Model for Compensation of Systematic Errors in Measurements of Two Radars

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Abstract
This article examines a model compensation of systematic errors in the measurements of two three-dimensional radars observing a group of objects. This model is based on the recursive estimation of systematic errors using the least squares method for radar measurement parameters. The issue of estimating and compensating systematic errors in azimuth, distance, and elevation when operating with two sensors is considered, based on the use of the Kalman filter. The next development stage involves creating a combinatorial algorithm for compensating systematic errors in the observation of a group of objects by a group of radars. It is assumed that the observed section of object movement is linear with constant speed.

Keywords
Radar stations, systematic errors, Kalman filter.

1. Introduction

The classical approach to estimating systematic errors (SE) involves increasing the dimensionality of the system's state vector by including the vector of systematic errors into the state vector. This is achieved by implementing the Kalman filter with augmented states (ASKF). The problem with this approach is that including the states of all observed objects in one ASKF filter may be computationally unfeasible. Additionally, numerical issues may arise during the implementation of such an algorithm, mainly for poorly conditioned systems. Friedland [1] proposed the idea of implementing two parallel filters of lower order instead of using the ASKF algorithm. Alouani, Rice, and Blair [2] argued that algebraic simplifications might be too restrictive in practice. An indirect proof of this is that all practical implementations of reduced-order filters are suboptimal. Van Doorn and Blom [3] obtained an exact solution to the Kalman filter problem with augmented states (ASKF) but then divided the equations, applying certain approximations (simplifications) to make the implementation of such an algorithm computationally feasible. A similar approach is used in [4, 5] – to separate the state estimation filter and the systematic error estimation filter at the cost of reducing the accuracy of the obtained estimates.

Lin, Kirubarajan, and Bar-Shalom [6] managed to obtain an exact solution to the problem of estimating systematic errors for two (or more) sensors.

They demonstrated that the computation of systematic errors in dynamics can be ensured based on the state estimates of objects from local filters. This is achieved by manipulating the estimates of local filters in such a way that they provide pseudo-measurements of systematic errors and an

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additive white noise with zero mean and easily computable covariance [7-8]. The results of statistical modeling confirm a significant increase in accuracy with a root mean square (RMS) error compensation of 60-80% compared to the widely used decoupled Kalman filter. Moreover, the developed algorithm is statistically efficient. The obtained estimation of systematic errors turns the Cramer-Rao Lower Bound (CRLB) inequality into an equality [9-16]. The variance of the estimate is the smallest possible, meaning it is, in a certain sense, better than all others. The next section presents a model for forming radar measurements that contain systematic errors.

2. Model of constant systematic sensor errors

Consider M sensors (M = 2) measuring distance, azimuth, and elevation for N objects simultaneously. It is assumed that the coordinates of each sensor are precisely known. The model for radar measurements with constant systematic errors in a polar coordinate system for the i-th sensor at time t_j is as follows

$$\boldsymbol{z}_i^p(t_j) = \begin{bmatrix} D_i(t_j) \\ A_i(t_j) \\ B_i(t_j) \end{bmatrix}, \tag{1}$$

where D_i – distance, A_i – azimuth, B_i – elevation angle:

$$\begin{aligned} D_i(t_j) &= D_i^t(t_j) + b_i^D(t_j) + w_i^D(t_j) \\ wA_i(t_j) &= A_i^t(t_j) + b_i^A(t_j) + w_i^A(t_j) \\ B_i(t_j) &= B_i^t(t_j) + b_i^B(t_j) + w_i^B(t_j). \end{aligned} \tag{2}$$

Let $D_i^t(t_j)$, $A_i^t(t_j)$ and $B_i^t(t_j)$ denote the true distance, azimuth, and elevation angle, respectively. $b_i^D(t_j)$, $b_i^A(t_j)$, $b_i^B(t_j)$ – constant systematic errors in distance, azimuth, and elevation angle. $w_i^D(t_j)$, $w_i^A(t_j)$ and $w_i^B(t_j)$ – white noise terms with zero mean and covariances σ_D^2, σ_A^2, σ_B^2, which are considered mutually independent of each other. Let us denote the vector of systematic errors of the i-th sensor at time t_j:

$$\beta_i(t_j) \triangleq \begin{bmatrix} b_i^D(t_j) \\ b_i^A(t_j) \\ b_i^B(t_j) \end{bmatrix}. \tag{3}$$

Then,

$$\boldsymbol{z}_i^p(t_j) = \begin{bmatrix} D_i^t(t_j) \\ A_i^t(t_j) \\ B_i^t(t_j) \end{bmatrix} + \beta_i(t_j) + \begin{bmatrix} w_i^D(t_j) \\ w_i^A(t_j) \\ w_i^B(t_j) \end{bmatrix}. \tag{4}$$

After recalculating the measurements from a polar coordinate system to a rectangular one, the measurement equations of the i-th sensor take the form

$$\boldsymbol{z}_i(t_j) = H_i(t_j)\boldsymbol{x}(t_j) + J_i(t_j)\beta_i(t_j) + w_i(t_j), \tag{5}$$

where $\boldsymbol{x}(t_j) \triangleq \begin{bmatrix} x(t_j) & \dot{x}(t_j) & y(t_j) & \dot{y}(t_j) & z(t_j) & \dot{z}(t_j) \end{bmatrix}'$ – state vector of the observed object;
$H_i(t_j)$ – measurement matrix,

$J_i(t_j)$ – is a nonlinear function of conversion from the true distance, azimuth, and elevation angle. Using the measured (or estimated) distance $\hat{D}_i(t_j)$, azimuth $\hat{A}_i(t_j)$, elevation angle $\hat{B}_i(t_j)$ from the *i*-th sensor, the matrix $J_i(t_j)$ in (5) can be calculated as

$$J_i(t_j) = \begin{bmatrix} cos(\hat{B})\,sin(\hat{A}) & \hat{D}\,cos(\hat{A})\,cos(\hat{B}) & -\hat{D}\,sin(\hat{A})\,sin(\hat{B}) \\ cos(\hat{A})\,cos(\hat{B}) & -\hat{D}\,cos(\hat{B})\,sin(\hat{A}) & -\hat{D}\,cos(\hat{A})\,sin(\hat{B}) \\ sin(\hat{B}) & 0 & \hat{D}\,cos(\hat{B}) \end{bmatrix} \tag{6}$$

the time index (t_j) at the measurements is omitted for simplification of the expression.

$w_i(t_j)$ – measurement noise with a covariance matrix in a rectangular coordinate system

$$R_i(t_j) = J_i(t_j) \cdot diag[\sigma_D^2, \sigma_A^2, \sigma_B^2] \cdot \left(J_i(t_j)\right)'. \tag{7}$$

3. Dynamic system model used

Let's take the following dynamic equation of motion as a model of object movement

$$\boldsymbol{x}(k+1) = F(k)\boldsymbol{x}(k) + v(k), \tag{8}$$

where /$F(k)$ – transition matrix of the system

$$F(k) = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{9}$$

$v(k)$ – white noise in a process model with zero mathematical expectation and covariance matrix $Q(k)$.

It is assumed that the velocity in the x, y, z directions does not change over a short period of time, i.e., it is constant over the interval Δt.

Measurement equation of the i-th sensor without taking into account systematic errors

$$\boldsymbol{z}_i(k) = H_i(k)\boldsymbol{x}(k) + w_i(k)\,. \tag{10}$$

Note the difference in equations (5) and (10). In equation (10), there is no term for systematic sensor errors. In order to account for and estimate the systematic errors of the sensors, it is obviously necessary to use a sensor measurement model as in equation (5).

3.1. The vector of pseudo-measurements of systematic errors

In this section, we will derive expressions for the pseudo-measurements of systematic errors for the case of M=2 sensors. It is assumed that systematic errors are unknown constants. According to the adopted model of sensor measurements (5) and the model of object movement (8), we write the equation of measurements of sensor 1 on k+1 measurements

$$\boldsymbol{z}_1(k+1) = H_1(k+1)F(k)\boldsymbol{x}(k) + H_1(k+1)v(k) + J_1(k+1)\beta_1(k+1) + w_1(k+1). \tag{11}$$

By analogy, the measurement of sensor 2 at the moment of k+1

$$\boldsymbol{z}_2(k+1) = H_2(k+1)F(k)\boldsymbol{x}(k) + H_2(k+1)\upsilon(k) + J_2(k+1)\beta_2(k+1) + w_2(k+1). \quad (12)$$

Note that the true state vector $x(k)$ and the process noise $\upsilon(k)$ in equations (11) and (12) are the same. The measurement matrices H_1 and H_2 are the same for both sensors. Accordingly, we define the vector of pseudo-measurements of systematic errors z_b as subtraction $z_1 - z_2$

$$\boldsymbol{z}_b(k+1) \triangleq \boldsymbol{z}_1(k+1) - \boldsymbol{z}_2(k+1). \quad (13)$$

Then

$$\boldsymbol{z}_b(k+1) = J_1(k+1)\beta_1(k+1) + w_1(k+1) - J_2(k+1)\beta_2(k+1) - w_2(k+1). \quad (14)$$

Hence, we obtain the equation of the pseudo-measurement vector of systematic sensor errors

$$\boldsymbol{z}_b(k+1) = H(k+1)\boldsymbol{b}(k+1) + \widetilde{w}(k+1), \quad (15)$$

where the pseudo-measurement matrix $H(k+1)$, the systematic error vector $b(k+1)$ and the pseudo-measurement noise $\widetilde{w}(k+1)$ are defined as

$$H(k+1) = [J_1(k+1), \quad -J_2(k+1)]. \quad (16)$$

$$\boldsymbol{b}(k+1) \triangleq \begin{bmatrix} \beta_1(k+1) \\ \beta_2(k+1) \end{bmatrix}. \quad (17)$$

$$\widetilde{w}(k+1) \triangleq w_1(k+1) - w_2(k+1). \quad (18)$$

The noise of pseudo-measurements $\widetilde{w}$ is white with zero mathematical expectation and covariance matrix

$$R(k+1) = R_1(k+1) + R_2(k+1). \quad (19)$$

The white noise property in (18) is the key to an accurate solution to the problem of estimating systematic errors - no simplifications (approximations) are required. Note that no approximations were made at all in deriving (15)-(19). This means that this method, unlike [8, 9, 10], is accurate.

3.2. Recursive estimation of systematic errors

If the parameters of the vector of systematic errors $b(k)$ are modeled (correspond to unknown constants), then a recursive least-squares method based on the equation of the vector of pseudo-measurements of systematic errors (15) can be used to estimate them. The implementation of the recursive method for updating the systematics estimate is written as follows, at time k for each observed object $t = 1,\ldots,N$:

1. Obtain a new pseudo-measurement $\boldsymbol{z}_{b,t}(k) = H_t(k)\boldsymbol{b}(k) + \widetilde{w}_t(k)$, the measurement matrix $H_t(k)$, using expression (16) and the measurement noise covariance matrix $R_{\ t}(k)$, using (19).
2. Calculate the gain matrix and the residual vector:

$$G_t(k) = \Sigma_{t-1}(k)H_t(k)'[H_t(k)\Sigma_{t-1}(k)H_t(k)' + R_{\ t}(k)]^{-1}, \quad (20)$$

$$r_t(k) = \boldsymbol{z}_{b,t}(k) - H_t(k)\widehat{\boldsymbol{b}}_{t-1}(k). \quad (21)$$

3. Update the estimate of the systematic error vector and its covariance matrix:

$$\widehat{\boldsymbol{b}}_t(k) = \widehat{\boldsymbol{b}}_{t-1}(k) + G_t(k)r_t(k), \quad (22)$$

$$\Sigma_t(k) = \Sigma_{t-1}(k) - \Sigma_{t-1}(k)H_t(k)'[H_t(k)\Sigma_{t-1}(k)H_t(k)' + R_t(k)]^{-1}H_t(k)\Sigma_{t-1}(k). \quad (23)$$

4. When the update with the latest pseudo-measurement is performed, we get the final adjusted estimate

$$\hat{\boldsymbol{b}}_0(k+1) \triangleq \hat{\boldsymbol{b}}_N(k), \quad (24)$$

$$\Sigma_0(k+1) \triangleq \Sigma_N(k). \quad (25)$$

4. Results of the modeling

4.1. Description of scenarios for research

For the research, a scenario was created where two radars are used as measurement tools, with a data update period of 4 seconds. The parameters of the standard deviations of the sensors (RMS):

Distance error – σD = 50 m;

Azimuth error – σA = 0.3 degrees;

Elevation angle error – σB = 0.3 degrees.

The scenario includes 11 individual objects moving on different courses, at different altitudes and speeds:

Object № 28 – airplane, moving at a speed of 936 km/h; course 281 degrees at an altitude of 11 km.

Object № 29 – helicopter, moving at a speed of 150 km/h; course 180 degrees at an altitude of 0.5 km.

Object № 30 – helicopter, moving at a speed of 150 km/h; course 258 degrees at an altitude of 1.5 km.

Object № 31 – airplane, moving at a speed of 800 km/h; course 176 degrees at an altitude of 11 km.

Object № 32 – airplane, moving at a speed of 1000 km/h; course 250 degrees at an altitude of 10 km.

Object № 33 – airplane, moving at a speed of 700 km/h; course 60 degrees at an altitude of 3.5 km.

Object № 34 – airplane, moving at a speed of 1000 km/h; course 300 degrees at an altitude of 10 km.

Object № 35 – airplane, moving at a speed of 800 km/h; course 270 degrees at an altitude of 4 km.

Object № 36 – airplane, moving at a speed of 800 km/h; course 90 degrees at an altitude of 4 km.

Object № 37 – helicopter, moving at a speed of 50 km/h; course 230 degrees at an altitude of 2.5 km.

Object № 38 – airplane, moving at a speed of 900 km/h; course 240 degrees at an altitude of 1 km.

The overall view of the scenario is presented in fig. 1.

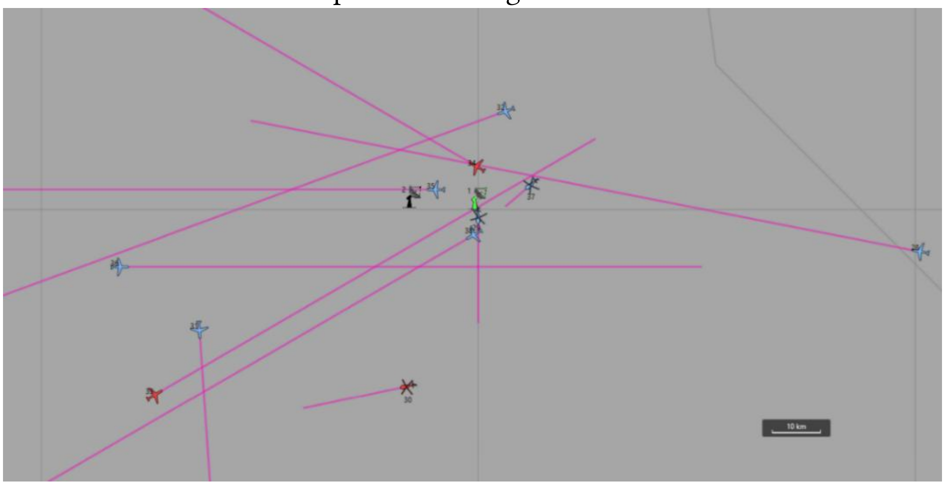

Figure 1: General view of the scenario for conducting research.

Data on the initial position, characteristics, and motion parameters of objects are presented in fig. 2.

Main					Coordinates			Velocity			Other	
Trajecto...	Number	Type	Membership	Plan number	Latitude, deg	Longitude, deg	Altitude, km	Course, deg	Speed, km/h	Vertical course, deg	Radar cross-sectio...	Max detection pro...
	28	airplane	alien	-1	25.339	70.748	11.000	281.00	936.00	0.00	3.00	0.00
	29	helicopter	alien	-1	25.417	69.743	0.500	180.00	150.00	0.00	1.00	0.00
	30	helicopter	own	-1	25.063	69.587	1.500	258.00	150.00	0.00	5.00	0.00
	31	airplane	alien	-1	25.178	69.108	11.000	176.00	800.00	0.00	3.00	0.00
	32	airplane	alien	-1	25.627	69.805	10.000	250.00	1000.00	0.00	3.00	0.00
	33	airplane	own	-1	25.044	69.006	3.500	60.00	700.00	0.00	2.00	0.00
	34	airplane	own	-1	25.513	69.740	10.000	300.00	1000.00	0.00	2.00	0.00
	35	airplane	alien	-1	25.466	69.646	4.000	270.00	800.00	0.00	3.00	0.00
	36	airplane	alien	-1	25.307	68.927	4.000	90.00	800.00	0.00	1.00	0.00
	37	helicopter	alien	-1	25.480	69.867	2.500	230.00	50.00	0.00	5.00	0.00
	38	airplane	alien	-1	25.374	69.733	1.000	240.00	900.00	0.00	2.00	0.00

Figure 2: Table with data on the initial parameters of object movement.

Research was conducted for identical object behavior, while modeling different systematic errors in sensor operations and various radar placements. The Monte Carlo method was used to obtain statistically stable results for estimating systematic sensor errors. Simulations were conducted for 100 repetitions for each of the studied scenarios.

4.2. The impact of systematic errors in distance, azimuth, and elevation angle, each corresponding to a specific standard deviation of a sensor

The study investigates how systematic errors in distance, azimuth, and elevation angle present in the measurements of one of the radars affect the accuracy of the obtained estimation of these systematic errors. For this study, in the scenario described in section 4.1, the following systematic errors were added to the measurements of Radar № 2:

Systematic distance error = 50 m;

Systematic azimuth error = -0.30 degrees;

Systematic elevation angle error = 0.30 degrees.

The distance between platforms was also varied in the scenario: 1 km, 2 km, 3 km, 5 km, 10 km, 15 km. Thus, a group of 6 scenarios was considered.

In one variant, processing was conducted without compensation for systematic errors, and in the second variant, processing was conducted with compensation for systematic errors. After simulating the processing process for 600 seconds, the following comparative graphs were constructed over 100 repetitions (fig. 3 - fig. 5). The dashed line on the graphs shows the residual systematic errors for the processing variant without compensation for systematic errors at a distance of 15 km between platforms. Other distance variants between platforms are not shown as they differ insignificantly. The solid line shows the residual systematic errors for the processing variant with compensation for systematic errors and at different distances between platforms.

Fig. 3 shows the graph of the residual systematic azimuth error estimation relative to the input standard deviation of the sensors. As seen from the graphs, the accuracy of estimating and compensating for the systematic azimuth error in measurements increases with the distance between radars. For a baseline distance between radars of more than 3 km, the residual systematic azimuth error does not exceed 15% relative to the corresponding root mean square deviation of the azimuth measurement.

Fig. 4 shows the graph of the residual systematic distance error estimation relative to the input standard deviation of the sensors. From the graphs, it is seen that for radars located more than 3 km apart, the accuracy of systematic distance error estimation converges to 25%-30% relative to its standard deviation.

Fig. 5 shows the graph of the residual systematic elevation angle error estimation relative to the input standard deviation of the sensors. Analysis of the graphs showed that the accuracy of estimating and compensating for the systematic elevation angle error in measurements also increases with the distance between radars. For a baseline distance between radars of more than 3 km, the residual systematic elevation angle error does not exceed 25% relative to its corresponding root mean square deviation.

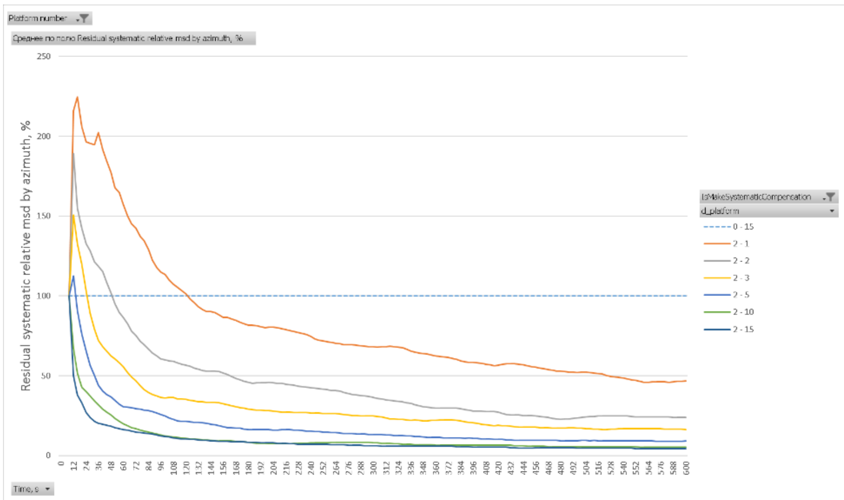

Figure 3: Residual systematic azimuth error relative to the input standard deviation of the sensors.

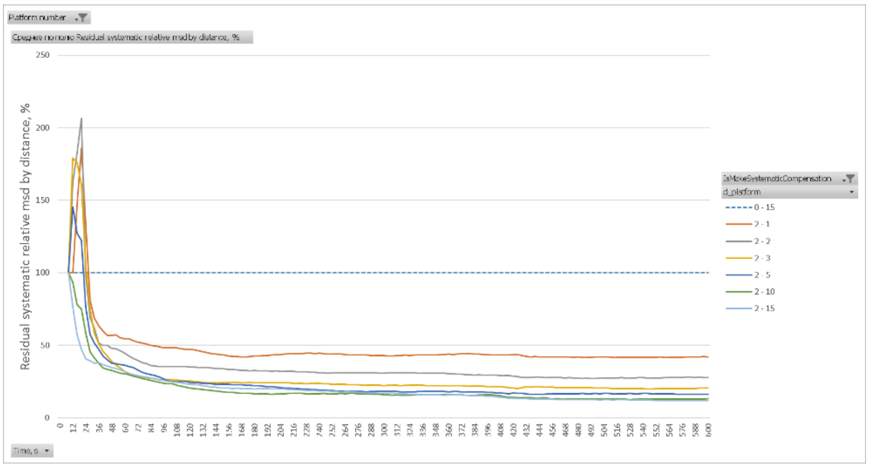

Figure 4: Residual systematic distance error relative to the input standard deviation of the sensors.

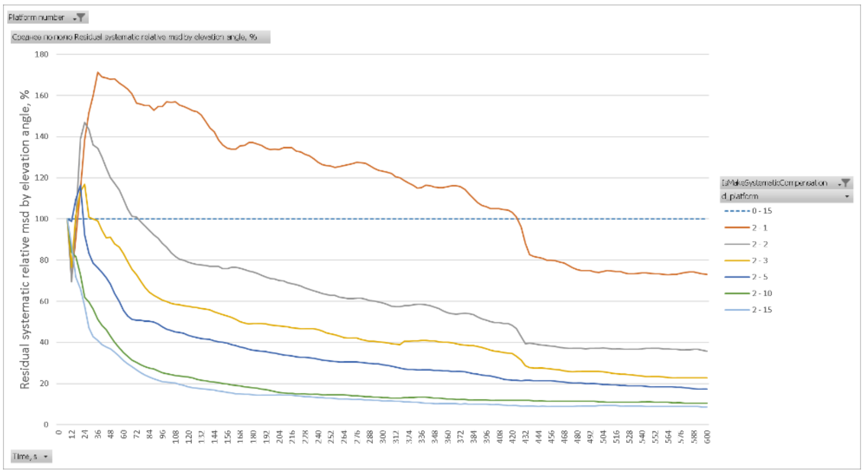

Figure 5: Residual systematic elevation angle error relative to the input standard deviation of the sensors.

4.3. The impact of systematic azimuth errors corresponding to one nominal RMS of the sensor

In this subsection, research is presented for the case where a systematic error is present only in the azimuth measurements of one of the radars. The accuracy of determining this systematic azimuth error is assessed, as well as the impact of the systematic error compensation algorithm on the accuracy of determining other parameters (distance and elevation angle), in which systematic error was not modeled. For this study, systematic errors were added to the measurements of Radar № 2 in the scenario described in section 4.1: Systematic azimuth error = -0.30 degrees. The distance between platforms was also varied: 1 km, 2 km, 3 km, 5 km, 10 km, 15 km. Thus, a group of 6 scenarios was considered. In one variant, processing was performed without systematic error compensation; in the second variant, processing was performed with systematic error compensation. After modeling the processing for 600 seconds, and with 100 repetitions, the following comparative graphs were constructed (fig. 6 – fig. 8). The dashed line on the graphs represents the residual systematic errors for the processing variant without systematic compensation at a distance of 15 km between platforms. The solid line represents the residual systematic errors for the processing variant with systematic compensation and at different distances between platforms. Fig. 6 shows the graph of the residual azimuth systematic error relative to the input standard deviation of the sensors.

As seen in the figure, the accuracy of estimating and compensating for systematic error in azimuth measurements increases with the distance between radars. For a baseline distance between radars greater than 3 km, the residual systematic azimuth error does not exceed 15% relative to its root mean square deviation.

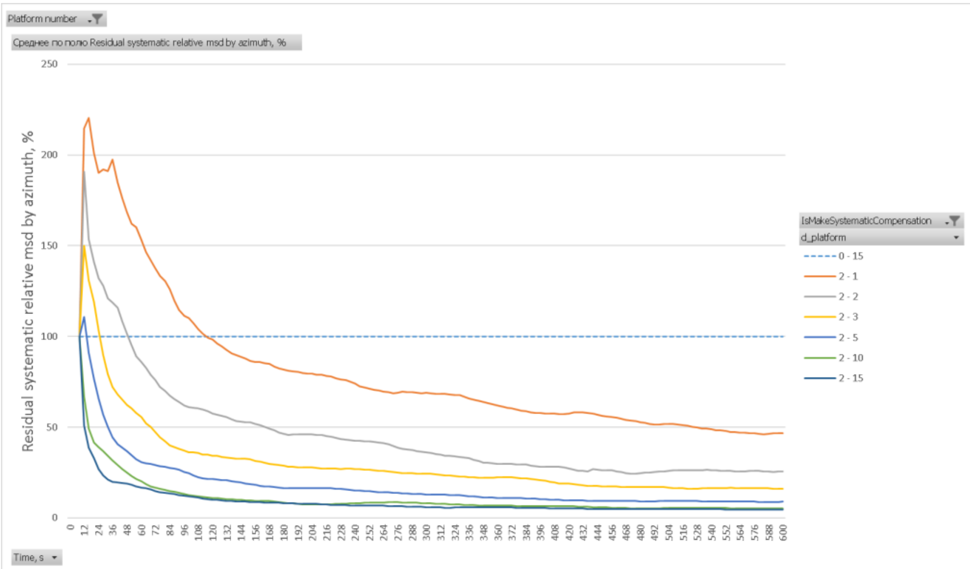

Figure 6: Residual systematic azimuth error relative to the input RMS of the sensors.

Fig. 7 shows a graph of the estimation of residual systematic distance error relative to the input RMS of sensors. According to the graphs, it is evident that despite the systematic error in distance not being modeled (equal to 0 m), the compensation algorithm estimated it at about 20% relative to the RMS for radars spaced more than 3 km apart and about 40% for radars spaced 1 and 2 km apart. This may lead to corresponding errors in distance measurements.

In fig. 8, a graph of the estimation of the residual systematic error of the elevation angle relative to the input standard deviation of the sensors is shown. The analysis of the figure revealed that in this case, a similar situation arose as with the distance. The systematic error in the elevation angle was not modeled (equaled 0 degrees). However, when the compensation algorithm was applied, it was estimated at about 25% relative to the RMS for radars spaced more than 3 km apart. For a distance between radars of 2 km, the systematic estimation of the elevation angle was calculated at about 40%.

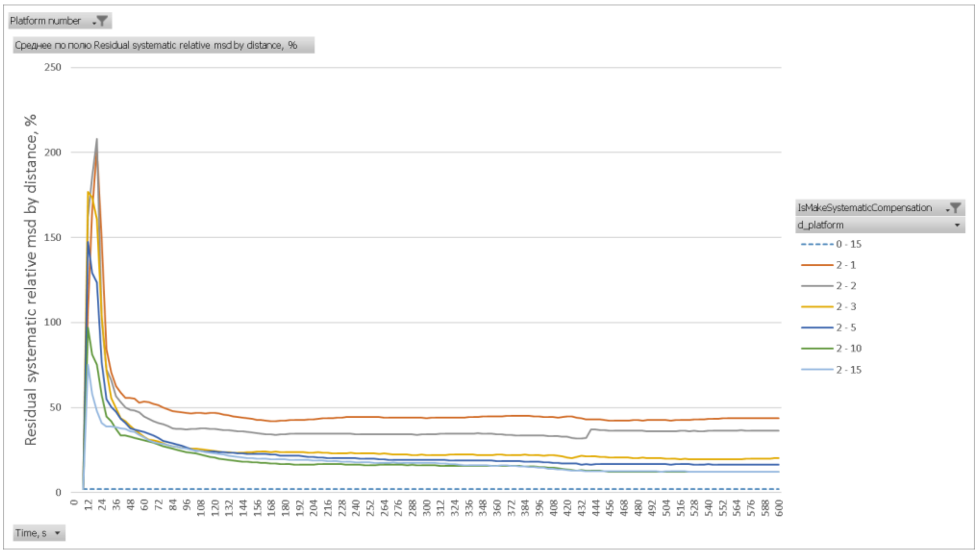

Figure 7: Residual systematic distance error relative to the input RMS of sensors.

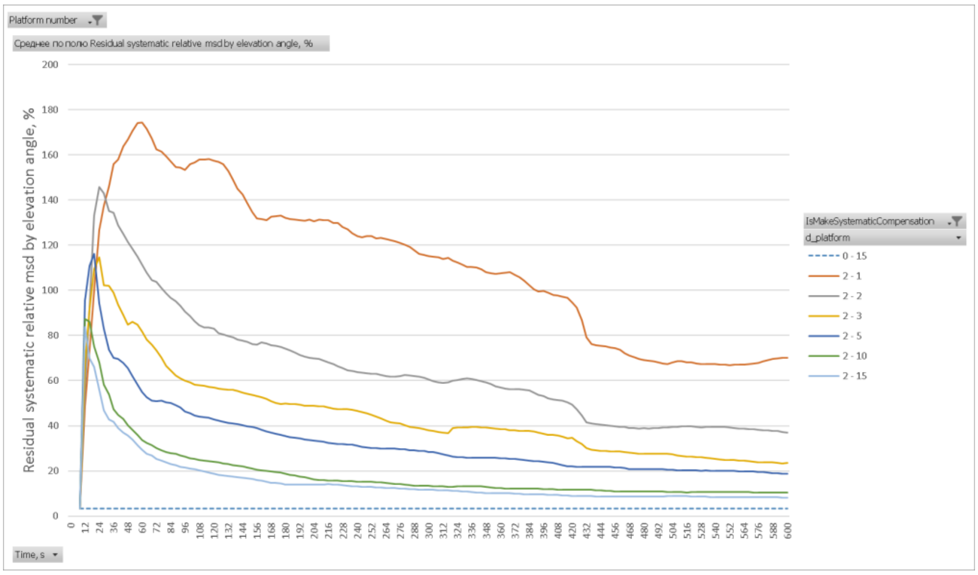

Figure 8: Residual systematic error of the angle of elevation relative to the input RMS of the sensors.

Conclusions:

1. The main features and advantages of this approach:

- obtained solution is accurate, requiring no approximations or simplifications, except for the linearization of nonlinear measurements;
- algorithm is implemented recursively, making it computationally efficient and applicable for real-time systematic error estimation;
- statistical modeling results confirm that the developed algorithm is statistically efficient;
- with a baseline distance between radars of at least 5 km, the accuracy of systematic error estimates in angular parameters of azimuth and elevation is no less than 15% of the RMS errors of the sensors, and for distance, no worse than 30% of the RMS error in distance determination.

2. The main factors affecting the accuracy of the algorithm:

- ratio of systematic error magnitudes to the nominal RMS of the sensors;
- number of observed objects used for systematic estimation;

- geometric factor of object placement - whether objects are distributed or concentrated in a single group (cluster);
- distance from the radar to the observed objects.

3. The minimum number of objects required for systematic estimation should be no less than two objects. As the number of objects increases, the accuracy of the algorithm improves.
4. A situation where the objects used for systematic error estimation are located far from each other is more favorable than when objects are located in a single cluster.
5. When the algorithm estimates systematic errors in cases where some radar measurements lack systematic error, the algorithm may erroneously identify its presence. This can lead to the introduction of additional errors to these measurements when compensating for systematic measurement errors. However, as statistical studies have shown, for a baseline distance between radars greater than 3 km, the error that may be added does not exceed 20% of the RMS of the sensors. At the same time, the existing systematic error is compensated at a level of 80%. In our opinion, this is an acceptable result.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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