited to standard cases such as Euclidean betweenness. Therefore, such approaches are a good example why it is helpful to consider abstract betweenness relations as conceptual blending approaches. Additionally, the path-based view allows understanding the blending process and judging the quality of the blend based on the smoothness of the path.

However, these approaches still have some pitfalls. First, the approaches rely on the fact that there are actually creative aspects represented in the latent space (and not only images which are part of or similar to the training data) and, if such creative representations exist, they need to be actually findable, thus in between the input representations based on the chosen betweenness relation. Next to these technical problems, there are basic problems due to the fact that the blending is done solely on a subsymbolic level, thus, conceptual information is not considered. By examining the example blend in Figure 5, it can be seen that not only the boat is blended but also the background. From an open water background in the first image, it is getting more and more hilly and forested in the course of the next pictures. This is, however, an irrelevant information for the houseboat-blend. Therefore, it is necessary to have a conceptual foundation of the blending and also of the betweenness relations, as until now, these approaches achieve visual blends (thus combining images) but not *visual conceptual blends* [27], thus the combination of images with the help of conceptual information.

An option would be to consider two betweenness relations, one for the blend and one for the generic space (the second version discussed in Section 3.2) and use the generic space-betweenness to guide the blend betweenness. Another interesting option would be to generalize from this instance-based view of these latent spaces to consider, e.g., sets of instances to define a blend on concept level. Such approaches do, however, still not act on a conceptual level, as latent spaces do not, in general, incorporate conceptual information. One option is to stick to betweenness on a latent space level but to equip the latent space with some conceptual information, e.g., by modeling concepts as geometric objects in such spaces and logical operations as geometric operations between them, e.g., with the help of spheres [28]. Another option would be to actually consider the generic space on a symbolic and the blending betweenness on a subsymbolic level, thus interpreting the conceptual blending process as a hybrid approach.

However, note that this blending with latent spaces is only one application of the idea of doing conceptual blending with the help of betweenness relations. Due to the nature of the latent space, it invites to consider numerical betweenness relations which are widely studied and nicely applicable. However, it is also possible to consider more abstract non-numeric betweenness relations, e.g., graph-based betweenness or variants of intersection betweenness (thus, following the first version discussed in Section 3.2), to tackle other variants of conceptual blending in a betweenness-focused setting.

5. Conclusion and Outlook

The example of visual blending shows that interpreting blending as betweenness relation is worth a try, as it helps to determine suitable properties of the blending and and enables to choose adequate interpolation techniques. However, as the considerations on betweenness axioms showed, betweenness is not only helpful for the visual blending case but can also be used to discuss conceptual blending approaches in general. This leads to a number of interesting open questions: How could the betweenness be considered in a setting that is not as geometric in nature as the visual blending case? How could the two betweenness relations (of the blend and of the generic space) be combined? Also in the visual blending setting further examinations are possible, namely, how to enhance the approach to not only considering visual blending but actually visual conceptual blending?

To conclude, in this paper, it has been shown that betweenness is an at least implicit image schematic notion which can not only be used for spatio-temporal reasoning but also for abstract, metaphorical considerations. These abstract notions can be used as a basic building block for creativity in the sense of conceptual blending. Through its strong connection to embodied cognition, this approach seems to be well justified and therefore worthwhile to be investigated further.

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