KLM-style Defeasible Reasoning on Concepts

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Abstract
In this paper, we introduce a KLM-style framework for defeasible reasoning about formal concepts. This framework can be used both for theoretical developments and in applications of non-monotonic reasoning about formal concepts.

Keywords
Formal Concept Analysis, Non-monotonic reasoning, KLM framework

1. Introduction

Non-monotonic logics are a class of logics which allow for inference relation to be non-monotonic, i.e. such that adding more knowledge or preferences can lead to some inferences to be retracted. These logical frameworks are intended to formally account for forms of reasoning which allow for exceptions and revision of conclusions. Non-monotonic logics play a crucial role in several fields of artificial intelligence, such as common-sense reasoning [1], ethical AI [2], and argumentation theory [3]. Various formal frameworks for non-monotonic reasoning have been developed, including Default Logic [4], AGM Belief Revision [5], Defeasible Entailment Reasoning [6], Conditional Logic [7], Circumscription [8], Autoepistemic logic [9].

Formal Concept Analysis (FCA) [10] is an established mathematical framework used in Knowledge Representation and Reasoning to study FCA hierarchies. The basic structures in FCA, namely formal contexts and their associated concept lattices, have been systematically linked with—and used as semantic environments of—a large family of lattice-based propositional logics, prominent examples of which are lattice-based modal logics, and their theory has been developed as a family of logics for reasoning about (formal) concepts in the context of data structures and information theory [11, 12, 13, 14]. Each logic in this family is defined in terms of a *monotone* consequence (or entailment) relation $C_1 \vdash C_2$ between concepts, which is semantically interpreted as 'C_1 is a subconcept of C_2', that is, 'all the objects in the extension of C_1 are in the extension of C_2', or equivalently, 'all the features in the intension of C_2 are in the intension of C_1'. On the basis of this framework, various more sophisticated logical frameworks have been proposed, including epistemic logic for categories and categorization endowed with a 'common knowledge' operator accounting for prototypicality [12], a basic environment for a Dempster-Shafer theory of concepts [15], a unifying environment for Rough Set Theory and FCA [16], many-valued logics accounting for vague categories [17], a specifically FCA-based description logic for FCA [18, 19], and various proof-theoretic frameworks laying the foundations of the computational theory of these logics [20, 21].

Deciding whether some concept inclusion is entailed by a given FCA knowledge base (e.g. a set of concept inclusions) is an important reasoning task which can be efficiently carried out by lattice-based

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propositional logics such as those mentioned above. However, in many applications, especially in context of large data, as well as in many real-life situations, a part of the available knowledge may be *defeasible* (i.e. presented in form of concept-inclusions which allow for exceptions). Studying defeasible entailment on concepts would allow us to infer knowledge from knowledge consisting of defeasible concept-inclusions, and to capture, and hence implement, common sense reasoning about concepts. For example, if a generic object (animal) a is in thecategory of 'mammals', then we can reasonably conclude that a is viviparous. However, if we receive additional information that the animal is a platypus, then we can conclude it is not viviparous.

Formally, we can define the following three defeasible counterparts of the monotone entailment relation $\vdash$ discussed above: (1) Relation $\mid\!\sim_A$ interpreted as 'all the objects in C_1, with some possible exceptions, are in C_2' or 'typical objects of C_1 are in C_2', (2) relation $\mid\!\sim_X$ interpreted as 'all the objects C_1 have all the features shared by C_2, with some possible exceptions' or 'all the objects of C_1 have typical features of C_2', and (3) relation $\mid\!\sim_{AX}$ interpreted as 'all the objects C_1, with some possible exceptions, have all the features of C_2 with some possible exceptions' or 'all the typical objects of C_1 have all the typical features of C_2'. For example, let C_1 and C_2 represent the concepts of 'mammals' and 'viviparous animals', respectively. Since mammals are typically viviparous, we have $C_1 \mid\!\sim_A C_2$. However, if we introduce C_3, representing the concept of 'echidnas', which are a kind of oviparous mammal, we find that $C_3 \vdash C_1$ (i.e., all echidnas are mammals), hence $C_3 \mid\!\sim_A C_1$, but $C_3 \not\mid\!\sim_A C_2$ (i.e., typically, echidnas are not viviparous).

In the present paper, we propose to extend the framework of Kraus, Lehmann, and Magidor (commonly referred to as the KLM framework) [6] to formalize defeasible entailment between concepts.

Since FCA does not have a natural notion of negation on concepts, the KLM framework cannot be directly applied to the FCA environment. Nonetheless, it can be suitably extended to FCA. Specifically, we define the FCA-counterparts of classical non-monotonic entailment relations such as the *cumulative entailment* $\mathbf{C}$, and the *cumulative entailment with loop* $\mathbf{CL}$. These counterparts are the three defeasible entailment relations $\mid\!\sim_A$, $\mid\!\sim_X$, and $\mid\!\sim_{AX}$ mentioned above. We do not include the preferential entailment system $\mathbf{P}$, as the counterpart of the rule OR in classical defeasible reasoning depends on the fact that the semantic counterpart of classical disjunction is the set-theoretic union, while in FCA disjunction is interpreted as the closure of the union. In fact, unlike what is the case in the classical setting, the FCA-counterpart of $\mathbf{C}$ is already complete w.r.t. the class of FCA preferential models (cf. Theorems 1, 2, 2). Moreover, as the language of FCA does not have a natural notion of negation for concepts, FCA-counterparts of axioms such as rational monotonicity are not available. It would be interesting for future research to explore whether some FCA counterparts of such rules can be defined.

Open directions on the front of semantic investigations concern the definition of the FCA-counterparts of cumulative models, cumulative ordered models, preferential models, and preferential ordered models and the proof of completeness theorems for different reasoning systems w.r.t. these classes of models. An interesting aspect of this research concerns exploring the similarities and differences between—as well as the relationships among—the defeasible consequence relations $\mid\!\sim_A$, $\mid\!\sim_X$, and $\mid\!\sim_{AX}$.

2. KLM framework for reasoning on concepts

In [22], the first steps were taken for developing the KLM framework in the setting of FCA, by introducing only the defeasible entailment relation $\mid\!\sim_A$ on formal concepts. Here, we start by recalling this framework and the results proved there.

To generalize the cumulative reasoning to the FCA setting, we modify the original framework [6] as follows: In [6], the language of underlying logic is assumed to be closed under all the classical connectives including negation and implication. However, lattice-based propositional logic does not have negation and implication in its language. Thus, we replace the formula $\phi \to \psi$ in the rules and axioms of $\mathbf{C}$ with the sequent $\phi \vdash \psi$, which encodes the entailment at a meta-logical level, rather than at the object language level. For any formal context $\mathbb{P} = (A, X, I)$, a *model* based on $\mathbb{P}$ is a tuple $\mathbb{M} = (\mathbb{P}, V)$ s.t. $V : \mathcal{L} \to \mathbb{P}^+$ is a homomorphism from the term algebra $\mathcal{L}$ of the propositional logic of lattices into the concept lattice $\mathbb{P}^+$ associated with $\mathbb{P}$. For any $\phi \in \mathcal{L}$, we let $[\![\phi]\!]_{\mathbb{M}}$ (resp. $(\![\phi]\!)_{\mathbb{M}}$) denote the extension (resp. intension) of $V(\phi)$ (dropping the subscripts when the context is clear), and $\mathbb{M} \models \phi \vdash \psi$ iff $[\![\phi]\!]_{\mathbb{M}} \subseteq [\![\psi]\!]_{\mathbb{M}}$ iff $(\![\psi]\!)_{\mathbb{M}} \subseteq (\![\phi]\!)_{\mathbb{M}}$.

A *lattice-based cumulative logic* consists of an *entailment relation*, i.e. a set of $\mathcal{L}$-sequents $\phi \vdash \psi$ closed under all axioms and rules of lattice-based propositional logic, and a *cumulative entailment relation*, i.e. a set of $\mathcal{L}$-sequents $\phi \mid\!\sim_A \psi$ closed under the Reflexivity axiom $\phi \mid\!\sim_A \phi$ and the rules

Left Logical Equivalence (LLE) $\dfrac{\phi\vdash\psi \quad \psi\vdash\phi \quad \phi\mid\!\sim_A\chi}{\psi\mid\!\sim_A\chi}$ $\dfrac{\phi\vdash\psi \quad \chi\mid\!\sim_A\phi}{\chi\mid\!\sim_A\psi}$ Right Weakening (RW)

Cautious Monotonicity (CM) $\dfrac{\phi\mid\!\sim_A\psi \quad \phi\mid\!\sim_A\chi}{\phi\wedge\psi\mid\!\sim_A\chi}$ $\dfrac{\phi\wedge\psi\mid\!\sim_A\chi \quad \phi\mid\!\sim_A\psi}{\phi\mid\!\sim_A\chi}$ (Cut).

From (LLE) and (RW) it follows that logically equivalent formulas are $\mid\!\sim_A$-entailed by the same formulas. A cumulative entailment relation $\mid\!\sim_A$ is *loop-cumulative* if it satisfies the following rule.

$$\frac{\phi_0\mid\!\sim_A\phi_1 \quad \phi_1\mid\!\sim_A\phi_2 \quad \dots \quad \phi_{n-1}\mid\!\sim_A\phi_n \quad \phi_n\mid\!\sim_A\phi_0}{\phi_0\mid\!\sim_A\phi_n}\ \text{(Loop)}$$

Let us define the FCA-counterparts of the models of defeasible reasoning by suitably adapting the approach used in [23] to define KLM-style modal logics.

A *pointed model* is a tuple $\mathbb{M}_a = (\mathbb{P}, V, a)$, where $\mathbb{M}$ is a model, and $a \in A$. Let $\mathcal{M} = (S, l, \prec)$ be a tuple s.t. S is a non-empty set (of *states*), $l : S \to \mathcal{P}(\mathcal{U})$ maps each state to a set of pointed models, and $\prec$ is a binary relation on S. For any $\phi \in \mathcal{L}$ and $s \in S$, $s \models \phi$ iff $a \in [\![\phi]\!]_{\mathbb{M}}$ for all $\mathbb{M}_a \in l(s)$. $\mathcal{M}$ is a *cumulative model* if, for any $\phi \in \mathcal{L}$, the set $\widehat{\phi} := \{s \mid s \in S, s \models \phi\}$ is *smooth* (i.e. for any $t \in \widehat{\phi}$, either t is $\prec$-minimal in $\widehat{\phi}$, or $s \prec t$ for some $\prec$-minimal element $s \in \widehat{\phi}$). A cumulative model $\mathcal{M} = (S, l, \prec)$ is *strong* if $\prec$ is asymmetric (i.e. $s \prec t$ implies $t \not\prec s$ for all $s, t \in S$) and $\widehat{\phi}$ has a minimum for every $\phi \in \mathcal{L}$; is *ordered* if $\prec$ is a strict partial order; is *preferential* if l assigns a single pointed model to each state. Any cumulative model $\mathcal{M}$ defines a *cumulative entailment* $\mid\!\sim_{\mathcal{M}}$ by: $\phi_1 \mid\!\sim_{\mathcal{M}} \phi_2$ iff for any s, if s is minimal in $\widehat{\phi_1}$, then $s \in \widehat{\phi_2}$.

It is easy to check that $\mid\!\sim_{\mathcal{M}}$ is a cumulative entailment relation. Reflexivity follows from $\min(\widehat{\phi}) \subseteq \widehat{\phi}$. (LLE) holds since $\widehat{\phi} = \widehat{\psi}$ implies $\min(\widehat{\phi}) = \min(\widehat{\psi})$. (RW) holds since $\min(\widehat{\chi}) \subseteq \phi$ and $\widehat{\phi} \subseteq \widehat{\psi}$ imply that $\min(\widehat{\chi}) \subseteq \psi$. As to (CM), if $\min(\widehat{\phi}) \subseteq \widehat{\psi}$, $\min(\widehat{\phi}) \subseteq \widehat{\chi}$, and $s \in \min(\widehat{\phi \wedge \psi})$, then, if $s \notin \min(\widehat{\phi})$, by smoothness, $s' \prec s$ for some $s' \in \min(\widehat{\phi})$. Hence, as $\min(\widehat{\phi}) \subseteq \widehat{\psi}$, $s' \in \widehat{\phi \wedge \psi}$, contradicting the minimality of s. The soundness of (Cut) is shown similarly.

Theorem 1. *(cf. [22]) A consequence relation is cumulative (resp. loop-cumulative) iff it coincides with $\mid\!\sim_{\mathcal{M}}$ for some* strong *(resp.* ordered*) cumulative model $\mathcal{M}$, iff it coincides with $\mid\!\sim_{\mathcal{M}}$ for some* preferential *(resp.* preferential ordered*) cumulative model $\mathcal{M}$.*

The defeasible entailment $\mid\!\sim_X$ can be characterized by dualizing the rules for $\mid\!\sim_A$, using the well known fact that the order on concepts is defined by reverse inclusion on their intensions.

A *lattice-based dually cumulative logic* consists of the entailment relation $\vdash$ of a lattice-based propositional logic, and a *dually cumulative entailment relation*, i.e. a set of $\mathcal{L}$-sequents $\phi \mid\!\sim_X \psi$ closed under the Reflexivity axiom $\phi \mid\!\sim_X \phi$ and the rules

Right Logical Equivalence (RLE) $\dfrac{\phi\vdash\psi \quad \psi\vdash\phi \quad \chi\mid\!\sim_X\phi}{\chi\mid\!\sim_X\psi}$ $\dfrac{\phi\vdash\psi \quad \psi\mid\!\sim_X\chi}{\phi\mid\!\sim_X\chi}$ Left Weakening (LW)

Dual Cautious Monotonicity (DCM) $\dfrac{\psi\mid\!\sim_X\phi \quad \chi\mid\!\sim_X\phi}{\chi\mid\!\sim_X\phi\vee\psi}$ $\dfrac{\chi\mid\!\sim_X\phi\vee\psi \quad \psi\mid\!\sim_X\phi}{\chi\mid\!\sim_X\phi}$ Dual Cut (DCut).

The rules above are obtained from the rules for $\mid\!\sim_A$ by switching the order of the consequence relation and interchanging $\vee$ and $\wedge$. This corresponds to the idea that the lattice of set of concept intensions under set inclusion forms a complete lattice dual to the concept lattice. From (RLE) and (LW) it follows that logically equivalent formulas $\mid\!\sim_X$-entail the same formulas.

Note that the rule loop is invariant under dualizing. A dually cumulative entailment relation is *loop-cumulative* if it satisfies the rule Loop.

$$\frac{\phi_0\mid\!\sim_X\phi_1 \quad \phi_1\mid\!\sim_X\phi_2 \quad \dots \quad \phi_{n-1}\mid\!\sim_X\phi_n \quad \phi_n\mid\!\sim_X\phi_0}{\phi_0\mid\!\sim_X\phi_n}\ \text{(Loop)}$$

We can define models for the various types of dually cumulative relations (i.e. *dually cumulative models* and their *strong*, *ordered*, and *preferential* subclasses) by replacing pointed models with *dually pointed models*, i.e. tuples $\mathbb{M}_x := (\mathbb{M}, x)$ s.t. $\mathbb{M}$ is a model and $x \in X$. All other parts of the definitions remain unchanged, including the dually cumulative entailment $\mid\!\sim_{\mathcal{M}}$ associated with a dual cumulative model $\mathcal{M}$. We can show soundness of all the above rules w.r.t. these models in a manner analogous to soundness proof of $\mid\!\sim_A$ rules. The proof of the following completeness theorem is similar to the previous one.

Theorem 2. *A consequence relation is dually cumulative (resp. dually loop-cumulative) iff it coincides with $\mid\!\sim_{\mathcal{M}}$ for some* strong *(resp.* ordered*) dually cumulative model $\mathcal{M}$, iff it coincides with $\mid\!\sim_{\mathcal{M}}$ for some* preferential *(resp.* preferential ordered*) dually cumulative model $\mathcal{M}$.*

A lattice-based *bi-cumulative logic* consists of an entailment relation $\vdash$ for lattice-based propositional logic, a cumulative entailment relation $\mid\!\sim_A$ and a dually cumulative entailment relation $\mid\!\sim_X$. Such a logic is *loop-cumulative* when both $\mid\!\sim_A$ and $\mid\!\sim_X$ are. Semantic models for these logics can be defined as tuples $\mathcal{M}_{AX} = (\mathcal{M}_A, \mathcal{M}_X)$, s.t. $\mathcal{M}_A$ is a cumulative model and $\mathcal{M}_A$ is a dually cumulative model; the corresponding (*strong*, *ordered*, and *preferential*) subclasses are defined by imposing the corresponding conditions on $\mathcal{M}_A$ and $\mathcal{M}_X$, and the bi-cumulative logic associated with $\mathcal{M}_{AX}$ is specified by $\mid\!\sim_{\mathcal{M}_A}$ and $\mid\!\sim_{\mathcal{M}_X}$.[1] The following is a straightforward corollary of the previous completeness results.

Theorem 3. *A pair of entailment relations defines a (loop-cumulative) bi-cumulative logic iff it arises from some (ordered) strong bi-cumulative model, iff it arises from some preferential (resp. preferential ordered) bi-cumulative model.*

Finally, we consider *expanded bi-cumulative logics* as bi-cumulative logics endowed with a third type $\mid\!\sim_{AX}$ of defeasible entailment, closed under the following rules except (Loop); when satisfying also (Loop), such a logic is *loop-cumulative.*

$$\text{(LLE)}\quad \frac{\phi\vdash\psi \quad \psi\vdash\phi \quad \phi\mid\!\sim_{AX}\chi}{\psi\mid\!\sim_{AX}\chi} \qquad \frac{\phi\vdash\psi \quad \psi\vdash\phi \quad \chi\mid\!\sim_{AX}\phi}{\chi\mid\!\sim_{AX}\psi}\quad \text{(RLE)}$$

$$Comb_A\quad \frac{\phi\mid\!\sim_{A}\psi}{\phi\mid\!\sim_{AX}\psi} \qquad \frac{\phi\mid\!\sim_{X}\psi}{\phi\mid\!\sim_{AX}\psi}\quad Comb_X$$

$$\text{(CM}_A\text{)}\quad \frac{\phi\mid\!\sim_{A}\psi \quad \phi\mid\!\sim_{AX}\chi}{\phi\wedge\psi\mid\!\sim_{AX}\chi} \qquad \frac{\psi\mid\!\sim_{X}\phi \quad \chi\mid\!\sim_{AX}\phi}{\chi\mid\!\sim_{AX}\phi\vee\psi}\quad \text{(CM}_X\text{)}$$

$$\text{(Cut}_A\text{)}\quad \frac{\phi\wedge\psi\mid\!\sim_{AX}\chi \quad \phi\mid\!\sim_{A}\psi}{\phi\mid\!\sim_{AX}\chi} \qquad \frac{\chi\mid\!\sim_{AX}\psi\vee\phi \quad \psi\mid\!\sim_{X}\phi}{\chi\mid\!\sim_{AX}\phi}\quad \text{(Cut}_X\text{)}$$

$$\text{(Loop)}\quad \frac{\phi_0\mid\!\sim_{AX}\phi_1 \quad \phi_1\mid\!\sim_{AX}\phi_2 \quad \ldots \quad \phi_{n-1}\mid\!\sim_{AX}\phi_n \quad \phi_n\mid\!\sim_{AX}\phi_0}{\phi_0\mid\!\sim_{AX}\phi_n}$$

The intuition behind these rules can be explained in the following manner.

- **(LLE) and (RLE)**: These rules simply say that $\mid\!\sim_{AX}$ respects logical equivalence. Note that $\mid\!\sim_{AX}$ is not assumed to be monotonic in either argument. This is consistent with the intended interpretation of $C_1 \mid\!\sim_{AX} C_2$ as 'typical objects of C_1 have typical features of C_2'. As typicality, which is a non-monotonic operator, is applied both to C_1 and C_2, it is natural to allow $\mid\!\sim_{AX}$ to be non-monotonic in both arguments.
- $\mathbf{Comb}_A$ **and** $\mathbf{Comb}_X$: These rules are sound under the intended interpretations of $\mid\!\sim_A$, $\mid\!\sim_X$, and $\mid\!\sim_{AX}$.
- $\mathbf{CM}_A$ **and** $\mathbf{CM}_X$: These rules state that the condition $\phi \mid\!\sim_A \psi$ (resp. $\psi \mid\!\sim_X \phi$) is enough to ensure the monotonicity of $\mid\!\sim_{AX}$ in the second (resp. first) argument.
- $\mathbf{Cut_A}$ **and** $\mathbf{Cut_X}$: We can perform a cut on the formula which is the second (resp. first) argument in a sequent containing $\mid\!\sim_{AX}$ using a sequent containing $\mid\!\sim_A$ and $\mid\!\sim_{AX}$.
- **Loop**: The loop rule behaves analogously to the loop rule for $\mid\!\sim_A$ or $\mid\!\sim_X$.

We believe that a further justification for these rules will be given by the completeness theorem for the expanded FCA bi-cumulative logic and FCA bi-cumulative ordered logic w.r.t. natural models for such systems conjectured below.

An entailment relation $\mid\!\sim_{\mathcal{M}_{AX}}$ can be associated with any bi-cumulative model $\mathcal{M}$ as follows: for any ϕ_1, ϕ_2, $\phi_1 \mid\!\sim_{\mathcal{M}_{AX}} \phi_2$ iff aIx for any $s_1 \in S_A$ and $s_2 \in S_X$, and all pointed models $\mathbb{M}_a \in l(s_1)$,

[1] Note that we do not assume any relationship between the partial orders on $\mathcal{M}_A$ and $\mathcal{M}_X$. However, in many applications these two orders have some relationship which needs to be formalized. Studying logics with such relationships would be an interesting future direction for this project.

$\mathbb{M}_x \in l(s_2)$ based on the same formal context $\mathbb{P} = (A, X, I)$ and valuation V on it. This corresponds to the idea that a typical object of ϕ_1 should have a typical feature of ϕ_2 when described in same (formal) context. We finish with the following conjecture.

Conjecture 1. *A triple of of entailment relations defines an expanded (loop-cumulative) bi-cumulative logic iff there exists a (ordered) strong bi-cumulative model $\mathcal{M}$, iff there exists some preferential (resp. preferential ordered) bi-cumulative model $\mathcal{M}$, such that $\mid\!\sim_A = \mid\!\sim_{\mathcal{M}_A}$, $\mid\!\sim_X = \mid\!\sim_{\mathcal{M}_X}$, and $\mid\!\sim_{AX} = \mid\!\sim_{\mathcal{M}_{AX}}$.*

3. Conclusion and future directions

In this work, we take first steps in defining a KLM style framework for defeasible reasoning on concepts. This opens several directions for future research and applications:

Formally modelling scenarios involving defeasible concept inclusions: Several real-life scenarios involving reasoning about concepts include defeasible reasoning. Our framework can be used to formally model these scenarios. A toy example (consisting only of $\mid\!\sim_A$) is discussed in [22].

Reasoning from defeasible knowledge bases: As discussed in the introduction, one of the primary aim of this work is to develop a framework for reasoning from knowledge given in the form of conceptual inclusions. In this direction, it would be interesting to study the complexity of various reasoning systems described in the present paper. In the classical setting, it is known that the complexity of defeasible reasoning is same as the complexity of the underlying logic [24]. As reasoning about conceptual inclusions is known to be polynomial-time, showing a similar result in the FCA-setting would show that reasoning in these logics is computationally efficient.

Belief revision for conceptual knowledge: In several applications, we are interested in scenarios where the reasoner may need to incorporate new possibly inconsistent knowledge with existing beliefs of the agents. In the classical setting, non-monotonic logics have been used to define belief revision operators [25]. It would be interesting to define and study revision operators in the setting of FCA using the non-monotonic reasoning systems introduced in the present paper.

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Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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