Linear operators for filtering digital images based on two-dimensional local spline of the fifth order

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Abstract

The paper explores the development and application of linear operators for filtering digital images, particularly through the use of fifth-order polynomial splines. The authors emphasize the importance of high-quality, high-speed image processing in enhancing cybersecurity systems, which require accurate and real-time data analysis for applications such as military intelligence and UAV navigation. They propose using B-splines, which offer precise approximation capabilities while maintaining computational efficiency. The paper outlines a method for representing these splines in a manner that reduces computational complexity, making them suitable for real-time processing in environments with high data throughput demands. The research highlights the advantages of using wider window filters and linear operators to maintain image accuracy and fidelity. It demonstrates how these techniques can be effectively applied to a range of signal processing tasks beyond image filtering, including audio and telecommunications. The experimental results presented in the paper indicate that the proposed filters significantly improve the detection and accuracy of special points in digital images, which is crucial for UAV navigation and other critical applications. The authors conclude that the use of these advanced filtering techniques can lead to more efficient and sophisticated image processing systems, with potential applications across various fields. In addition to the practical applications, the paper also discusses the theoretical underpinnings of spline-based filters, including the derivation of low- and high-frequency filter masks. The authors propose that these filters can be used to perform both sub-band filtering and multi-scale analysis of digital images, leading to improved data processing and analysis capabilities. The research contributes to the ongoing development of efficient image processing techniques that are essential for modern technology and various real-time applications.

Keywords

digital image filtering, polynomial splines, linear operators, B-spline, UAV navigation, real-time processing, low-frequency filters, high-frequency filters, image approximation

1. Introduction

Digital images play a crucial role in cybersecurity, providing various opportunities for protecting information and systems. From anomaly detection in video surveillance to user authentication via biometric data, image processing becomes an indispensable tool for modern cybersecurity solutions. The use of high-quality and high-speed image processing algorithms can significantly enhance the effectiveness of cybersecurity systems, ensuring reliable protection against threats and intrusions. Through facial recognition, behavioral pattern analysis, and network activity monitoring, image processing technologies help create comprehensive protection systems capable of adapting to new challenges and threats.

The application of these advanced techniques is extremely important in various fields, such as military intelligence, processing images obtained from unmanned aerial vehicle (UAV) cameras, and any area where high resolution and real-time processing are required [1, 2]. For instance, in military

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intelligence, the ability to quickly and accurately process high-resolution images can lead to more effective decision-making on the battlefield, more precise enemy location detection, and better planning of military operations. Similarly, in processing images from UAVs, the ability to handle large volumes of high-resolution data can improve real-time monitoring and analysis, which is critical for rapid response and strategic planning [3, 4].

One of the main advantages of using linear operators and wider window filters is their ability to maintain high accuracy in processed images. By effectively managing local variations and preserving important details, these techniques ensure that processed images remain true to the original, which is critically important in applications where accuracy is paramount.

Moreover, the development of new mathematical methods and computational algorithms continues to enhance the efficiency and effectiveness of these processing techniques. Researchers are constantly exploring new ways to optimize these algorithms, reduce computational complexity, and improve the overall performance of image processing systems. This continuous innovation is essential for keeping up with the growing demands of modern applications and the ever-increasing volumes of data.

In addition to improving the quality and speed of image processing, these advancements also contribute to the development of more sophisticated tools for image analysis. Enhanced filtering techniques can be combined with machine learning and artificial intelligence to create powerful systems capable of automatic image interpretation and decision-making. This integration of advanced filtering methods with AI can lead to breakthroughs in various fields, providing more accurate predictions, better insights, and more effective interventions.

The importance of real-time processing cannot be overstated, especially in critical applications such as autonomous driving, where quick and accurate image analysis is vital for safety. The ability to process images in real-time ensures that systems can respond promptly to dynamic conditions, making timely decisions that can prevent accidents and improve overall safety.

Furthermore, the versatility of these advanced filtering techniques makes them applicable to a wide range of signal processing tasks beyond image processing. For example, in audio signal processing, these methods can enhance the clarity and quality of sound recordings, improve noise reduction techniques, and facilitate more accurate sound analysis. Similarly, in telecommunications, they can improve signal clarity and reduce interference, leading to better communication quality.

Thus, the pursuit of increased processing speed and efficiency in digitized sequence processing, particularly in image processing, leads to significant advancements in this field. The development and application of linear operators, wider window filters, and advanced mathematical methods are key to achieving these goals. These innovations not only improve the quality and speed of image processing but also open up new possibilities for various applications across different fields. With ongoing research and development, we can expect even more powerful and efficient processing methods that will further expand the capabilities of modern technology.

2. Analysis of research and problem statement

Without dismissing the possibilities of applying methods for processing digital signals as outlined in other publications, it is proposed to consider the use of operators obtained by involving special cases of local polynomial splines close to interpolating on average based on fifth-order B-splines. The reasoning for such an approach is outlined in [5].

The technology of saving (or transmitting) a digital image assumes that each element p of the two-dimensional sequence can be represented as such a sum:

$$p = \bar{p} + \varepsilon,$$

where $\bar{p}$ – the average value; ε – error.

Regarding the appropriateness of $\bar{p}$ it should be noted: the resolution of the frame is essentially decisive in how much information can be concentrated per unit area of the capturing matrix. Thus, it involves an integral characteristic, the discrete analogue of which is precisely the magnitude $\bar{p}$. There is a sufficiently thoroughly researched mathematical apparatus for processing the mentioned data, which meets the requirement of the speed of the relevant computational procedures. These are various types of local splines [6, 7], close to interpolating on average. In works [8–12], examples are provided of using partial cases of spline operators for one and two variables in tasks of sub-band filtering, contrast enhancement, and scaling of one- and two-dimensional sequences, which can be generalized to cases where the width of the local carrier is increased.

Let the division of the real axis be given in steps $\Delta_h: t_i = ih$, $i \in Z$, at each point of which the value of some continuous function is obtained $p(t) \in C^r$, $r \geq 2$, defined on $R_1(-\infty;\infty)$. Information about the function $p(t)$, which is subject to reproduction, specified in partition nodes Δ_h in the form of an integral $\bar{p}_i = \frac{1}{h}\int_{(i-0,5)h}^{(i+0,5)h} p(t)dt$, while the true value of the function $p(t)$ in nodes is defined as follows:

$$p_i = \bar{p}_i + \varepsilon_i, i \in Z. \tag{1}$$

To approximate the function $p(t)$ by values of type (1) in the partition nodes Δ_h, a polynomial spline is introduced based on the B-spline of the fifth order, which is close to the interpolation one on average [13]:

$$S_{5,0}(p,t) = \sum_{i \in Z} p_i B_{5,h}(t - ih),$$

where $B_{5,h}(t)$ is defined as in [12]. Representing a spline in the form of a linear combination of B-splines is not very convenient for implementation in a computing environment. To reduce the computational complexity, it is possible to submit the spline in an explicit form. For example, if you enter a replacement

$$x = \frac{2(t-(i+0,5)h)}{h}, |x| \leq 1,$$

then spline $S_{5,0}(p,t)$ takes the form:

$$\begin{aligned} S_{5,0} = {} & \frac{1}{3840}(-p_{i-2} + 5p_{i-1} - 10p_i + 10p_{i+1} - 5p_{i+2} + p_{i+3})x^5 + \\ & + \frac{1}{768}(p_{i-2} - 3p_{i-1} + 2p_i + 2p_{i+1} - 3p_{i+2} + p_{i+3})x^4 + \\ & + \frac{1}{384}(-p_{i-2} - 3p_{i-1} + 14p_i - 14p_{i+1} + 3p_{i+2} + p_{i+3})x^3 + \\ & + \frac{1}{384}(p_{i-2} + 21p_{i-1} - 22p_i - 22p_{i+1} + 21p_{i+2} + p_{i+3})x^2 + \\ & + \frac{1}{768}(-p_{i-2} - 75p_{i-1} - 154p_i + 154p_{i+1} + 75p_{i+2} + p_{i+3})x + \\ & + \frac{1}{3840}(p_{i-2} + 237p_{i-1} + 1682p_i + 1682p_{i+1} + 237p_{i+2} + p_{i+3}). \end{aligned} \tag{2}$$

If $\|S_{5,0}(p,t)\| = \sup_{|\varepsilon_i|} \max_t |S_{5,0}(\varepsilon,t)|$ – the norm of the spline operator $S_{5,0}(p,t)$, then the statement is true

$$\|S_{5,0}(p,t)\| = \|p(t)\|.$$

Value $\|S_{5,0}(p,t)\|$ characterizes how many times the error can increase during the reproduction of the function with the help of a spline, if the values p_i are specified with an error. Therefore, the norm of the spline operator characterizes the stability of the reproduction of the function $p(t)$.

The error in reproducing the function $p(t)$ using a spline $S_{5,0}(p,t)$ is evidenced by the following statement:

$$\|p(t) - S_{5,0}(p,t)\| = \frac{7}{24}h^2\|p''(t)\| + \varepsilon\|p(t)\| + o(h^2).$$

A two-dimensional spline close to the interpolation spline on average is defined as follows [9].

Let's fix two partitions Δ_{h_t} Δ_{h_q}, axes T and Q points $t_i = ih_t, i \in Z, h_t > 0, q_j = jh_q, j \in Z, h_q > 0$, according to which the partition Δ_{h_t,h_q} of the real plane R_2 is given. Let the partition nodes Δ_{h_t,h_q} have the value of some function $p(t,q) \in C^{r,r}$, $r_1, r_2 \geq 2$: $p_{i,j}$, $i,j \in Z$, and it is assumed that

$$p_{i,j} = \bar{p}_{i,j} + \varepsilon_{i,j},$$

where $\varepsilon_{i,j}$ is error;

$$\bar{p}_{i,j} = \frac{1}{h_t h_q} \int_{(i-1)h_t}^{ih_t} \int_{(j-1)h_q}^{jh_q} p(t,q)dtdq.$$

Then a two-dimensional polynomial spline of the fifth order, close to the interpolated spline on average, can be defined as follows:

$$S_{5,0}(p,t,q) = \sum_{i\in Z} \sum_{j\in Z} p_{i,j} B_{5,h_t}(t - ih_t) B_{5,h_q}(q - jh_q).$$

If

$$\left\| S_{5,0}(p,t,q) \right\| = \sup_{|\varepsilon_{i,j}|} \max_{t,q} \left| S_{5,0}(\varepsilon,t,q) \right|$$

the norm of the spline $S_{5,0}(p,t,q)$, then $\|S_{5,0}(p,t,q)\| = \|p(t,q)\|$.

In addition, for $\forall p(t,q) \in C^{5,5}$ i $\forall \varepsilon > 0$ is true

$$\|p(t,q) - S_{5,0}(p,t,q)\| \le \frac{7h_t^2 \|p''_{t^2}(t,q)\|}{24} +$$

$$+7\frac{h_q^2\|p''_{q^2}(t,q)\|}{24} + \frac{49h_t^2 h_q^2 \|p^{(4)}_{t^2q^2}(t,q)\|}{576} + \varepsilon \cdot \|p(t,q)\| + o(h^4),$$

where $h = max\{h_t, h_q\}$.

Having stated the known provisions about polynomial splines, which are close to interpolation splines in the average, we will set the goal in the following presentation to show the possibility of obtaining one- and two-dimensional filters based on the algorithmizing of computational schemes of splines of the corresponding dimension, thereby extending the properties of the latter to the desired or already known procedures.

3. Obtaining filter masks

Example (2), the unfolded sale of a spline, makes it clear that the operator under consideration is indeed a polynomial. So, for a one-dimensional spline based on a B-spline of the fifth order, close to the interpolation one on average, the following representation is valid:

$$S_{5,0}(p,t) = \sum_i p_i \sum_{c=0}^{r} \gamma_{i,c}^{(5,0)} x^c, \tag{3}$$

where

$$x = \frac{2(t-ih)}{h},\ |x| \le 1; \tag{4}$$

$$\gamma^{(5,0)} = \frac{1}{3840}\begin{pmatrix} 1 & -5 & 10 & -10 & 5 & -1 \\ 237 & -375 & 210 & -30 & -15 & 5 \\ 1682 & -770 & -220 & 140 & 10 & -10 \\ 1682 & -770 & -220 & -140 & 10 & 10 \\ 237 & -375 & 210 & 30 & -15 & -5 \\ 1 & -5 & 10 & 10 & 5 & 1 \end{pmatrix}, \tag{5}$$

for two-dimensional spline–

$$S_{5,0}(p,t,q) = \sum_{i\in Z} \sum_{j\in Z} p_{i,j} \sum_{c_t=0}^{r} \sum_{c_q=0}^{r} \gamma_{i,c_t}^{(5,0)} \gamma_{j,c_q}^{(5,0)} x^{c_t} y^{c_q}, \tag{6}$$

where

$$x = \frac{2(t-ih_t)}{h_t},\ |x| \le 1;\ y = \frac{2(q-jh_q)}{h_q},\ |y| \le 1;\ \gamma_{a,b}^{(5,0)}$$

are determined from the expressions (4) and (5).

Taking into account the quality of spline approximation of smooth functions given in the analysis, to find low-frequency filters we will be primarily interested in the values of spline operators at the partition nodes Δ_h and Δ_{h_t,h_q}. In this case, for $x = 1$ submission (3) takes the form

$$S_{5,0}(p,ih) = \sum_{j=i-2}^{i+2} \gamma\text{H}_{1,j}^{(S_{5,0})} p_j, \tag{7}$$

where

$$\gamma_{H_1}^{(S_{5,0})} = \frac{1}{120}\begin{pmatrix}1\\26\\66\\26\\1\end{pmatrix},$$

and representation (6) gives the functionals:

$$S_{5,0}(p, ih_t, jh_q) = \sum_{i_t=i-2}^{i+2}\sum_{j_q=j-2}^{j+2}\gamma_{H_{2,i_t,j_q}}^{(S_{5,0})} p_{i_t,j_q}, \tag{8}$$

where

$$\gamma_{H_2}^{(S_{5,0})} = \frac{1}{14400}\begin{pmatrix}1 & 26 & 66 & 26 & 1\\26 & 676 & 1716 & 676 & 26\\66 & 1716 & 4356 & 1716 & 66\\26 & 676 & 1716 & 676 & 26\\1 & 26 & 66 & 26 & 1\end{pmatrix}. \tag{9}$$

To obtain high-speed computing schemes, it is sufficient to present expressions (7), (8) with the least number of arithmetic operations.

High-frequency filters based on the considered splines are not difficult to obtain from equality

$$p_i = p_{H_i} + p_{B_i},\ i \in Z,$$

where p_{H_i}, p_{B_i} are low- and high-frequency components. If p_{H_i} choose the value of the splines in the nodes of the partitions Δ_h and Δ_{h_t,h_q},

$$p_{H_i} = S_{5,0}(p, ih) = p_{H_i}^{(S_{5,0})},$$

$$p_{H_{i,j}} = S_{5,0}(p, ih_t, jh_q) = p_{H_{i,j}}^{(S_{5,0})},$$

$p_{B_i}^{(S_{5,0})} = \sum_{j=i-2}^{i+2}\gamma_{B_{1,j}}^{(S_{5,0})} p_j$ in the one-dimensional case, where

$$\gamma_{B_1}^{(S_{5,0})} = \frac{1}{120}\begin{pmatrix}-1\\-26\\54\\-26\\-1\end{pmatrix}.$$

In the two-dimensional case, given the expression $p_{B_{i,j}} = p_{i,j} - p_{H_{i,j}},\ i,j \in Z$, we get:

$$p_{B_{i,j}}^{(S_{r,0})} = \sum_{i_t=i-2}^{i+2}\sum_{j_q=j-2}^{j+2}\gamma_{B_{2,i_t,j_q}}^{(S_{r,0})} p_{i_t,j_q},$$

where

$$\gamma_{B_2}^{(S_{5,0})} = \frac{1}{14400}\begin{pmatrix}-1 & -26 & -66 & -26 & -1\\-26 & -676 & -1716 & -676 & -26\\-66 & -1716 & 10044 & -1716 & -66\\-26 & -676 & -1716 & -676 & -26\\-1 & -26 & -66 & -26 & -1\end{pmatrix}. \tag{10}$$

We will show the possibility of obtaining a contrast filter.

Therefore, when low-frequency filtering is performed using a linear filter, the possibility of obtaining the inverse transformation is provided by solving an elementary algebraic problem.

Let after applying a linear spline filter $S_{5,0}(p,t)$ sequence is obtained

$$P_H^{(S_{5,0})} = \left\{p_i^{(S_{5,0})}\right\}_{i\in Z}.$$

Then expressions for arbitrary indices $i-2, \ldots i+2$ sequences P will be as follows:

$$p_{H_{i-2}}^{(S_{5,0})} = \frac{1}{120}p_{i-4} + \frac{26}{120}p_{i-3} + \frac{66}{120}p_{i-2} + \frac{26}{120}p_{i-1} + \frac{1}{120}p_i,$$

$$p_{H_{i-1}}^{(S_{5,0})} = \frac{1}{120}p_{i-3} + \frac{26}{120}p_{i-2} + \frac{66}{120}p_{i-1} + \frac{26}{120}p_i + \frac{1}{120}p_{i+1},$$

$$p_{H_i}^{(S_{5,0})} = \frac{1}{120}p_{i-2} + \frac{26}{120}p_{i-1} + \frac{66}{120}p_i + \frac{26}{120}p_{i+1} + \frac{1}{120}p_{i+2},$$

$$p_{H_{i+1}}^{(S_{5,0})} = \frac{1}{120}p_{i-1} + \frac{26}{120}p_i + \frac{66}{120}p_{i+1} + \frac{26}{120}p_{i+2} + \frac{1}{120}p_{i+3},$$

$$p_{H_{i+2}}^{(S_{5,0})} = \frac{1}{120}p_i + \frac{26}{120}p_{i+1} + \frac{66}{120}p_{i+2} + \frac{26}{120}p_{i+3} + \frac{1}{120}p_{i+4}.$$

Let's find the coefficients A, B, C, D, E of the inverse transformation, which ensures obtaining the sequence $P\text{к}^{(S_{5,0})} = \left\{p\text{к}_i^{(S_{5,0})}\right\}_{i \in Z}$:

$$p\text{к}_i^{(S_{5,0})} = A \cdot p\text{н}_{i-2}^{(S_{5,0})} + B \cdot p\text{н}_{i-1}^{(S_{5,0})} + C \cdot p\text{н}_i^{(S_{5,0})} + D \cdot p\text{н}_i^{(S_{5,0})} + E \cdot p\text{н}_i^{(S_{5,0})}, i \in Z, \quad (11)$$

so that if possible

$$p\text{к}_i^{(S_{5,0})} = p_i, i \in Z.$$

In other words:

$$p_i = \frac{A}{120} p_{i-4} + \frac{26A + B}{120} p_{i-3} + \frac{66A + 26B + C}{120} p_{i-2} +$$
$$+ \frac{26A + 66B + 26C + D}{120} p_{i-1} + \frac{A + 26B + 66C + 26D + E}{120} p_i +$$
$$+ \frac{B + 26C + 66D + 26E}{120} p_{i+1} + \frac{C + 26D + 66E}{120} p_{i+2} + \frac{D + 26E}{120} p_{i+3} + \frac{E}{120} p_{i+4}.$$

Assuming the uniqueness of the inverse operation, we obtain the following system of linear algebraic equations:

$$\begin{cases} \frac{A}{120} = 0, \\ \frac{26A + B}{120} = 0, \\ \frac{66A + 26B + C}{120} = 0, \\ \frac{26A + 66B + 26C + D}{120} = 0, \\ \frac{A + 26B + 66C + 26D + E}{120} = 1, \\ \frac{B + 26C + 66D + 26E}{120} = 0, \\ \frac{C + 26D + 66E}{120} = 0, \\ \frac{D + 26E}{120} = 0, \\ \frac{E}{120} = 0. \end{cases}$$

This system is incompatible, unlike the following:

$$\begin{cases} 66A + 26B + C = 0, \\ 26A + 66B + 26C + D = 0, \\ A + 26B + 66C + 26D + E = 120, \\ B + 26C + 66D + 26E = 0, \\ C + 26D + 66E = 0, \end{cases}$$

solution is

$$\begin{cases} A = \frac{73080}{160574}, \\ B = -\frac{202800}{160574}, \\ C = \frac{449520}{160574}, \\ D = -\frac{202800}{160574}, \\ E = \frac{73080}{160574}. \end{cases}$$

Therefore, the expression (10) takes the form

$$p\kappa_i^{(S_{5,0})} = \frac{73080}{160574} p\text{н}_{i-2}^{(S_{5,0})} - \frac{202800}{160574} p\text{н}_{i-1}^{(S_{5,0})} + \\ + \frac{449520}{160574} p\text{н}_i^{(S_{5,0})} - \frac{202800}{160574} p\text{н}_{i+1}^{(S_{5,0})} + \frac{73080}{160574} p\text{н}_{i+2}^{(S_{5,0})}, \ i \in Z. \quad (12)$$

It is not difficult to make sure that it is an error δ_i, $i \in Z$ after low-pass filtering and inverse transformation is equal to

$$\delta_i = \frac{1}{160574}(609p_{i-4} + 14144p_{i-3} + 14144p_{i+3} + 609p_{i+4}).$$

Considering that

$$\left|\frac{1}{160574}(609p_{i-4} + 14144p_{i-3} + 14144p_{i+3} + 609p_{i+4})\right| \le |\delta_i|,$$

expression (12) can be presented as follows:

$$p\kappa_i^{(S_{5,0})} = -\frac{609}{160574} \cdot p\text{н}_{i-4}^{(S_{5,0})} - \frac{14144}{160574} \cdot p\text{н}_{i-3}^{(S_{5,0})} + \\ + \frac{73080}{160574} \cdot p\text{н}_{i-2}^{(S_{5,0})} - \frac{202800}{160574} \cdot p\text{н}_{i-1}^{(S_{5,0})} + \\ + \frac{449520}{160574} \cdot p\text{н}_{i}^{(S_{5,0})} - \frac{202800}{160574} \cdot p\text{н}_{i+1}^{(S_{5,0})} + \\ + \frac{73080}{160574} \cdot p\text{н}_{i+2}^{(S_{5,0})} - \frac{14144}{160574} \cdot p\text{н}_{i+3}^{(S_{5,0})} - \frac{609}{160574} \cdot p\text{н}_{i+4}^{(S_{5,0})}.$$

Finally, it can be written as:

$$p\kappa_i^{(S_{r,0})} = \sum_{j=i-4}^{i+4} \gamma\kappa_j^{(S_{r,0})} p\text{н}_j^{(S_{r,0})},$$

where

$$\gamma\kappa_1^{(S_{5,0})} = \frac{1}{160574}\begin{pmatrix} -609 \\ -14144 \\ 73080 \\ -202800 \\ 449520 \\ -202800 \\ 73080 \\ -14144 \\ -609 \end{pmatrix}.$$

Similarly, we get the expression for the two-dimensional case:

$$p\kappa_{i,j}^{(S_{5,0})} = \sum_{i_t=i-4}^{i+4} \sum_{j_q=j-4}^{j+4} \gamma\kappa_{2,i_t,j_q}^{(S_{5,0})} p\text{н}_{i_t,j_q}^{(S_{5,0})},$$

$$\gamma\kappa_2^{(S_{5,0})} = \begin{pmatrix} 0{,}000014 & 0{,}000334 & -0{,}00173 & 0{,}00479 & -0{,}01062 & \dots \\ 0{,}000334 & 0{,}007759 & -0{,}04009 & 0{,}111247 & -0{,}246587 & \dots \\ -0{,}00173 & -0{,}04009 & 0{,}207132 & -0{,}5748 & 1{,}274081 & \dots \\ 0{,}00479 & 0{,}111247 & -0{,}5748 & 1{,}595091 & -3{,}53563 & \dots \\ -0{,}01062 & -0{,}246587 & 1{,}274081 & -3{,}53563 & 7{,}836959 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

and others are determined taking into account the symmetry of the matrix $\gamma\kappa_2^{(S_{5,0})}$.

4. Experimental studies

Since the computational schemes built on the basis of the proposed linear functionals will satisfy the requirement of the software functioning in real time, experimental findings were conducted. We will apply low-frequency filtering of the image in the task of finding special points of the terrain during UAV navigation along the optical channel. We are talking about SIFT-like methods based on differential invariants [7]. Let the continuous model of a two-dimensional image use the model as a function of the impulse call

$$S_{r,0}(p,t,q) = \sum_{i \in Z} \sum_{j \in Z} p_{i,j} B_{r,h_t}(t - ih_t) B_{r,h_q}(q - jh_q), r = 2,3, \dots \quad (13)$$

A set of derivatives for (14) and their analogues in scale-position space up to the order at a given point of the image and at a given scale is called a K-jet and corresponds to a truncated distribution of Taylor for a locally smoothed image fragment [12, 14, 15]. These derivatives together describe the basic types of features in the scale–position space and compactly represent the local structure of the image. For $k = 2$, at the selected scale, 2–jet contains derivatives

$$\left(S'_{r,0}(p,t,q)_t, S'_{r,0}(p,t,q)_q, S''_{r,0}(p,t,q)_{tt}, S''_{r,0}(p,t,q)_{qq}, S''_{r,0}(p,t,q)_{tq}\right). \quad (14)$$

Of the five components of 2-jet for each of the models (13) of order $r = 2,3, \ldots$ four differential invariants with respect to local rotations can be constructed - the magnitude of the gradient $|\nabla S_{r,0}|$, laplacian $\nabla^2 S_{r,0}$, determinant of the Hessian $det\, H_{r,0}$ i the curvature of the scaling curve $\tilde{k}_{r,0}$ (with precision up to the notation of operators of different order):

$$|\nabla S| = {S'}_t^2 + {S'}_q^2, \quad (15)$$

$$\nabla^2 S = S''_{tt} + S''_{qq}, \quad (16)$$

$$det\, H = S''_{tt} S''_{qq} - {S''}_{tq}^2, \quad (17)$$

$$\tilde{k} = {S'}_t^2 S''_{qq} + {S'}_q^2 S''_{tt} - 2S'_t S'_q S''_{tq}. \quad (18)$$

It is worth noting that digital images, in particular photographs, can be considered as the implementation of a discretized function of illumination intensity (raster), which has a multimodal form with pronounced local and global features, and the location of such features on the raster for each individual photo is the value random Let's also take into account that a feature that can be determined on a certain scale of the center of gravity may not be useful during further processing due to the fact that such a feature may not be found on other scales. The value of a specific differential invariant (16) - (19) can be a measure that determines the "usefulness" of this or that feature for further work. Therefore, in the further explanation, we will consider examples and analysis of distributions of such invariants when processing aerial photography data (Figure 1).

Figure 1. Examples of terrain elements based on aerial photography data.

The Table 1 shows the statistical value of coincidences of special points of the test image after smoothing to the number of special points of the reference image, for six test images after applying smoothing by the operator with a mask (9). The left column of the table contains the percentage of the original number of special points.

Table 1
The Percentage of Coincidence of the Locations of Special Points of the Invariant (18) after Digital Image Smoothing by an Operator with a Mask (9)

% of special points at the image	Image **a**	Image **b**	Image **c**	Image **d**	Image **e**	Image **f**
1	16.67	35.71	19.05	25.64	7.69	26.09
2	22.73	34.48	27.27	22.37	10	17.78
4	25.53	30.65	28.4	19.86	14.52	21.98
6	26.67	27.78	25.52	25.79	18	22.92
8	32.35	25.21	41.28	36.93	31.62	24.87
20	35.1	48.47	35.03	27.03	24.39	36.8

As can be seen, after the CC is smoothed, the percentage of coincidences of the number of singular point positions is significantly higher than that of the original image and the smoothed image. This is completely consistent with the well-known approach of SIFT-like methods to select special points by conducting a large-scale analysis of the central nervous system, the essence of which is to compare features at different scales and with different degrees of smoothing and leave those that "appear" at all scales. In contrast to the known approach, we propose to select points for invariants, after one or two smoothings of the original digital image with the mask operator (9), as the one that provides the greatest degree of smoothing.

This approach is less computationally burdensome, and the percentage of stable singularities is quite high for operators (16)–(18) – about 60%.

We also note that an increase in the percentage of the number of invariant values selected at the tails of distributions by more than 8% is not justified, because the number of "features" increases, and their stability actually does not increase, or changes slightly. Leaving 1-2% of the calculated invariants is also inappropriate - there is high variability and not always a high percentage of matches. The number of points at the level of 4-6% should be considered quite acceptable and optimal, according to quality and quantity criteria.

5. Conclusions

In the work, contrast, low- and high-frequency filters of one- and two-dimensional discrete data were obtained on the basis of the algorithmization of computational schemes of splines of the appropriate dimension based on B-splines of the fifth order, close to interpolation ones in the middle one. The justification for introducing the mentioned filters is the smoothing and approximating properties of the spline operators considered in the work. The obtained results can be used when solving tasks of subband two-band filtering and multiple-scale analysis, for digital processing of signals and images.

Testing of the obtained low-frequency filter for digital images was carried out in the task of finding special points of the terrain during UAV navigation along the optical channel.

Experimental studies have shown the expediency of using the filters proposed in the work.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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