System-information models of digital twins with elements of controlled self-organization

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Abstract

The article considers an approach to modeling the processes of controlled self-organization of a digital twin based on a single system-information space. Information processes are the basic basis for the manifestation of self-organization of systems. The structure of energy and material connections reflects the properties of objects of the external world and the internal environment of a self-organizing system. These connections with significant structural features are information connections. The tasks of controllability of self-organization of a digital twin are the tasks of dynamics with the definition of the system-information structure of an object with information stability. A dynamic system based on system-information models of a digital twin, in addition to the classical one, considers the uncertainty of parameters as an indicator that affects information stability, optimality, and the degree of self-organization. System-information criteria for solving problems of the dynamics of a digital twin with elements of controlled self-organization allow for the choice of structures of systems with information stability that satisfy the conditions of self-organization. The controllability of self-organization of a digital twin is achieved by selecting the function of informational coordination of uncertainty of system elements, ensuring the emergence of stable structures of connections in a dynamic system. The article considers the issues of analysis of elements of controllability of self-organization, from the position of the system-information approach, models of system information of digital twin data with elements of controlled self-organization, as well as examples of solving problems of the dynamics of the process of self-organization of a digital twin.

Keywords

digital twin, controlled self-organization, system information, system-information models

1. Introduction

Manufacturing technologies in their development have gone through stages of evolution starting from elementary – manual labor, mechanized, automated, automatic, digital, and further to the self-organization of unmanned production. The development of the concept of a "digital twin" with elements of self-organization is an urgent need of our time to create unmanned production outside of human habitation. Self-organization of production is designed to help find non-standard solutions to problems faster, predict their results more accurately, and ensure the production of high-quality products. A digital twin with elements of self-organization is a digital copy of a process, system, or asset that expands the capabilities for transforming new, previously unforeseen solutions to achieve set goals [1-4].

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The digital twin as an information system is characterized by classical problems of the general theory of systems, which require the development of principles for formalizing system problems and solving them based on system-information models. This is theoretically based on the concept of the term system information of objects. In production, digital twins with elements of self-organization can be created for production technologies, specific production lines, the final product, or any other object within the production process [5].

The term self-organization is not new. There is a special discipline in science – "synergetic" [6], which aims to find, together with other sciences, the principles of self-organization, according to which the processes of formation of order from chaos are realized in the universe. The main concepts of synergetic are the bifurcation point, attractor, dissipative processes, and fractals. Hermann Haken gave the following definition of self-organization within the framework of synergetic [6]: "Self-organization is a process of ordering (spatial, temporal, or spatio-temporal) in an open system, due to the coordinated interaction of many elements of its components." However, in recent years, this concept has increasingly and persistently appeared in a variety of scientific applications. Self-organization occurs in many physical, chemical, biological, robotic, mathematical, virtual, and cognitive systems and is an information process. Self-organization is associated with the concept of emergence [7].

The concept of controlled self-organization began to form in 2008. This approach aims to regulate self-organization for specific purposes so that a dynamic system can achieve certain attractors or results [7]. Concerning self-organization of production, the system is characterized by regulated self-organization of the production and technological process, the operation of equipment and its software, and has the properties of self-healing, self-adjustment, self-management, self-adaptation of production systems, etc.

The above characteristics of real production are provided at the virtual level by a digital twin with elements of self-organization based on a single system information space [8]. System tasks of digital twins based on system-information models relate to methods of managing the uncertainty of parameters of processes and systems of real production. The solution of system tasks of a digital twin in a single information space is implemented based on system-information models taking into account the presence of uncertainty. The systemic tasks of a digital twin are questions of systems theory: analysis, synthesis, identification, observability, forecasting, evaluation, solvability, control, stability, dynamics, optimization, and others.

The single system-information space is based on the methodology of system-information modeling [9, 10] of processes and systems. System-information modeling complements modern information theory by introducing a new concept and its mathematical justification – system information. System information is characterized by a quantitative indicator of the communication ability of an object to exchange information with the environment [11, 12]. It takes into account the presence of uncertainty in the parameters of the state of objects and their information coordination in the system based on the formulated system-information laws. In the process of exchanging system information, objects change their state by an amount multiple of the sensitivity threshold to the influencing object.

The basic foundation of system-information models are:

1. Formulated information laws for the transformation of system information of physical quantities (parameters).
2. Modeling of the information measure and norm of system information of a physical quantity (parameter).
3. Use of Planck units of physical quantities in models, which ensures their high accuracy.
4. Modeling of objects in a single system-information space, which ensures their information virtual interaction.

The mathematical apparatus of system-information modeling adequately reflects the production processes of both real production and its virtual copy - a digital twin. Analysis of methods for solving production problems based on system-information models showed additional possibilities for using intelligent information processing in virtual systems of digital twins. These are based on the developed algorithms of system information based on the information norm and measure [12].

The use of system-information models in digital twins with elements of self-organization allows solving the problems of controlled self-organization at the virtual level with the transfer of solutions to real production. Therefore, developing a methodology for creating digital twins with elements of controlled self-organization is a pressing task.

2. Analysis of elements of controlled self-organization from the position of the system-information approach

General principles of self-organization of systems follow from the regularities of hierarchical systems. These principles are decisive for the functioning of living and non-living organisms, and the strict evolutionary nature of their development. The concept of controlled self-organization aims to regulate self-organization for specific purposes so that a dynamic system can achieve certain attractors or results. Regulation limits the self-organizing process within a complex system, at the level of local interactions between system components, and does not follow an explicit control mechanism or a global design plan [13, 14]. Desired results, such as an increase in the resulting internal structure or functionality, are achieved by combining task-independent global goals that are task-dependent with constraints on local interactions.

An important ability of a self-organizing system is the ability to change under the influence of certain factors of the external world that are significant for this self-organizing system [7]. This is the essence of the manifestation of the sensitivity threshold in the processes of self-organization of the system. The process of self-organization of the system is dynamic, and the sensitivity threshold has a stochastic nature. By controlling the value of the sensitivity threshold of the system elements, the principle of controllability of the self-organization of the system is achieved.

The information system is the basic foundation for the manifestation of the principles of self-organization of systems. The distinctive property of the information system is that the structure of energy and material connections is essential for it, which is a reflection of the properties of objects of the external world and the internal environment of the self-organizing system. These connections with essential structural features are information connections. The elements of the structure of the information system are physical quantities of the SI system - parameters.

The basis of controlled self-organization is the system-information principles of reflection and the laws of transformation of system information [11, 12], which determine the types of algorithms for managing self-organization. Let's consider the system-information laws.

Axiom. A change in the state of an object occurs as a result of external (internal) influence, starting from the threshold of the object's sensitivity to this influence.

1. The law of identical reflection of the properties of objects X_i, Y_j:

$$I(X_i) = I(Y_j)\,. \tag{1}$$

2. The law of coordination of the uncertainty of the properties of objects X_i, Y_j of the reflection process of the reflection process:

$$\begin{aligned} log_2 \frac{\mu_i}{U_i} &= log_2 \frac{\mu_j}{U_j}\,; \\ log_2 \frac{X_i}{\Delta x_i} &= log_2 \frac{Y_i}{\Delta x_j}\,. \end{aligned} \tag{2}$$

where μ_i, μ_j is the mathematical expectation, U_i, U_j is the expanded uncertainty, X_i, Y_i is the properties of objects, Δx, Δy is the sensitivity threshold.

3. The law of systemic properties of stationary, equilibrium reflection space:

$$\sum_{i=1}^{n} I_{S_i}(t) = const \,, \quad (3)$$

where n is the number of system elements, s_i is the element of system S, I_{S_i} is the amount of system information.

4. Additivity of system properties of the reflection space (consequence):

$$I_{Y_j} = \sum_{i=1}^{t} I_{X_i}, \ \ t = \overline{1, n}\,, \quad (4)$$
$$\mu_j = U_j \cdot n^{\sum_{i=1}^{t} I_i}\,,$$

where n is the base of the logarithm, U is the expanded uncertainty, μ is the mathematical expectation, and I_i is the system information of the i-th property.

Based on the presented system-information laws, information structures of a digital twin with controlled self-organization are formed.

3. Models of system information of a digital twin with controlled self-organization

System-information modeling of a single information space of a digital twin is based on the principles of superposition. The sum of the system's reactions to individual disturbances from identical initial states is equal to the reaction of this system to the total impact from the same initial state. The elementary reaction of the system to an external disturbance is the threshold of the system's sensitivity to the impact.

One of the solutions to information system problems is to assess the uncertainty value of the parameters of a technical (closed) system based on a system-information model of the dynamic processes of a digital twin, which characterizes the stability of the system and the optimization of its state.

The object of this study is a stochastic system – a set of elements *X*, which are in information links with each other and form a certain integrity and unity. System information, possessing elements of set *X*, is characterized by the interval of the upper X_{max} and lower X_{min} limits of its manifestation, as well as the uncertainty $2U_x = \mu_x - X_{min}/n$, where $\mu_x = nU + X_{min}$ – discrete variable value on the interval $X_{max} - X_{min}$, $2U_x$ – uncertainty/sensitivity threshold.

The structure of a single system-information space of a digital twin with elements of controlled self-organization consists of three main blocks [4]. The central block is a fragment of the reality of a set of properties of an object, the so-called intensity of properties. The intensity of properties is characterized by the maximum number of perceived properties and their values max and min. The modeled factor is the scale of the intensity of properties, which is characterized by the sensitivity threshold.

The other two blocks are the duration block (time) and the extension block (length). These blocks are characterized by the extreme boundaries of duration and extension, the sensitivity thresholds of duration and extension, the scales of duration variability, and extension heterogeneity. The combination of intensity, duration, and extension block elements in various configurations forms layers of reality fragments.

The source of the content of system information of the probable physical quantity xi is the mathematical expectation of the manifestation of the properties of processes and systems

$$\mu = \sum_{i=1}^{n} x_i p_i \, . \tag{5}$$

The expression for calculating the amount of system information of the assumed physical quantity (PQ) is determined based on the logarithmic indicator of communication capacity:

$$I = log_2 \frac{\mu}{\text{U}} = log_2 \frac{\sum_{i=1}^{n} x_i p_i .}{\text{K}\sqrt{D}} \, , \tag{6}$$

where D is the variance, U is the expanded uncertainty, K is the coverage factor, μ is the mathematical expectation, p is the probability of the event.

In the algorithms of dynamic processes of digital twins with controlled self-organization, models of objects with an information *measure* and *norm* are used [15, 16].

The information measure characterizes the system information of an object |I(X)|, which is a function of the absolute value of the qualitative or quantitative proportion of the relationship. The information measure |(X)| is a function of the ratio of the value of the interval of the object's feature to its sensitivity threshold:

$$|I(X)| = f\left(\frac{X_{max} - X_{min}}{2U_x}\right) . \tag{7}$$

The amount of system information of the *first kind* will be calculated based on the information measure:

$$log_2 \, |I(X)| = \; log_2 \, \left(\frac{X_{max} - X_{min}}{2U_x}\right) , \tag{8}$$

where X_{max}, X_{min} are the upper and lower boundaries of the interval of values of the object's properties, $2nU_x$ – uncertainty/sensitivity threshold.

The information norm $\|(X)\|$ is a function of the ratio of the interval value of an object's feature to its particular variable value:

$$\|(X)\| = f\left(\frac{X_{max} - X_{min}}{\mu_x}\right) = \left(\frac{X_{max} - X_{min}}{2nU_x + X_{min}}\right) . \tag{9}$$

The amount of system information of the *second kind* will be calculated based on the information norm:

$$log_2 \, \|(X)\| = \; log_2 \, \left(\frac{X_{max} - X_{min}}{\mu_x}\right) = log_2 \, \left(\frac{X_{max} - X_{min}}{2nU_x + X_{min}}\right) , \tag{10}$$

where $\mu_x = nU + \text{X}_{min}$.

The presented stochastic models of system information of a digital twin in combination with system-information methods are used in solving problems of controlled self-organization.

4. General approach to solving the problems of the dynamics of the self-organization process of a digital twin

The solution of systemic problems of a digital twin based on system-information models refers to methods of managing the self-organization of parameters of processes and systems of real production [14]. The tasks of managing the self-organization of a digital twin are problems of dynamics with the definition of the system-information structure of an object with information stability. A dynamic system based on system-information models of parameters of processes and systems of a digital twin, in addition to the classical one, considers the uncertainty of parameters as an indicator that affects the information stability, optimality, and self-organization of the system. System-information criteria for solving problems of the dynamics of a digital twin with self-organization management allow selection structures with information stability that satisfy the conditions of the problem. Information stability of information links of system elements is characterized by three states [16]:

- $\Delta x/U(x) = 1$ – sufficient stability;
- $\Delta x/U(x) < 1$ – insufficient stability;
- $\Delta x/U(x) > 1$ – excess stability.

The safety factor of the information stability of the parameter is determined by the expression:

$$K_{is} = \Delta x/U(x) - 1,$$

where Δx is the parameter sensitivity threshold, $U(x)$ is the expanded uncertainty of the parameter.

Example 1.

The conditions of the system-information process have the form:

$$x_i(t),\ y(t),\ I_Y = \sum_{i=1}^{N} I_{X_i},\ \Delta x_i = U_{X_i},\ \Delta y = U_y,$$

at the same time

$$\frac{x_1}{U_1} = \frac{x_2}{U_2},\quad x_1 = \frac{U_1}{U_2},\ \ dx_1 = d\left(\frac{U_1}{U_2}\right),$$

where: x_1, x_2 – elements of the system, U_1, U_2 – expanded uncertainty.

The increment of system information $I(dx_1)=I(dx_2)$ determines the characteristics of the dynamics of the information connection of the elements $x_1(t)$ and $x_2(t)$ depending on the values U_1, U_2, which can be a function of time $U_i(x_i)$.

The task is set: to formalize the dynamics of the information system [12] based on the uncertainty-matching function of the elements:

$$I_Y(t) = \sum_{i=1}^{N} I_{X_i}(t),$$
$$log_2\frac{y(t)}{\Delta y} = \sum_{i=1}^{N} log_2\frac{x_i(t)}{\Delta x_i}. \quad (11)$$

The problem of system information dynamics is solved in several stages.

1. A matrix of information links of elements $x_i(t)$ is built (Table 1).

The time variable $x_i(t)$ can be an argument of the function (x_1^t), then you need to solve the equation and take positive roots as a separate element $x_i(t)$.

Table 1
A matrix of Information Links of Elements

N/N	x_1	x_2	....	x_N
x_1	$K_{11} = x_1/x_1$	$K_{12} = x_1/x_2$	....	$K_{12} = x_1/x_N$
x_2	$K_{21} = x_2/x_1$	$K_{22} = x_2/x_2$	....	$K_{22} = x_2/x_N$
...	....	....	....	....
x_N	$K_{N1} = x_N/x_1$	$K_{N2} = x_N/x_2$	....	$K_{N2} = x_N/x_N$

2. We compose a system of equations of dynamics for each pair of elements, such pairs will be equal to $N \times N$:

$$\Delta x_i(k) = x_i(k+1) - x_i(k),\qquad i = 1, \dots, N,$$

where: k – discrete time, Δx_i – sensitivity threshold;

$$a_{12} = \frac{\Delta x_1}{\Delta x_2}, \dots, a_{1N} = \frac{\Delta x_1}{\Delta x_N}, \dots,\ a_{N1} = \frac{\Delta x_N}{\Delta x_1}, \dots, a_{NN} = \frac{\Delta x_N}{\Delta x_N},$$

$$x_1(0), \dots, x_N(0), \quad \frac{\Delta x_1}{\Delta x_N} = \frac{x_1}{x_N}, \quad \Delta x_1 = \Delta x_N \frac{x_1}{x_N},$$

$$\left\{\begin{array}{c} x_1(k+1) - x_1(k) = a_{11}x_1(k) + a_{12}x_2(k) + \dots + a_{1N}x_N(k), \\ x_2(k+1) - x_2(k) = a_{21}x_1(k) + a_{22}x_2(k) + \dots + a_{2N}x_N(k), \\ \dots \\ x_N(k+1) - x_N(k) = a_{N1}x_1(k) + a_{N2}x_2(k) + \dots + a_{NN}x_N(k), \end{array}\right\},$$

or

$$\left\{\begin{array}{c} x_1(k+1) = (1+a_{11})x_1(k) + a_{12}x_2(k) + \dots + a_{1N}x_N(k), \\ x_2(k+1) = a_{21}x_1(k) + (1+a_{22})x_2(k) + \dots + a_{2N}x_N(k), \\ \dots \\ x_N(k+1) = a_{N1}x_1(k) + a_{N2}x_2(k) + \dots + (1+a_{NN})x_N(k), \end{array}\right\},$$

or finally

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \dots \\ x_N(k+1) \end{bmatrix} = \begin{bmatrix} (1+a_{11}) & a_{12} \ \dots & a_{1N} \\ a_{21} & (1+a_{22}) \ \dots & a_{2N} \\ & \dots & \\ a_{N1} & a_{N2} \ \dots & (1+a_{NN}) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \dots \\ x_N(k) \end{bmatrix}. \quad (12)$$

The solution of the given equations of the dynamics of the information system (12) relative to the information stability of the connections of each pair of elements of the system allows us to determine a set of stable structures of connections of elements of the information system. The choice of the structure of the information system that satisfies the conditions of the problem is the process of managing the self-organization of the digital twin.

Example 2.

Solve the system of dynamic equations for the elements x_1 and x_2 of the information system based on the uncertainty matching function of the elements:

$$\begin{gathered} \frac{x_1(t)}{dt} = \ell x_2, \quad \frac{x_2(t)}{dt} = \varphi x_1, \\ \ell = K_{12} = \frac{\Delta x_1}{\Delta x_2} = \frac{U_1}{U_2}, \quad \varphi = K_{12} = \frac{\Delta x_2}{\Delta x_1} = \frac{U_2}{U_1}, \\ \Delta x_1, \Delta x_2 = const, \ min \le x_i \le max, \end{gathered} \quad (13)$$

where: x_i – variables, Δx_i – sensitivity threshold, U_i – expanded uncertainty, K_{ij} – information link coefficient. Then we have:

$$\begin{gathered} x_1(n+1) = x_1(n) + \ell x_2(n), \\ x_2(n+1) = x_2(n) + \varphi x_2(n), \quad x_1(0), x_2(0). \end{gathered} \quad (14)$$

We apply the discrete Z-Laplace transform:

$$x(z) = \sum_{n=0}^{\infty} x(n) z^{-n}. \quad (15)$$

or

$$\begin{bmatrix} x_1(z) \\ x_2(z) \end{bmatrix} = \frac{z}{z^2 - 2z + 1 - \ell\varphi} \cdot \begin{bmatrix} z-1 & \ell \\ \varphi & z-1 \end{bmatrix} \cdot \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}. \quad (16)$$

Let's put $\sigma^2 = l \cdot \varphi \ge 0$, then we can determine the roots of the equation:

$$\begin{gathered} z^2 - 2z + 1 - \sigma^2 = 0, \\ \big(z - (1-\sigma)\big) \cdot \big(z - (1+\sigma)\big) = 0, \\ z_1 = 1 - \sigma, z_2 = 1 + \sigma. \end{gathered} \quad (17)$$

Solution (17) takes the form:

$$\begin{gathered} x_1(z) = \frac{z^2 - z}{z^2 - 2z + 1 - \sigma^2} \cdot x_1(0) + \frac{\ell\varphi}{z^2 - 2z + 1 - \sigma^2} \cdot x_2(0), \\ x_2(z) = \frac{\varphi z}{z^2 - 2z + 1 - \sigma^2} \cdot x_1(0) + \frac{z^2 - z}{z^2 - 2z + 1 - \sigma^2} \cdot x_2(0). \end{gathered} \quad (18)$$

Find the inverse transformation to the temporary variable *n*:

$$\frac{z^2 - z}{z^2 - 2z + 1 - \sigma^2} = \frac{1}{2} \cdot \left(\frac{z}{z - (1 - \ell)} + \frac{z}{z - (1 + \ell)}\right) \Rightarrow$$
$$\Rightarrow \frac{1}{2} \cdot (1 - \sigma)^n + \frac{1}{2} \cdot (1 + \sigma)^n,$$
$$\frac{z}{z^2 - 2z + 1 - \sigma^2} = \frac{\sigma - 1}{2\sigma} \frac{1}{z - (1 - \sigma)} + \frac{\sigma + 1}{2\sigma} \frac{1}{z - (1 + \sigma)} \Rightarrow \tag{19}$$
$$\Rightarrow \frac{1}{2\sigma} \cdot (1 + \sigma)^n + \frac{1}{2\sigma} \cdot (1 - \sigma)^n.$$

It follows that

$$x_1(n) = \frac{1}{2} \left(x_1(0) - \frac{\ell}{\sigma} x_2(0)\right) (1 - \sigma)^n + \frac{1}{2} \left(x_1(0) + \frac{\ell}{\sigma} x_2(0)\right) (1 + \sigma)^n,$$
$$x_2(n) = \frac{1}{2} \left(x_2(0) - \frac{\varphi}{\sigma} x_1(0)\right)(1 - \sigma)^n + \frac{1}{2} \left(x_2(0) + \frac{\varphi}{\sigma} x_1(0)\right) (1 + \sigma)^n. \tag{20}$$

Thus, in the closed information system of the digital twin, the stability of the dynamic process is ensured by a limitation:

$$min \leq x_i \leq max\,, \tag{21}$$

and the condition

$$\sum_{i=1}^{N} \frac{x_i}{\Delta x_i} = const\,. \tag{22}$$

The analysis of the dynamics of the self-organization process of the digital twin showed that the dynamics of the systemic connections of the elements $\{x_2 \longrightarrow x_1,\ x_1 \longrightarrow x_2\}$ of the system is self-developing and represents an evolutionary spiral, and the dynamics of the systemic connections of the elements $\{1/x_2 \longrightarrow x_1,\ 1/x_2 \longrightarrow x_1\}$ of the system is damped. In a stable information system of real production, there are connections between the elements of both dynamics.

5. Examples of system-information analysis of controlled self-organization processes

Example 3. Let us consider an example of constructing a system-information model of a dynamically stable process of a digital twin with the definition of the coefficients of the transfer matrix A_{NN} and the function of the sensitivity thresholds $\Delta x_i = f(x_i)$, $\Delta x_j = f(x_j)$, ensuring the dynamic stability of the digital twin.

Given:

$$log_2 \frac{x_i}{\Delta x_i} = log_2 \frac{x_j}{\Delta x_j},\ i,j = \overline{1,N}\,, \sum_{i=1}^{N} \frac{x_i}{\Delta x_i} = const\,,$$
$$\frac{x_i}{\Delta x_i} = \frac{x_j}{\Delta x_j},\ x_i = x_j \frac{\Delta x_i}{\Delta x_j}, k_{ij} = \frac{\Delta x_i}{\Delta x_j},$$
$$x_1(0), x_2(0), \dots, x_n(0), \qquad x_1(t), x_2(t), \dots, x_n(t)\,,$$
$$k_{ij} = \frac{\Delta x_i}{\Delta x_j},\ k_{ij} = 1/k_{ji}\,, k_{ij}(t) = \varphi_{ij}(t - t_0)\,.$$

Find: $\varphi_{ij}(t - t_0), \Delta x_i = f(x_i), \Delta x_j = f(x_j)$.

It is necessary to solve the system of equations

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} \varphi_{11}(t - t_0) & \varphi_{12}(t - t_0) & \dots & \varphi_{1N}(t - t_0) \\ \varphi_{21}(t - t_0) & \varphi_{22}(t - t_0) & \dots & \varphi_{2N}(t - t_0) \\ & \dots & & \\ \varphi_{11}(t - t_0) & \varphi_{12}(t - t_0) & \dots & \varphi_{1N}(t - t_0) \end{bmatrix} \begin{bmatrix} x_1(t_0) \\ x_2(t_0) \\ \dots \\ x_N(t_0) \end{bmatrix} \tag{23}$$

to determine the roots $\lambda_i\ \ i = \overline{1,N}$.

When the conditions are met $\lambda_i < 0,\ i = \overline{1,N}$ – the system is dynamically stable.

When the conditions are met $\lambda_i \geq 0,\ i = \overline{1,N}$ – the system is dynamically unstable.

Let's consider a special case when $N = 2$. Then we have:

$$K_{2\times2} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}, \; k_{11} = k_{22} = 1, \; k_{12} = \varphi, \; k_{22} = \frac{1}{\varphi}.$$

Then

$$K_{2\times2} = \begin{bmatrix} 1 & \varphi \\ \frac{1}{\varphi} & 1 \end{bmatrix}, \; \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 1 & \varphi \\ \frac{1}{\varphi} & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where

$$det\left|A_{x_1} - \lambda\, E_{x_2}\right| = 0, \; \begin{bmatrix} 1-\lambda & \varphi \\ \frac{1}{\varphi} & 1-\lambda \end{bmatrix} = 0,$$

$$(1-\lambda)^{\,2} - \varphi \frac{1}{\varphi} = 0, \quad (\lambda^2 - 2\lambda) = 0.$$

From here we find the roots of the system: $\lambda_1 = 0$, $\lambda_2 = 2$.

By analyzing the obtained roots of system $N_{2\times2}$, we can conclude that this system is unstable.

Note that this system-information model of the digital twin of order $N_{2\times2}$ represents the evolutionary spiral of development of a dynamic system. The dynamically stable state of the system is achieved due to two processes when $x_i \longrightarrow 1/x_j$ and when the stochastic function of the dependence of the sensitivity threshold on the parameter $\Delta x_N = f(x_N)$ of the digital twin is determined.

Example 4. Let us now consider an example of determining the information stability of digital twin processes. The systemic information approach to the analysis of the information stability of digital twin processes allows the establishment of quantitative links between the assessment of the accuracy tolerance for the size and the uncertainty of the parameter. The criterion of information stability is the condition when the expanded uncertainty is equal to or less than the accuracy tolerance of the parameter

$$\mathrm{U}\,(x_i) \leq \mathrm{IT}\,(x_i).$$

$$log_2 \frac{\mathrm{y}}{\Delta y_{\mathrm{U}_y}} = log_2 \frac{\mathrm{x}_1}{\mathrm{U}(x_1)} + log_2 \frac{\mathrm{x}_2}{\mathrm{U}(x_2)} + \dots + log_2 \frac{\mathrm{x}_n}{\mathrm{U}(x_n)},$$

$$log_2 \frac{\mathrm{y}}{\Delta y_{\mathrm{IT}_y}} = log_2 \frac{\mathrm{x}_1}{\mathrm{IT}(x_1)} + log_2 \frac{\mathrm{x}_2}{\mathrm{IT}(x_2)} + \dots + log_2 \frac{\mathrm{x}_n}{\mathrm{IT}(x_n)}, \tag{24}$$

$$log_2 \frac{\mathrm{y}}{\Delta y_{\mathrm{IT}_y}} = log_2 \frac{\mathrm{x}_1}{\mathrm{IT}(x_1)} + log_2 \frac{\mathrm{x}_2}{\mathrm{IT}(x_2)} + \dots + log_2 \frac{\mathrm{x}_n}{\mathrm{IT}(x_n)},$$

$$\frac{\mathrm{y}}{\Delta y_{\mathrm{IT}_y}} = \prod_{i=1}^{n} \frac{\mathrm{x}_i}{\mathrm{IT}(x_i)}, \quad \Delta y_{\mathrm{IT}_y} = \prod_{i=1}^{n} IT(x_i), \quad \Delta y_{\mathrm{U}_y} = \sum_{i=1}^{n} U_{\mathrm{x}_i},$$

$$k_y = \frac{\sum_{i=1}^{n} U_{\mathrm{x}_i}}{\prod_{i=1}^{n} \mathrm{IT}(x_i)}.$$

When the conditions are met $k_y \leq 1$, – the system is information stable

When the conditions are met $k_y > 1$, – the system is information unstable.

The conducted system-information analysis of the controlled self-organization of the digital twin examines the information stability, which provides the self-organizational structure with the management of the quality of the parameters of the digital twin. It creates the basis for a logical and consistent approach to the problem of decision-making from the position of the dynamic and information stability of the digital twin. The peculiarity of the system-information approach to the process of controlled self-organization of the digital twin is that the categories of processes of dynamic and information systems are considered. The effectiveness of solving problems using system-information analysis is determined by constructing the self-organization structure of the solved problems of the digital twin. The result of system-information research is the definition of the structure of the parameters of the digital twin.

6. Conclusions

The concept of modeling digital twins with elements of controlled self-organization based on a single system-information space is presented. It aims to regulate self-organization for specific purposes, providing a dynamic system with the ability to achieve certain attractors or required results. System-information regulation limits the self-organizing process inside a complex system at the level of information functions for coordinating the uncertainty of local interactions between components of a dynamic system. Desired results, such as increased internal structure or functionality, are achieved by combining tasks independent of global goals with tasks that restrict local interactions.

The tasks of controllability of self-organization of a digital twin are the tasks of dynamics with the definition of the system-information structure of an object with information stability. System-information criteria for solving the tasks of dynamics of a digital twin with elements of controlled self-organization allow for the choice of structures of systems with information stability that satisfy the conditions of self-organization.

The results of solving the problems show the dependence of the dynamic system reaching the attractor point on the information function of coordinating the uncertainty of the system elements. By choosing the information function of coordinating uncertainties, the self-organization of the digital twin is controlled to achieve the required goals. In general, the methodology of system-information modeling takes into account "fluctuations and other statistical phenomena" in the dynamics problems and allows developing methods for managing the information function of coordination when solving problems of self-organization of a digital twin, due to parameter uncertainty in the model.

An example of constructing a system-information model of a dynamically stable process of a digital twin is given, and the conditions of its dynamic stability are defined. An example of determining the information stability of processes of a digital twin is also given, and the conditions of its information stability are defined.

Declaration on Generative AI

The authors have not employed any Generative AI tools

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