Chaotic data processing generator design by using biangular transformation

Roman Voliansky[1,*,†], Nina Volianska[2,†] and Arief Bramanto Wicaksono Putra[3,†]

[1] National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Polytechnichna Str., 37, Kyiv, 03056, Ukraine

[2] Taras Shevchenko National University of Kyiv, Volodymirska Str, 60, Kyiv, 01033, Ukraine

[3] Politeknik Negeri Samarinda, Str. Jl. Cipto Mangun Kusumo, Samarinda, 75242, Indonesia

Abstract

The paper is devoted to the development of a multichannel chaotic dynamical system. Our study is based on the design of theoretical backgrounds to transform a phase portrait of the generalized second-order dynamical system from cartesian coordinates into biangular ones. Such a transformation is defined by two nonlinear functions, which depend on the system's position and the positions of some base points. The system angular position is considered as the relative one respectively to base points. The differentiating of these functions and considering the initial system dynamic in the cartesian plane allows us to construct differential equations that define system dynamics in the novel state space. Since in the most general case the considered system can be quite nonlinear the obtained equations become enough complex. To reduce their complexity, we offer to replace the system nonlinearities as well as transformation nonlinearities with piecewise dependencies to approximate nonlinearities. Such an approach gives us the possibility to design a highly formal matrix-based approach to transform the piecewise linear dynamical system from cartesian coordinates into biangular ones. We show the example of this approach usage by designing a chaotic generator which is based on the well-known Duffing equations. The developed novel chaotic system can be easily implemented by using finite difference approximations for derivative operators with modern digital devices. Simulation results prove the significant difference between the designed and known systems.

Keywords

chaotic system, biangular coordinates, differential equations, data generating

1. Introduction

In today's world, chaotic systems are widely used in various scientific and engineering applications due to their unique ability to produce unpredictable signals [1, 2, 3]. These systems are utilized to study and predict biological [4, 5, 6], meteorological, and financial [10, 11, 12] processes, and to design control systems for different technical systems, devices, and networks [13, 14, 15].

At the same time, the primary application of chaotic systems is in data encryption [16, 17, 18] and secure data transmission [19, 20, 21]. The increasing need for highly secure communication systems is driven by the necessity to transmit large amounts of data for critical infrastructure cyber-physical systems [22, 23, 24], while restricting access to unauthorized persons. This interest is further fueled by the emergence of Industry 4.0 and Industry 5.0.

Consequently, numerous chaotic systems have been developed [25, 26, 27], with many authors studying chaotic systems in the real domain and designing channels to provide desired system features. However, these systems are often considered as single-channel dynamical systems, which limits their ability to perform multichannel data transmission.

Considering a dynamical system with multichannel observability equation can automatically solve this issue [28, 29] and facilitate the design of a multichannel chaotic system for parallel data

WDA'25: International Workshop on Data Analytics, January 6, 2025, Kyiv, Ukraine

* Corresponding author.

† These authors contributed equally.

voliansky@ua.fm (R. Voliansky); ninanin@i.ua (N. Volianska); ariefbram@gmail.com (A.B.W. Putra)

0000-0001-5674-7646 (R. Voliansky); 0000-0001-5996-2341 (N. Volianska); 0000-0003-1187-5040 (A.B.W. Putra)

transmission [30, 31, 32]. This approach also eliminates the need for subjective design of new chaotic systems as it involves transforming known systems [33, 34]. We demonstrate our method by considering a well-known Duffing system, but it can easily be extended to any system with chaotic and regular dynamics [35, 36].

The paper is organized as follows: firstly, we consider the generalized 2nd order nonlinear differential equation and transform it from cartesian coordinates into biangular ones. Then, we show the practical usage of our approach by replacing the system nonlinearities with piecewise linear functions. Such an approach allows us to design highly-formalized mathematical background for system designing. Thirdly, we illustrate the use of our approach by considering a well-known chaotic system based on the Duffing pendulum equations. Finally, we make a conclusion.

2. Method

2.1. Exact nonlinear model

Let us consider the generalized 2nd order dynamical system which motion is given in normal form as follows

$$\dot{x} = f_1(x,y); \quad \dot{y} = f_2(x,y), \tag{1}$$

where x and y are system state variables, $f_1(.)$ and $f_2(.)$ are some nonlinear functions.

Here and further, we consider a free motion of closed-loop dynamical system and in more complex case some external control signal should be taken into account while the considered system is modeled.

It is clear that the system motion can be given by some trajectory in the phase plane (Figure 1).

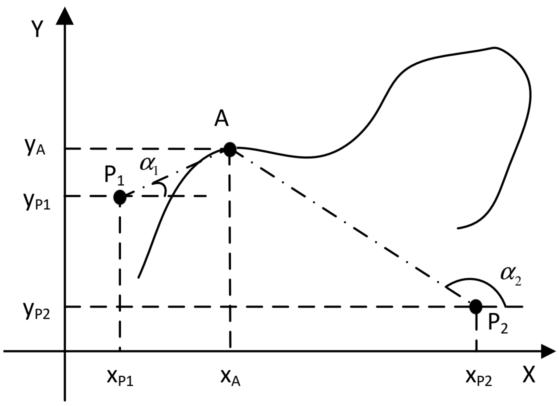


Figure 1: System phase trajectory.

Let us assume that in the considered phase plane the two points P_1 and P_2 are defined and their coordinates are known. We call these points as the base points.

In this case one can define the system current position given by point A in some biangular coordinates α_1 and α_2. These coordinates are defined by trivial formulas

$$tan(\alpha_1) = \frac{y_A - y_{P1}}{x_A - x_{P1}}, \quad tan(\alpha_2) = \frac{y_A - y_{P2}}{x_A - x_{P2}}, \tag{2}$$

which solution allows to define coordinates of point A in such a way

$$x_A = \frac{(y_{P1} - y_{P2}) \cos\alpha_1 \cos\alpha_2 + x_{P2} \cos\alpha_1 \sin\alpha_2 - x_{P1} \sin\alpha_1 \cos\alpha_2}{sin(\alpha_2 - \alpha_1)};$$
$$y_A = \frac{(-x_{P1} + x_{P2}) \sin\alpha_1 \sin\alpha_2 - y_{P2} \sin\alpha_1 \cos\alpha_2 + y_{P1} \cos\alpha_1 \sin\alpha_2}{sin(\alpha_2 - \alpha_1)}. \tag{3}$$

From the control theory viewpoint one can consider (2) and (3) as direct and inverse observability equations which interrelate system position in non-cartesian and cartesian coordinates.

Analysis of (3) shows that system position depends on multiplications of harmonic functions of its biangular position.

We offer to take into consideration three aliases for nonlinear harmonic functions of two variables

$$cc(\alpha_1,\alpha_2) = \cos\alpha_1 \cos\alpha_2\,; \quad cs(\alpha_1,\alpha_2) = \cos\alpha_1 \sin\alpha_2\,; \quad ss(\alpha_1,\alpha_2) = \sin\alpha_1 \sin\alpha_2, \tag{4}$$

which derivatives are defined in the following way

$$\begin{gathered} D\big(cc(\alpha_1,\alpha_2)\big) = -\dot{\alpha}_1 cs(\alpha_2,\alpha_1) - \dot{\alpha}_2 cs(\alpha_1,\alpha_2); \\ D\big(cs(\alpha_1,\alpha_2)\big) = -\dot{\alpha}_1 ss(\alpha_1,\alpha_2) + \dot{\alpha}_2 cc(\alpha_1,\alpha_2); \\ D\big(ss(\alpha_1,\alpha_2)\big) = \dot{\alpha}_1 cs(\alpha_1,\alpha_2) + \dot{\alpha}_2 cs(\alpha_2,\alpha_1). \end{gathered} \tag{5}$$

The use of (4) allows to simplify motion representation of (3)

$$\begin{aligned} x_A &= \frac{(y_{P1} - y_{P2})\, c\,c\,(\alpha_1,\alpha_2) + x_{P2}\, c\,s\,(\alpha_1,\alpha_2) - x_{P1}\, c\,s\,(\alpha_2,\alpha_1)}{c\,s(\alpha_2,\alpha_1) - c\,s(\alpha_1,\alpha_2)}; \\ y_A &= \frac{(-x_{P1} + x_{P2})\, s\,s\,(\alpha_1,\alpha_2) - y_{P2}\, c\,s\,(\alpha_2,\alpha_1) + y_{P1}\, c\,s\,(\alpha_1,\alpha_2)}{c\,s(\alpha_2,\alpha_1) - c\,s(\alpha_1,\alpha_2)}. \end{aligned} \tag{6}$$

Let us assume that in the most general case all factors in (6) are time-depended variables. Such an assumption means the motions not only the considered system but also the base points which can be defined similar to (1)

$$\begin{gathered} \dot{x}_{P1} = f_{1_{P1}}(x_{P1}, y_{P1}); \quad \dot{y}_{P1} = f_{2_{P1}}(x_{P1}, y_{P1}); \\ \dot{x}_{P2} = f_{1_{P2}}(x_{P2}, y_{P2}); \quad \dot{y}_{P2} = f_{2_{P2}}(x_{P2}, y_{P2}). \end{gathered} \tag{7}$$

In other words, angles α_1 and α_2 can be considered as components of angular position of the studied dynamical system respectively to two ones. Here we consider the simplest way of independent base points 2D motions but one can easy take into account possible depended high-order motions by using crosslinks between base points motions and high derivatives to define these motions. If one substitutes (6) into (1), following equations in the implicit form can be written down

$$\begin{gathered} -\frac{(\dot{\alpha}_1 - \dot{\alpha}_2) cs(\alpha_2,\alpha_1)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big) x_{P1}}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} + \\ + \frac{(\dot{\alpha}_1 - \dot{\alpha}_2) cs(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big) x_{P2}}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} + \\ + \frac{(\dot{\alpha}_1 - \dot{\alpha}_2) cc(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big) y_{P1}}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} - \\ - \frac{(\dot{\alpha}_1 - \dot{\alpha}_2) cc(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big) y_{P2}}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} + \\ + \frac{cs(\alpha_2,\alpha_1) f_{1P1}(x_{P1}, y_{P1}) - cs(\alpha_1,\alpha_2) f_{1P2}(x_{P2}, y_{P2})}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)} - \\ - \frac{cc(\alpha_1,\alpha_2)(f_{2P1}(x_{P1}, y_{P1}) - f_{2P2}(x_{P2}, y_{P2})))}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)} - \\ \frac{(\dot{\alpha}_1 - \dot{\alpha}_2) ss(\alpha_2,\alpha_1)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big)(x_{P2} - x_{P1})}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} + \\ + \frac{(\dot{\alpha}_1 - \dot{\alpha}_2) cs(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2) - ss(\alpha_1,\alpha_2)\big) y_{P1}}{\big(cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)\big)^2} - \\ - \frac{f_{2P1}(x_{P1}, y_{P1}) cs(\alpha_1,\alpha_2) - f_{2P2}(x_{P2}, y_{P2}) cs(\alpha_2,\alpha_1)}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)} - \\ - \frac{f_{1P1}(x_{P1}, y_{P1}) ss(\alpha_1,\alpha_2) - f_{1P2}(x_{P2}, y_{P2}) ss(\alpha_1,\alpha_2)}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)} - \\ -f_2\left(\begin{gathered} \frac{(y_{P1} - y_{P2}) cc(\alpha_1,\alpha_2) + x_{P2} cs(\alpha_1,\alpha_2) - x_{P1} cs(\alpha_2,\alpha_1)}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)}, \\ \frac{(-x_{P1} + x_{P2}) ss(\alpha_1,\alpha_2) - y_{P2} cs(\alpha_2,\alpha_1) + y_{P1} cs(\alpha_1,\alpha_2)}{cs(\alpha_2,\alpha_1) - cs(\alpha_1,\alpha_2)} \end{gathered} \right) = 0. \end{gathered} \tag{8}$$

Analysis of (8) shows that the fourth first summands in each equation define its motion by using current values of base points position as well as the fifth and sixth summands define influence of the base points motions on the system dynamic. This fact allows to rewrite (8) in more specific way for the case of the stationary base points

$$
\begin{aligned}
&-\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cs(\alpha_2,\alpha_1)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)x_{P1}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}+\\
&+\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cs(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)x_{P2}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}+\\
&+\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cc(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)y_{P1}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}-\\
&-\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cc(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)y_{P2}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}-\\
&-f_1\left(\begin{array}{c}\dfrac{(y_{P1}-y_{P2})cc(\alpha_1,\alpha_2)+x_{P2}cs(\alpha_1,\alpha_2)-x_{P1}cs(\alpha_2,\alpha_1)}{cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)},\\ \dfrac{(-x_{P1}+x_{P2})ss(\alpha_1,\alpha_2)-y_{P2}cs(\alpha_2,\alpha_1)+y_{P1}cs(\alpha_1,\alpha_2)}{cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)}\end{array}\right)=0;\\
&\frac{(\dot{\alpha}_1-\dot{\alpha}_2)ss(\alpha_2,\alpha_1)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)(x_{P2}-x_{P1})}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}+\\
&+\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cs(\alpha_1,\alpha_2)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)y_{P1}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}-\\
&-\frac{(\dot{\alpha}_1-\dot{\alpha}_2)cs(\alpha_2,\alpha_1)\big(cc(\alpha_1,\alpha_2)-ss(\alpha_1,\alpha_2)\big)y_{P2}}{\big(cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)\big)^2}-\\
&-f_2\left(\begin{array}{c}\dfrac{(y_{P1}-y_{P2})cc(\alpha_1,\alpha_2)+x_{P2}cs(\alpha_1,\alpha_2)-x_{P1}cs(\alpha_2,\alpha_1)}{cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)},\\ \dfrac{(-x_{P1}+x_{P2})ss(\alpha_1,\alpha_2)-y_{P2}cs(\alpha_2,\alpha_1)+y_{P1}cs(\alpha_1,\alpha_2)}{cs(\alpha_2,\alpha_1)-cs(\alpha_1,\alpha_2)}\end{array}\right)=0.
\end{aligned}
\tag{9}
$$

Equations (8) and (9) are given in the implicit form. One can rewrite them into explicit normal form by solving these equations for derivatives of system angular position components. Such an approach allows us to rewrite notion equations in the normal form as follows

$$\dot{\alpha}_1=g_1(\alpha_1,\alpha_2,x_{P1},x_{P2},y_{P1},y_{P2});\ \dot{\alpha}_2=g_2(\alpha_1,\alpha_2,x_{P1},x_{P2},y_{P1},y_{P2}), \tag{10}$$

here $g_1(.)$ and $g_2(.)$ shows nonlinear functions which are solutions (8) and (9). Due to the complexity of the general solution of these equations we do not give them here and think that functions g_1 () and g_2() differ for (8) and (9).

Thus, one can consider (10) as the model of the considered dynamical system in biangular coordinates. This model shows angular system position relatively some base points. It is clear that in the plane two base points allows us to define the exact system angular position.

2.2. Approximated piecewise linear model

Analysis of the above-given formulas their enough complex structure which makes usage of the proposed approach quite difficult. That is why we offer to replace a nonlinear functions in the initial dynamical system model (1) as well as transformation equations (3), and motion equations for the base points (7) with some piecewise linear functions. Such an approach gives us the possibility to rewrite the above-mentioned equations as follows

$$\dot{x}=a_{11}x+a_{12}y+a_{10};\ \dot{y}=a_{21}x+a_{22}y+a_{20}, \tag{11}$$

$$x_A\ =\ b_{11}\alpha_1+b_{12}\alpha_2+b_{13}x_{p1}+b_{14}y_{p1}+b_{15}x_{p2}+b_{16}y_{p2}+b_{10}; \tag{12}$$

$$y_A = b_{21}\alpha_1 + b_{22}\alpha_2 + b_{23}x_{p1} + b_{24}y_{p1} + b_{25}x_{p2} + b_{26}y_{p2} + b_{20},$$
$$\dot{x}_{P1} = c_{11}x_{p1} + c_{12}y_{p1} + c_{10}; \quad \dot{y}_{P1} = c_{21}x_{p1} + c_{22}y_{p1} + c_{20}. \tag{13}$$
$$\dot{x}_{P2} = d_{11}x_{p2} + d_{12}y_{p2} + d_{10}; \quad \dot{y}_{P2} = d_{21}x_{p2} + d_{22}y_{p2} + d_{20},$$

here a_{ij}, b_{ij}, c_{ij}, and d_{ij} are factors of piecewise linear approximation of nonlinear functions.

Contrary to the above-considered case of nonlinear system such an approach allows us to define highly formalized equations by using matrix and operational calculus.

Let us take into account system initial conditions and rewrite (11)-(13) into matrix operator form

$$s\boldsymbol{X} - s\boldsymbol{X0} = \boldsymbol{AX} + \boldsymbol{A0};\ s\boldsymbol{Y} - s\boldsymbol{Y0} = \boldsymbol{C_D Y} + \boldsymbol{C_D 0};\ \boldsymbol{X} = \boldsymbol{B1\alpha} + \boldsymbol{B2Y} + \boldsymbol{B0}, \tag{14}$$

$$\boldsymbol{\alpha} = (\alpha_1 \quad \alpha_2)^T; \quad \mathbf{X} = (x \quad y)^T; \quad \boldsymbol{Y} = (x_{p1} \quad y_{p1} \quad x_{p2} \quad y_{p2})^T;$$
$$\boldsymbol{\alpha 0} = (\alpha_{10} \quad \alpha_{20})^T; \quad \mathbf{X0} = (x_0 \quad y_0)^T; \quad \boldsymbol{Y0} = (x_{0p1} \quad y_{0p1} \quad x_{0p2} \quad y_{0p2})^T;$$
$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix};\ \boldsymbol{A0} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix};\boldsymbol{B1} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix};\boldsymbol{B2} = \begin{pmatrix} b_{13} & b_{14} & b_{15} & b_{16} \\ b_{23} & b_{24} & b_{25} & b_{26} \end{pmatrix}; \tag{15}$$
$$\boldsymbol{B0} = \begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix};\boldsymbol{C0} = \begin{pmatrix} c_{10} \\ c_{20} \end{pmatrix};\boldsymbol{D0} = \begin{pmatrix} d_{10} \\ d_{20} \end{pmatrix};\boldsymbol{C1} = \begin{pmatrix} c & c_{12} \\ c_{21} & c_{22} \end{pmatrix};\boldsymbol{B1} = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix};$$
$$\boldsymbol{C_D} = \begin{pmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D} \end{pmatrix};\boldsymbol{C_D 0} = \begin{pmatrix} \boldsymbol{C0} \\ \boldsymbol{D0} \end{pmatrix},$$

here s is a Laplace operator, **Y0**, **X0**, and **α0** are initial condition's matrices.

If one substitutes the last equation of (14) into the first one, the dynamical system motion in terms of angular coordinates can be written down

$$s\boldsymbol{B1\alpha} + s\boldsymbol{B2Y} + s\boldsymbol{B0} - s\boldsymbol{X0} = \boldsymbol{AB1\alpha} + \boldsymbol{AB2Y} + \boldsymbol{AB0} + \boldsymbol{A0}. \tag{16}$$

here we consider the third and fourth summands in the left-hand expression as weighted δ-functions to define the generalized system initial conditions which are caused by piecewise linearity of the considered system.

The second summand in the left-hand expression (16) defines the components of system dynamic which are caused by motions of the base points. It is clear that in the case of motionless base points one can equals this summand to zero and consider only the one summand in tight-hand expression as a variable one and others are constants

$$s\boldsymbol{B1\alpha} + s\boldsymbol{B0} - s\boldsymbol{X0} = \boldsymbol{AB1\alpha} + \boldsymbol{AB2Y} + \boldsymbol{AB0} + \boldsymbol{A0}. \tag{17}$$

Thus, equations (16) and (17) can be considered as piecewise linear analogues for (8) and (9).

It is clear that left-hand expression in both of (16) and (17) have some weight matrix **B1** which makes system study more complex. That is why we offer to rewrite (17) in the normal form as follows

$$s\boldsymbol{\alpha} = \boldsymbol{B1^{-1}}(\boldsymbol{AB1\alpha} + \boldsymbol{AB2Y} + \boldsymbol{AB0} + \boldsymbol{A0} - s\boldsymbol{B0} + s\boldsymbol{X0}). \tag{18}$$

Equation (18) define system motion in the biangular coordinates with the motionless base points. Analysis of matrices in (18) shows that dynamic of a transformed system depends on initial system matrices as well as transformation ones. Now we turn our attention into the case of the moved base points and define their position as the result of solution the second equation in (14)

$$\boldsymbol{Y} = (\boldsymbol{E}s - \boldsymbol{C_D})^{-1}(\boldsymbol{C_D 0} + s\boldsymbol{Y0}), \tag{19}$$

where **E** is 4x4 identity matrix.

Substitution of (19) in (16) makes it possible to redefine the transformed system dynamic as follows

$$s\boldsymbol{\alpha} = \boldsymbol{B1^{-1}}(\boldsymbol{AB1\alpha} + \boldsymbol{AB2}(\boldsymbol{E}s - \boldsymbol{C_D})^{-1}(\boldsymbol{C_D 0} + s\boldsymbol{Y0}) + \boldsymbol{AB0} + \boldsymbol{A0} - \tag{20}$$
$$-s\boldsymbol{B2}(\boldsymbol{E}s - \boldsymbol{C_D})^{-1}(\boldsymbol{C_D 0} + s\boldsymbol{Y0}) + s\boldsymbol{B0} - s\boldsymbol{X0}).$$

It is understood that in the most general case system motion in the biangular coordinates depend on its initial conditions and approximation factors which are defined as piecewise linear functions of system state variables in the cartesian and biangular coordinates.

Since this equation is defined by using inverse characteristic matrix of second equation in (14), one can rewrite it in terms of characteristic polynomial-adjugated matrix

$$s\boldsymbol{\alpha} \cdot \mathbf{det}\,(\boldsymbol{E}s - \boldsymbol{C_D}) = \boldsymbol{B1^{-1}}(\boldsymbol{AB1\alpha} \cdot \mathbf{det}\,(\boldsymbol{E}s - \boldsymbol{C_D}) +$$
$$+\boldsymbol{A0} \cdot \boldsymbol{det}(\boldsymbol{E}s - \boldsymbol{C_D}) - s\boldsymbol{B2} \cdot \boldsymbol{adj}(\boldsymbol{C_D} - \boldsymbol{E}s)(\boldsymbol{C_D 0} + s\boldsymbol{Y0}) + \tag{21}$$
$$+s\boldsymbol{B0} \cdot \boldsymbol{det}(\boldsymbol{E}s - \boldsymbol{C_D}) - s\boldsymbol{X0} \cdot \boldsymbol{det}(\boldsymbol{E}s - \boldsymbol{C_D})).$$

If one takes into account an order of matrices **E** and $\mathbf{C_D}$, he finds that (21) define the dynamic of system with moved base points as 5th order motion trajectory. This trajectory can be defined in the normal form if one puts only the highest derivatives of system position's components in left-hand expression of (21) the and writes its lower derivatives in the right-hand-expression. Aa a result, after some trivial transformations one can rewrite (21) as follows

$$s^5\boldsymbol{\alpha} = \sum_{i=1}^{4} s^i \boldsymbol{B_i \alpha} + (\boldsymbol{AB2} - s\boldsymbol{B2}) \cdot \boldsymbol{adj}(\boldsymbol{E}s - \boldsymbol{C_D})(\boldsymbol{C_D 0} + s\boldsymbol{Y0}) + \\ +(\boldsymbol{AB0} + \boldsymbol{A0} + s\boldsymbol{B0} - s\boldsymbol{X0}) \cdot \boldsymbol{det}(\boldsymbol{E}s - \boldsymbol{C_D}). \tag{22}$$

We offer to consider (22) as a matrix piecewise linear motion equation in a biangular coordinates. Due to the use of piecewise constant elements in the matrices one should use this equation to perform cyclic calculations of system angular position, then define its position in cartesian coordinates, and at last use both of them to specify the elements of system matrices.

3. Results and discussions

3.1. Duffing pendulum model in biangular coordinates with motionless base points

Let us show the use of the proposed approach by modeling and simulating the well-known Duffing pendulum which can be considered as 2nd order nonlinear dynamical system

$$\dot{x}_A = y_A; \quad \dot{y}_A = -a_1 x_A - a_2 y_A - a_3 x_A^3 + a_4 \cos a_5 t, \tag{23}$$

here x_1 and x_2 are system state variables, a_i are system factors, and t is a system operating time.

The simplest way to perform a piecewise linear approximation for pendulum nonlinearity which can be highly formalized and implemented with various MCU/FPGA/CPU is using secant lines for the nonlinearity. Such an approximation allows us to replace system nonlinearity with piecewise linear function as follows

$$x_A^3 \approx kx_A + y_0,\ \ x_A \in (x_i, x_{i+1}), i = 1..N\,, \\ k = \frac{x_i^3 - x_{i-1}^3}{x_i - x_{i-1}} = x_i^2 + x_i x_{i-1} + x_{i-1}^2, y_0 = x_i^3 - kx_i = -x_i x_{i-1}(x_i + x_{i-1}) \tag{24}$$

here N is a number of fracture points, x_i is a coordinate of i-th fracture point.

It is clear that such an approach allows us replace nonlinear function with piecewise linear one, which is designed with some lines with the same parameters between neighbor fracture points.

In a similar way we approximate nonlinear surfaces (3) by some planes in sixth dimensional state space. Equation of such a plane can be given in matrix form as follows

$$\begin{vmatrix} \alpha_1 - \alpha_{1ij} & \alpha_2 - \alpha_{2ij} & x_{P1} - x_{P1ij} & x_{P2} - x_{P2ij} & y_{P1} - y_{P1ij} & y_{P2} - y_{P2ij} \\ \alpha_{1(i-1)j} - \alpha_{1ij} & \alpha_{2(i-1)j} - \alpha_{2ij} & x_{P1(i-1)j} - x_{P1ij} & x_{P2(i-1)j} - x_{P2ij} & y_{P1(i-1)j} - y_{P1ij} & y_{P2(i-1)j} - y_{P2ij} \\ \alpha_{1i(j-1)} - \alpha_{1ij} & \alpha_{2i(j-1)} - \alpha_{2ij} & x_{P1i(j-1)} - x_{P1ij} & x_{P2i(j-1)} - x_{P2ij} & y_{P1i(j-1)} - y_{P1ij} & y_{P2i(j-1)} - y_{P2ij} \end{vmatrix} = 0, \tag{25}$$

here indices i and j means ij-fracture point in plane.

Intersections of two neighbor planes allows us to define some line which bound the plane and define some triangulars. Equation for such boundaries can be written down similar to (25)

$$\begin{vmatrix} \alpha_1 - \alpha_{1ij} & \alpha_2 - \alpha_{2ij} & x_{P1} - x_{P1ij} & x_{P2} - x_{P2ij} & y_{P1} - y_{P1ij} & y_{P2} - y_{P2ij} \\ \alpha_{1(i-1)j} - \alpha_{1ij} & \alpha_{2(i-1)j} - \alpha_{2ij} & x_{P1(i-1)j} - x_{P1ij} & x_{P2(i-1)j} - x_{P2ij} & y_{P1(i-1)j} - y_{P1ij} & y_{P2(i-1)j} - y_{P2ij} \end{vmatrix} = 0; \\ \begin{vmatrix} \alpha_1 - \alpha_{1ij} & \alpha_2 - \alpha_{2ij} & x_{P1} - x_{P1ij} & x_{P2} - x_{P2ij} & y_{P1} - y_{P1ij} & y_{P2} - y_{P2ij} \\ \alpha_{1i(j-1)} - \alpha_{1ij} & \alpha_{2i(j-1)} - \alpha_{2ij} & x_{P1i(j-1)} - x_{P1ij} & x_{P2i(j-1)} - x_{P2ij} & y_{P1i(j-1)} - y_{P1ij} & y_{P2i(j-1)} - y_{P2ij} \end{vmatrix} = 0 \tag{26}$$

Thus, for the considered case the problem of approximation multidimensional transformation expressions (3) can be considered as the triangulation problem in the 6D space.

Replacing the system and transformation nonlinearities with the (24) and (25) gives us the possibility to write down following differential-algebraic equations

$$\dot{x}_A = y_A; \quad \dot{y}_A = -(a_1 + a_3 k)x_A - a_2 y_A - a_3 y_0 + a_4 \cos a_5 t, \\ x_A = b_{11}\alpha_1 + b_{12}\alpha_2 + b_{13}x_{p1} + b_{14}y_{p1} + b_{15}x_{p2} + b_{16}y_{p2} + b_{10}; \\ y_A = b_{21}\alpha_1 + b_{22}\alpha_2 + b_{23}x_{p1} + b_{24}y_{p1} + b_{25}x_{p2} + b_{26}y_{p2} + b_{20}. \tag{27}$$

One can consider these equations as some state space equations, where the first and second equations are motion equations and the third and fourth equations are observability equations.

If one substitutes the observability equations into the motion ones and takes into account assumption about motionless base points, the following equations can be written down

$$\begin{gathered}b_{11}\dot{\alpha_1}+b_{12}\dot{\alpha_2}=b_{21}\alpha_1+b_{22}\alpha_2+b_{23}x_{p1}+b_{24}y_{p1}+b_{25}x_{p2}+b_{26}y_{p2}+b_{20};\\ b_{21}\dot{\alpha_1}+b_{22}\dot{\alpha_2}=-\big((a_3k+a_1)b_{11}+a_2b_{21}\big)\alpha_1-\big((a_3k+a_1)b_{12}+a_2b_{22}\big)\alpha_2-\\ -\big((a_3k+a_1)b_{13}+a_2b_{23}\big)x_{P1}-\big((a_3k+a_1)b_{15}+a_2b_{25}\big)x_{P2}-\\ -\big((a_3k+a_1)b_{14}+a_2b_{24}\big)y_{P1}-\big((a_3k+a_1)b_{16}+a_2b_{26}\big)y_{P2}-(a_3k+a_1)b_{10}-\\ -a_2b_{20}-a_3y_0+a_4\cos a_5\,t.\end{gathered}\tag{28}$$

Analysis of (28) shows that both of equations depend on each of derivatives of angular positions. Such a form of differential equations is not a conventional one, that is why we solve (28) for these derivatives

$$\begin{gathered}\dot{\alpha}_1=\frac{a_3b_{11}b_{12}k+a_1b_{11}b_{12}+a_2b_{12}b_{21}+b_{21}b_{22}}{b_{11}b_{22}-b_{12}b_{21}}\alpha_1+\frac{a_3b_{12}^2k+a_1b_{12}^2+a_2b_{12}b_{22}+b_{22}^2}{b_{11}b_{22}-b_{12}b_{21}}\alpha_2+\\ +\frac{a_3b_{12}b_{13}k+a_1b_{12}b_{13}+a_2b_{12}b_{23}+b_{22}b_{23}}{b_{11}b_{22}-b_{12}b_{21}}x_{P1}+\frac{a_3b_{12}b_{15}k+a_1b_{12}b_{15}+a_2b_{12}b_{25}+b_{22}b_{25}}{b_{11}b_{22}-b_{12}b_{21}}x_{P2}+\\ +\frac{a_3b_{12}b_{14}k+a_1b_{12}b_{14}+a_2b_{12}b_{24}+b_{22}b_{24}}{b_{11}b_{22}-b_{12}b_{21}}y_{P1}+\frac{a_3b_{12}b_{16}k+a_1b_{12}b_{16}+a_2b_{12}b_{26}+b_{22}b_{26}}{b_{11}b_{22}-b_{12}b_{21}}y_{P2}+\\ +\frac{a_3b_{10}b_{12}k+a_1b_{10}b_{12}+a_2b_{12}b_{20}+a_3b_{12}y_0+b_{20}b_{22}}{b_{11}b_{22}-b_{12}b_{21}}-\frac{\cos(a_5t)a_4b_{12}}{b_{11}b_{22}-b_{12}b_{21}};\\ \dot{\alpha}_1=-\frac{a_3b_{11}^2k+a_1b_{11}^2+a_2b_{11}b_{21}+b_{21}^2}{b_{11}b_{22}-b_{12}b_{21}}\alpha_1-\frac{a_3b_{11}b_{12}k+a_1b_{11}b_{12}+a_2b_{11}b_{22}+b_{21}b_{22}}{b_{11}b_{22}-b_{12}b_{21}}\alpha_2-\\ -\frac{a_3b_{11}b_{13}k+a_1b_{11}b_{13}+a_2b_{11}b_{23}+b_{21}b_{23}}{b_{11}b_{22}-b_{12}b_{21}}x_{P1}-\frac{a_3b_{11}b_{15}k+a_1b_{11}b_{15}+a_2b_{11}b_{25}+b_{21}b_{25}}{b_{11}b_{22}-b_{12}b_{21}}x_{P2}-\\ -\frac{a_3b_{11}b_{14}k+a_1b_{11}b_{14}+a_2b_{11}b_{24}+b_{21}b_{24}}{b_{11}b_{22}-b_{12}b_{21}}y_{P1}-\frac{a_3b_{11}b_{16}k+a_1b_{11}b_{16}+a_2b_{11}b_{26}+b_{21}b_{26}}{b_{11}b_{22}-b_{12}b_{21}}y_{P2}+\\ -\frac{a_3b_{10}kb_{11}+a_1b_{10}b_{11}+a_2b_{20}b_{11}+a_3y_0b_{11}+b_{20}b_{21}}{b_{11}b_{22}-b_{12}b_{21}}+\frac{\cos(a_5t)a_4b_{11}}{b_{11}b_{22}-b_{12}b_{21}}.\end{gathered}\tag{29}$$

We call (29) as Duffing pendulum piecewise linear equation in the biangular coordinates with motionless base points. Since of the using constant coordinates of base points, one can consider this system as reduced system because its dynamic is defined by only the two differential equations. Contrary to the conventional approach (29) define the system dynamic in the normal form in which each state variable α_i depends on other one as well as coordinates of base points and external signal. The main feature of such system, while it is implemented, is necessary to define two angular coordinates. One can find that such an approach is not convenient.

It is a well-known fact that the use of systems which dynamic are defined in the canonical space allows us to use only one angular coordinates and speed of its changing. Let us transform (29) into canonical form by assuming that only angle α_1 is used

$$\begin{gathered}\dot{\alpha}_1=\omega_1;\\ \dot{\omega}_1=-(a_3k+a_1)\alpha_1-a_2\omega_1-\frac{(a_3k+a_1)(b_{13}b_{22}-b_{12}b_{23})}{b_{11}b_{22}-b_{12}b_{21}}x_{P1}-\frac{(a_3k+a_1)(b_{15}b_{22}-b_{12}b_{25})}{b_{11}b_{22}-b_{12}b_{21}}x_{P2}-\\ -\frac{(a_3k+a_1)(b_{14}b_{22}-b_{12}b_{24})}{b_{11}b_{22}-b_{12}b_{21}}y_{P1}-\frac{(a_3k+a_1)(b_{16}b_{22}-b_{12}b_{26})}{b_{11}b_{22}-b_{12}b_{21}}y_{P2}+\\ +\frac{\cos(a_5t)a_4b_{22}+\sin(a_5t)a_4a_5b_{12}}{b_{11}b_{22}-b_{12}b_{21}}+\frac{((-b_{10}k-y_0)a_3-a_1b_{10})b_{22}+b_{12}b_{20}(a_3k+a_1)}{b_{11}b_{22}-b_{12}b_{21}}\end{gathered}\tag{30}$$

As one can see, the motion equations in a canonical form (30) is simpler than in a normal one. The main feature of these equations is dependence of system output only from one angular position. So, the second angle should not be defined. Nevertheless, (30) clear depend from coordinates of both base points. It can cause some misunderstanding in system structure, which we offer to avoid by considering the summands with coordinates of the second base as components of the first base point's coordinates

$$\begin{gathered}\dot{\alpha}_1=\omega_1;\\ \dot{\omega}_1=(-a_3k+a_1)\alpha_1-a_2\omega_1-\left(\frac{(a_3k+a_1)(b_{13}b_{22}-b_{12}b_{23})}{b_{11}b_{22}-b_{12}b_{21}}+\frac{(a_3k+a_1)(b_{15}b_{22}-b_{12}b_{25})}{b_{11}b_{22}-b_{12}b_{21}}\frac{x_{P2}}{x_{P1}}\right)x_{P1}-\end{gathered}\tag{31}$$

$$-\left(\frac{(a_3\text{k}+\text{a}_1)(b_{14}b_{22}\text{-b}_{12}b_{24})}{b_{11}b_{22}\text{-b}_{12}b_{21}}+\frac{(a_3\text{k}+\text{a}_1)(b_{16}b_{22}\text{-b}_{12}b_{26})}{b_{11}b_{22}\text{-b}_{12}b_{21}}\frac{y_{\text{P2}}}{y_{\text{P1}}}\right)y_{\text{P1}}+$$
$$+\frac{\cos(a_5t)a_4b_{22}+\sin(a_5t)a_4a_5b_{12}}{b_{11}b_{22}\text{-b}_{12}b_{21}}+\frac{((\text{-b}_{10}\text{k-y}_0)a_3\text{-a}_1b_{10})b_{22}+\text{b}_{12}b_{20}(a_3\text{k}+\text{a}_1)}{b_{11}b_{22}\text{-b}_{12}b_{21}}$$

We call (31) as Duffing pendulum equations in the canonical form.

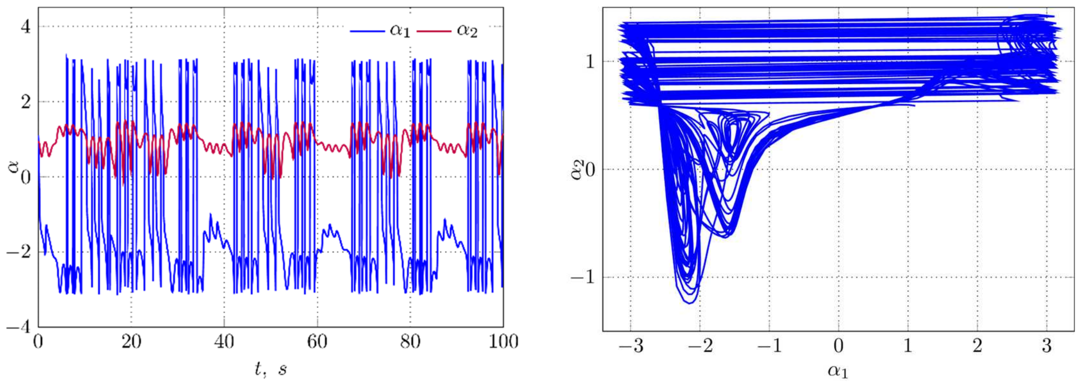
Thus, one can define pendulum dynamic in biangular coordinates in canonical form and obtain equations which are similar to the initial pendulum piecewise linear equations. One can consider these equations as some generalization for the known ones because of adding some summands with piecewise constant factors only.

It is clear that one can implement above-given canonical and normal pendulum equations in the discrete time domain by using known approximations for the derivative operator. We use the Tustin approximation in our studies

$$\dot{x}\approx\frac{2}{T}\cdot\frac{x-z^{-1}x}{x+z^{-1}x}, \quad (32)$$

here z^{-1} is a shit operator and T is a discretization time.

The usage (32) allows to consider the generated by (29)-(31) continuous-time chaotic signals as some chaotic signals sequences which can be easy implemented in various FPGA/MCU devices and boards. Results of numerical solution of (29) and (31) with Arduino Due board are shown in Figure 2 and Figure 4 for normal and canonical systems. In Fig.3 we show numerical solution of classical Duffing equations. We use following pendulum parameters a_1=1; a_2=0.02, a_3=5, a_4=8, a_5=0.5. Base point coordinates are x_{P1}=1, y_{P1}=1, x_{P2}=-2, y_{P2}=-3. As one can see transformation of Duffing pendulum equations in the non-cartesian domain changes its dynamic significantly.



a) System outputs b) System attractor

Figure 2: Numerical solution of normal reduced Duffing pendulum equations.

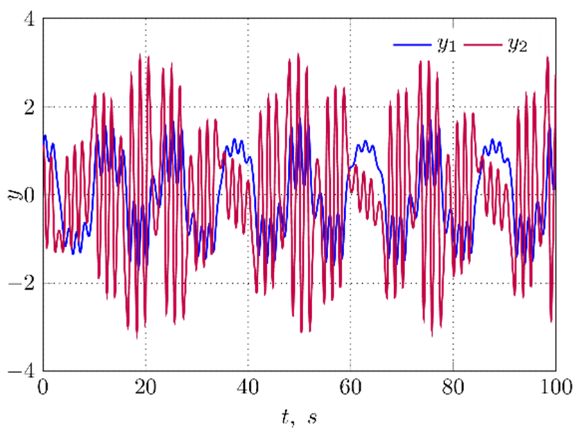


Figure 3: Simulation results of classical Duffing pendulum.

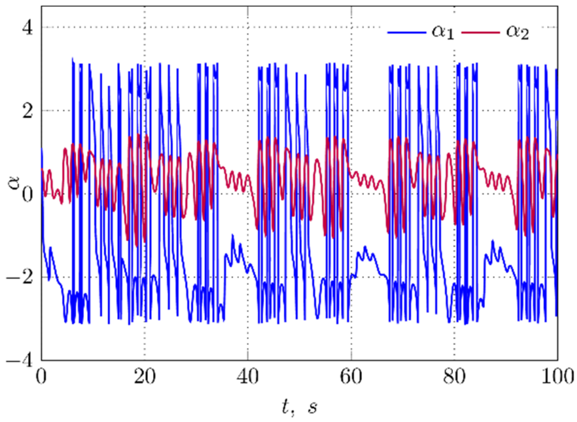


Figure 4: Numerical solution of canonical equations.

3.2. Duffing pendulum model in biangular coordinates with moved base points

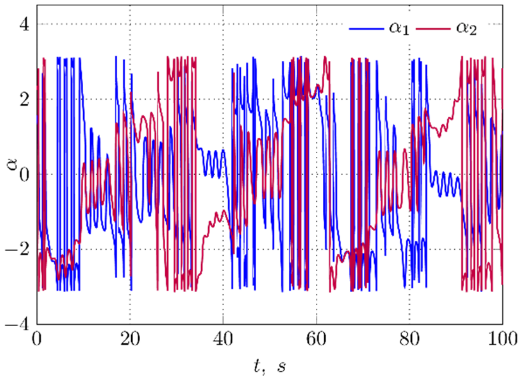
The above-given differential equations are obtained for the case of constant coordinates of base points. Since in the most general case these points can move, we generalize our equations by taking into account of speeds of the base points. We consider the case when both of base points have cyclic trajectories which can be defined by following equations

$$\dot{x}_{P1} = y_{P1};\ \dot{y}_{P1} = -\omega_1^2 x_{P1}; \dot{x}_{P2} = y_{P2};\ \dot{y}_{P2} = -\omega_2^2 x_{P2}, \tag{33}$$

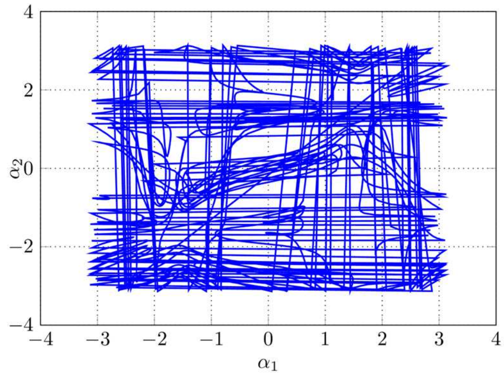
which are used when observability equations in (27) are substituting in the system equations. The solution of obtained equations for the derivatives of system angular position, allows us to rewrite Duffing pendulum equations with moved base points as follows

$$\begin{aligned}
&\dot{x}_{P1} = y_{P1};\ \dot{y}_{P1} = -\omega_1^2 x_{P1}; \dot{x}_{P2} = y_{P2};\ \dot{y}_{P2} = -\omega_2^2 x_{P2};\\
\dot{\alpha}_1 = &\frac{a_3 b_{11} b_{12} k + a_1 b_{11} b_{12} + a_2 b_{12} b_{21} + b_{21} b_{22}}{b_{11} b_{22} - b_{12} b_{21}} \alpha_1 + \frac{a_3 b_{12}^2 k + a_1 b_{12}^2 + a_2 b_{12} b_{22} + b_{22}^2}{b_{11} b_{22} - b_{12} b_{21}} \alpha_2 + \\
&+ \frac{a_3 b_{12} b_{13} k + a_1 b_{12} b_{13} + a_2 b_{12} b_{23} + b_{22} b_{23} - b_{12} b_{24} \omega_1^2 + b_{14} b_{22} \omega_1^2}{b_{11} b_{22} - b_{12} b_{21}} x_{P1} + \\
&+ \frac{a_3 b_{12} b_{15} k + a_1 b_{12} b_{15} + a_2 b_{12} b_{25} + b_{22} b_{25} - b_{12} b_{26} \omega_2^2 + b_{16} b_{22} \omega_2^2}{b_{11} b_{22} - b_{12} b_{21}} x_{P2} + \\
&+ \frac{a_3 b_{12} b_{14} k + a_1 b_{12} b_{14} + a_2 b_{12} b_{24} + b_{12} b_{23} - b_{13} b_{22} + b_{22} b_{24}}{b_{11} b_{22} - b_{12} b_{21}} y_{P1} + \\
&+ \frac{a_3 b_{12} b_{16} k + a_1 b_{12} b_{16} + a_2 b_{12} b_{26} + b_{12} b_{25} - b_{15} b_{22} + b_{22} b_{26}}{b_{11} b_{22} - b_{12} b_{21}} y_{P2} + \\
&+ \frac{b_{12} a_3 b_{10} k + b_{12} a_1 b_{10} + b_{12} a_2 b_{20} + b_{12} a_3 y_0 + b_{20} b_{22}}{b_{11} b_{22} - b_{12} b_{21}} - \frac{b_{12} a_4 \cos(a_5 t)}{b_{11} b_{22} - b_{12} b_{21}};\\
\dot{\alpha}_2 = &- \frac{a_3 b_{11}^2 k + a_1 b_{11}^2 + a_2 b_{11} b_{21} + b_{21}^2}{b_{11} b_{22} - b_{12} b_{21}} \alpha_1 - \frac{a_3 b_{11} b_{12} k + a_1 b_{11} b_{12} + a_2 b_{11} b_{22} + b_{21} b_{22}}{b_{11} b_{22} - b_{12} b_{21}} \alpha_2 - \\
&- \frac{a_3 b_{11} b_{13} k + a_1 b_{11} b_{13} + a_2 b_{11} b_{23} + b_{21} b_{23} - b_{11} b_{24} \omega_1^2 + b_{14} b_{21} \omega_1^2}{b_{11} b_{22} - b_{12} b_{21}} x_{P1} + \\
&- \frac{a_3 b_{11} b_{15} k + a_1 b_{11} b_{15} + a_2 b_{11} b_{25} + b_{21} b_{25} - b_{11} b_{26} \omega_2^2 + b_{16} b_{21} \omega_2^2}{b_{11} b_{22} - b_{12} b_{21}} x_{P2} + \\
&- \frac{a_3 b_{11} b_{14} k + a_1 b_{11} b_{14} + a_2 b_{11} b_{24} + b_{11} b_{23} - b_{13} b_{21} + b_{21} b_{24}}{b_{11} b_{22} - b_{12} b_{21}} y_{P1} - \\
&- \frac{a_3 b_{11} b_{16} k + a_1 b_{11} b_{16} + a_2 b_{11} b_{26} + b_{11} b_{25} - b_{15} b_{21} + b_{21} b_{26}}{b_{11} b_{22} - b_{12} b_{21}} y_{P2} - \\
&- \frac{a_3 b_{10} k b_{11} + a_1 b_{10} b_{11} + a_2 b_{20} b_{11} + a_3 y_0 b_{11} + b_{20} b_{21}}{b_{11} b_{22} - b_{12} b_{21}} + \frac{b_{11} a_4 \cos(a_5 t)}{b_{11} b_{22} - b_{12} b_{21}}.
\end{aligned} \tag{34}$$

We call (34) as full Duffing pendulum normal equations in the biangular coordinates. Contrary to the reduced ones the order of these equations equals to six. Moreover, analysis of terms near state variables in the last equations shows general pattern in terms determination. According to this pattern the terms in system with moved base points are defined as sum of terms in motionless system and some terms which are cause by base points motions. Simulation results for the considered system are shown in Figure.5. We assume that ω_1=0.1 s^{-1} and ω_1=0.2 s^{-1}.



a) System coordinates



b) System attractor

Figure 5: Duffing pendulum dynamic in normal form with moved base points

Analysis of given curves shows that motion of base points changes system motion.

4. Conclusions

Applying the nonlinear transformations to a chaotic system allows us to change its dynamic significantly. Such a transformation gives us the possibility to change system state variables and state space domain where the motions of the considered system are defined. Nevertheless, the initial system's nonlinearities and the proposed formulas can be defined in an analytical way without using any numerical methods. At the same time, the use of piecewise linear functions gives us the possibility to simplify its motion equations and design novel chaotic systems.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

References

[1] H. Bian, et.al., Parameter inversion of high-dimensional chaotic systems using neural ordinary differential equations, in: Proceedings of 13th Data Driven Control and Learning Systems Conference (DDCLS), IEEE, Kaifeng, China, 2024, pp. 400–405. doi: 10.1109/DDCLS61622.2024.10606602.

[2] P. Liu, et.al., Dynamic analysis of novel memristor chaotic systems with influence factors, in: Proceedings of International Conference on Electrical, Automation and Computer Engineering, IEEE, Changchun, China, 2023, pp. 118–122. doi: 10.1109/ICEACE60673.2023.10441899.

[3] M. F. A. Elzaher, M. Shalaby, Two-level chaotic system versus non-autonomous modulation in the context of chaotic voice encryption, in: Proceedings of International Telecommunications Conference, IEEE, Alexandria, Egypt, 2021, pp. 1–6. doi: 10.1109/ITC-Egypt52936.2021.9513947.

[4] O. Kuzenkov, et.al, Nonlinear analysis of bifurcatory properties of mathematical model of subpopulation dynamics in the case of a single niche for subpopulation, in: Proceedings of 3rd International Conference on System Analysis & Intelligent Computing (SAIC), IEEE, Kyiv, Ukraine, 2022, pp. 1–5. doi: 10.1109/SAIC57818.2022.9923003.

[5] B. Pandurangi, et.al., Comparison of bio-inspired and transform based encryption algorithms for satellite images, in: Proceedings of International Conference on Electrical, Electronics, Communication, Computer, and Optimization Techniques (ICEECCOT), IEEE, Msyuru, India, 2018, pp. 1412-1417. doi: 10.1109/ICEECCOT43722.2018.9001344.

[6] T. H. Pham, B. Raahemi, Bio-inspired feature selection algorithms with their applications: a systematic literature review, IEEE Access 11 (2023) 43733-43758. doi: 10.1109/ACCESS.2023.32725.
[7] Y. Mai, et.al., A new short-term prediction method for estimation of the evaporation duct height, IEEE Access 8 (2020) 136036-136045. doi: 10.1109/ACCESS.2020.3011995.
[8] H. R. Rangel, et.al., A brief comparison of different learning methods for wind speed forecasting, in: Proceedings of IEEE International Autumn Meeting on Power, Electronics and Computing (ROPEC), IEEE, Ixtapa, Mexico, 2020, pp. 1-6. doi: 10.1109/ROPEC50909.2020.9258733.
[9] Z. Hu, et.al., Prediction of PM2.5 based on Elman neural network with chaos theory, in: Proceedings of 35th Chinese Control Conference (CCC), IEEE, Chengdu, China, 2016, pp. 3573-3578. doi: 10.1109/ChiCC.2016.7553908.
[10] W. Shi, et.al., Computer simulation analysis of a new hyperchaotic financial system, in: Proceedings of International Conference on Inventive Research in Computing Applications, IEEE, Coimbatore, India, 2021, pp. 1148-1151. doi: 10.1109/ICIRCA51532.2021.9544736.
[11] M. Roshdy, et.al, FPGA implementation of delayed fractional-order financial chaotic system, in: Proceedings of 16th International Computer Engineering Conference (ICENCO), IEEE, Cairo, Egypt, 2020, pp. 51-54. doi: 10.1109/ICENCO49778.2020.9357375.
[12] L. Kirichenko, et.al., Forecasting weakly correlated time series in tasks of electronic commerce, in: Proceedings of 12th International Scientific and Technical Conference on Computer Sciences and Information Technologies (CSIT), IEEE, Lviv, Ukraine, 2017, pp. 309-312. doi: 10.1109/STC-CSIT.2017.8098793.
[13] M. Beisenbi, et.al., Control of deterministic chaotic modes in power systems, in: Proceedings of IEEE International Conference on Smart Information Systems and Technologies (SIST), IEEE, Astana, Kazakhstan, 2023, pp. 516-521. doi: 10.1109/SIST58284.2023.10223493.
[14] Y. Leng, et.al, A chaotic FHSS based power line communication method in parallel buck converters, in: Proceedings of IEEE 4th International Electrical and Energy Conference (CIEEC), IEEE, Wuhan, China, 2021, pp. 1-6. doi: 10.1109/CIEEC50170.2021.9510580.
[15] P. Mukherjee, et.al., Multi-dimensional Lorenz-based chaotic waveforms for wireless power transfer, IEEE Wireless Communications Letters 10(12) (2021) 2800-2804. doi: 10.1109/LWC.2021.3118114.
[16] H. Feng, et.al., Chaotic encryption method for network privacy data based on dynamic data mining, in: Proceedings of 6th International Conference on Smart Grid and Electrical Automation, IEEE, Kunming, China, 2021, pp. 361-365. doi: 10.1109/ICSGEA53208.2021.00088.
[17] G. Mehta, et.al.,Biometric data encryption using 3-D chaotic system, in: Proceedings of 2nd International Conference on Communication Control and Intelligent Systems (CCIS), IEEE, Mathura, India, 2016, pp. 72-75. doi: 10.1109/CCIntelS.2016.7878203.
[18] A. Sultan, et.al., Physical-layer data encryption using chaotic constellation rotation in OFDM-PON, in: Proceedings of 15th International Bhurban Conference on Applied Sciences and Technology (IBCAST), IEEE, Islamabad, Pakistan, 2018, pp. 446-448. doi: 10.1109/IBCAST.2018.8312262.
[19] X. Zheng, Design of parallel big data stream transmission system based on chaotic migration learning, in: Proceedings of 5th International Conference on Electronics, Communication and Aerospace Technology (ICECA), IEEE, Coimbatore, India, 2021, pp. 1432-1435. doi: 10.1109/ICECA52323.2021.9676157.
[20] Z. Yang, et.al., Chaotic optical communication over 1000 km transmission by coherent detection, Journal of Lightwave Technology 38(17) (2020) 4648-4655. doi: 10.1109/JLT.2020.2994155.
[21] H. A. Naser, et.al., Utilizing a high-sensitive and secure communication system for data transmission, in: Proceedings of Second International Conference on Advanced Computer Applications (ACA), IEEE, Misan, Iraq, 2023, pp. 1-5. doi: 10.1109/ACA57612.2023.103468.
[22] A. P. Neto et al., A cyber-physical system for energy efficiency and indoor air conditioning of multiple office rooms, in: Proceedings of Symposium on Internet of Things (SIoT), IEEE, São Paulo, Brazil, 2022, pp. 1-4. doi: 10.1109/SIoT56383.2022.10070204.

[23] C. Sun, et.al., Cyber-physical systems for real-time management in the urban water cycle, in: Proceedings of International Workshop on Cyber-physical Systems for Smart Water Networks (CySWater), IEEE, Porto, Portugal, 2018, pp. 5-8. doi: 10.1109/CySWater.2018.00008.
[24] X. Feng, S. Hu, Cyber-physical zero trust architecture for industrial cyber-physical systems, IEEE Transactions on Industrial Cyber-Physical Systems, 1 (2023) 394-405. doi: 10.1109/TICPS.2023.3333850.
[25] V. Rusyn, et. al., Non-autonomous two channel chaotic generator: computer modelling, analysis and practical realization, in: C.H. Skiadas, Y. Dimotikalis, (Eds.), 14th Chaotic Modeling and Simulation International Conference. CHAOS 2021. Springer Proceedings in Complexity. Springer, Cham, Switzerland, 2021, pp.1-10. doi: 10.1007/978-3-030-96964-6_25.
[26] V. Rusyn, et. al., Computer modelling, analysis of the main information properties of memristor and its application in secure communication system, CEUR Workshop Proceedings 3702 (2024) 216-225.
[27] R. Voliansky, et al., Variable-structure interval-based duffing oscillator, in: Proceedings of 42nd International Conference on Electronics and Nanotechnology (ELNANO), IEEE, Kyiv, Ukraine, 2024, pp. 581-586. doi: 10.1109/ELNANO63394.2024.10756860.
[28] R. Voliansky, N. Volianska, V. Kuznetsov, M. Tryputen, A. Kuznetsova, M. Tryputen, The generalized chaotic system in the hyper-complex form and its transformations. In: Z. Hu, S. Petoukhov, F. Yanovsky, M. He (Eds.), Advances in Computer Science for Engineering and Manufacturing. ISEM 2021, volume 463 of Lecture Notes in Networks and Systems, Springer, Cham, 2022, pp. 350–359. doi: 10.1007/978-3-031-03877-8_31.
[29] R. Volianskyi, V. Kuznetsov, V. Kuznetsov, O. Ostapchuk, V. Artemchuk, N. Volianska, Modeling of dynamical objects with hypercomplex numbers for railway non traction consumers with renewable energy sources, in: Proceedings of International Conference on Electrical, Communication, and Computer Engineering (ICECCE), IEEE, Kuala Lumpur, Malaysia, 2021, pp. 1-6. doi: 10.1109/ICECCE52056.2021.9514151.
[30] L.-y. Kong, et.al., The control of chaotic attitude motion of a perturbed spacecraft, in: Proceedings of Chinese Control Conference, IEEE, Harbin, China, 2006, pp. 166-170. doi: 10.1109/CHICC.2006.280796.
[31] H. Shen, et.al., Data-driven near optimization for fast sampling singularly perturbed systems, IEEE Transactions on Automatic Control 69(7) (2024) 4689-4694. doi: 10.1109/TAC.2024.3352703.
[32] R. Voliansky, et.al., The interval perturbed motion of the generalized nonlinear dynamical plants, in: Proceedings of 4th International Conference on Modern Electrical and Energy System (MEES), IEEE, Kremenchuk, Ukraine, 2022, pp. 1-6. doi: 10.1109/MEES58014.2022.10005720.
[33] V. Rusyn, et al., Analysis and experimental realization of the logistic map using Arduino Pro Mini, in: Proceedings of The Third International Workshop on Computer Modeling and Intelligent Systems (CMIS-2020), Zaporizhzhia, Ukraine, 2020, pp.300-310.
[34] M. Zaliskyi, et al., Methodology for substantiating the infrastructure of aviation radio equipment repair centers, CEUR Workshop Proceedings 3732 (2024) 136–148. URL: https://ceur-ws.org/Vol-3732/paper11.pdf.
[35] O. Solomentsev, et al., Efficiency analysis of current repair procedures for aviation radio equipment, in: I. Ostroumov, M. Zaliskyi (Eds.), Proceedings of the 2nd International Workshop on Advances in Civil Aviation Systems Development. ACASD 2024, volume 992 of Lecture Notes in Networks and Systems, Springer, Cham, 2024, pp. 281–295. doi: 10.1007/978-3-031-60196-5_21.
[36] O. Holubnychyi, et al., Self-organization technique with a norm transformation based filtering for sustainable infocommunications within CNS/ATM systems, in: I. Ostroumov, M. Zaliskyi (Eds.), Proceedings of the 2nd International Workshop on Advances in Civil Aviation Systems Development. ACASD 2024, volume 992 of Lecture Notes in Networks and Systems, Springer, Cham, 2024, pp. 262–278. doi: 10.1007/978-3-031-60196-5_20.