Navigation aids network performance estimation with geospatial data analysis

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Abstract

Networks of navigation aids are widely used in civil aviation to measure navigation parameters. Networks of Distance measuring equipment (DME) and VHF omnidirectional range (VOR) provide services performance of each depending on the geographic location of the airplane. The network provides specific values of the number of available navigation aids, the number of pairs available for positioning, and the accuracy of positioning for each point of airspace. In the paper, we study the application of a global geospatial indexing system to use in geospatial data analysis of navigation aid performance. The geospatial indexing system provides partitioning of ellipsoidal shape into a grid with a particular cell shape. Also, the geospatial indexing system specifies the global addressing of each cell, which supports a hierarchical structure. We study the application of hierarchical hexagonal Spatial Index (regular hexagonal cell shape) and open location codes (rectangular cell shape) in the task of navigation aids network performance evaluation. The navigation aids network of Poland has been used for numerical demonstration of the proposed algorithm of geospatial data analysis. The performance of the navigation aid network is estimated based on the accuracy of positioning by pairs of DME/DME, VOR/DME, and VOR/VOR.

Keywords

geospatial data, air navigation, distance measuring equipment, navigation aids, performance of positioning, spatial index

1. Introduction

Civil aviation uses a wide network of navigational aids to provide airplane positioning and navigation [1, 2]. Navigation aids are specific ground-based equipment that is used during airplane flight to perform measuring navigation parameters (ranges and angles) to specific waypoints associated with a place of equipment installation. The most commonly used navigation aids include Distance Measuring Equipment (DME), VHF Omnidirectional Range (VOR), and Non-Directional Beacons (NDB) [3, 4]. Ground-based beacons of this equipment are placed at waypoints with precisely known coordinates. Each equipment uses a specific radio channel to perform measurements of navigation data. Measured navigation data includes ranges (provided by DME) and angles in the horizontal plane. Measured ranges and angles are used by the onboard Flight Management System (FMS) for airplane position calculation [5, 6]. Positioning by navigation aids is considered as a backup system in case if satellite navigation system and Inertial navigation system are unavailable for coordinates measurements [7, 8].

Each navigation aid has a particular service volume inside of which the beacon could be used for parameter measurement [9, 10]. This service volume is hardly fixed in geographic position. During the flight, airplane equipment simultaneously uses ground beacons to identify its position. The performance of positioning by a pair of navigation aids is geospatial distributed. It means that in

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each point of airspace, the performance of positioning is different based on the configuration of the navigation aids network. Air navigation service providers have to configure network topology to provide efficient network operation. It requires computer simulation of network performance and geospatial data analysis of obtained results [11].

The performance of the navigation aids network is demonstrated by parameters of availability, number of pairs, accuracy of positioning, and correspondence to requirements of navigation specifications [12, 13]. The distribution of all these parameters is different for each position. Simulation of all these parameters distribution and geospatial data analysis of obtained results helps to understand navigation aids performance which could be useful for airspace users for flight planning tasks and air navigation service providers to identify possible gaps in service.

A few studies consider the application of generic algorithms to minimize navigational aids network configuration [14, 15]. Most studies consider optimization of the network based on maximization accuracy of positioning and efficient ground beacon distribution over a wide area. Modern requirements for ground infrastructure require incurring a particular level of not only accuracy but number of available beacons and number of pairs, due to its participation in system redundancy and reliability analysis [16, 17].

All approaches of geospatial analysis of navigation network configuration ground on partitioning considered airspace into a set of elementary cells in which estimated parameters are considered constant [18, 19]. This elementary cell is a result of partitioning a particular geographical area specified as a big box with maximum and minimum boundaries in latitude and longitude. The partitioning process is based on the equal distribution of elementary cells. Therefore, this approach requires the specification of a number of cells and the calculation of the geographic coordinates of cell centers. Coordinates of cell centers in latitude and longitude provide geographic markup of estimated network parameters. Due to the spherical nature of latitude and longitude cell size will vary on the ellipsoidal surface, which introduces bias in geospatial data analysis.

Global geospatial indexing (GGI) is a reach tool for data analysis. GGI uses a specific algorithm for partitioning the ellipsoidal surface into a set of elementary cells [20, 21]. Each cell has a specific index (code). The size of the cell is changed based on required precision. Also, each children cell has an address that includes the addresses of all parents at lower precision levels. This property makes GGI highly useful in vary of applications in geospatial data analysis [22, 23]. Also, most GGI standards operate with approximately equal cell geometry of some regular shape. GGI provides a hard connection of some data with unique cell indexes that have defined coordinates of corners.

GGI is used in vary of applications as an example analysis of urban settlements' global distribution (mapping hierarchical urban boundaries for global urban settlements). GGI is also used for analysis of tidal elevation based on global ocean altitude data [24]. Global hexagonal spatial indexing was used for ocean surface currents estimated from satellite remote sensing data [25].

In the paper, we use geospatial data analysis for navigation aid network performance estimation. A GGI is used as a primary tool for network performance estimation. Two types of GGI: Hexagonal Hierarchical Geospatial Indexing System and Open Location Codes are used.

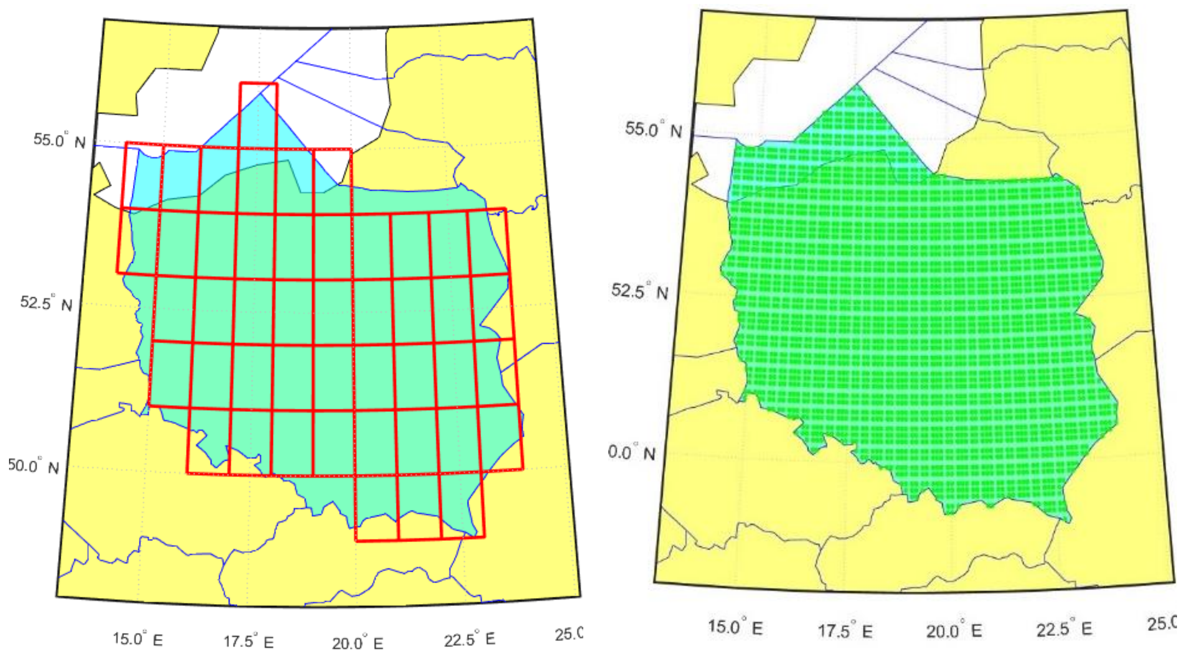
Geospatial data analysis based on GGI grid system provides an efficient computation process based on input level of precision. Also, regular cell size gives approximately equal performance for any geographic location around the globe.

2. Global geospatial indexing

GGI is a grid of particular cell shapes that partition the surface of the ellipsoidal model. Each cell has a unique index or address. This index is generated based on addressing logic applied in a particular reference system. Each standard of GGI has its unique index generation algorithm. Moreover, a hierarchal indexing system provides a reach feature for data aggregation and statistical analysis, due to holding addresses of each parent cell in the children cell index. Aggregation of data in such case could be based on simple statistical analysis of data which is grounded on prefix selection in indexes. Thus, it does not require any geospatial calculation with coordinates but requires only logical operation with indexes. In this case, it will reduce computation power and make possible practical implementations with devices with low hardware performance.

Reach flexibility is provided by the regular shape size of the cell in GGI. There are three commonly used types of cell shapes in GGI: regular triangular, rectangular, and regular hexagonal. Each of the types has advantages for particular application tasks. Triangular is suitable for relief data visualization, where the studied parameter is a monotonic function. Rectangular size is the best for addressing areas instead of specification geographic coordinates (latitude and longitude). The regular hexagonal shape provides the best for geospatial data analysis, which supports simple data aggregation and visualization based on the required precision level. Commonly used GGI which grounds on rectangular cell shape are C-squares, World Geographic Reference System (GEOREF), geohash, Open Location Code (OLC), and MapCode. There are only two GGI grounds on regular hexagonal cells Hexbin and Global Hierarchical hexagonal Spatial Index (H3).

Open Location Code (OLC) uses a rectangular cell shape with a partition the shape of WGS84 ellipsoidal model (Figure 1). Basic level partitions range of latitude into 9 equally distributed parallels and 18 cells are used in each ring [26].



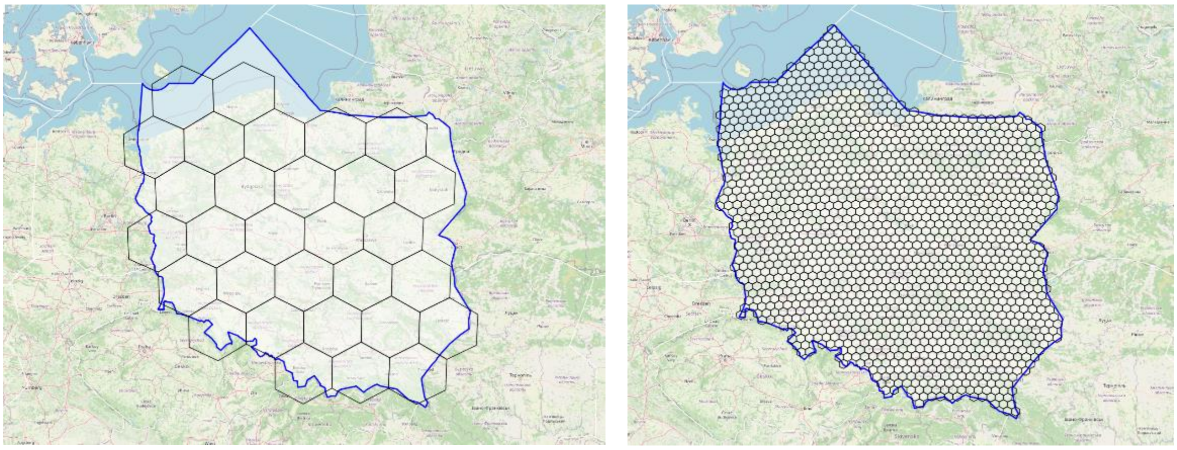
a) Second resolution level *b) Third resolution level*

Figure 1: Visualization of Airspace of Poland with OLC on different precision levels.

This approach is one of the valuable disadvantages of OLC due to changing grid area based on parallel number. In the equatorial plane, it reaches its maximum in the pre-polar parallels its minimums. Each cell has 20 children with identical logic of partition. OLC shortcodes consider only five levels of precision which start from 2213 km squared cell length and end with 13.86 m length on the fifth level of precision.

OLC uses the following simple address format AABBCCDD+EE which is referred AA to the first level of precision prefix, BB is associated with the second level, CC is the third, DD is the fourth, and EE is the highest level. There are few algorithms for cell address generation however the most useful one is grounded on coordinates transformation to system with base twenty. As an example, Figure 1 illustrates the partition of airspace of Poland with OLC of different resolution levels. The second resolution level required only 46 cells to represent airspace of Poland, however next third resolution level required 12439 cells. Unfortunately, between second and third resolution levels is not possible to get any additional sublevels. Thus, the second level is low precision and the third level requires too much computation power.

Hexagonal Hierarchical Geospatial Indexing System (H3) grounds on the hexagonal shape of the cell. H3 uses gnomonic projection applied for each side of a regular icosahedron [27]. Regular icosahedron includes twenty equilateral triangles. This model provides a small distortion and provides approximately equal cell size around the globe. Small distortion of cell size is one of the main advantages of H3 model. Each cell includes seven children cells at the next precision level. H3 provides sixteen levels of precision which support mapping of data with 1281 km edge length at the initial resolution level up to 0.58 m at the highest resolution level. Also, initial precision level includes 122 cells. The cell index is generated by a specific H3 coding algorithm with 15 hex numbers. As an example, a visualization of airspace of Poland with H3 grid of different resolution levels is shown in Figure 2. A third precision level includes only 30 cells to provide geospatial addressing of airspace of Poland. Fifth precision level partitions into 1472 cells.



a) Third precision level *b) Fifth precision level*

Figure 2: Visualization of Airspace of Poland with H3 on different precision levels.

A simple comparison of OLC and H3 shows three main advantages of H3:

- 12 resolution levels of H3 provide more flexible tune-in resolution levels than 5 resolution levels of OLC.
- Each cell of H3 includes 7 children, 6 of them are equally distanced from the center of the parental cell. OLC uses a regular partition of squared cells into 20 children. Thus, aggregation of data from lower to higher resolution levels in H3 is a mean value from equally distanced cells. In the case of OLC is a mean value from a linear grid.
- The shape of H3 cell holds its shape for any location. In OLC the shape is changed from squared to trapezial which influences data visualization, aggregation, and computational performance.

These three advantages make H3 much more useful for geospatial data processing than OLC.

3. Performance of positioning by navigation aids

There are three common types of navigation aids used in civil aviation: DME, VOR, and NDB. Ground beacons form a network for particular parameters measuring. On-board equipment of DME interrogates ground DME station to perform range measuring. VOR and NDB ground stations transmit a specific radio signal which is used by on-board part of VOR and Automatic Direction Finder for angles of relative location measurements. Measured navigation data by a network of navigation aids are used by onboard FMS to calculate the coordinates of airplane position. FMS includes algorithms of positioning by pairs of navigation aids: DME/DME, VOR/DME, VOR/VOR, and NDB/NDB. The performance of positioning depends on the ground network configuration and geometry of the airplane location [28].

DME/DME. Performance of positioning by pair of DME/DME is estimated as follows:

$$\sigma_{DME/DME} = \frac{\sqrt{2\sigma_{sis}^2 + \sigma_{DME\ Air\ A}^2 + \sigma_{DME\ Air\ B}^2}}{sin(\alpha)} \quad (1)$$

where $\sigma_{sis}$is a signal in space propagation error; $\sigma_{DME\ Air}$ is a bias introduced by on-board DME interrogator; α is an inclusion angle between directions from airplane to DMEs ground stations.

Signal in space error σ_{sis} is connected with radio wave propagation and accumulates losses introduced with signal traveling in space. Normative documents specify hard peril for a maximum value of σ_{sis}= 0.05NM.

An error of on-board interrogator is mostly caused by bias introduced during time measuring between the ending interrogation and the received reply. In common case value of $\sigma_{DME\ Air}$ is considered proportional to the range R between airplane and ground DME beacon [29]:

$$\sigma_{DME\ Air} = maximum[0.085; 0.125R] \quad (2)$$

VOR/DME. An error of positioning by a pair of co-located VOR/DME could be estimated by a simplified formula [19]:

$$\sigma_{VOR/DME}^2 = \sigma_{DME}^2 + \rho_h^2\sigma_{VOR}^2, \quad (3)$$

$$\sigma_{DME}^2 = \sigma_{sis}^2 + \sigma_{DME\ Air}^2 \quad (4)$$

where σ_{DME}^2 is an error of range measuring in DME; σ_{VOR}^2 is an error of angle measuring in VOR (given in radians); and ρ_h is a horizontal range between the airplane and VOR/DME ground station.

VOR/VOR. The performance of VOR/VOR positioning method is identically calculated to NDB/NDB. The following simplified formula could be used [19]:

$$\sigma_{VOR/VOR}^2 = \frac{D\sigma_{VOR}^2(sin^2\alpha_A + sin^2\alpha_B)}{sin^2(\alpha_A + \alpha_B)}, \quad (5)$$

where D is the base distance between VOR positions in the pair; α_A and α_B are angles measured from the airplane to VOR A and VOR B (counted clockwise from the North side).

Performance analysis requires identification of the number of available navigation aids at each point of airspace based on specific geometrical descriptions of navigation aids service volume. The number of available equipment forms a particular number of pairs. For each pair, performance is estimated for DME/DME by (1), for VOR/DME by (3), and for VOR/VOR by (5). The pair which provides the highest accuracy is chosen as efficient. The value of accuracy which could be given by an efficient pair is used as network performance level guaranteed at a particular cell. Results of the analysis are provided in the form of a heat map for each positioning method separately. The structure scheme of the proposed algorithm of data analysis is shown in Figure 3.

As input algorithm required specification of airspace boundary to identify addresses of elementary cells of the chosen geospatial indexing system. A boundary line is specified as a set of latitude and longitude. The commonly used TopoJSON format provides efficient data storage with minimization of stored data. It is grounded on the sequential process using only changes in latitude and longitude between nearest points. This data format could be easily integrated into arrays in any programming language. A boundary line is used to generate a set of addresses of cells inside of this boundary. Most GGI libraries include specific functions of efficient cell index generation for a given geographic area.

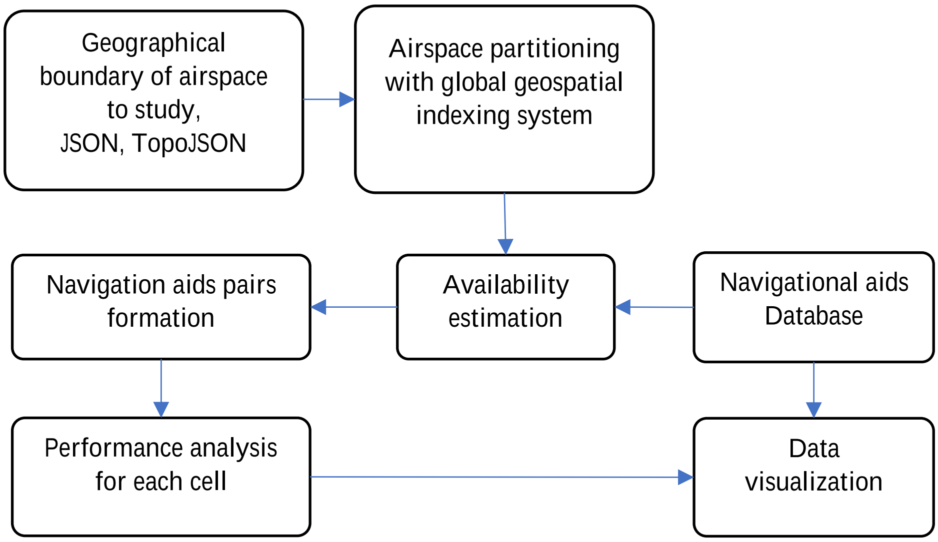


Figure 3: Structure scheme of navigational aids performance analysis.

Results visualization is different based on the library used. In the case of JavaScript front-end software development data visualization could be done with OpenStreetMap with a Leatlef, OpenLayers, or MapBox libraries. In the case of providing geospatial data analysis in Matlab computation environment a Mapping Toolbox could be useful.

4. Numerical demonstration

In a numerical study, we analyze the performance of the navigational aids network of the air navigation service provider of Poland. The ground network of navigational aids includes 42 DMEs and 23 co-located VORs [30]. Parameters of navigation aids have been used based on published official information valid on Jul 2024 [30]. Navigation aids network configuration is shown in Figure 4. We use H3 and OLC GGIs to perform performance analysis. The third resolution level of OLC is used in performance analysis in Matlab computation environment. Thus, it required 12439 cells for performance estimation (Figure 1). We use the coordinates of each cell center to obtain parameter values. Geospatial data analysis with regular hexagons has been done in JavaScript with H3 library and OpenLayer visualization library. The results of DME availability analysis with H3 are shown in Figure 5.

Results of the geospatial data analysis of the Polish navigation aids network with OLC are shown in Figures 6-8. Results of availability estimation in a number of navigation aids that could be used for parameter estimation are shown in Figure 6 for DME/DME and VOR/VOR positioning. Availability analysis shows that at each point of this airspace, at least 5 DMEs and 3 VORs are available for navigation. A number of navigation aid pairs available for coordinate measuring are shown in Figure 7. The result of the analysis for flight level 290 is that at each cell at least 5 DME/DME pairs and about 2 VOR/VOR pairs are available for airplane positioning. Results of accuracy estimation by (1) for DME/DME and (5) for VOR/VOR are given in Figure 8.

Obtained results show that both GGI could be useful in geospatial analysis of navigational aid performance distribution over the airspace. However, the identified advantages of H3 made it more useful for efficient computation power consumption than OLC. The results of the analysis indicate that the Polish network of navigation aids is well configured to guarantee the required precision level of navigation by pairs of DME/DME. The performance of positioning by VOR/VOR is poorer.

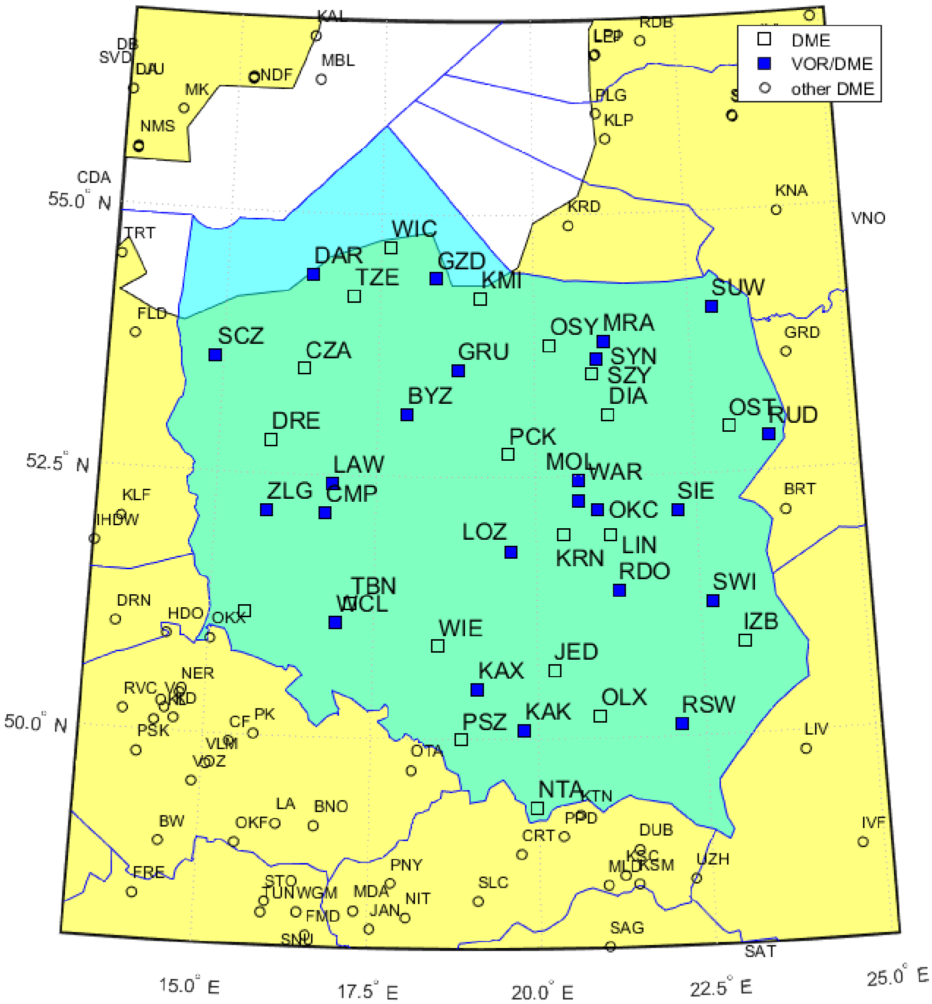


Figure 4: Configuration of Navigation aids network of Poland.

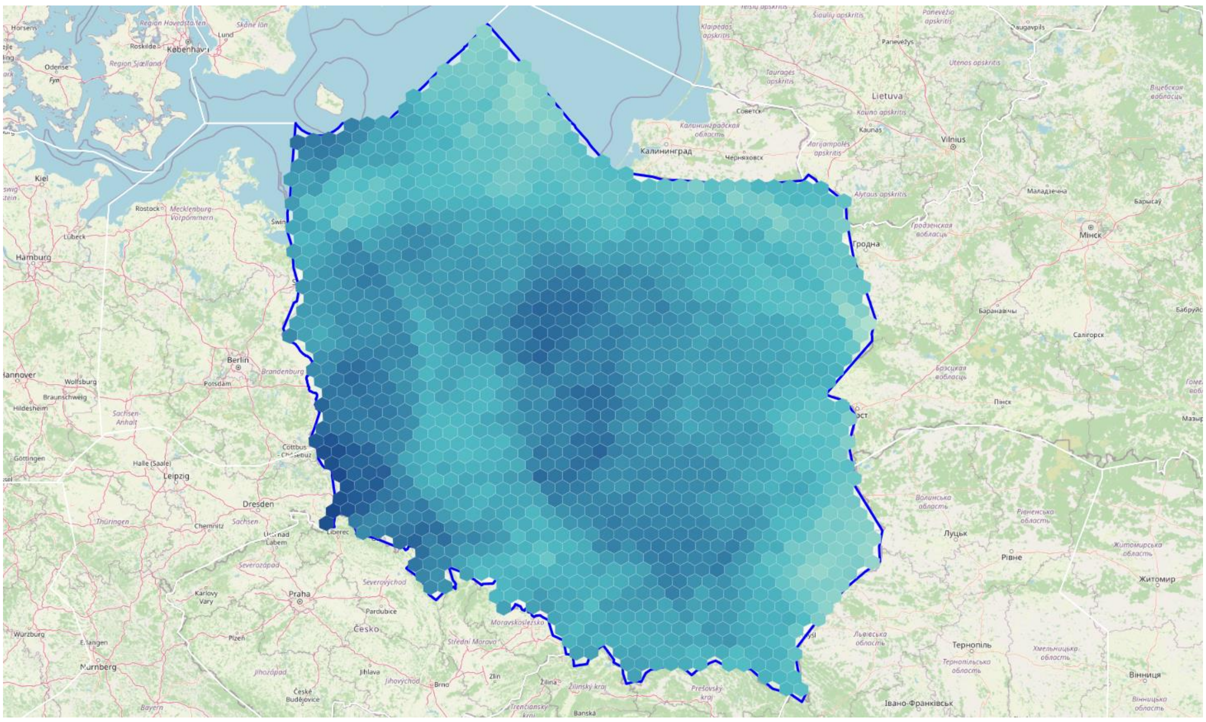


Figure 5: Results of availability of DME estimation in H3.

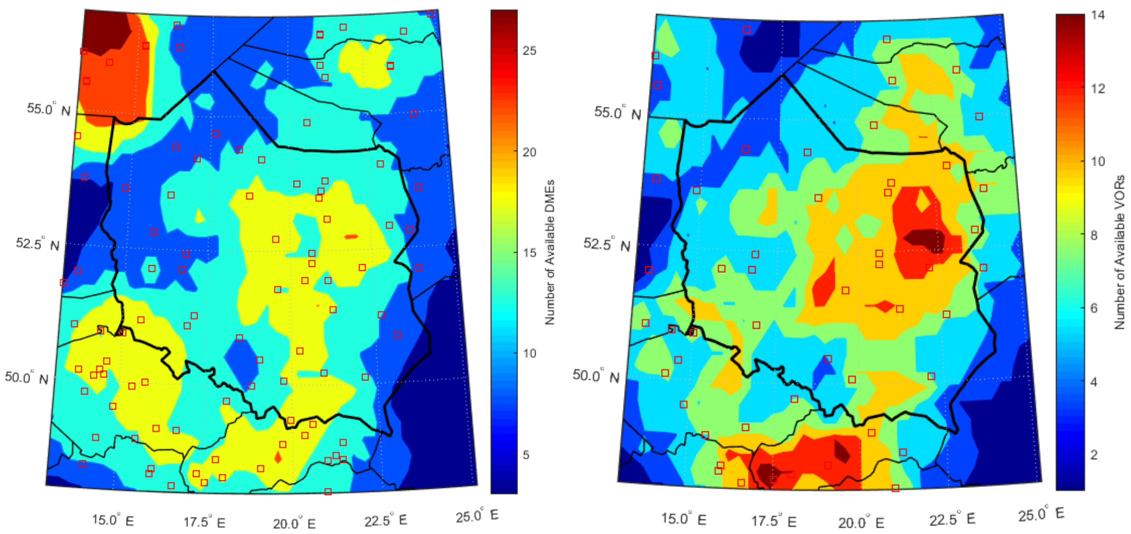


Figure 6: Number of available navigation aids.

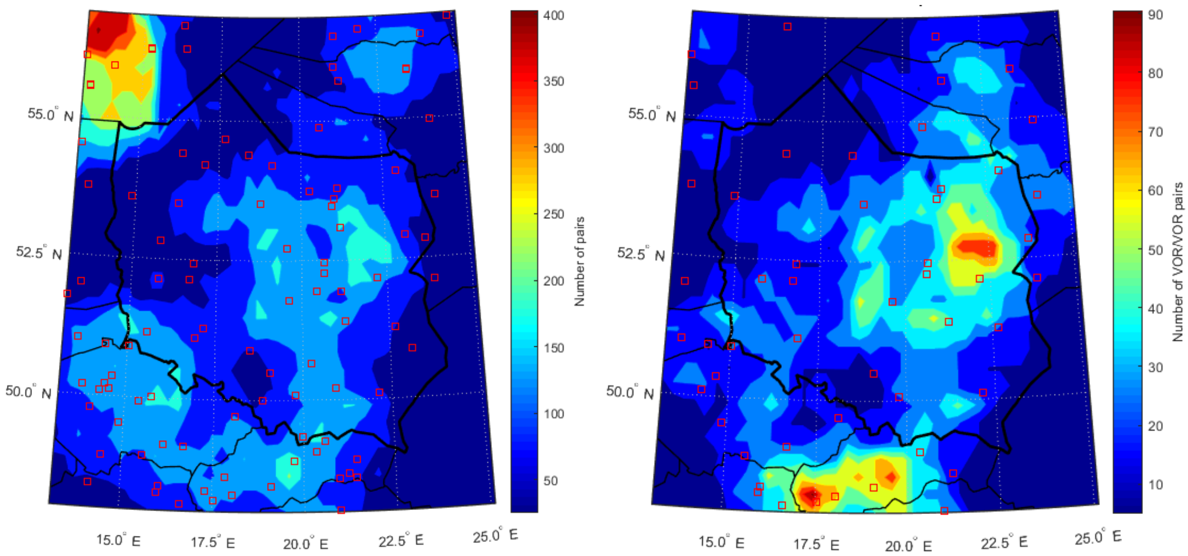


Figure 7: Number of available navigation aids pairs for DME/DME and VOR/VOR.

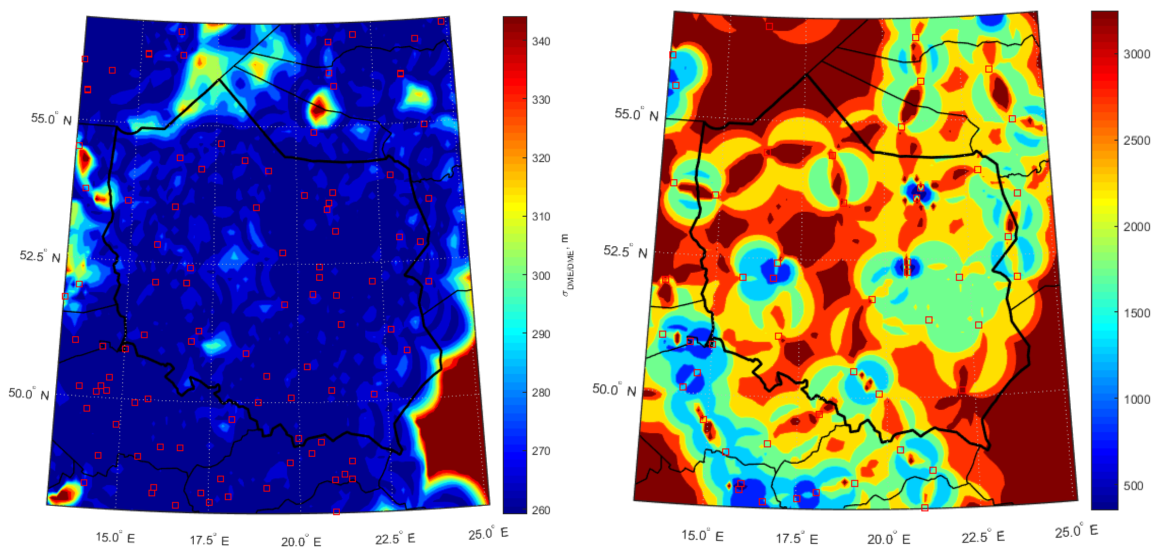


Figure 8: Accuracy of positioning service of DME/DME and VOR/VOR.

5. Conclusions

GGI provides a reach feature for geospatial data analysis. The performance of a navigation aids network depends on the geometry and is estimated based on particular parameter distribution over a geographic area. GGI helps to identify a set of cells for geospatial data analysis. Identified advantages of hexagonal cell shape H3 make this GGI highly useful for geospatial analysis of globally distributed data. The 12 resolution levels accurately tune computation precision, which minimizes required computation power. Also, the hexagonal shape of the cell provides a nearly equally distributed grid of points for geospatial data processing.

Results of geospatial data analysis of the navigation aids network of Poland indicate about well configured for positioning by a pair of DME/DME. The network provides accuracy of positioning less than 280m for most areas. VOR/VOR positioning methods show poor accuracy with multiple gaps in service. Poor accuracy is a result of low performance of angle measuring.

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