Identification and Analysis of the Structure of Rhythmic Variability of Electricity Consumption Time Series for Estimating and Forecasting Energy Flows*

Ihor Javorskyj[1,2], Valentyna Pleskach[3,*], Roman Yuzefovych[1,4], Oleh Lychak[1]

[1] Karpenko Physico-Mechanical Institute of the NAS of Ukraine, 79060, 5 Naukova Str., Lviv, Ukraine
[2] Bydgoszcz University of Science and Technology, 85796, 7 Al. Prof. S. Kaliskiego, Bydgoszcz, Poland
[3] Taras Shevchenko National University of Kyiv, Volodymyrs'ka Str. 64/13, Kyiv, 01601, Ukraine
[4] Lviv Polytechnic National University, 79013, 12 Bandera Str., Lviv, Ukraine

Abstract

This article is more of an overview, but it shows the importance of analysing the structure of the rhythmic variation of the time series of electricity consumption. The integration of renewable energy sources and the development of multi-directional smart grids present challenges for the stability and management of modern energy networks. Unlike traditional unidirectional systems, smart grids incorporate feedback loops and distributed energy generation, introducing high-frequency fluctuations that can propagate through the network, so This study identifies and analyzes the rhythmic variability in electricity consumption time series using deterministic and probabilistic approaches, including Fourier and spectral-correlation theories. An application of Periodically Correlated Random Processes (PCRP) is proposed to model and forecast energy flows, offering a robust framework for identifying hidden variability structures. The findings support enhanced decision-making for network regulation and the optimization of smart grids architectures, ensuring resilience and reliability in energy distribution.

Keywords

Smart grids, renewable energy integration, rhythmic variability, PCRP, stochastic modeling, energy flow forecasting

1. Introduction

The development of the Green transmission concept and the adoption within the EU of a number of regulatory acts within the framework of the European Green Deal, which oblige to integrate into a single energy network practically all available energy sources down to the level of individual households, the integration of low-power renewable energy sources into common "global" consumption networks have created serious challenges for the organization and management of the operation of such a network.

This kind of network, unlike previous, classical, essentially unidirectional networks of the type "from power generation – through distribution networks – to consumers", is transformed into a multi-stream multi-directional network (Smart Grids) on the type of modern Internet data transmission networks.

This network itself is a generating multi-level distributed network, since it provides for reverse energy flows, and each, even the smallest of its nodes is a separate dynamic system with feedback, i.e. it can also potentially generate consumption-generation fluctuations. Minor fluctuations in consumption are not critical in the case of a unidirectional system, although there have been significant accidents (blackouts) that have led to major losses in power grids that arose as a result of minor "hidden" fluctuations. It should be noted separately that a significant component of electricity consumption is chaotic, and known low-frequency (daily, weekly, monthly) consumption cycles are known and non-critical, they are "global", i.e. they cover a significant part (or parts) of the unidirectional distribution network. In the case of a generating multi-level network with feedback in nodes, short-term high-frequency fluctuations may occur, primarily local, which can quickly lead to local oscillations and, as it was in the case of "unidirectional"

8th International Scientific and Practical Conference Applied Information Systems and Technologies in the Digital Society AISTDS'2024, 2024, October 1, Kyiv, Ukraine

*Corresponding author.

javor@pbs.edu.pl (I. Javorskyj); v.pleskach64@gmail.com (V. Pleskach); roman.yuzefovych@gmail.com (R. Yuzefovych); olehlychak2003@yahoo.com (O. Lychak)

0000-0003-0243-6652 (I. Javorsky); 0000-0003-0552-0972 (V. Pleskach); 0000-0001-5546-453X (R. Yuzefovych); 0000-0001-5559-1969 (O. Lychak)

networks, spread over a short period of time to significant fragments of the entire “global” network. It is clear that scenarios of this type are mostly probabilistic, but ignoring them, given the obvious ability of active networks or electrical circuits with complex feedback loops to generate oscillations, including those of a chaotic nature, is at least unreasonable. Therefore, identifying and analyzing the structure of the hidden rhythmic variability of time series of energy consumption in networks can become a serious safeguard against possible accidents on the one hand and an important tool for supporting network regulation decisions and even a basis for optimizing its structure. This research is highly significant for the future of energy development due to its alignment with key trends and challenges in the energy sector. It is important is integration of renewable energy sources; smart grid evolution, decentralization of energy systems. Future energy systems will heavily rely on renewable sources like solar and wind, which are inherently variable and unpredictable.

By analyzing rhythmic patterns in energy consumption and generation, this research helps to integrate these intermittent sources more seamlessly into the grid, ensuring stability and efficiency.

As energy systems evolve into smart grids, the complexity of managing bidirectional energy flows increases. This research contributes to developing intelligent systems capable of predicting energy demands and generation patterns, crucial for optimizing the operation of smart grids.

The shift towards decentralized energy systems, where individual households and small-scale producers contribute energy, requires advanced forecasting and analysis tools.

Understanding consumption rhythms aids in managing these decentralized networks effectively, minimizing losses and maximizing energy use.

Not to be overlooked also energy security and resilience, so the shift towards decentralized energy systems, where individual households and small-scale producers contribute energy, requires advanced forecasting and analysis tools and understanding consumption rhythms aids in managing these decentralized networks effectively, minimizing losses and maximizing energy use. With the increasing complexity of energy systems, the risk of disruptions and blackouts rises.

This research can provide tools to identify and mitigate hidden fluctuations and anomalies that could compromise energy security.

Forecasting energy flows accurately allows for better planning and allocation of resources, reducing waste and operational costs. This efficiency is critical for the sustainable growth of energy infrastructure and keeping energy affordable.

Meeting global climate goals, such as those outlined in the Paris Agreement and European Green Deal, requires innovative solutions for reducing carbon emissions.

By optimizing energy systems and integrating renewable resources, this research supports the transition to low-carbon and sustainable energy models. The methods developed can drive technological innovation in energy analytics, smart sensors, and IoT applications, laying the groundwork for future advancements in energy management systems.

2. Approaches to the research of oscillatory processes

During studying of oscillatory processes two main attempts are conditionally distinguished. The first one is grounded on the so-called phenomenological models, which is based on the principles, formulated by Fourier: it is assumed that arbitrary oscillations can be combined from simple harmonic ones, that is, a complex oscillatory process is interpreted as the result of superimposing of oscillations of individual harmonic oscillators.

The second one is grounded on the construction of physical and mathematical models of the systems that generate oscillations, and the analysis of their properties on this basis. Both of these attempts were closely joined during their development. Despite the mathematical complexity of the analysis of system oscillations, associated primarily with their nonlinearity and the dependence of parameters on time, this methodology is successfully developing and important results have been obtained in it during the study of the properties of oscillatory motion.

The first studies of the properties of oscillations mostly used a deterministic approach, according to which the time changes of the function, describing the oscillations occur according to certain established dependencies, which actually represent their defining feature – repetition. Among the deterministic oscillations two classes are distinguished: oscillations with limited power and oscillations with limited energy [1, 2]. Among the first class, a special place is occupied by periodic and almost periodic oscillations, the properties of which are analyzed using Fourier series. Oscillations belonging to the second class are also called transient or vanishing. Methods of their analysis and processing are based on the theory of Fourier integrals. Determination of Fourier coefficients or Fourier images that correspond to functions given in one way or another is called harmonic analysis [3]. Harmonic analysis methods transfer the study of oscillations from the time domain to the frequency domain, in which their properties are characterized by the amplitude and phase spectra, as well as the power spectrum or energy spectrum. Note that the power spectrum does not depend on the phase relations between the harmonic components that form the oscillations, so it cannot be used to describe their shape in the time domain. The spectra of periodic and almost periodic oscillations are discrete, and the spectra of vanishing oscillations are continuous. The first ones do not belong to the class of oscillations with limited energy, so they cannot be represented by the Fourier-Riemann integral. Combining the discrete and continuous cases allows obtaining a representation in the form of the Fourier-Stieltjes integral, which can be used to describe the sum of periodic or almost periodic oscillations and vanishing oscillations. However, non-vanishing oscillations with a continuous spectrum remain outside the scope of such a model.

Mathematically rigorous theory of non-vanishing oscillations with a continuous spectrum was developed by N. Wiener [4, 5] and was called by the author himself generalized harmonic analysis. N. Wiener moved away from the analysis of individual oscillatory processes and come on to the study of their correlation functions, defined by means of time averaging. The Fourier transform of such correlation functions characterizes the power of the harmonic components of the oscillation. However, when directly transferring such an approach to the analysis of real oscillatory processes, researchers encountered a number of problems that manifested themselves as instability of the estimates of the spectra obtained during the processing of experimental data. Similar problems appear when using a purely spectral approach to the analysis of non-vanishing oscillations, introduced by A. Schuster [6–8] and called periodogram analysis. Therefore, the instability of the results can be explained primarily, as noted in the work of A. Einstein [9], by fluctuating time changes in the studied quantities. Fluctuating components can be added to regular oscillations, modulate oscillations as a whole, as well as its individual components. In many situations, the power of fluctuation changes is significant if compared to the power of regular oscillations and even may exceed it. In this case methods for analyzing and processing of oscillations should be developed on the basis of their models in the form of random processes. Such an approach is also necessary because of it is the probabilistic characteristics of random processes that those properties of oscillations, that are essential for determining their physical nature and describing the state of the objects that generate them are mostly associated.

3. Random oscillations analysis methods

The theory of random processes and methods of their statistical analysis are developed based on their division into classes.

The methods of stationary random processes and certain types of nonstationary ones are developed and mostly used in practice: random processes with evolutionary nonstationarity, including locally stationary ones, and random processes with the so-called rhythmic structure. The latter include periodically nonstationary random processes and their generalizations: bi-, poly- and almost periodically nonstationary random processes. In the second-order theory (also called spectral-correlation theory), the properties of a random process are described by the mathematical expectation, correlation function and its Fourier transform - spectral density. The foundations of the spectral-correlation theory of stationary random processes were performed in the works of A.Ya. Khinchin [10, 11]. The theorem on the spectral decomposition of the correlation function proved by him is actually another expression of S. Bochner's theorem on the harmonic analysis of positive definite functions [12]. N. Wiener's condition on the existence of the correlation function obtained by means of time averaging was replaced by A.Ya. Khinchin with the condition of stationarity of the random process. In the case of ergodic stationary random processes, A.Ya. Khinchin's theorem follows from N. Wiener's theorem, therefore the theorem on the spectral decomposition of the correlation function is often called the Wiener-Khinchin theorem [13]. The results obtained by A.Ya. Khinchin were developed by A.M. Kolmogorov [14, 15] and G. Kramer [16, 17], who substantiated the harmonic decompositions of the random processes. Non-correlating of harmonics in this expansion is a consequence of the stationarity of the random process. These results are actually a transfer to random processes of the Fourier-Stiltjes transform, while the transition to the ordinary Fourier integral in this case is impossible. The pioneer of empirical spectral analysis, i.e., the estimation of the spectral density of a stationary random process from experimental data, is considered to be J. Tukey [18]. To calculate the spectral density, he used smoothed estimates of the correlation function. The smoothing procedure itself, i.e., the use of the correlation function estimates of certain windows in calculating the Fourier transform, made it possible to obtain valid estimates of the spectral density, and therefore led to stable results of empirical spectral analysis. This method of nonparametric spectral analysis was called the Blackman-Tukey method [19]. After smoothing, the estimates of the spectral density obtained using the periodogram method of A. Schuster [6–8], which can be considered as a special case of the Blackman-Tukey correlogram method. It differs from the latter only in the method of calculating the spectral density.

The mathematical expectation of a stationary random process determines the regular component of the oscillation, and the correlation function characterizes the relationships between the values of the oscillatory process at time points and depends only on the difference. The power spectral density describes the distribution of the oscillation power in harmonics, therefore it is called the power spectral density. This value, as already noted, is not affected by the phase relations between the harmonic components, therefore it cannot characterize time structure of the oscillatory process, namely its repeatability, which, although in an idealized form, is represented by deterministic models. If the oscillations contain regular time changes, then within the framework of the stationary model they are analyzed in terms of the correlation function and the power spectral density, which is not logical and creates methodological problems as well as difficulties in interpreting the results of statistical processing of experimental data. This was first noted by E. Slutsky [20], who showed that peak values of spectral density can be the result of a certain type of relations of time series, but not only the presence of a regular periodic component. A natural resolve of this situation is to isolate regular oscillations in the form of a time-varying mathematical expectation function and to study it using deterministic function methods. The mathematical model of oscillation then will be a random process nonstationary in terms of mathematical expectation. However, as the results of real data processing show [20–30], in many cases nonstationarity is also inherent in higher order moment functions, and then it is necessary to move on to the analysis of the properties of oscillations based on their models in the form of periodically nonstationary random processes.

4. Methods of analysis of periodically nonstationary random processes (PNRP)

The first step in the development of the theory of nonstationary random oscillations can be considered an extension of the conditions for the existence of a harmonic decomposition, which were introduced by M. Loev [31].

Random processes for which such a decomposition exists were called harmonized. In the general case, individual harmonics of the decomposition are correlated, therefore their property in the frequency domain is described by the two-frequency spectral density. The two-frequency spectral density takes on a diagonal form, that is, becomes a function of one argument, in the case of a stationary random process, when the harmonics of the decomposition are uncorrelated. When determining the two-frequency spectral density from experimental data, a number of difficulties arise, therefore, in practice, for the analysis of nonstationary random oscillations, a time-dependent spectral density, which is called a variable (instantaneous) spectrum [32], as well as the spectral density of their stationary approximation, are often used. The probabilistic characteristics of a stationary approximation are the time-averaged probabilistic characteristics of a nonstationary random process. The correlation function of a stationary approximation has all the properties of the correlation function of a stationary random process, so its Fourier transform is called the power spectral density of the stationary approximation. This quantity describes the frequency distribution of the time-averaged power of the oscillations.

A more complete and deep study of the structure of nonstationary random oscillations allows specifying the type of time changes in their probabilistic characteristics. In this way, we arrive at models of oscillations in the form of locally stationary random processes and random processes with a rhythmic structure. The latter are called cyclostationary in literature [23, 29, 30]. We note that belonging to processes with a rhythmic structure does not exclude the properties of local stationarity.

Locally stationary random processes are characterized by the fact that their probability characteristics change significantly over a relatively large time interval, and this interval is much larger than the correlation interval, which is determined by the behavior of the correlation function with respect to the shift. At the same time, for any moment in time, the probability characteristics of such processes have the properties of stationary random processes (hence the term local stationarity). Although the correlation function is variable in time, it is a positive definite function for each, and the change in spectral density is non-negative and is interpreted as an instantaneous power spectral density. Under such conditions, when studying the probability structure of real fluctuations, statistical methods of stationary random processes can be used. The main issue here is determining the length of the processing interval, since due to the presence of two trends in the behavior of the evaluation efficiency characteristics with increasing length – increasing bias and decreasing variance – there is a certain optimal length of the processed segment that provides the minimum total processing error.

5. PCRP approach to analysis of the PNRS

A special place in the development of the theory and methods of analysis of stochastic oscillations belongs to their models in the form of random processes with a rhythmic structure.

In the second-order theory, they are defined as processes with a periodic or almost periodic change in time of the mathematical expectation, correlation function, spectral density. They are called periodically correlated random processes (PCRP) and their generalization - almost periodically correlated random processes (APCRP), as well as subclasses of the latter: bi- and poly-periodically correlated processes [22, 24, 25, 33, 34]. Research, based on models of nonstationary random processes belonging to these classes naturally combines and develops deterministic and probabilistic approaches, which are based, respectively, on the theory of periodic and almost periodic functions and the theory of stationary random processes. PCRP models and their generalizations make it possible more adequately describe the structure of the oscillatory process, covering as separate extreme cases the above-mentioned representations, as well as models known in the literature, in which attempts have been made to combine the features of repeatability and stochasticity: additive, multiplicative, their combinations, poly-harmonic, quadrature, etc. All this creates a basis for the analysis of stochastic oscillations not only by special means characteristic of each model, but also in terms common to all of them.

Random processes with periodic and almost periodic in time probabilistic characteristics were first considered in the work of L.I. Koronkevych [35]. This was done when studying solutions of differential equations with periodic and almost periodic coefficients and a force in the form of a stationary random process. This work is referred to by E.G. Gladyshev [36], analyzing the properties of the Fourier coefficients of the correlation function, their representations, and the issue of harmonization of these classes of nonstationary random processes. E.G. Gladyshev first used the terms "periodically correlated" and "almost periodically correlated" random processes. U. Bennett [37] and R.L. Stratonovich [38] considered random processes with periodically varying characteristics as a models (V. Pleskach [39]), suitable for describing of telecommunication signals and fluctuation oscillations in telecommunication systems. The first author introduced the term "cyclostationary", and the second – "periodically nonstationary". The latter term was also used in signal analysis in a later work by L. Franks [40]. The first studies on the theory of estimating probabilistic characteristics of the PCRP were carried out by L.I. Gudzenko [41, 42]. He obtained the conditions for the consistency of estimates of the Fourier coefficients of the mathematical expectation and the correlation function, as well as estimates of these characteristics themselves, which are based on the first estimates. The author showed the asymptotic convergence of the estimates calculated in this way to those found by averaging the PCRP samples taken over the correlation period. As L.I. Gudzenko noted, the validity of the latter estimates is a consequence of the fact that this type of reference sequences are stationary and stationary related. The properties of reference sequences were analyzed more fully in the work of Ya.P. Dragan [43]. Estimates of correlation components, as well as estimates of spectral components of the PCRP constructed on their basis, were studied by H. Hurd [44, 45].

The two-frequency spectral representation of the correlation function was considered by A. Papoulis [46], S.M. Rytov [47], H. Ogura [48]. The latter obtained a representation of the PCRP through stationary random processes, which are selecting using frequency shift and linear bandpass filtering.

Such stationary components have a finite spectrum. Mathematically rigorous representations through stationary random processes with a finite spectrum for PCRPs harmonized according to M. Loev were analyzed by H. Hurd [49, 50]. V. Gardner and L. Franks [51] systematized a number of properties of PCRPs, and also considered the problem of optimal linear filtering of PCRPs using filters with periodically varying parameters.

The basics of the spectral-correlation theory of PCRP with limited average power were developed by Ya.P. Dragan [22, 25, 33, 34, 52, 53]. He was the first to establish a representation for PCRP obtained using a filter with periodically varying parameters in terms of stationary random processes with a infinite spectrum [54]. Ya.P. Dragan [53, 54] outlined the class of D-harmonized random processes and showed that this class of processes and the class of processes with limited average power are equivalent. For D-harmonized PCRP, transformation was obtained in the general case through stationary random processes, and a one-to-one correspondence between transformation with finite and infinite spectra was established.

A number of works by W. Gardner and colleagues [23, 28, 55–57] were motivated by telecommunications problems. W. Gardner [57] developed a non-probabilistic approach to the analysis of so-called cyclostationary processes, which can be considered a further development of N. Wiener's generalized harmonic analysis. On the initiative of W. Gardner, a monograph [23] was published in 1994, which collected the main results in the field of PCRP analysis and its applications obtained by well-known specialists in this field. This monograph mainly highlights the results, published in the English-language literature. The results of research by a wider range of specialists, including those from Ukraine and the former Soviet Union, are briefly reviewed in [28]. This work also provides an extensive list of literature (786 titles) on the theory, statistics of PCRP and generalizations, as well as applications in various fields of science and technology. It should be noted that the first studies by scientists from Eastern Europe in the English-language literature were characterized in the well-known monograph by A.M. Yaglom [58]. Among the latest monographic publications is the book by H. Hurd and A. Miami [29], devoted to the analysis and statistics of periodically correlated random sequences and their applications. It develops the results obtained in the initial works of E.G. Gladyshev [40, 59], Ya.P. Dragan [25, 33].

One of the first works on research on the use of PKRP methods for the analysis of real stochastic oscillations, along with the already mentioned work of L.I. Koronkevych [35] in the field of mechanics, was the work of L.I. Gudzenko on the study of the properties of fluctuations in self-oscillating systems. S.M. Rytov [47] indicated the use of PCRP for the study of cyclic magnetization reversal noise, O.F. Romanenko and T.O. Sergeyev [60] noted the possibility of describing by these processes the turbulent flow of water adjacent to the ship's propeller, temporal changes in electricity consumption, and physiological phenomena. V.I. Kolesnikova and A.S. Monin [61] emphasized the expediency of studying the seasonal and daily variability of meteorological phenomena from the PCRP perspective, which is constrained, in the authors' opinion, by insufficient elaboration of theoretical and methodological issues.

Ya.P. Dragan, in collaboration with K.S. Voychyshyn [62], and then independently [63, 64], considered the general properties of the stochastic rhythmic model within the framework of the PCRP model, which is interpreted as the result of a regular in the probabilistic sense repetition of cycles - an interconnected sequence of phases in the development of the system under study. These works formulated the main requirements for the rhythmic model: a description of its stochasticity, decomposition into harmonics and repeatability of properties. K.S. Voychyshyn [65] made the first attempts to analyze the daily rhythmicity of some geophysical processes based on such a model, and Ya.P. Dragan and I.M. Javorskyj applied this approach to the study of sea wind waves [25, 66, 67]. In the first case, the rhythmicity analysis was carried out at the level of the time dependence of the estimates of the mathematical expectation and dispersion, in the second case, estimates of correlation characteristics were included in the analysis. It should be noted that even earlier, the PCRP model, even in a somewhat broader aspect (with the involvement of histograms and estimates of correlation functions), was used to study the daily changes of air temperature and humidity, as well as soil temperature [68–73]. P.Ya. Groisman considered the correlation properties of precipitation series within the framework of PCRP [74]. Other properties PCRP are considered in [75, 76].

The best modern methods of analysis and demodulation of signals, representing PNRS are based on the application of the Hilbert transform and the construction of an analytical signal. It is necessary due to complex mutual amplitude- and phase- modulations between signal components. A significant advantage of applying the Hilbert transform to the analysis of diagnostic signals is the ability to analyze all types of carrier's modulations: amplitude, frequency and phase, as well as their effective separation [77, 78]. However, frequent neglecting of known significant limitations in the application of the Hilbert transform to the processing of modulated signals formulated by the Bedrosian and Nuttall theorems [79, 80] leads to errors in detecting defects and significant errors in their classification as well as assessment of the degree of development. To avoid such troubles, the decomposition of the PNRS by empirical methods into a number of narrow-band so-called "eigenmode functions" and the application of the Hilbert transform to analyze the structure of each of these modes separately is used [Feldman, Huang]. The most widely used are various versions of the empirical method of decomposition (EMD) (so-called Hilbert-Huang method) [78, 81]. This empirical method involves a cyclic procedure of signal analysis with the selection and subtraction of "average modes" from the signal until the rest bacame purely stochastic. The difference between the various implementations of the method is mainly reduced to the use of different stopping criteria for mode selection and methods of signal interpolation between peaks (except for the original cubic spline method proposed by Huang). In the works [82-84], the correlation structure of the PNRS was analyzed theoretically for different types of carrier signal modulations. Based on this study, a model for such signal was proposed and a rigorous theoretical analysis of its application under the conditions of various types of carrier modulations was carried out. A method for narrow-band filtration and demodulating of the quadratures of the modulating components was developed and verified. Its effectiveness for real diagnostic vibration signal data processing was demonstrated. In particular, it was proposed to estimate the degree of defect development in the system under investigation based on higher-order joint correlations between different signal components. For this purpose, a map of correlations between different signal components was constructed. The dependence of defect development degree estimates on signal filteration parameters was shown and conditions for avoiding the leakage effect during signal processing were established.

6. Conclusion

Use of the PCRP model and its generalizations for the analysis of stochastic oscillations is based on a data processing methodology grounded on the theory of evaluating the entire complex of probabilistic characteristics of a given class of nonstationary random processes.

The development of such a methodology and its use for studying the structure of variability of periodically nonstationary random oscillations in electrical networks will provide an important basis for the development and reliable operation of modern "smart power grids".

The research is crucial due to its relevance in modern energy systems, especially with the shift toward smarter and more sustainable grids. The transition from unidirectional traditional energy systems to bidirectional, multi-level smart grids has introduced complexities. Analyzing rhythmic variations helps in understanding and optimizing these flows, minimizing inefficiencies and disruptions. Smart grids, with their feedback loops, are more susceptible to high-frequency and localized fluctuations that can propagate and cause system-wide failures. Identifying hidden patterns in energy consumption can provide early warnings for such risks. Understanding the rhythmic variability of electricity consumption enables better forecasting and regulation, which is critical for balancing supply and demand, so supports the stability and reliability of power systems, especially as renewable energy sources are intermittent. Accurate modeling and forecasting of energy flows reduce operational costs, improve resource allocation, and enhance the economic feasibility of renewable energy such projects.

The study bridges deterministic and probabilistic approaches, offering a robust methodology to analyze and forecast energy flows in modern power systems, thereby addressing critical challenges in energy management and sustainability.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

References

[1] Papoulis A. The Fourier Integral and Its Applications. New York: McGraw-Hill, 1962.
[2] Szabatin I. Podstawy teorii sygnalow. Warszawa: Wyd-wa Komunikacij i Łączności, 1990. 499 s.
[3] Lawrence Marpl S. Digital spectral analysis. Second Edition. Mineola. New York : Dover Publications. 2019. 432 p.
[4] Wiener N. Generalized harmonic analysis. Acta Mathematica. 1930. Vol. 55. P. 117–258.
[5] Wiener N. The Fourier Integral and Certain of its Applications (Dover, New York, 1932).

[6] Shuster A. On lunar and solar periodicities of earthquakes. A Proceedings of the Royal Society of London. 1897. Vol. 61. P. 455–465.
[7] Shuster A. On the investigation of hidden periodicities with application to supposed 26 day period of meteorological phenomena. Terrestrial Magnetism and Atmospheric Electricity. 1898. № 3. P. 13–41.
[8] Shuster A. On the periodicities of sunspots. Transactions of the Royal Society of London. 1906. Series A. Vol. 206. P. 69–100.
[9] Yaglom A.M. Einstein's Work on Methods for Processing Fluctuating Series of Observations and the Role of these Methods in Meteorology, Bull. Acad. Sci. SSSR, Atmospheric and Oceanic Physics, 22, 1, pp. 101-107, 1986.
[10] Khinchine A. Korrelations Theorie der Stationare Stochastichen Processes. Math. Ann. 1934. 109. P. 604–615.
[11] Khinchin A.Ya. Theory of correlation of stationary stochastic processes, Uspekhi Mat. Nauk, 1938, № 5, P. 42–51.
[12] Bochner S., Monotone Funktionen Stieltjessische und Harmonische Analyse. Math. Ann. 1933.108. P. 378-379.
[13] Yaglom A.M. Correlation theory of stationary random functions, L.: Gidrometeoizdat, 1981.280 p.
[14] Kolmogorov A.N. Curves in Hilbert space which are invariant with respect to a one-parameter group of motions, Dokl. Akad. Nauk SSSR, 26, 1940. P. 6–9.
[15] Kolmogorov A.N. Static theory of oscillations with non-perturbative spectrum. Anniversary collection Akad. Nauk SSSR, p. 1. M.: Izdatelstvo Akad. Nauk SSSR. 1947. P. 242–249.
[16] Cramer H. On harmonic analysis in certain functional spaces. Arkiv Mat., Astr. och Fysik. 1942. 28 B. № 12.
[17] Cramer H. On the theory of stochastic processes. Proceedings of the Tenth Congress of Scandinavian. Mathematician. Copengagen, 1946. P. 28–39.
[18] Blackman R.B., Tukey J.W. The measurements of power spectra from the point of view of communications engineering. Bell Labs Techn. J. 1958. Vol. 33. P. 185–282.
[19] Kay S.M. Modern Spectral Estimation: Theory and Application. New Yersey: Prentice Hall Englewood Cliffs, 1988. 543 p.
[20] Slutsky E.E. Selected works (probability theory and mathematical statistics). M.: Izdatelstvo Akad. Nauk SSSR. 1956. 292 p.
[21] Rozhkov V.A. Methods of probabilistic analysis of oceanological processes. L.: Gidrometeoizdat, 1974. 280 p.
[22] Dragan Ya.P., Rozhkov V.A., Yavorsky I.N. Methods of Probabilistic Analysis of Oceanological Processes Rhythmics. L.: Gidrometeoizdat, 1987. 320 p.
[23] Cyclostationarity in Communications and Signal Processing. Ed. by W.A. Gardner. New York: IEEE Press, 1994. 504 p.
[24] Voychyshyn K.S., Dragan Ya.P., Kuksenko V.I., Mikhailovsky V.N. Information links of bioheliogeophysical phenomena and elements of their forecasting. Kyiv: Naukova dumka, 1974. 208 p.
[25] Dragan Ya.P., Yavorsky I.N. Rhythmics of sea wave and underwater acoustic signals. Kyiv: Naukova dumka, 1982. 247 p.
[26] Mykhaylyshyn V.Yu., Fligel D.S., Yavorsky I.N. Statistical analysis of wave packets of geomagnetic pulsations of PC1 type by methods of periodically correlated random processes. Geomagnetizm i aeronomiya. 1990. Vol. 30. № 5. P. 757–764.
[27] Mykhaylyshyn V.Yu., Yavorsky I.N. Probabilistic structure of seasonal variability of air temperature.Meteorologiya и gidrologiya. 1994. № 2. P. 20–35.
[28] Gardner W.A., Napolitano A., Paural L. Cyclostationarity: Half century of research. Signal Processing. 2006. 86. P. 639–697.
[29] Hurd H.L., Miamee A. Periodically Correlated Random Sequences. Spectral Theory and Practice. New Jersey: Wiley-Interscience, 2007. 353 p.
[30] Antoni J. Cyclostationarity by examples. Mechanical Systems and Signal Processing. 2009. Vol. 23. P. 987–1036.
[31] Loèv M. Probability Theory. M.: Izdatelstvo inostr. lit., 1962. 719 p.
[32] Kharkevich A.A. Spectra and analysis. M.: Fizmatgiz, 1962. 134 p.

[33] Dragan Ya.P. Structure and representation of stochastic signal models. Kyiv: Naukova dumka, 1980. 384 p.
[34] Dragan Ya.P. Energy theory of linear models of stochastic signals. Lviv: Centre for Strategic Studies of Eco-Bio-Technical Systems, 1997. 361 p.
[35] Koronkevych O.I. Linear dynamic systems under the action of random forces. Naukovi zapysky Lvivskogo universytetu. 1957. Vol. 44. Bul. 8. P. 175–183.
[36] Gladyshev E.G. Periodically and almost-periodically correlated processes with continuous time. Probability theory and its applications. 1963. Vol. 3. Bul. 2. P. 184–189.
[37] Bennet W.R. Statistics of regenerative digital transmission. Bell Labs Techn. J. 1958. Vol. 38. № 6. P. 1501–1502.
[38] Stratonovich R.L. Selected questions on the theory of fluctuations in radio engineering. M.: Sov. radio, 1961. 559 p.
[39] Pleskach V.L. Modelling of financial and economic processes. Monograph, Kyiv, Kyiv National University of Trade and Economics, 2010. 242 p.
[40] Franks L.E. Signal Theory. New York: Prentice Hall, Englewood Cliffs, 1969. 317 p.
[41] Gudzenko L.I. About periodically nonstationary processes. Radiotechnika & elektronika. 1959. Vol. 4. Bul. 6. P. 1062–1064.
[42] Gudzenko L.I. Generalisation of the ergodic theorem for nonstationary random processes. Radiofizika. 1961. Vol. 4. № 2. P. 265–274.
[43] Dragan Ya.P. Properties of stochasts of periodically correlated random processes. Otbor i peredacha information. 1972. № 33. P. 9–12.
[44] Hurd H.L. An investigation of periodically correlated stochastic processes. PhD dissertation. Duke University department of Electrical Engineering, 1969.
[45] Hurd H.L. Nonparametric time series analysis for periodically correlated processes. IEEE Trans. Inf. Theory. 1989. IT. 35. P. 350–359.
[46] Papoulis A. Probability, Random Variables and Stochastic Processes. New York: Mc. Grow-Hill, 1965. 583 p.
[47] Rytov S.M. Introduction to Statistical Radiophysics (part 1). M.: Nauka, 1976. 495 p.
[48] Ogura H. Spectral representation of a periodic nonstationary random process. IEEE Trans. Inf. Theory. 1971. IT. 17. № 2. P. 143–149.
[49] Hurd H.L. Periodically correlated processes with discontinuous correlation functions.Theory Probab. Appl. 1974. P. 834–838.
[50] Hurd H.L. Representation of strongly harmonizable periodically correlated random processes and their covariances. J. Multivariate Anal. 1989. 29. P. 53–67.
[51] Gardner W.A., Franks L.E. Characterisation of cyclostationary random processes. IEEE Trans. Inf. Theory. 1975. IT. 21. P. 4–14.
[52] Dragan Ya.P. About representation of a periodically correlated random process through stationary components. Otbor i peredacha information. 1975. № 45. P. 7–20.
[53] Dragan Ya.P. Harmonisation and spectral decomposition of random processes with finite mean power. Dop. Akad. Nauk USSR. 1978. Ser. A. № 8. P. 679–684.
[54] Dragan Ya.P. On periodically correlated random processes and systems with periodically varying parameters. Otbor i peredacha information. 1969. № 22. P. 22–33.
[55] Gardner W.A. Introduction to Random Processes with Application to Signals and Systems. New York: Macmillan, 1985. 434 p.
[56] Gardner W.A. The spectral correlation theory of cyclostationary time-series. Signal Processing. 1986. Vol. 11. № 1. P. 13–16.
[57] Gardner W.A. Statistical Spectral Analysis: A Nonprobabilistic Theory. New York: Pronstice Hall, Englewood Cliffs, 1987. 566 p.
[58] Yaglom A.M. Correlation Theory of Stationary and Related Random Functions. New York: Spronger-VerLag, 1987. 526 p.
[59] Gladyshev E.G. About periodically correlated random sequences. Dokl. Akad. Nauk SSSR.1961. 137. № 5. P. 2236–2239.
[60] Romanenko A.F., Sergeyev G.A. Problems of applied analysis of random processes. M: Sov. radio, 1968. 255 p.

[61] Kolesnikova V.N., Monin A.S. On the spectra of meteorological field oscillations // Izv. of the USSR Academy of Sciences. Ser. Atmosphere and Ocean Physics. 1965. Vol. 1. № 7. P. 653–669.
[62] Voychyshyn K.S., Dragan Ya.P. About a simple stochastic model of natural rhythmic processes. Otbor i peredacha information. 1971. № 29. P. 7–15.
[63] Dragan Ya.P. To substantiation of the stochastic model of rhythmic phenomena. Otbor i peredacha information. 1972. № 34. P. 21–27.
[64] Dragan Ya.P. General properties of the stochastic model of rhythmics. Otbor i peredacha information. 1975. № 44. P. 3–14.
[65] Voychishin K.S. Questions of statistical analysis of nonstationary (rhythmic) phenomena in relation to some tasks of geophysics. Avtoref. dissert. na sosisk. uchet. degree kand. fiz.-mat. nauk. M.: Izd-vo Institute of Physics of the Earth, 1975. 26 p.
[66] Dragan Ya.P., Yavorsky I.N. To the description of the rhythmics of sea waves. Proceedings of the 6th All-Union school-seminar on statistical hydroacoustics. Nsk: Institute of Mathematics, SO AS USSR, 1975. P. 197–206.
[67] Dragan Ya.P., Yavorsky I.N. Description of the rhythm of sea waves. Bulletin of the Academy of Sciences of the USSR. 1977. No. 2. P. 26–36.
[68] Zhukovsky E.E., Kosenkov I.I., Mandelstam S.M. and etc. Investigation of statistical characteristics of relative air humidity. Collected works on agronomic physics. 1969. Bul. 20. P. 3–28.
[69] Kiseleva T.L., Chudnovskiy A.F. Statistical study of the diurnal course of air temperature. Bulletin of scientific and technical information on agronomic physics. 1968. № 11. P. 17–38.
[70] Mamontov N.V. Root-mean-square deviations and coefficients of air temperature asymmetry in the south-east of the West Siberian Plain. Trudy NIIK. 1968. Bul. 54. № 4. P. 29–34.
[71] Mamontov N.V. Study of relative humidity distribution statistics. Trudy NIIK.1960. Vol. 91. P. 15–28.
[72] Mishchenko Z.A. Daily variation of air temperature and thermoperiodism of plants. Trudy GGO. 1960. № 91. P. 15–28.
[73] Mishchenko Z.A. Daily air temperature variation and its agroclimatic significance. L.: Gidrometeoizdat, 1966. 200 p.
[74] Groisman P.Y. Estimation of autocorrelation matrices of precipitation series considered as periodically correlated random processes. Trudy GGI. 1977. Bul. 247. P. 119–127.
[75] Javorskyj I., Yuzefovych R., Lychak O., Slyepko R., Semenov P. Detection of distributed and localized faults in rotating machines using periodically nonstationary covariance analysis of vibrations. Meas. Sci. Technol., 34, 2023, 065102.
[76] Javorskyj I., Yuzefovych R., Kurapov P. Periodically nonstationary analytic signals and their properties", IEEE 13th International Scientific and Technical Conference on Computer Sciences and Information Technologies, Lviv, 2018, pp. 191–194.
[77] Feldman M. Hilbert transform applications in mechanical vibration. John Wiley. 2011.
[78] Huang N.E., Shen Z., Long S.R, et al. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. London: Proceedings of the Royal Society of London. Series A. 1998. 454. 903–995.
[79] Bedrosian E. A product theorem for Hilbert Transform. Proceedings of the IEEE, 1963, 51, 868-869.
[80] Nuttall A.H., Bedrosian E. On the quadrature approximation to the Hilbert Transform of modulated signal. Proceedings of the IEEE, 1966, 54(10), 1458-1459
[81] Huang N., Wu Z. A review on Hilbert-Huang transform: Method and its applications to geophysical studies. Reviews of Geophysics, 2008, 46, 1–23.
[82] Javorskyj I., Yuzefovych R., Lychak O., Matsko I. Hilbert transform for covariance analysis of periodically nonstationary random signals with high-frequency modulation. ISA Transactions, 2024, 144, 452–481.
[83] Javorskyj I., Yuzefovych R., Lychak O., Trokhym G., Varyvoda M. Methods of periodically non-stationary random processes for vibrations monitoring of rolling bearing with damaged outer race. Digital Signal Processing: A Review Journal, 145, 2024, 104343.
[84] Javorskyj I., Yuzefovych R., Lychak O., Torba Yu., Sbrodov Ye., Komarnytskyi B. Correlation matrix for analysis of the covariance and spectral structures of PNRP. Proceedings of 14th International Conference on Advanced Computer Information Technologies, Ceske Budejovice, Czech Republic, 19-21 September 2024. P. 158–161.