Cognitive context and syllogisms from ontologies for handling discrepancies in learning resources

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Abstract. The deployment of learning resources on the web by different experts has resulted in the accessibility of multiple viewpoints about the same topics. In this work we assume that learning resources are underpinned by ontologies. Different formalizations of domains may result from different contexts, different interpretation of terminology, different vocabularies to define concepts, incomplete knowledge and conflicting knowledge of the experts deriving the ontologies. We define the notion of *cognitive learning context* that refers to multiple and possibly inconsistent ontologies about a single topic. We then discuss how this notion relates to the cognitive states of *ambiguity* and *inconsistency*. Discrepancies in viewpoints can be identified via the inference of conflicting arguments from consistent subsets of statements. Two types of arguments are discussed, namely arguments inferred directly from taxonomic relations between concepts and arguments about the necessary and jointly sufficient features that define concepts.

1 Introduction

Learning resources are becoming increasingly available to the learners on the web. As a result a learner may have access to multiple learning resources about the same topic. We assume that each learning resource is underpinned by an ontology. Ontologies of the same domain may be represented at various degrees of abstraction and granularity. Reasons can be traced to different points of view and experience of the experts deriving the ontologies. It can also be due to different degrees of completeness of ontologies. The learner may not be able to determine whether discrepancies in ontologies arise due to incompleteness of knowledge, due to disagreement between ontologies, or due to differences in the perspectives giving rise to different viewpoints.

This paper's purpose is twofold. Firstly, to formalize the cognitive state of ambiguity and inconsistency arising when a learner encounters incomplete ontologies of learning resources about a topic. In order to address the problem of cognitive ambiguity and confusion of learners we allow resources with conflicting or different information to be part of the same cognitive context. We assume that the context is related to the goal of the learning activity (referred to as the *focus* of the learning activity) rather than on the compatibility of the resources referred to by the context. As a consequence, the context may involve multiple domains, if multiple domain points of view are relevant to the learning topic. For example, the topic may involve the points of view of multiple domains like psychology, social science and anthropology in order to form a particular position.

Secondly, we propose a proof-theoretic approach to the automatic derivation of arguments from ontologies. Out of the resources available to the learner, she only needs to consider those subsets of statements that are relevant to her reasoning. We use the notion of *sub-ontology* to describe consistent subsets of ontologies that can be used to form the cognitive context of the learner. We also suggest that differences in ontologies can be identified via the use of argumentation and we formalize two different types of arguments that are useful in learning. These are syllogistic arguments following from hierarchical relations in ontologies and arguments about necessary and jointly sufficient features of concepts. The rest of this paper is outlined as follows. Section 2 reviews related work on the definition of context based on the locality assumption and paraconsistent logics modeling inferences from inconsistent theories. In section 3 we discuss the notions of cognitive learning context, cognitive ambiguity and inconsistency arising from the resources taking part in a learning activity. Section 4 discusses our approach to defining syllogistic arguments and arguments from necessary and jointly sufficient features for the definition of concepts. Section 5 summarizes the main issues discussed and briefly outlines current research.

2 Related Work

The Local Model Semantics [6] provide a foundation for reasoning with contexts which is based on two main principles: the principle of locality and the principle of compatibility. The first states that reasoning requires only a part of what is potentially available [6]. The principle of compatibility states that there is compatibility among the kinds of reasoning performed in different contexts [6], thus assumes a relatedness between different contexts by some meaningful relation of subsets of local models. In this paper we focus on the cognitive context of a learner in a learning situation. The principle of locality, discrepancies in the ontologies of learning resources and the assumption that the available information may be incomplete affect the way learners interpret the information and can be used to model the cognitive state of the learner.

The assumption of possible inconsistency in theories or knowledge bases has been addressed via a number of logics. For example paraconsistent, many-valued logics and modal logics are among the ones used widely to model inconsistency. Notable uses of paraconsistent and possible world semantics to model mental models and epistemic states can be traced to the works of [5] and [9]. Fagin and Halpern [5] consider each agent as a society of minds rather than a single mind. Lokhorst [8], inspired by the local reasoning models of Fagin and Halpern [5], developed a two-valued split-patients local reasoning model as a structure: $M = \langle W, w_0, \Psi, S, R, V \rangle$, where W is a set of possible worlds, w_0 is the actual world in W, Ψ is a set

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of "minds", S is a function from W into the set of non-empty subsets of Ψ (i.e. S maps a world to the set of minds in which this world is possible) and R is a function from Ψ into $W \times W$. If we consider each mind in Lockhorst's model as a different ontology obtained independently of each other we might argue that the above model is suitable to be allied as a local reasoning model of a learner. However, this model would imply that the learner is unable to associate information from different resources.

The paraconsistent logic LEI is based on the idea of multiple observers having diverging views about a certain state of affairs. It extends classical logic with the formula $p?$ where $p?$ is satisfied whenever p holds in all plausible worlds. Unlike the traditional modal logics approach to modeling necessity and possibility, the LEI employs two satisfaction relations: the credulous and the skeptical approach. Martins et al. [9] provided a multiple world semantics to the above idea where each plausible world corresponds to a particular view of the world. The above approach is useful in comparing beliefs derived by the credulous vs. skeptical entailment relation which is different from the focus of this paper.

Unlike the above model, we assume that the learner is able to compare information obtained from different ontologies for its relevance, its validity, and for drawing inferences. Where the relevance between concepts used in different ontologies cannot be established the learner is bound to feel confused. We therefore need to combine two levels of reasoning: a local reasoning level which considers each ontology locally and the meta-epistemic level, at which the agent compares inferences drawn locally in each ontology and determines compatibility with other ontologies.

Our work is influenced by significant relevant work in the area of representing and combining information from different ontologies using context formalisms, e.g. [2, 7, 3]. In this paragraph we briefly discuss how our work relates to the use of ontologies to represent context. Bouquet et al. [2] introduced the notion of *contextual ontology* where the contents of an ontology are kept local and explicit mappings are used to associate the contents (e.g. concepts, roles, etc) of one ontology to the content of another. He addressed the fact that each ontology may represent its own local domain rather than a unique shared domain and provided the semantics of *local domains*. In line with this approach, we also assume that each ontology (representing a learning resource) used in the learning activity has its own local domain and interpretation function. Although correspondences between ontologies may be represented via bridge rules [6] or simple default rules where applicable, these may not always be known to the learner. The above assumptions are important in the construction of derivations whenever assumptions from one ontology can be combined to make inferences in another ontology.

The notion of *context space* addressed in [2] is similar to the idea upon which the *cognitive learning context* of a learner is based in this paper, namely that a context consists of a set of resources. However, the focus of our work is on the representation of the notions of cognitive ambiguity and inconsistency rather than on modeling the mappings of concepts between different ontologies. Of particular importance to us then are the relevance of the ontologies being used in the learning task and the plausible epistemic alternatives in case of information incompleteness.

3 Cognitive Learning Context, Ambiguity and Inconsistency

In this paper we represent each epistemology via its underlying ontology. In this project we use $OWL-DL$ as an ontology representation language because it is a decidable fragment of description logic and expressive enough to satisfy our need for the representation of concepts, roles and hierarchies that give rise to the type of arguments formalized in this work.

3.1 Ontology

An Ontology in this paper is described as a structure $\langle T, A \rangle$ where T denotes a DL TBox (i.e. a set of terminological) axioms and A denotes a DL ABox (i.e. a set of grounded assertions). Each ontology has its own signature consisting of a disjoint set of relation names, concept names and constant names of individuals. We denote the signature of an OWL ontology O_i by $Sig(O_i) \equiv R \cup C \cup N$, where R denotes the relation names, C the concept names and N the set of individual names. The interpretation I_i of $Sig(O_i)$ is the structure $\langle D_i, \cdot^{I_i} \rangle$ where D_i is the domain of the ontology and $\cdot^{I_i}$ is the interpretation function such that: $C^{I_i} \subseteq D_i$, $R^{I_i} \subseteq {D_i}^n$ (in OWL is $D_i \times D_i$). Assume a set of resources $R_1, \dots, R_n$ underpinned by a set of ontologies $E_1, \dots, E_n$, respectively. Then we define the local learning context of a learner as follows:

3.2 Cognitive Learning Context

The local reasoning learning context of a learner L is defined as a structure

$$\Upsilon \equiv \langle E, W, \delta, \eta, sit, \sigma, I^* \rangle$$

where
$E = \{O_1, \dots O_n\}$ is the set of possibly inconsistent ontologies referred to by the learner, where each O_i has its own vocabulary. W is a non-empty set of epistemic alternatives (possible worlds). A subset of terminological axioms and assertions selected from an ontology O_i is referred to as *sub-ontology* and is denoted by $Sub(O_i)$. The function δ associates each sub-ontology (set of statements selected from an ontology), $Sub(O_i)$, to a set of *compatible epistemic alternatives* in W. The phrase *compatible epistemic alternatives* refers to the possible epistemic states such that if the associated resource contains p or infers p, each of the epistemic alternatives also infers or entails p. We assume that the *focus,* η, of the learning activity is either is a proposition. Also sit denotes the actual situation the learner is in during the learning activity. The relevance function σ accesses subsets of ontologies that are relevant to the focus of the epistemic activity. Therefore σ maps an ontology and a proposition denoting the focus of the learning activity to a set of propositions of the ontology (sub-ontology) relevant to the focus. Assume that Φ denotes the set of all possible propositions and SO_i is the set of sub-ontologies that can be created out of each ontology. Then $\sigma : O_i \times \Phi \rightarrow SO_i$. The axiomatization of relevance is currently under development and not provided in this paper. I^* is an interpretation function on the joint vocabulary

$$V = \bigcup_{i=1}^{n} Sig(O_i)$$

such that:

1. For each axiom $\alpha_i \in T_i$, $\alpha^{I^*} = \alpha^{I_i}$
2. For each axiom $\alpha_j \notin T_i$ and $\alpha_j \in T_j$ there exist $Sub(O_i)$-compatible subsets of epistemological alternatives $W_1 \subseteq W$ and $W_2 \subseteq W$ such that $W_1 \models \alpha_j$ and $W_2 \not\models \alpha_j$. Note that $W \models \phi$ if and only if for each world $w \in W$ we have $w \models \phi$ for any formula ϕ.

Using the above definition of the cognitive state of a learner, we are now able to discuss the cognitive states of ambiguity, ignorance and inconsistency.

3.3 Cognitive Ambiguity

Intuitively, a learner reaches a cognitive state of ambiguity whenever she has access to more than one plausible epistemic alternatives and the learner is unable to choose one. The Oxford English Dictionary defines ambiguity as: *wavering of opinion, hesitation, doubt, uncertainty, as to one's course, or, capable of being understood in two or more ways, or, doubtful, questionable, indistinct, obscure, not clearly defined and lastly, admitting more than one interpretation or explanation; of double meaning or several possible meanings* (in [4]). The notion of ambiguity in our case refers to the interpretation of incompleteness of information contained in learning resources by the learner. We assume that a learner becomes aware of the incompleteness of a learning resource when she compares it with her background knowledge or with another resource. The set of resources relevant to the subject of the learning activity may change in each situation according to the focus of the learning activity. Assume a unified signature Σ which consists of the union of all the signatures $Sig(O_i^{'})$ (defined as above). To simplify matters, we assume that any two identical non-logical symbols of two resources R_1 and R_2 are considered the same unless there is evidence to the contrary. The following defaults enable us to draw inferences based on default correspondences between identical symbols across ontologies.

$$\frac{[R_1 : C(x)] : [R_2 : C(x)] \leftrightarrow [R_1 : C(x)]}{[R_2 : C(x)]} \tag{1}$$

Default rule 1 states that if there is no inference inconsistent to $[R_2 : C(x)] \leftrightarrow [R_1 : C(x)]$ in R_2 then $R_2 : C(x)$ can be asserted in R_2. A similar default inference rule is used for relations between concepts and names of individuals.

$$\frac{[R_1 : R(x, y)] : [R_2 : R(x, y)] \leftrightarrow [R_1 : R(x, y)]}{[R_2 : R(x, y)]} \tag{2}$$

The biconditional used in the inference rules aims to maintain consistency with mappings of terms between different vocabularies. For example, when two people (P_1 and P_2 say) are viewing a scene from opposite sites then $P_1 : right \leftrightarrow P_2 : left$. Further assume that $P_i : right \rightarrow \neg P_i : left$ holds for each person. Then obviously, it is inconsistent to assume that $P_1 : right \leftrightarrow P_2 : right$. Note that the intended meaning of the notions of $P_i : right$ and $P_i : left$ for each $i \in \{1, 2\}$ is independent of the situation of P_i. However the actual assignment of terms is dependent on their situation.

Let us consider $O_1^{'} = Sub(O_1)$ and $O_2^{'} = Sub(O_2)$ of two different ontologies O_1 and O_2 as above. We use the notation $[O_i^{'} \setminus O_j^{'}]_T$ to denote all the terminological axioms of $O_i^{'}$ that are not included in $O_j^{'}$. For example, assume $O_1^{'} = \{ENC \sqsubseteq OOL, VB \sqsubseteq ENC\}$ and $O_2^{'} = \{INH \sqsubseteq OOL, VB \sqsubseteq INH\}$. Then, $[O_1^{'} \setminus O_2^{'}]_T = \{ENC \sqsubseteq OOL\}$.

Now assume that O_2 does not include any axiom associating the concepts of ENC and INH. If there was an association (e.g. $Disjoint(ENC, INH)$) then $\{ENC \sqsubseteq OOL\}$ might not be a possibility at all. However, since there is no information associating the two concepts in O_2, then $O_2^{'}$ is compatible with two sets of epistemic alternatives: the first set is the one in which the axiom holds and the second in which it doesn't. Using this approach we define cognitive ambiguity as the situation in which the learner can see possible epistemic alternatives of a resource which are compatible with the resource but inconsistent with each other.

3.4 Cognitive Inconsistency (Confusion)

Intuitively, we assume that Cognitive inconsistency arises when in the actual world of the learner, information about a topic is conflicting. This is evidenced by conflicting information from different resources. It is different from cognitive ambiguity in that cognitive ambiguity appears as a consequence of possible epistemic alternatives due to lack of knowledge. The cognitive state of inconsistency can be explained via the existence of conflicting arguments from different learning resources. The cognitive state of ambiguity arises from the *possibility* of inconsistency between incomplete resources due to absence of information to the contrary.

In the next section we argue that the method of argumentation can be used to determine inconsistencies between conflicts or ambiguities between ontologies. Inconsistencies are determined via the derivation of refuting arguments from different resources related to the *focus* of the learning activity.

4 Syllogistic Arguments and Ontological Taxonomic Relations.

The process of argumentation is important during interaction with a learner in order to determine discrepancies in conceptualizations of the learner and the tutor or the learner and the learning resources related to the focus of the learning activity. It is also important for the recognition of differences or inconsistencies in ontologies automatically. In the next section we discuss the formalization of two types of arguments that can be inferred from ontologies, namely syllogisms and arguments about necessary and jointly sufficient features associated to the definition of concepts.

An Ontology may include one or more hierarchies of concepts that can be used to infer categorical statements.

4.1 Concept hierarchy

A *concept hierarchy* is a structure $\mathcal{H} = \langle C_{\mathcal{H}}, R_{\mathcal{H}} \rangle$ where $C_{\mathcal{H}}$ is a set of concepts, st. $C_{\mathcal{H}} \subseteq C$ of the ontology O, and $R_{\mathcal{H}} = \{Disjoint, SubclassOf, Intersects, ComplementOf\}$ and every concept in $C_{\mathcal{H}}$ is associated with another concept via a relation in $R_{\mathcal{H}}$. OWL-DL provides all of relations in $R_{\mathcal{H}}$ and therefore a hierarchy can be represented in it. We are interested in those interpretations of a hierarchy that satisfy all the taxonomic relations within the hierarchy. A model, $\mathcal{M}_{\mathcal{H}}$ of $\mathcal{H}$ is an interpretation I of $\mathcal{H}$ where all the taxonomic relations in $R_{\mathcal{H}}$ are satisfied. The semantics of ontological primitives used in a taxonomic hierarchy are as follows: If $C_1, C_2 \in C_{\mathcal{H}}$ then $subclassOf(C_1, C_2)$ if and only if $C_1^I \subseteq C_2^I$, $Disjoint(C_1, C_2)$ if and only if $C_1^I \cap C_2^I = \emptyset$, $Intersects(C_1, C_2)$ if and only if $C_1^I \cap C_2^I \neq \emptyset$, and $ComplementOf(C_1) = \mathcal{U} \setminus C_1^I$. Obviously, $\mathcal{M}_{\mathcal{H}}$ is a sub-model of $\mathcal{M}$ and therefore any entailment of $\mathcal{M}_{\mathcal{H}}$ is an entailment of $\mathcal{M}$.

The above set-theoretic semantics of taxonomic primitives are used to represent syllogisms and arguments from necessary and jointly sufficient properties for the representation of concepts. Bennett[1] showed (see 4.4) that set-equations can be translated to equivalent propositional formulae subject to certain constraints. Consequently syllogisms can be tested for their validity against a propositional theorem prover.

4.2 Categorical statements

Generalized statements of the form: *Every X is a Y* or *Every X has the property of Y* can be inferred from taxonomic hierarchies and can be combined to form *syllogistic arguments*. These statements are referred to as *categorical statements*. A syllogism [11] is a particular type of argument that has two premises and a single conclusion and all statements in it are categorical propositions.

4.2.1 Individuals

In ontologies, a distinction is made between individuals and classes. In the consequent we argue that the set equations that can be used to represent ontological primitives can be translated to propositional logic formulae that can be used to test validity of arguments. To simplify computation and to prove whether an individual belongs to a class (or a refutation that an individual belongs to a class) we represent individuals as singular sets consisting of that individual only. In this way we treat individuals as classes during inference. An ontology may include one or more hierarchies of concepts that can be used to infer syllogisms.

4.2.2 Syllogisms

Syllogisms form a particular type of arguments that are constructed from generalized statements (categorical statements). There are four basic categorical statements which can be combined to produce 64 patterns of Syllogistic Arguments. These are shown below together with the corresponding ontological primitives:

Categorical Statement	**Ontological Primitive**
Every S is a P	SubclassOf(S, P)
No S is a P	SubclassOf(S, ComplementOf(P))
Some S is a P	Intersects(S, P)
Some S is not P	Intersects(S, ComplementOf(P))

However, only 27 of them are valid syllogisms. This suggests the need to check the validity of syllogisms constructed from ontologies and exchanged during interaction with the learner.

4.3 Necessary and Sufficiency Conditions Arguments.

The classical view of the representation of concepts states that the features representing a concept are *singly necessary* and *jointly sufficient* to define a concept. In line with the above view we propose the following definitions for the *necessary* and *jointly sufficient* features representing a concept.

4.3.1 Necessary Features for the Representation of a Concept

Intuitively, a feature ϕ is *singly necessary* for the definition of C if and only if existence of C implies existence of ϕ. Assume a feature ϕ. We define a set Φ consisting of all individuals of the domain which have property ϕ (e.g. via the onProperty restriction in OWL-DL). Then, ϕ is a necessary property for the representation of concept C if and only if $C^I \subseteq \Phi$. An example of a refutal to the assumption that ϕ is a necessary feature for C is the derivation of an individual that belongs to C and to a class disjoint with Φ.

4.3.2 Jointly Sufficient Features for the Representation of a Concept

Let $\{\Phi_1, \dots, \Phi_n\}$ represent the set of concepts corresponding to features $\phi_1, ..., \phi_n$ respectively. Then $\phi_1, ..., \phi_n$ are jointly sufficient for the representation of concept C if and only if $\{\Phi_1 \cap, \dots, \cap \Phi_n\} \subseteq C^I$. An example of a refutal (i.e. an attacking argument) to the above assumption would be the existence of an individual that has these properties but does not belong to C. Conflicting arguments about these notions can be used to differentiate concept definitions between different ontologies.

4.4 Bennett's theory

Bennett [1] proved that set equations can be translated to equivalent universal equations which can in turn be converted to propositional logic formulae and can be tested for their validity with a Gentzen theorem prover. The theorem expressing the correspondence between set equations to universal equations is called *classical entailment correspondence theorem.* Although his theory was intended primarily for reasoning with mereological relations it is applicable in our case for reasoning with the type of arguments described above. This is because the mereological relations being represented using this theory closely resemble the set-theoretic semantics attributed to the ontological primitives describing associations between concepts in ontologies. Based on Bennett's *classical entailment correspondence theorem* we were able via a small adaptation to derive a *taxonomic entailment correspondence theorem* which is very similar to the theorem described above but concerns hierarchical relations. This is stated as follows:

$$M_H \models \phi \, if \; and \; only \; if \; M_{C+} \models \tau = \mathcal{U} \tag{3}$$

where $\mathcal{U}$ is the universe of discourse. The *Taxonomic entailment correspondence theorem* shows the correspondence between taxonomic relations and universal set equations. As in [1], we can avoid unintended taxonomic relations captured during the translation from universal set equations to propositional formulae, by the use of *entailment constraints* [1]. In order to avoid excessive technical details which are beyond the scope of this paper, we focus on the use of the above theory to our work. In particular, it can be used to convert each categorical statement in a syllogistic argument to its corresponding propositional form which can be tested efficiently against a propositional theorem prover.

4.5 Conflicts between arguments

Intuitively, a set of arguments consists of a minimal set of premises (here categorical statements) used in the derivation of a claim. In this paper we focus on strict arguments that are inferred via the classical entailment relation. Two arguments conflict with each other (attack) if either (i) the claim of one argument is inconsistent with the claim of the other argument (i.e. *rebutal* [10]) or (ii) the claim of one argument is inconsistent with one of the other premises of the other argument (i.e. *undercutting* [10]) or (iii) one argument's premises are inconsistent with the other argument's premises. Since a syllogism is defined entirely in terms of categorical expressions then two syllogistic arguments conflict each other if any expression in one argument is inconsistent with an expression in the other argument.

5 Conclusion and Future Work

In this paper we introduced the notion of cognitive learning context that refers to multiple and possibly inconsistent ontologies. Differences in ontologies can be identified via arguments that can be inferred from consistent subsets of ontologies. We show that syllogistic arguments can be inferred from ontological primitives and we represent the necessary and sufficient properties of concepts used to argue in learning situations. Current work focuses on the axiomatization of relevance of modules, the argumentation theory and its use in the representation of the cognitive state of the learner.

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