# Indoor Position Anti-jam Via Robust IPNCM

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**Abstract**

Indoor position in interference and multipath environment need adaptive beamforming to realize interference filtering and navigation signal demodulation. However, module mismatches include steer mismatch and array mismatch occurs frequent and will cause adaptive beamforming performance serious degradation. Using uncertainty set to constrains mismatch error is robust but the set size is hard to decide. Aim at this problem, we propose a novel robust IPNCM type algorithm without any uncertainty set constrains but mismatch fix matrixes. The key idea of the new algorithm is constructing a maximum SINR optimal problem then using an iterative direction set to solve it. By using the verification data in the signal of interested, the SINR can be estimated. By analyze the subspace character, a finite iterative direction set can be found, and the NP hard maximum SINR optimal problem can be solved. Unlike most of previous algorithms, the proposed algorithm is much more robust to module mismatches as it adaptive achieve fix matrixes to reduce the whole estimation error include the interference steer vector and the signals of interested steer vector. Numerical results verified that the new algorithm is much more robustness to large mismatch error to the others.

**Keywords**

robust adaptive beamforming, IPNCM, module mismatch, gain and phase error

## 1. Introduction

Indoor position via wireless electromagnetic signal like communication, MIMO all experience performance decline in interference and multipath environment. Adaptive beamforming which adaptive forming beam in the signal of interested (SOI) direction and forming null in the interference direction is a classic research topic in array signal processing. It has been widely applied in mobile communication, and MIMO [1, 2]. While in engineering application, DOA mismatch of SOI caused by module mismatch will lead adaptive beamforming performance decline, especially when the SNR of SOI is large than 0, the SOI will also be filtered out.

To solve the mismatch problem, lots robust adaptive beamforming algorithm have been proposed, but few are suitable to mismatch caused by failed sensors in large array.Dialog loading is always robust to any kind of mismatch by suppress the target signal lower than the adding noise [3, 4, 5]. However, the loading factor is hard to choose in engineering. And the dialog loading algorithms are always loss interference filtering performance. Linear and/or quadratic constrains adaptive beamforming will achieve robust performance to module mismatch by adding extra constrains. This kind of algorithm is target at mismatch caused by steer mismatch or array mismatch which usually are small mismatch [6, 7, 8, 9, 10, 11, 12]. Weight constrains algorithm as robust to outliers as dialog loading, and have the same drawbacks include the constrains are hard to choose and may loss interference filtering performance as in the weight constrains may not exist a global optimal solution [13, 14]. GSC (Generalized sidelobe cancellation) using blocking matrix to obtain the pure interference-noise signals. So, GSC will received robust beamforming performance

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while the SNR of SOI is large than 0 [15, 16]. However only if the blocking matrix designed robust enough to steer mismatch and array mismatch, the SOI will not pass though blocking matrix and will not be filtered out which is hard to realize [17, 18]. IPNCM (interference-plus-noise covariance matrix reconstruction) algorithm robust to the mismatch by rebuilding pure interference-noise matrix based on interference DOA estimation. The module mismatch includes DOA error, array sensor gain and phase error will lead both direction steer vector estimation error of SOI and interference. Both steer vector errors will lead the beamformer performance degrade seriously [29]. By Assuming the true direction steer lies entirely in IPNCM estimated, the true steer vector can be estimated by calculate the intersection of signal subspaces eigen decomposition from array data covariance matrix and IPNCM. Then the estimation error can be departed into 2 subparts: one is perpendicular to signal subspace part, and anther is parallel to signal subspace part. Aim to reduce the estimation error effects, lots robust IPNCM algorithms have been proposed in recent years [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]. However, most of these algorithms only address the parallel to signal subspace part.

The problem studied in this paper is to realize a robust IPNCM type algorithm to both steer vector error parallel to signal subspace parts and perpendicular to signal subspace parts. We show in this paper that by using verification data in SOI, a maximum SINR optimal problem can be constructed and solved by interactive methods in section IV. And we prove that, the algorithm converges to optimal solution with no extra assume. In section V, the proposed algorithm is compared with LCMV, Worst-Case, IPNCM, VSP-IPNCM proposed in [23], RCB-IPNCM proposed in [27], and the robust IPNCM algorithm proposed in [29]. The simulation results shown that the proposed algorithm achieve a much more robust performance than the others.

## 2. Problem statement

To set up the problem, consider a scenario where $\mathrm{K}+1$ far-field narrowband source signals impinge upon an M-sensor linear array from directions-of-arrival (DOAs) $\{\theta_k, k = 1, \ldots, K+1\}$. Without loss of generality, the last source signal is taken as the SOI, while the remaining K signals are regarded as the interferences. The array sensor collected data at time t is denoted by the following $\mathrm{M} \times 1$ vector:

$$\boldsymbol{x}(t) = \underbrace{\tilde{\boldsymbol{a}}(\theta_s)s(t)}_{\triangleq \boldsymbol{x}_s(t)} + \underbrace{\tilde{\boldsymbol{A}}\boldsymbol{i}(t)}_{\triangleq \boldsymbol{x}_i(t)} + \boldsymbol{n}(t) \tag{1}$$

where $\theta_s = \theta_{K+1}$, $\tilde{a}(\theta_s)$ represents the $\mathrm{M} \times 1$ practical (mismatched) steering vector of the SOI, $s(t)$ represents the signal waveform of the SOI, $\tilde{A} = [\tilde{a}(\theta_1), \ldots, \tilde{a}(\theta_K)]$ represents the $\mathrm{M} \times \mathrm{K}$ mismatched array response matrix of the interferences, $\tilde{a}(\theta_k)$ represents the $\mathrm{M} \times 1$ mismatched steering vector of the kth interference, $i(t) = [i_1(t), \ldots, i_K(t)]^T$ is the $\mathrm{K} \times 1$ waveform vector of the interferences, and $n(t) = [n_1(t), \ldots, n_M(t)]^T$ is the $\mathrm{M} \times 1$ additive noise vector. $(*)^T$ is transpose.

Here, the array mismatch is modeled by unknown gain-phase uncertainties of sensors. In this case, the mismatched steering vectors can be expressed by left multiplying an $\mathrm{M} \times \mathrm{M}$ complex diagonal matrix G to the corresponding ideal steering vectors, i.e.,

$$\tilde{\boldsymbol{a}}(\theta) = \mathbf{G}\boldsymbol{a}(\theta) \tag{2}$$

where $a(\theta)$ represents the $\mathrm{M} \times 1$ steering vector of the fully calibrated array with respect to a source signal at DOA $\theta$. For a linear array, $a(\theta)$ is given as

$$\boldsymbol{a}(\theta) = [\, e^{-j\frac{2\pi}{\lambda}d_1 sin\theta} \quad \ldots \quad e^{-j\frac{2\pi}{\lambda}d_m sin\theta} \ldots \quad e^{-j\frac{2\pi}{\lambda}d_M sin\theta}]^T \tag{3}$$

where $d_m$ represents the location of the mth sensor.

The present problem is to find an $\mathrm{M} \times 1$ weighting vector w such that the signal to interference and noise ratio (SINR) is maximized. Toward this end, the following assumptions are made:

1. The source DOAs are pairwise distinct, i.e., $\theta_k \neq \theta_l, \forall k \neq l$.
2. The value $K + 1 < M$ is correctly determined.
3. The source signals are uncorrelated from each other.

The additive noise is complex white Gaussian and is uncorrelated from the source signals.

## 3. Robust IPNCM algorithm via DA

Indoor position via communication, MIMO all usually using a set of training data known to both transmitter and receiver to complete data synchronization [38, 39], which defined as closed-form data aided (DA) scenario [40].

In interference and multipath environment, first using traditional IPNCM algorithm to improve signal receive performance. The signal after IPNCM process can be written as:

$$y_s(t) + y_i(t) = \mathrm{s}(t)e^{-j\theta_s} + \boldsymbol{w}^{\boldsymbol{H}}(\tilde{\boldsymbol{A}}\boldsymbol{i}(t) + \boldsymbol{n}(t)) \tag{4}$$

Then the ML algorithm [41] can use the data aided to estimate the time delay $\tau$ and phase delay $\theta_s$ from transmitter to receiver. The corresponding logarithmic likelihood formula is written as:

$$\boldsymbol{L}(\hat{\theta}_s \quad \hat{\tau}) = \boldsymbol{Re}\int_{\boldsymbol{T}} \mathbf{w}^{\boldsymbol{H}}\mathbf{x}(t)s_v^H(t+\hat{\tau})\, e^{j\hat{\theta}_s}dt \tag{5}$$

In (5), the ***Re*** is take real part from complex data operation, T is sampling interval. Define the data aided is $1 \times N$ row vector $s_v$, N is the data length and the typical value is 200[64].Then the logarithmic likelihood formula can be changed as

$$\boldsymbol{L}(\hat{\theta}_s \quad \hat{\tau}) = \boldsymbol{Re}\sum \mathbf{w}^{\boldsymbol{H}}\mathbf{x}(n)s_v^H(n+\hat{\tau})e^{j\hat{\theta}_s} \tag{6}$$

After time delay $\tau$ and phase delay $\theta_s$ estimated, using data aided $s_v$ can filter out the data aided received by antenna $s_v^H(n+\hat{\tau})e^{j\hat{\theta}_s}$. Because the data aided is uncorrelated with the interference signals and noise, so the filter out data aided process will not distort the interference signals and noise.Then after data aided signals filtered out, the pure interference signals and noise are obtained. Define the pure interference signals and noise signals received by the mth antenna is $\tilde{x}_m(n)$, the new signals $\tilde{x}$ can be written as:

$$\tilde{x}_m(n) = x_m(n) - \frac{\sum x_m(n)s_v^H(n+\hat{\tau})}{\sum s_v(n+\hat{\tau})s_v^H(n+\hat{\tau})}s_v(n+\hat{\tau}) \tag{7}$$

The new antenna array siganls $\tilde{x}$ is:

$$\tilde{\boldsymbol{x}} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_M \end{bmatrix} \tag{8}$$

By using the subspace spanned by the $K + 1$ largest principal eigenvectors of $R = E\{\tilde{x}(t)\tilde{x}^H(t)\}$, a more accurate interference signals space estimator $\tilde{V}_s$ can be calculated. As the signals transmission rate is far faster than indoor position terminals, the interference signals space estimator $\tilde{V}_s$ calculated in train data time period is fitted to position data time period.

**Figure 1:** signal subspace update.

### 3.1. IPNCM

The new signals subspace $\widetilde{\mathrm{V}}_{\mathrm{s}}$ belong to $\mathrm{V}_{\mathrm{s}}$ and more accurate. So applying the new signal subspace projection matrix $\widetilde{\mathrm{V}}_{\mathrm{s}}\widetilde{\mathrm{V}}_{\mathrm{s}}{}^{\mathrm{H}}$ will achieve a better performance:

$$\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\mathbf{a}(\theta) = \tilde{\mathbf{a}}(\theta) + \tilde{e}_{\parallel}(\theta) \tag{9}$$

Written the signals subspace as column vector:

$$\widetilde{\boldsymbol{V}}_{s} = [\boldsymbol{v}_{\mathbf{1}}; \cdots; \boldsymbol{v}_{\mathrm{K}}] \tag{10}$$

In(10), $\mathrm{v}_{\mathrm{i}}$ is the ith eigenvector. $\widetilde{\mathrm{V}}_{\mathrm{s}}\widetilde{\mathrm{V}}_{\mathrm{s}}{}^{\mathrm{H}}\mathrm{a}(\theta) = \tilde{\mathrm{a}}(\theta) + \tilde{\mathrm{e}}_{\parallel}(\theta)$ all can be linear represented by signals subspace $\widetilde{\mathrm{V}}_{\mathrm{s}}$:

$$\tilde{\mathbf{a}}(\theta) + \tilde{e}_{\parallel}(\theta) = \sum_{i=1}^{\mathrm{K}+1} \boldsymbol{k}_{\boldsymbol{i}}(\theta)\boldsymbol{v}_{\boldsymbol{i}} \tag{11}$$

In (11), $\mathrm{k}_{\mathrm{i}}(\theta)$are combination coefficient correspond to $\tilde{\mathrm{a}}(\theta) + \tilde{\mathrm{e}}_{\parallel}(\theta)$, and which are $\mathrm{K} \times 1$ column vector:

$$\boldsymbol{K}(\theta) = \begin{bmatrix} \boldsymbol{k}_1(\theta) \\ \vdots \\ \boldsymbol{k}_K(\theta) \end{bmatrix} \tag{12}$$

Then, the ideal DOA vector after subspace projection can be written as:

$$\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\boldsymbol{A} = \widetilde{\boldsymbol{V}}_{\boldsymbol{s}}[\boldsymbol{K}(\theta_1) \quad \cdots \quad \boldsymbol{K}(\theta_K)] = \widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\boldsymbol{K}_{\boldsymbol{kk}} \tag{13}$$

The $\mathrm{K} \times \mathrm{K}$ matrix $\mathrm{K}_{\mathrm{kk}}$ are composed by combination coefficient column vectors. By using the he ideal DOA vector after subspace projection $\widetilde{\mathrm{V}}_{\mathrm{s}}\widetilde{\mathrm{V}}_{\mathrm{s}}{}^{\mathrm{H}}\mathrm{A}$ to calculate the interference-noise subspace, the IPNCM algorithm can form nulls in the direction of interference signals more accurately:

$$\boldsymbol{R}_{i+n} = \sum_{i=1}^{K} \frac{\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\mathbf{a}(\theta_i)(\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\mathbf{a}(\theta_i))^{\boldsymbol{H}}}{(\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\mathbf{a}(\theta_i))^{\boldsymbol{H}}\widehat{\boldsymbol{R}}^{-\mathbf{1}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}\widetilde{\boldsymbol{V}}_{\boldsymbol{s}}{}^{\boldsymbol{H}}\mathbf{a}(\theta_i)} \tag{14}$$

### 3.2. SOI DOA estimation

As the new interference signals subspace do not contain SOI, so the SOI DOA estimation using the orthogonal subspace projection to calculate:

$$\widehat{\boldsymbol{a}}(\theta_s) = (\boldsymbol{I} - \widetilde{\boldsymbol{V}}_s\widetilde{\boldsymbol{V}}_s^H)\boldsymbol{a}(\theta_s) \tag{15}$$

## 4. Numerical simulations

Chose a linear antenna array with 10 sensors. Distance between adjacent sensors is $\lambda/2$, where $\lambda$ denotes the wavelength corresponding to the frequency of SOI. In far field exist 1 signal and 2 interferences, defined as s, $\mathrm{j}_1$and $\mathrm{j}_2$ with azimuth and directions $71°$, $112°$and $151°$ respectively. The signal verification data use simple {1, -1} sequence. The interference and noise all meet Gaussian distribution. The JNR is 20dB. The number of verification data snapshots is 200 and the totally number of snapshots is 600. The compare algorithm includes LCMV algorithm [7], Worst-Case algorithm [12] with error range parameter 3, IPNCM algorithm [20], Huang's algorithm [29] with integrate number parameter 1024 and estimation error parameter 0.1, Yuan's algorithm [23] with subspace size parameter 0.9, Liu's algorithm [27] with subspace size parameter 0.8, Sun's algorithm [32] with parameter $\eta$ 0.1, Yang's algorithm [35] with parameter $\varepsilon$ 0.3 and $\rho$ 0.9. All the IPNCM-like algorithm with phase sector parameter 8°.

## 4.1. Mismatch by gain and phase error

Gain error meet Gaussian distribution (0,0.052) and phase error meet Gaussian distribution (0, (0.025 π)2). Change the SNR from -10dB to 30dB, the SINR of SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 2.

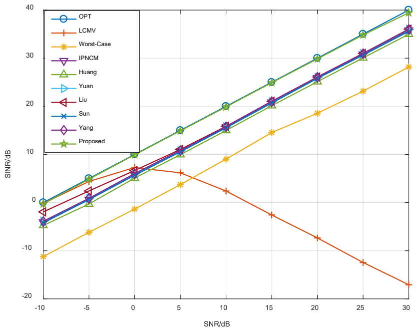

**Figure 2:** SOI SINR by different algorithms var SNR.

In Figure 2, the proposed algorithm achieves a much more robust adaptive beamforming performance and almost achieve the global optimal performance. Next, we exam the snapshot number effect to algorithm performance. Fix the SNR as 15dB, change the snapshots number from 150 to 900, and the verification data is changed from 50 to 300, the SINR of SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 3. In Figure 3, the proposed algorithm achieve robust adaptive beamforming performance while the snapshots are few.

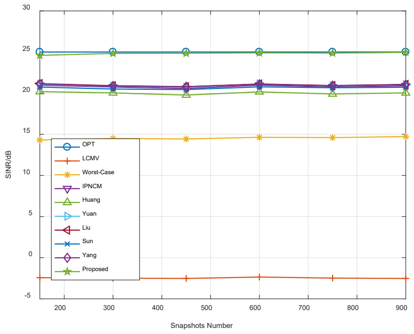

**Figure 3:** SOI SINR by different algorithms var different snapshots number.

## 4.2. Mismatch by signal direction bias

Direction biases of three signals are all meet uniform distribution in section of ±0.02 π. Change the SNR from -10dB to 30dB, the SINR of SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 4.

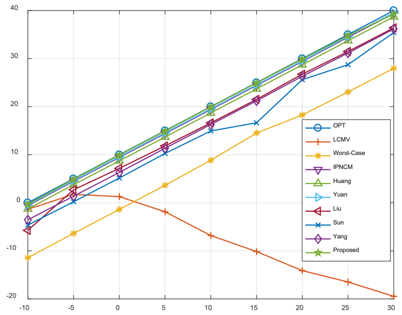

**Figure 4:** SOI SINR by different algorithms var SNR.

In Figure 4, the proposed algorithm almost achieves the global optimal performance. Next, we exam the snapshot number effect to algorithm performance. Fix the SNR as 15dB, change the snapshots number from 150 to 900, and the verification data is changed from 50 to 300, the SINR of SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 5. In Figure 5, the proposed algorithm will achieve a robust adaptive beamforming performance while the snapshots are few.

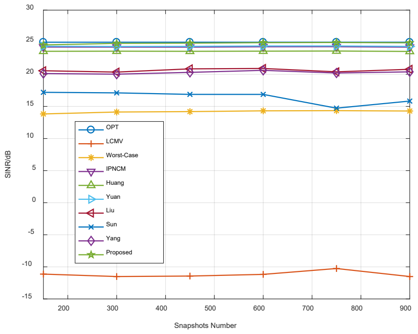

**Figure 5:** SOI SINR by different algorithms var different snapshots number.

### 4.3. Mismatch by array geometry error

Array sensor’s location errors are all meet uniform distribution in section of ±0.01λ. Change the SNR from -10dB to 30dB, the SINR of SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 6.

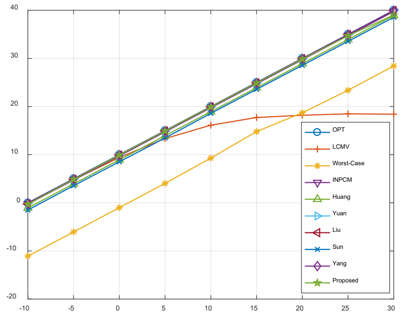

**Figure 6:** SOI SINR by different algorithms var SNR.

In Figure 6, the proposed algorithm almost achieves the global optimal performance. Next, we exam the snapshot number effect to algorithm performance. Fix the SNR as 15dB, change the snapshots number from 150 to 900, and the verification data is changed from 50 to 300, the SINR of

SOI by the proposed robust IPNCM algorithm and the others algorithm were calculated respectively across 200 Monte Carlo simulations and shown in Figure 7. In Figure 7, the proposed algorithm achieves robust adaptive beamforming performance while the snapshots are few.

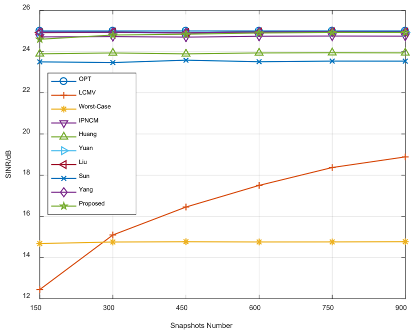

**Figure 7:** SOI SINR by different algorithms var different snapshots number.

## 5. Conclusions

In array signal process, IPNCM is very helpful to interference filtered out as the SNR of SOI is often greater than 0. In practical engineering applications, estimation error caused by gain error and phase error are always exist and will cause SOI loss in IPNCM. The estimation error perpendicular to signal subspace part can be filter out by using subspace algorithm. But the estimation error parallel to signal subspace part still remains and effect the IPNCM performance.Aim at this special problem to filter out parallel part, we proposed a new robust IPNCM type algorithm. With the aiding of the verification data, a more precise IPNCM can be estimated. The new IPNCM-type algorithm will achieve a much more robust performance to the others which is already proved by simulation results.

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