Analysis of the vibration signals based on PCRP representation

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Abstract
The characteristics of the methods and facilities of vibration diagnostics of rotating parts of mechanisms based on models of vibration signals in the form of periodically correlated random processes (PCRP) are given. Such models make it possible to detect and analyze the interaction of repeatability and stochasticity in vibration signal properties, which is characteristic of the appearance of defects. This approach significantly increases the efficiency of early detection of defects and establishment of their types. The main stages of statistical processing of vibration signals for the purpose of determining diagnostic features are described. .

Keywords
vibration diagnostics, statistical signal processing, periodically correlated random process, defect, bearing.

1. Introduction

Modern systems for monitoring the condition of complex mechanisms and systems are an important component of the process of supporting their life cycle management [1]–[4]. The assessment of the technical condition of rotating mechanisms is based on a structural analysis of the reliability of their components by means of dynamic methods of controlling changes in their vibration parameters [5]–[8]. First of during processing of the vibration signal it is divided into regular and random parts. The analysis of the regular part is grounded on original methods for development and selection of the hidden periodicities, developed by authors. As a rule, macro defects of mechanical systems are associated with the regular component of vibration signals, like imbalance, eccentricity, misalignment, beating, engagement, etc. Conclusions about the

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type of fault of the rotating element are made grounding on the analysis of the phases and amplitudes in spectrum of the regular part of signal.

The random (stochastic) part of signal as a rule contains data about the non-linear properties of rotating mechanism, which are associated, for example with inhomogeneous viscosity of lubricants, variations in friction forces, inhomogeneous of the parameters of surface roughness, etc. Vibration signal random part analysis, especially the periodic non-stationarity, makes it possible to develop faults in mechanism the early stages of their development. The random parts of vibration signal obtain its periodic non-stationarity due to modulation of the carrier harmonics with stochastic signals. Last one causes the correlation of the harmonic components of the spectrum [9]–[13] results a vibration signal as a periodically correlated random process (PCRP). This correlation is one of the most sensitive signatures of the appearance and early stage of development of defects.

2. Model of vibration signal

The model of the vibration signal $\zeta(t)$ of complex mechanisms is given in the following form

$$\zeta(t)=s(t)+\eta(t),$$

where $s(t)$ is the regular component of the vibration signal, $\eta(t)=\zeta(t)+\varepsilon(t)$ is the random component of the signal, where $\zeta(t)$ is the periodically non-stationary component, $\varepsilon(t)$ is the stationary background noise, the random processes $\varepsilon(t)$ and $\eta(t)$ are uncorrelated. The regular part $s(t)$is represented here as an almost periodical series

$$s(t)=\sum_{k=-M}^{M} c_k e^{i\omega_k t}$$

where M is a number of components, c_k is a complex amplitudes of each component, and ω_k is the cyclic frequency of component. The model of the non-stationary component$\zeta(t)$ is the PCRP, for which the harmonic representation is valid

$$\zeta(t)=\sum_{k\in Z}\zeta_k(t)e^{ik\omega_0 t}$$

where $\zeta_k(t)$ are, stochastically related stationary random processes that represent the amplitude and phase stochastic modulation of the basic harmonic components of the PCRP. Correlation and spectral characteristics of modulating processes $\zeta_k(t)$ are carrying the data about the fault types of rotating units. Features, used for diagnostic of mechanisms, are developed based on parameters of modulating processes or using the appropriate characteristics of the PCRP formed by the stationary components of $\zeta_k(t)$.

The mean function of PCRP $m(t)=E\zeta_k(t)$ and the correlation function $b(t,u)=E\zeta(t)\zeta(t+u)$, $\zeta(t)=\zeta(t)-m(t)$ are periodic functions in time

$$m(t)=m(t+T),\ b(t,u)=m(t+T,u)$$

and can be represented by their Fourier series:

$$m(t)=\sum_{k\in Z} m_k e^{ik\omega_0 t}, \qquad b(t,u)=\sum_{k\in Z} B_k(u) e^{ik\omega_0 t}$$

The instantaneous spectral density of the PCRP (Fourier transform of the correlation function) also changes periodically in time:

$$f(\omega,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} b(t,u)e^{-i\omega u}du=\sum_{k\in Z} f_k(\omega)e^{ik\omega_0 t}$$

here

$$f_k(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} B_k(u)e^{-i\omega u}du$$

The quantities $B_k(u)$ and $f_k(\omega)$, respectively, are called correlation and spectral components. The zeroth components $B_0(u)$ and $f_0(\omega)$ describe the properties of the stationary approximation of the PCRP, i.e. the averaged correlations and the time-averaged power spectral density of fluctuating oscillations.

The Fourier coefficients of the mathematical expectation function of the PCRP m_k are the mathematical expectations of the modulating processes in signal representation (1), i.e., $m_k(t)=E\zeta_k(t)$ their correlation components are determined by the auto- and cross-correlation functions and spectral components by the corresponding spectral densities of these processes

$$B_k(u)=\sum_{n\in Z} R_{n-k,n}(u)e^{¿\omega_0 u},\ f_k(u)=\sum_{n\in Z} f_{n-k,n}(\omega-n\omega_0)$$

$R_{kl}(u)=E\zeta_k(t)\zeta_l(t+u)$, $\zeta_k(t)=\zeta_k(t)-m_k$, “-”is a conjugation sign, and

$$f_{k,l}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{kl}(u)e^{-i\omega u}du$$

From relation (2) we can see that the random process (1) is a PCRP if and only if the stochastic processes which are modulating different harmonic components are correlated. For this, their spectral bands should at least in part overlap.

The general methodological scheme used to analyze the condition of bearing assemblies of rotating mechanisms presented on Fig. 1.

Original results in the field of theory and analysis of stochastic oscillations [14]–[16], in that – methods for the detection of hidden periodicities became the theoretical basis for the principles of building these systems, the justification of processing algorithms, and the creation of appropriate software. Coherent [9] and component [15] methods, least squares method [9], linear comb and bandpass filtering are used to calculate estimates of oscillation characteristics.

3. Mechanism condition indicators

Stochastic modulation is detecting and type of possible fault is identifying at the initial stage of investigation using diagnostic parameters, grounded on the first and second orders of periodic nonstationarity. To evaluate the degree of this non-stationarity, the Fourier coefficients of the function of mathematical expectation m_k and $B_k(u)$correlation components were used and the following two diagnostic parameters were considered:

$$I_1 = \frac{\frac{1}{2}\sum_{k=1}^{N_1} \left| m_k \right|^2}{\widehat{B}_0(0)} \quad I_2 = \frac{\frac{1}{2}\sum_{k=1}^{N_2} \left| B_k(0) \right|^{\square}}{\widehat{B}_0(0)}$$

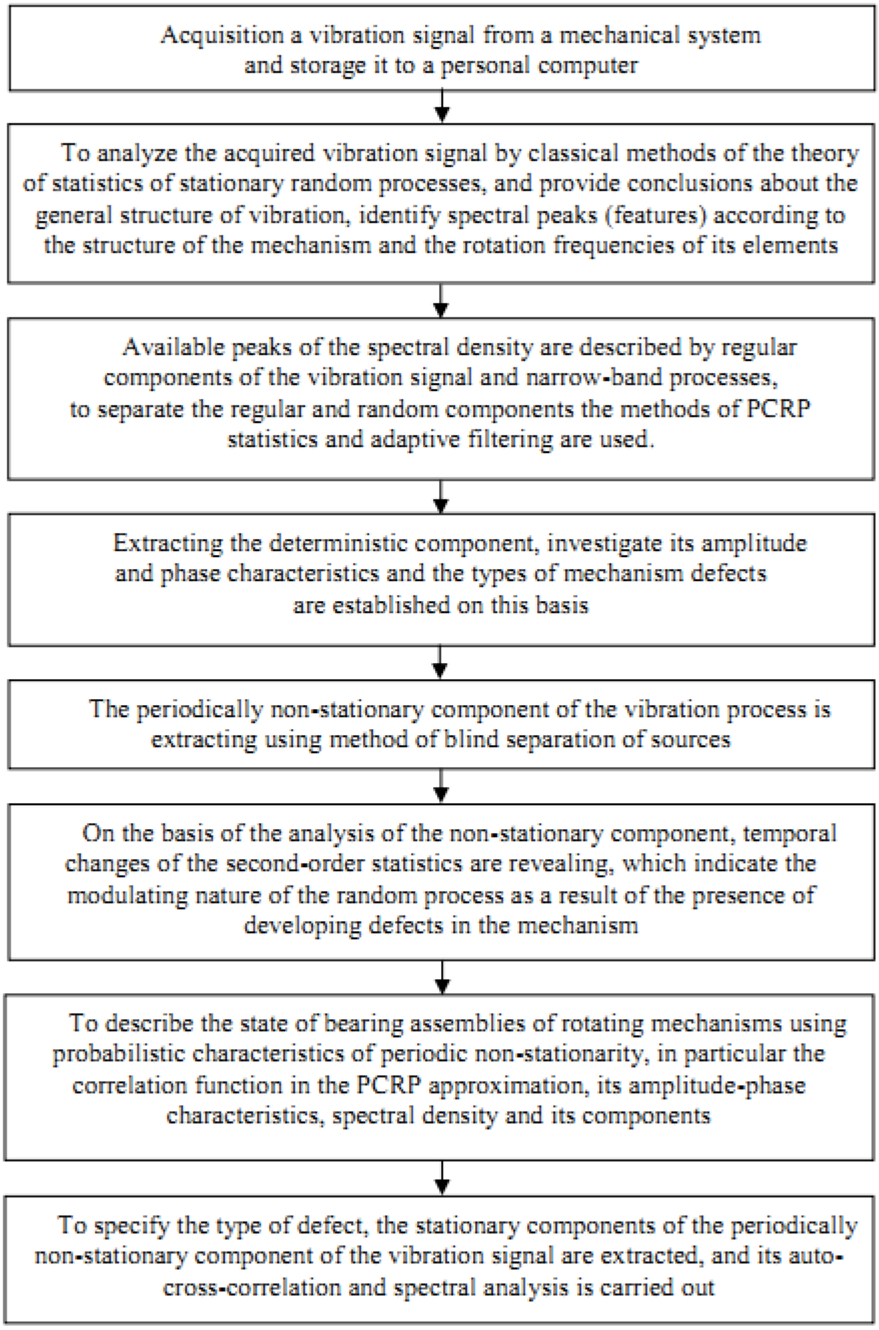

Fig. 1. General methodological scheme to analyze the vibration signals as PCRP

The first value determines the ratio of regular changes in the vibration signal power to the whole fluctuations power, averaged over signal realization. I_2 is a value of the power of fluctuating part of fluctuations divided by the power of fluctuation, averaged over signal realization. The introduced diagnostic parameters have the properties of a measure of the periodic non-stationarity; they grow monotonically with an increase in the power of regular and fluctuating vibrations of the vibration signal. In the case of a stationary centered random signal, when $m_k=0$ and $B_k(u)=0$ for all $k\neq 0$, the parameters I_1 and I_2 are equal to zero. It is obvious that the faults of the mechanisms also interfere on the nature of the decay of the correlations of the modulating stochastic processes. A third parameter is proposed to represent this effect

$$I_3^{(k)}=\frac{\int_{-\infty}^{\infty}\left|B_k(u)\right|du}{\int_{-\infty}^{\infty}\left|B_0(u)\right|du},\ k=\overline{1,N_2},$$

which is called a measure of periodic correlation. For the stationary case, we also have $I_3^{(k)}=0$.

The similar properties of vibration signals, but already in the frequency domain, are described by the spectral coherence function

$$C_k(\omega)=\frac{\left|f_k(\omega)\right|}{f_0(\omega)},\ k=\overline{1,N_2}.$$

Used here normalization of spectral components makes it possible to emphasize the relation between weak components from minor defects on the background of components that are not relevant to the identification of the defect, but have a much more power.

It would be appropriate to use also the spectral coherence function, which is defined in the spectral band $\left[\omega_1,\omega_2\right]$:

$$I_4^{(k)}=\int_{\omega_1}^{\omega_2}C_k(\omega)d\omega .$$

For the faults classification are effective features, obtained grounding on the correlation and spectral parameters of the stationary part of the PCRP-model of vibrations, in that function of the normalized cross-correlations:

$$r_{kl}(u)=\frac{R_{ll}(u)}{R_{nk}(0)R_{ll}(0)}$$

and coherence functions

$$\gamma_{kl}^2(\omega)=\frac{\left|f_{kl}(\omega)\right|^2}{f_{kk}(\omega)f_{ll}(\omega)}.$$

4. Methods of hidden periodicity estimation

The coherent method consists in averaging the signal readings taken over a period T :

$$\hat{m}(t)=\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nT)$$

$$\hat{b}(t,u)=\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nT)\xi(t+u+nT)-\hat{m}(t)\hat{m}(t+u+nT)$$

Component estimates have a form of trigonometric polynomials

$$\hat{m}(t)=\sum_{k=-N_1}^{N_1}\hat{m}_k e^{ik\frac{2\pi}{T}t},$$

$$\hat{b}(t,u)=\sum_{k=-N_2}^{N_2}\hat{B}_k(u)e^{ik\omega_0 t}$$

where N_i *i=1,2*, are the numbers of the highest harmonics. Coefficients of polynomials $\hat{m}_k$ and $\hat{B}_k(u)$ are determined on the basis of following statistics

$$\hat{m}_k=\frac{1}{\theta}\int_0^\theta \xi(t)e^{-ik\frac{2\pi}{T}t}dt$$

$$\hat{B}_k(u)=\frac{1}{\theta}\int_0^\theta\left[\xi(t)-\hat{m}(t)\right]\left[\xi(t+u)-\hat{m}(t+u)\right]e^{-ik\frac{2\pi}{T}t}dt,$$

here θ is the duration in time of the segment of vibration signal. Estimates of components are formulated grounding on a priori data of harmonic components number, obtained from the Fourier series for each characteristic that is calculated. They are more effective than coherent ones, especially if correlations decay rapidly with time lag increasing.

Least squares estimates [9] are found by minimizing the following functionals:

$$F_1\left[\hat{m}_0,\hat{m}_1^c,...,\hat{m}_{N_1}^c,\hat{m}_1^s,...,\hat{m}_{N_1}^s\right]=\int_0^\theta\left[\xi(t)-\hat{m}(t)\right]^2 dt,$$

$$F\left[\hat{B}_0(u),\hat{B}_1^c(u),...,\hat{B}_{N_2}^c(u),\hat{B}_1^s(u),...,\hat{B}_{N_2}^s(u)\right]=\int_0^\theta\left[\mathring{\xi}(t)\mathring{\xi}(t+u)-\hat{b}(t,u)\right]^2 dt.$$

Advantage of such estimates is the absence of seepage effects for all values of θ. The BlackmanTukey correlogram method was used to construct statistics of spectral characteristics. To do this, the cutting point of the correlogram is necessary set to u_m and the smoothing window *k(u)* was used.

Estimates of the instantaneous spectral density $f(\omega,t)$ as well as spectrum components $f_k(\omega)$ were calculated using the equations:

$$\hat{f}(\omega,t)=\frac{1}{2\pi}\int_{-u_m}^{u_m}\hat{b}(t,u)k(u)e^{-i\omega u}du$$

$$\hat{f}_k(\omega)=\frac{1}{2\pi}\int_{-u_m}^{u_m}\hat{B}_k(u)k(u)e^{-i\omega u}du$$

where $k(-u)=k(u), k(o)=1, k(u)\equiv 0$ at $|u|\geq u_m$. The selection of real signal processing parameters is carried out grounding on the statistical parameters of estimates (3)–(12) and appropriate quality criteria obtained analytically [9, 16].

Presented here methods of spectral- and correlation analysis of PCRP needs previously defined value T of the correlation period. Mainly, for rotating mechanical system the period of excitation of can be obtained grounding on its kinematic diagram, because of rotation frequency of the driving motor shaft is known. However, the values calculated in this way, have insufficient accuracy and have variations in real situations. Therefore, the value of the shaft drive period (frequency) should be found by means of processing of acquired vibration signal. The determination of the shaft drive rotation period grounded on the PCRP model of the structure of stochastic fluctuations. For this purpose, methods of hidden periodicities detecting can be considered. Since the hidden periodicities properties are not always developed as the peak values at the spectrum of vibration signal, calculated with assumption of its stationarity, some other methods, grounded on PCRP signal model have been developed to estimate the period. They are based on the detection of periodic temporal changes in probabilistic characteristics [9, 17–20]. For this, functionals were used, which have the form of estimates (3)–(9) with the difference that instead of the true value of the period$T$, some trial value $\hat{}$ was used in them. Estimates of the period T are then found at the extremum values points of these functionals. So, the component estimates of the period are based on the extreme values of the functionals

$$\hat{m}_k^c(\tau)=\frac{1}{\theta}\int_{-\theta}^{\theta}\xi(t)\cos k\frac{2\pi}{\tau}tdt$$

$$\hat{m}_k^s(\tau)=\frac{1}{\theta}\int_{-\theta}^{\theta}\xi(t)\sin k\frac{2\pi}{\tau}tdt$$

$$\hat{B}_k^c(t,u)=\frac{1}{\theta}\int_{-\theta}^{\theta}\mathring{\xi}(t)\mathring{\xi}(t+u)\cos k\frac{2\pi}{\tau}tdt$$

$$\hat{B}_k^s(t,u)=\frac{1}{\theta}\int_{-\theta}^{\theta}\mathring{\xi}(t)\mathring{\xi}(t+u)\sin k\frac{2\pi}{\tau}tdt$$

Estimates of the period determined in this way have great accuracy: the value of their bias is of the order of $O(N^{-2})$, and the variance is of the order of $O(N^{-3})$

Two methods have been developed for the extraction of modulating stationary components of signals. The first of them consists in the frequency shift of the signal by an amount $-k\,\omega_0$ and subsequent low-frequency

$$\xi_k(t)=\int_{-\infty}^{\infty}h(t-\tau)\xi(\tau)e^{-ik\omega_0\tau}d\tau$$

where $h(\tau)$ – impulse response of a low-pass filter

$$h(\tau)=\frac{\sin\left(\frac{\omega_0\tau}{2}\right)}{\pi\tau}.$$

The second method based on the band-pass filtering to extract the components whose spectra are concentrated in the ranges $\left[k\,\omega_0-\frac{\omega_0}{2},k\,\omega_0+\frac{\omega_0}{2}\right]$, and then their envelopes are founding using the Hilbert transformation. Such signal transformations make it possible to provide an analysis of the probabilistic characteristics of modulations of the carrier harmonics of the PCRP, as well as investigate cross-spectral and cross-correlation parameters of vibration signal components.

5. Conclusions

The use of PCRP methods opens qualitatively new opportunities for statistical analysis of vibration signals of bearing assemblies of rotating mechanisms. Methods for identifying the regular component in the vibration signal allow a detailed analysis of phase changes of processes in rotating mechanisms. The developed adaptive methods for evaluating of signal parameters minimize man-made influence on the processing process, which allows their use in automated diagnostic systems. The methods of extracting the periodically non-stationary component make it possible to extract that characteristic of the signal, corresponding to the responses of defects in the mechanical system, minimizing the impact of noise. The developed methods and means make it possible to analyze the condition of the bearing units of the rotation mechanisms, to identify and classify their defects in the early stages of their development.

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