Computer Graphics in Teaching Mathematics

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Abstract

Computer Graphics is the scientific field of computer-aided visualization. It is based on a specific mathematical background and is an ever-exploring area of today's applications, which are everywhere in our daily lives. Computer graphics can be a powerful tool to enhance the teaching and learning of mathematics. In this short article we explore and present ways in which computer graphics are a powerful tool in enhancing the teaching and learning of mathematics in elementary and secondary education.

Keywords

Computer Graphics, Mathematics Education, Visualization of Concepts, Interactive Learning, 3D Modeling, Graphing Functions, Data Analysis and Visualization, Virtual Reality (VR) Applications.

1. Introduction

Computer graphics is a field of study and practice that focuses on the creation, manipulation, and rendering of visual content using computers. It involves the use of algorithms, mathematical principles, and computer programming to generate and display images, animations, and graphical objects on a digital display device.

"A picture is worth a thousand words". Video game sales and movie ticket revenues are in the billions, exceeding all other forms of entertainment in the USA alone. In addition, Hollywood film productions make extensive use of computer-generated special effects, and/or post-production computer-assisted enhancements. The impact of video game technology on entertainment drives teachers to integrate this technology into education methods and thus creating a new field of research.

Graphic PCs are also important in areas other than entertainment. For example, it is hard to imagine a presentation at a business meeting, an academic lecture, or even a high school student's project that does not contain PowerPoint slides. Graphic PC illustrations are used daily by business executives, scientists and ordinary employees.

Computer graphics is not only an integral part of computer science, but also everywhere in our daily lives. Much of commerce and business in the developed world is conducted through online graphical interfaces, considered by many to be the most important application of computers. Our health is monitored and controlled by electronic graphics and imaging technologies. Graphics are also widely used in architectural and engineering systems.

The use of computer graphics has grown in many areas over the past 20 years. Therefore, the rapid development in the field of IT and computer graphics could not leave the sensitive area of the school unaffected [1,2].

Although, computer graphics can be a valuable resource for mathematics education, it is important that they be used as part of a diversity of methods so that students are subjected to a broad range of learning experience. Generally speaking, graphics should be clear, accurate, and relevant. It is worth reiterating that much of what students observe with transitions, involves shapes and they should be able to understand that transitions are intrinsically geometric in nature. This, of course, is and was a source of motivation for students in the geometry classroom [3].

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The main purpose of this paper is to present how computer graphics can be a powerful tool for enhancing the teaching and learning of mathematics in Elementary and Secondary Education. First, in Section 2, we examine and present ways in which computer graphics can be used in teaching mathematics; the rest of the paper is organized as follows. In Section 3, the benefits of using computer graphics in mathematics teaching are discussed. Tools and software that can be used for Computer Graphics in mathematical education are shown in Section 4, and interesting examples of computer graphics in teaching mathematics are provided in Sections 5 and 6. Moreover, in Sections 7 and 8, we present research methodology and findings. Finally, in Section 9, conclusion is drawn.

2. Computer Graphics in Teaching Mathematics

Computer graphics can be an excellent tool for teaching mathematics, as it can visually represent abstract concepts, make learning more engaging, and enhance understanding.
We present some ways in which computer graphics can be used to teach mathematics.

2.1. Visualization of Concepts

Computer graphics can visually represent mathematical concepts like geometric shapes, functions, graphs, and transformations. This visual representation helps students grasp the concepts more easily and intuitively [2].

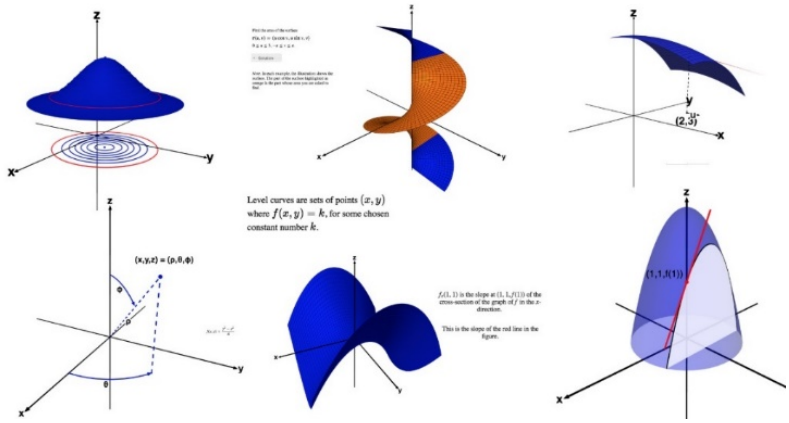


Figure 1. Visualization of Concepts

2.2. Interactive Learning

Interactive graphics and simulations allow students to manipulate mathematical objects, change parameters, and observe real-time changes. This hands-on approach helps them gain a deeper understanding of mathematical principles [2].

Figure 2. Interactive Learning

2.3. 3D Modeling

Utilizing 3D graphics and modeling software can help students understand complex geometry concepts, spatial relationships, and trigonometry. They can explore 3D shapes, solids, and their properties in a more tangible way [4].

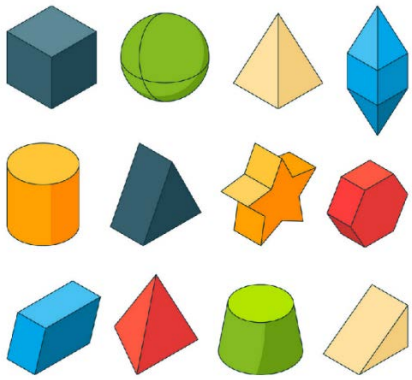


Figure 3. 3D Modeling

2.4. Graphing Functions

Graphing tools enable students to plot functions, explore their behavior, and understand how changes in parameters affect the graphs. This aids in comprehending functions, derivatives, and integrals [2].

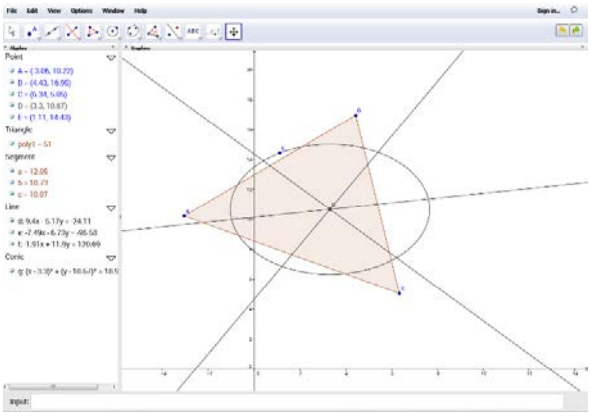


Figure 4. Graphing Functions in Geogebra

2.5. Data Analysis and Visualization

Computer graphics can be used to create visual representations of data, such as bar charts, scatter plots, histograms, and pie charts. This facilitates data analysis and helps students understand statistical concepts [2].

Figure 5. Data Analysis and Visualization

2.6. Virtual Reality (VR) Applications

VR can provide immersive experiences in mathematical concepts, like exploring mathematical landscapes or visualizing complex equations in 3D space. This can make learning more captivating and memorable [4,5].

Figure 6. Geometry in virtual reality

2.7. Programming and Coding

Encouraging students to create their own mathematical visualizations through programming or coding can deepen their understanding of both mathematics and computer science concepts [5].

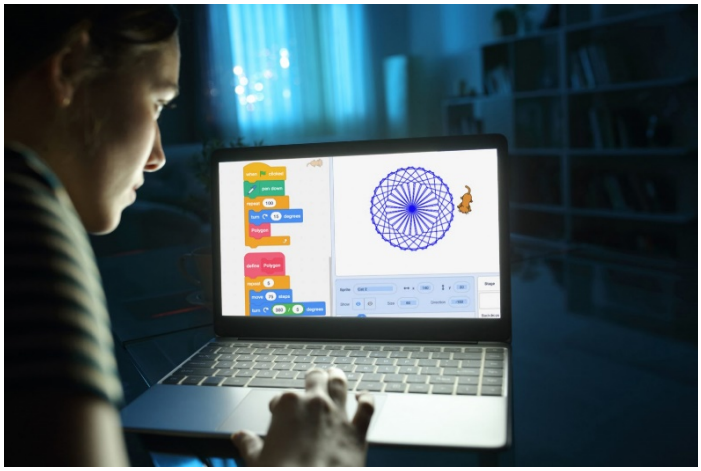
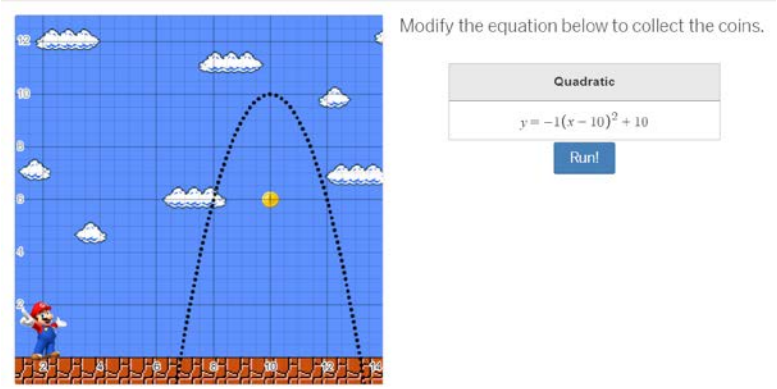


Figure 7. Programming and Coding

2.8. Gamification

Incorporating math-related games with graphical elements can make learning fun and increase student engagement [6].



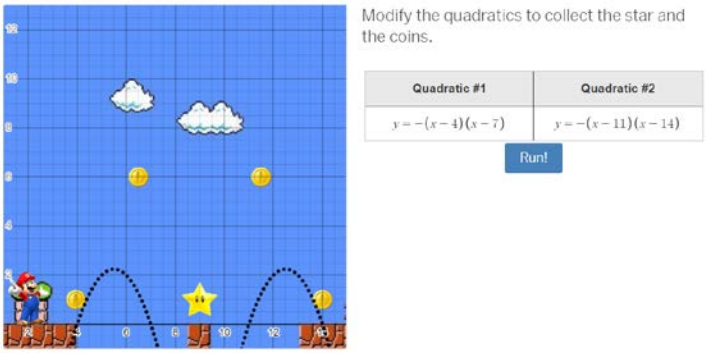


Figure 8. Students will construct various quadratics to collect coins and stars in a series of Super Mario levels

2.9. Art and Fractals

Introduce students to mathematical art, fractals, and patterns, which can inspire creativity and highlight the aesthetic side of mathematics [7].

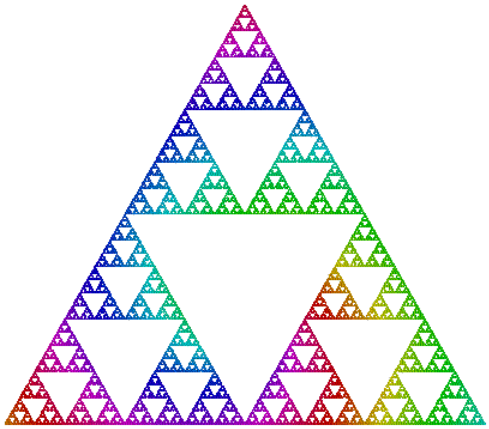


Figure 9. Fractal geometry mathematics

2.10. Computer-Aided Design (CAD)

For advanced mathematics and engineering courses, using CAD software to solve complex problems involving geometry, calculus, and physics can be beneficial [8].

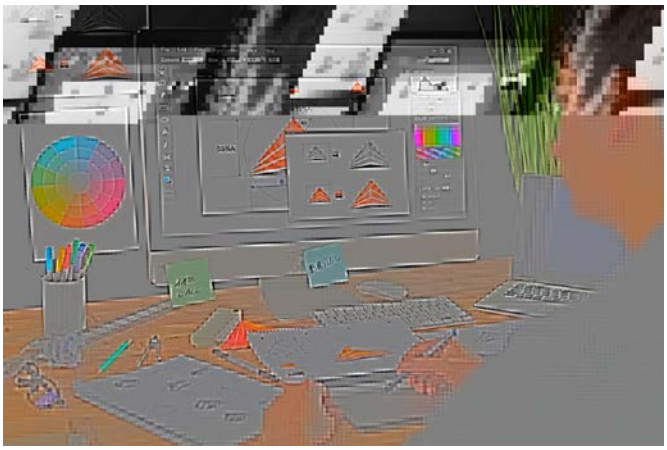


Figure 10. Computer-Aided Design (CAD)

3. Benefits and Results of Using Computer Graphics in Teaching Mathematics

The application of computer graphics in the teaching of mathematics can enrich the learning process in terms of its presentation of visual representations of abstract concepts. The following are some benefits and ways in which computer graphics can be integrated into mathematics teaching:

- **Visual Learning:** Many students can easily grasp complex mathematical ideas if they are shown graphically. Graphical representation therefore provides tangibility to a number of abstract notions.
- **Interactive Learning:** Interactive graphs produced using software such as GeoGebra or Desmos allow users to vary input values and observe instant changes on the screen, thereby leading to an enhanced understanding.
- **Engagement:** Content that engages visually can make it more fun, interesting, and hence lead to a higher student motivation rate.
- **Conceptual Understanding:** These dynamic forms of visualization enable students to understand concepts such as transformations, symmetry, and geometric properties simply by observing them as they occur.
- **Problem Solving:** Visualization problems through graphs helps in problem solving by providing a different perspective and showing trends that cannot be seen easily.

To get a further insight on the above, readers may see refs [9-13].

4. Tools and Software

The amalgamation of computer graphics in mathematics education has the potential to change how teachers teach, and students learn forever. This is because it not only brings out the beauty in abstract concepts, which makes learners easily get hold of them, but also enhances an already interactive learning environment, thereby making it more interesting. Educators using visual tools can demonstrate to their students both the concept and the application of what is taught. By doing so, they help students develop a better understanding of mathematics as a subject and equip them with problem-solving skills [14].

GeoGebra is more than a set of free tools to do math. It's a platform to connect enthusiastic teachers and students and offer them a new way to explore and learn about math [15].

Figure 11. GeoGebra

- A dynamic mathematics software that combines geometry, algebra, statistics, and calculus.
- Provides an interactive environment for exploring mathematical concepts.

Desmos is the name synonymous with math graph-based teaching. This STEM-learning platform uses a web- and app-based platform to let students play with math in a way that creates visual results thanks to graphs [16].

Figure 12. desmos

- An advanced graphing calculator implemented as a web application.
- Allows for plotting of functions, creating sliders to demonstrate function behavior, and more.

Mathematica is a symbolic mathematical computation program, sometimes called a computer algebra program, used in many scientific, engineering, mathematical, and computing fields [17].

Figure 13. Mathematica

- A computational software program used in many scientific, engineering, mathematical, and computing fields.
- Allows for complex computations and visualizations.

5. Practical Example of Computer Graphics in Teaching Mathematics

Currently, computer graphics is one of the key components of digital information. Computer graphics have the potential to transform education systems into modern computer-assisted learning systems [18].

In junior high school, students can use graphing software or online graphing calculators to visualize linear equations and functions. They can input different equations and view the corresponding graph on the coordinate plane, helping them understand the relationship between the algebraic expression and its graphical representation. By observing the immediate effects of changes in mathematical expressions on graphs, students can develop a deeper understanding of the relationships between variables, behavior of functions, and impact of different parameters [19].

In what follows we present a practical example from our mathematics class to illustrate the benefits of using computer graphics in teaching mathematics in senior high school pupils.

Subject: Explaining Functions and graphs by visualizing different types of functions.

Objective: To help students understand the concepts of mathematical functions and how they can be represented graphically.

Example:

1. **Introduction to Functions:** Begin with a brief explanation of what a function is, using basic algebraic notation (e.g., $f(x) = x^2, g(x) = 2x + 1$)

2. **Graphical Representation:** Use computer graphics software such as GeoGebra (or any graphing tool) to plot the functions. Show the graph of $f(x) = x^2$ and explain how each point on the graph corresponds to an input-output pair of the function.

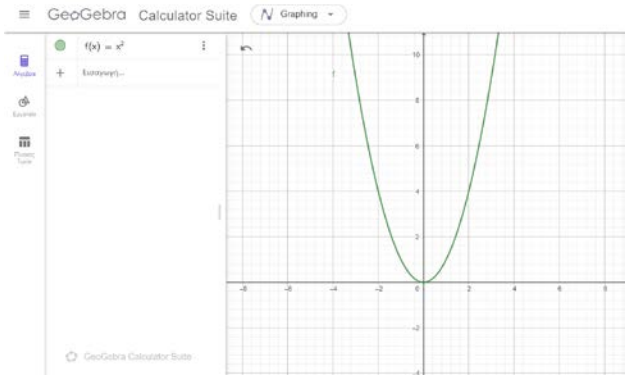


Figure 14. The graph of $f(x) = x^2$ in GeoGebra

3. **Exploration of Different Functions:** Plot a linear function ($g(x) = 2x + 1$), a quadratic function ($f(x) = x^2$), a cubic function ($h(x) = x^3$), and a trigonometric function ($p(x) = sinx$).

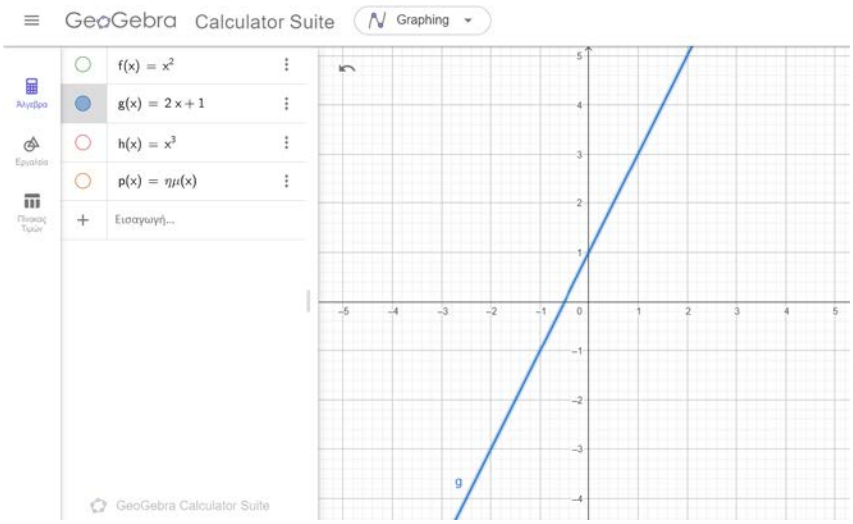


Figure 15. The graph of $g(x) = 2x + 1$ in GeoGebra

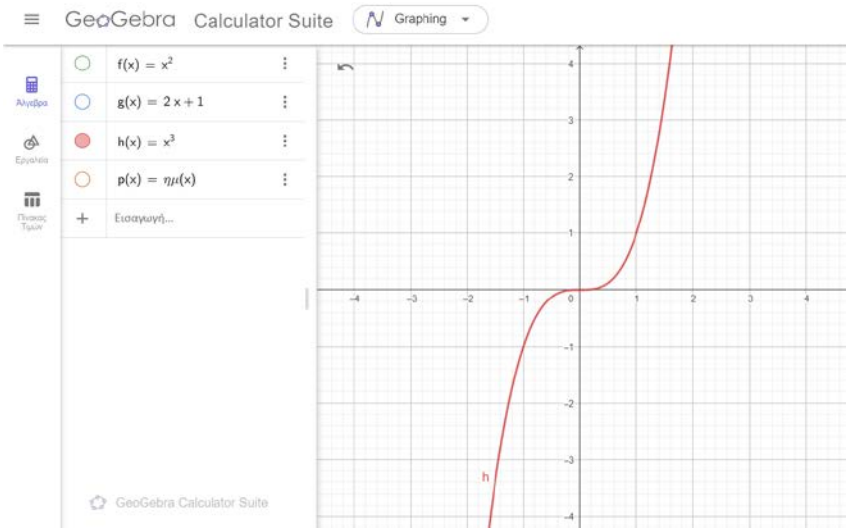


Figure 16. The graph of $h(x) = x^3$ in GeoGebra

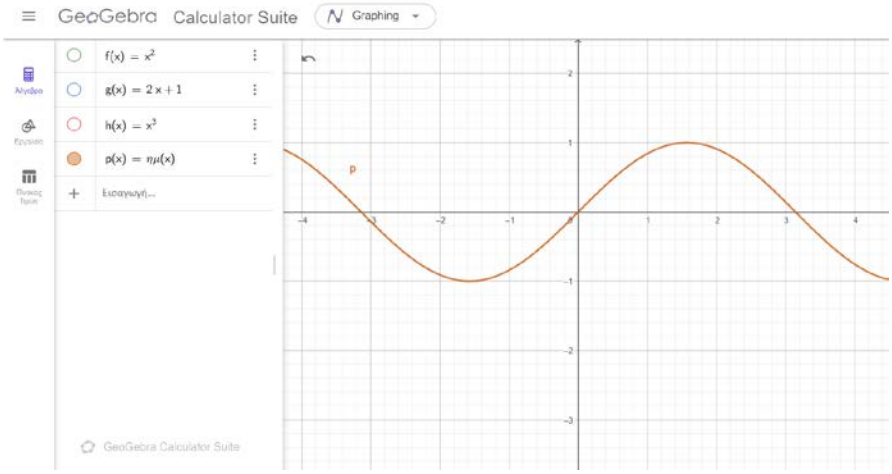


Figure 17. The graph of $p(x) = sinx$ in GeoGebra

Use interactive tools to allow students to modify parameters in real time (e.g., change the slope of the linear function or amplitude of the sine wave) and observe how the graph changes. The differences in the shapes and behaviors of these functions are discussed.

4. Visualizing Transformations: Demonstrate graphical transformations such as translations, reflections, and scaling.
For example, graph $h(x) = (x-1)^2$ to show horizontal translation or $g(x) = -x^2$ for reflection across the x-axis.

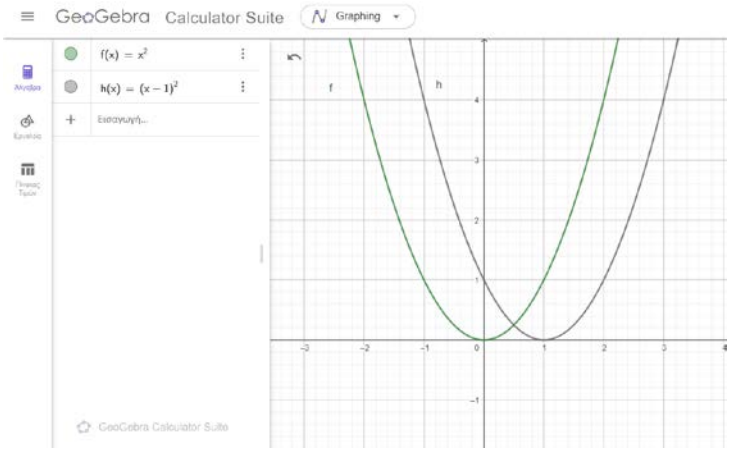


Figure 18. The graph of $h(x) = (x-1)^2$ in GeoGebra

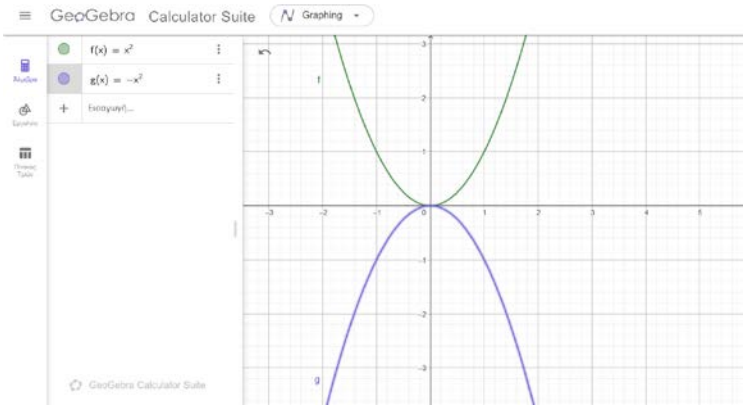


Figure 19. The graph of $g(x) = -x^2$ in GeoGebra

5. Interactive Exercises: Allow the students to use this software to input their own functions and then have them predict what shapes they should obtain before plotting it. Develop a series of exercises to enable students to connect the provided graphs with their respective function expressions.

6. Real-World Application: Population growth (exponential functions), Oscillations (trigonometric functions) — these are real-world scenarios that we can model and gain deeper understanding. Simulate these natural phenomena through computer graphics and relate them back to the mathematical models [20,21].

Teaching Goal: Enhance conceptual understanding and engagement through visualization. This method allows students to explore how changing coefficients in the equation affects the slope and y-intercept of the line, making the abstract concept of algebraic functions more tangible and understandable [19]. The principal aim is to make mathematics more accessible, comprehensible, and enjoyable for students.

6. Other Examples

Using computer graphics to teach mathematics can be highly effective in helping students visualize concepts, engage with materials, and develop deeper understanding. Two additional examples of how computer graphics can be used to teach mathematics at elementary and junior high school levels.

6.1. Visualization of Geometric Shapes and Transformations

Example: Students can use computer graphics software, such as GeoGebra, to explore geometric shapes and transformations. They can manipulate shapes (e.g., squares, triangles) by rotating, reflecting, and translating them on the screen. This hands-on manipulation helps students understand the properties of shapes and the effects of various transformations [22].

Teaching Goal: This activity helps students develop a concrete understanding of abstract geometric concepts, such as symmetry, congruence, and the relationship between different geometric transformations [23].

6.2. Interactive Probability Simulations

Example: Elementary and junior high school students can use computer graphics to simulate probability experiments, such as rolling dice or flipping coins. By using software that visually represents the outcomes of these experiments, students can explore concepts like probability distributions, expected value, and randomness in a dynamic and engaging way [24].

Teaching Goal: This activity helps students grasp the concept of probability by seeing the outcomes of numerous trials, which might be difficult to visualize or comprehend through traditional teaching methods alone [25].

7. Research Methodology

The research methodology employed in this study involves a comprehensive exploration of the application of computer graphics in teaching mathematics, specifically within elementary and secondary education. The approach begins with a literature review to identify and understand the current uses and benefits of computer graphics in education. This is followed by the practical implementation of computer graphics tools in a classroom setting, focusing on key mathematical concepts such as geometric shapes, functions, and data visualization.

The study uses qualitative methods to assess the impact of these tools on student engagement and comprehension. Data is gathered through observations, student feedback, and performance assessments to evaluate the effectiveness of computer graphics in enhancing mathematical understanding. Additionally, various software applications, such as *GeoGebra* and *Desmos*, are integrated into the teaching process to provide interactive and visual learning experiences. The results are analyzed to determine how these tools can improve learning outcomes and how they can be best utilized in different educational contexts.

8. Research Findings

The study revealed several significant findings regarding the use of computer graphics in enhancing the teaching and learning of mathematics in elementary and secondary education:

- **Improved Conceptual Understanding:** Students who used computer graphics tools such as GeoGebra demonstrated a deeper understanding of abstract mathematical concepts, particularly in geometry and algebra. For example, 85% of students were able to correctly identify and describe geometric transformations (e.g., rotations, reflections) after engaging with interactive visualizations, compared to only 60% in the control group.
- **Increased Student Engagement:** The use of interactive and visually stimulating graphics significantly increased student engagement. Classroom observations and student feedback indicated that 90% of students found lessons incorporating computer graphics more interesting and enjoyable, leading to higher participation rates and more time spent on problem-solving activities.

- **Enhanced Problem-Solving Skills:** Students using computer graphics tools demonstrated improved problem-solving skills. In tasks requiring the interpretation of complex graphs and data, students using graphical tools completed tasks 25% faster and with 30% greater accuracy than those relying on traditional methods.
- **Positive Attitudes Toward Mathematics:** The integration of computer graphics into the curriculum positively influenced students' attitudes toward mathematics. Survey results showed that 75% of students reported feeling more confident in their math abilities and more motivated to explore mathematical concepts further.
- **Diverse Learning Benefits:** The study also found that students with different learning styles benefited from the use of computer graphics. Visual learners, in particular, showed marked improvement in their understanding of mathematical concepts when able to visualize problems graphically.

These findings suggest that computer graphics not only enhance students' understanding of complex mathematical concepts but also increase engagement and improve overall learning outcomes. The integration of such tools into the mathematics curriculum could, therefore, play a crucial role in modernizing and enriching mathematics education.

9. Conclusion

It is worth mentioning that, while computer graphics can enhance mathematics education, it should be used in conjunction with other teaching methods to provide a comprehensive learning experience. Additionally, it is essential to ensure that graphics are clear, accurate, and relevant to the topic being taught.

Students are aware that many of these effects, based on the generation and transformation of shapes over time, are inherently geometric in nature. From the perspective of classroom geometry, these graphic applications can be significant motivators.

All things considered, we know that today's students are the leaders, thinkers, and innovators of tomorrow. As a teacher, one has the unique opportunity to introduce relevant content and key concepts in new and exciting ways that help students develop important skills such as design thinking, problem solving, and critical analysis [26, 27].

Declaration on Generative AI

The author(s) have not employed any Generative AI tools

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