Phase shift measurement method based on a half-period converter with amplitude-time conversion in the development of modern weapon systems

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Abstract

This paper presents a method for determining the phase shift based on the composite signal obtained by summing two harmonic signals after a dual half-period transformation. The proposed method can be categorised as a compensation-based measurement technique. The phase shift is determined by comparing a vector derived from the amplitude-time analog-to-digital conversion of the composite signal with a set of reference function vectors. The matching criterion is defined as the minimum of the sum of squared differences. An algorithm for finding the minimum value using the golden section search method is developed. The main sources of measurement error in the proposed phase shift estimation method are identified. Implementation of this method in artificial intelligence systems for diagnostics and condition monitoring of modern weapons and military equipment enables the reduction of requirements for measuring instruments without compromising accuracy.

Keywords

Compensation method, phase shift, harmonic signal, measurement, diagnostics, phase shift measurement, measurement error, extremum, correlation coefficient.

1. Introduction

At present, one of the key priorities of the state is to ensure national security. A crucial aspect of achieving this objective is equipping the Armed Forces of Ukraine and other military formations within the Defense Forces with advanced and high-tech models of weapons and military equipment. The main sources of such high-tech military assets include both supplies from partner states and production by domestic defense enterprises.

According to [1], the decision to authorize the supply of weapon systems developed and manufactured by the Ukrainian defense-industrial complex—or received from non-NATO partner countries, or models not officially adopted by NATO military formations, is made based on the results of testing. These results are then compared with the technical specifications outlined in regulatory

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and technical documentation for the prototype (such as technical specifications, technical requirements, and operating documentation).

A promising direction for improving the performance characteristics of such systems lies in the use of modern control and diagnostic systems. These systems integrate cutting-edge technologies based on computing and telecommunication equipment, artificial intelligence, and new diagnostic methods for assessing technical conditions [2, 3]. To reduce time and improve the quality of monitoring while enabling condition forecasting of weapons and military equipment, the use of intelligent diagnostic systems is essential [4, 5].

One of the most critical procedures during testing and diagnostics of complex technical systems is the accurate measurement of physical quantities that characterise system parameters and technical condition. For example, during the testing and operation of unmanned aerial and ground systems, it is necessary to measure accelerations, angles, time delays in control and data transmission channels, and distances to objects. The foundation of many such measurements relies on determining the phase shift between two harmonic signals as an intermediate quantity.

2. Related Work

Phase measurement methods and the information-measuring systems based on them make it possible to solve a wide range of scientific and technical tasks related to the precise measurement of distances, time intervals, angles, and analysis of signal field characteristics of various physical natures (electromagnetic, optical, acoustic). The transformation of diverse physical quantities and their values into a phase shift between two harmonic signals enables simplified measurement procedures while ensuring the required accuracy.

Phase measurement methods are widely used in various scientific and engineering domains such as radar and radio navigation, aerospace technology, unmanned aerial and ground systems, geodesy, mechanical engineering, telecommunications, and non-destructive testing [6–9]. The transformation of physical quantities into a phase shift has extended beyond traditional applications and is frequently utilized in experimental physics, radio physics, experimental medicine, and cutting-edge areas of science and technology [10].

In certain practical applications, it is necessary to measure phase shifts over a frequency range from infralow to ultrahigh, in the presence of noise and interference, across a broad dynamic range of signal amplitudes. For harmonic signals in measurement systems, key concepts include phase, initial phase, phase shift, and time delay. Currently, the most critical aspect in phasemetry is the accurate determination of the phase shift. According to normative documents, phase shift is defined as the absolute difference between the initial phases of two harmonic signals. Therefore, the scientific and technical task of improving existing methods and developing advanced approaches for phase shift measurement between two harmonic signals remains highly relevant.

The most comprehensive classification of phase shift measurement methods is presented in [11]. Based on the principle of phase shift evaluation, these methods are divided into compensation methods and methods that convert the phase shift into other quantities such as voltage, time interval, or geometric parameters of oscilloscope traces.

The compensation method is based on balancing (compensating) the phase shift between two harmonic signals, reducing it to zero by adjusting the phase of one or both signals using a controlled phase shifter (or a phase shift standard) [11, 12]. Measurements are performed at fixed intermediate frequencies, ensuring the correct operation of the phase shifter and phase deviation indication systems. This method provides high accuracy, close to that of the measuring phase shifters.

The conversion method determines the phase shift after it is transformed into an intermediate quantity such as voltage, current, beam deflection in an oscilloscope, or time interval. Key known implementations of this method are reviewed.

The additive method [11, 13] relies on vector addition of signals. When two harmonic signals are added, the amplitude of the resulting signal depends on the amplitudes of the input signals and their phase shift. Since the phase shift is inferred from the amplitude measurements of three harmonic signals, this method is often referred to as the "three-voltmeter method." To simplify the process, automatic adjustment of input signal levels is applied [14], making the output amplitude dependent only on the phase shift.

The multiplicative method (voltage multiplication) [11, 15] involves multiplying two harmonic signals to obtain a signal that includes a DC component and a harmonic component. The DC value depends on the amplitudes and the phase shift. As with the additive method, automatic signal level regulation is used to simplify the measurement process.

In the oscillographic method, as shown in [11, 16], the phase shift is determined by analyzing the shape and nature of oscilloscope traces. For linear sweep measurement, a multi-channel oscilloscope is required. In the sinusoidal sweep mode, one signal is applied to the horizontal deflection and the other to the vertical, resulting in an interference pattern (typically an ellipse) whose axes are rotated. The measured dimensions of the ellipse allow the phase shift to be calculated.

The time-delay conversion method, presented in [11], determines the phase shift based on the time delay between the signals. Measuring the time delay between characteristic points (e.g., zero-crossings with matching signal derivatives) allows the phase shift to be inferred from the signal frequency or period.

The main limitations of existing methods include [11–16]:

- Significant measurement error due to phase asymmetry in the signal transmission channels;
- Requirement for two analog-to-digital conversion channels, necessitating synchronization of their sampling clocks;
- High sensitivity to internal and external noise sources;
- Large computational load due to a wide initial uncertainty range for phase shift estimation;
- Non-linearity of the calibration characteristic.

The objective of this study is to propose methodological principles for implementing a compensation-based method for determining the phase shift between two harmonic signals using a signal obtained by summing the outputs of a dual half-period transformation applied to the original signals. This approach aims to measure the phase shift between the input harmonic signals.

As a multi-valued metric for phase shift estimation, the study explores the use of a set of reference base functions synthesized computationally. The zero-indicator is defined as the condition of minimizing the sum of squared deviations between the vector of the summed signal and the reference function. This approach is expected to significantly reduce the cost of the device and improve its metrological performance.

Furthermore, a methodology is proposed for determining the initial interval for phase shift estimation based on the analysis of the vector obtained after amplitude-time transformation of the composite signal. This operation is intended to reduce the number of computational operations required, thereby decreasing the overall measurement time.

The proposed approach to implementing the compensation method for phase shift measurement is expected to enhance the effectiveness of diagnostics for weapons and military equipment by ensuring the required level of parameter monitoring reliability while reducing the cost of measurement equipment used in intelligent diagnostic systems.

3. Measurement Procedure and Operational Steps

This section provides a detailed description of the measurement procedure and outlines the sequence of operations recommended for determining the phase shift between two harmonic signals using the dual half-period transformation method. To facilitate understanding, a structural diagram illustrating the implementation of the proposed phase shift measurement method is presented in Figure 1.

Harmonic signals u_{in1}(t) and u_{in2}(t), exhibiting a phase shift $\Delta\varphi$, within the interval [0,2π], are input into the measurement system. To enhance the accuracy of phase shift measurement, the input stage performs auxiliary operations, including hardware-based filtering to mitigate external noise and amplification of signals u_{f1}(t) and u_{f2}(t) and u_1(t) and u_2(t). This amplification ensures that the output amplitudes of the signals differ by no more than 20%, thereby improving the sensitivity of the phase shift measurement.

This foundational representation facilitates subsequent processing steps, including the dual half-period transformation and amplitude-time conversion, which are integral to the proposed phase shift measurement methodology.

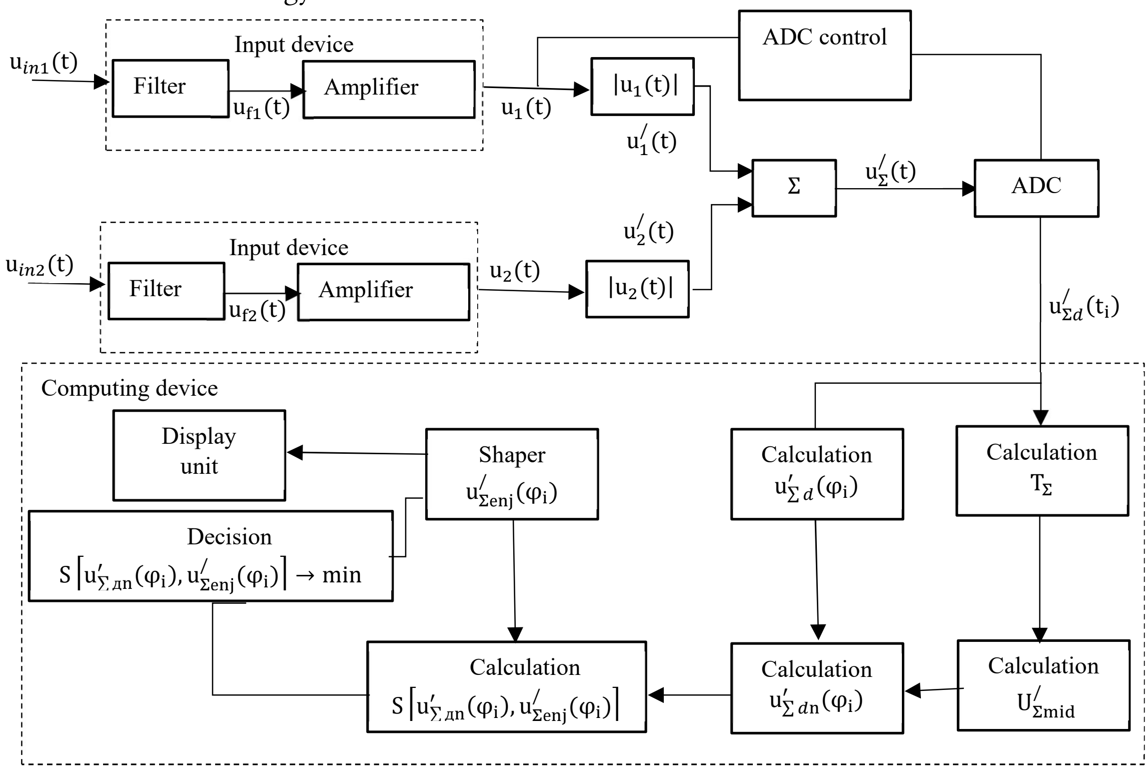

Figure 1: Structural diagram of the phase shift measurement system utilizing dual half-period transformation.

Harmonic signals $u_{in1}(t)$ and $u_{in2}(t)$, exhibiting a phase shift $\Delta\varphi$, within the interval [0,2π], are input into the measurement system. To enhance the accuracy of phase shift measurement, the input stage performs auxiliary operations, including hardware-based filtering to mitigate external noise and amplification of signals $u_{f_1}(t)$ and $u_{f_2}(t)$ and $u_1(t)$ and $u_2(t)$. This amplification ensures that the output amplitudes of the signals differ by no more than 20%, thereby improving the sensitivity of the phase shift measurement.

Considering that phase shift measurement is inherently a relative measurement, the variations in signals $u_1(t)$ and $u_2(t)$ can be expressed as:

$$u_1(t) = U_{m1}\cos(2\pi f t), \tag{1}$$

$$u_2(t) = U_{m2}\cos(2\pi f t + \Delta\varphi). \tag{2}$$

Where Um1 and Um2 are the amplitudes of the respective signals, $f = \frac{1}{t}$ is the angular frequency, and $\Delta\varphi$ is the phase shift between the signals.

The time diagram illustrating the harmonic signals $u_1(t)$ and $u_2(t)$, which exhibit a specific phase shift $\Delta\varphi$ relative to each other, is presented in Figures 2a and 2b.

Following the input stage, the signals $u_1(t)$ and $u_2(t)$ are directed to the dual half-period transformation unit. At the outputs of the transformation units, the resulting signals are:

$$u'_1(t) = |u_1(t)| = |U_{m1}\cos(2\pi f t) \quad (3)$$

$$u'_2(t) = |u_2(t)| = |U_{m1}\cos(2\pi f t + \Delta\varphi) \quad (4)$$

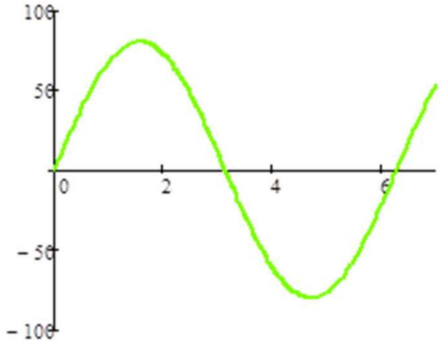

Figure 2a: Time Diagram of Signal $u_1(t)$.

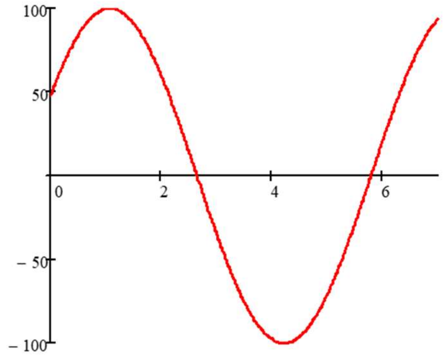

Figure 2b: Time Diagram of Signal $u_2(t)$.

The time diagrams for the signals u'1(t) and u'2(t) is presented in Figures 3a and 3b, respectively.

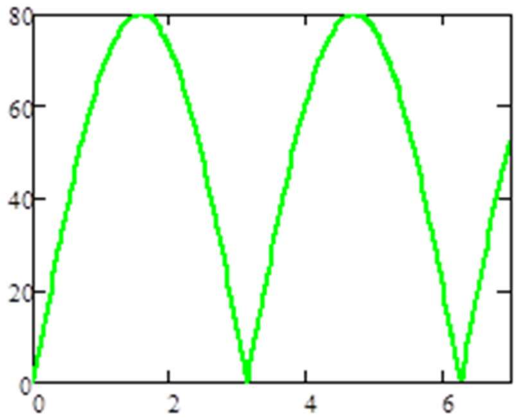

Figure 3a: Time Diagram of Signal $u_1'(t)$

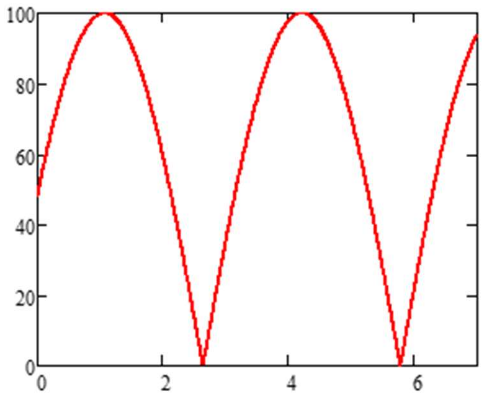

Figure 3b: Time Diagram of Signal $u_2'(t)$

From the output of the device for conducting a two-half-wave conversion, the signals $u_1'(t)$ and $u_2'(t)$ are fed to the summing device (adder). Summing $u_1'(t)$ and $u_2'(t)$ we obtain the signal $u'_{\Sigma}(t)$, which can be described by the following relation:

$$u'_{\Sigma(t)} = \left\langle \begin{matrix} U_{1mi} + \frac{U_{1min} - U_{2min}}{t_{1.2}} + \left(U'_{1m} - \frac{U_{1min} + U_{2min}}{2}\right) \times \sin(f_{1.2} t) for \ \ t_1 \le t < t_2 \\ U_{2min} + \frac{U_{2min} - U_{1mi}}{t_{2.1}} + \left(U'_{2max} - \frac{U_{1min} + U_{2min}}{2}\right) \times \sin(f_{2.1} t) \ for \ \ t_2 \le t < t_1 \end{matrix} \right\rangle \quad (5)$$

where U_{1mi} , U_{2mi} - points of discontinuity of the function on the interval from 0 to $\frac{T}{2}$;

$U'_{1max} = (U_{m1} + U_{m2})cos\frac{\Delta\varphi}{2}$- local maximum on the time interval $t_1 \le t < t_2$;

$U'_{2max} = (U_{m1} + U_{m2})sin\frac{\Delta\varphi}{2}$- local maximum on the time interval $t_2 \le t < t_1$.

The time diagram of the signal $u'_{\Sigma(t)}$ is shown in Fig. 4:

From the output of the summing device (adder), the signal enters the input of the analog-to-digital converter, which implements amplitude-time conversion.

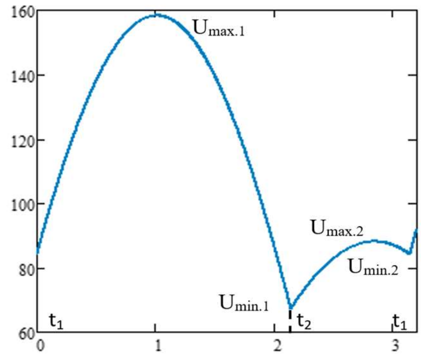

Figure 4: Time Diagram of Signal $u'_{\Sigma(t)}$

The control input of the analog-to-digital converter receives a signal indicating the start of analog-to-digital conversion and its end. In this case, we will introduce the assumption that the amplitude-time conversion begins under the condition of transition $u_1(t) = 0$ from a negative value to a positive value, and ends under the condition of transition $u_1(t) = 0$ from a positive value to a negative value. The minimum number of samples is selected taking into account the maximum signal frequency and the Nyquist frequency. After performing the specified operation, the signal $u'_{\Sigma}(t)$, according to [10], will be represented by a vector of instantaneous values $u'_{\Sigma d}(t_i)$ corresponding to certain moments of time ti., which can be written in the form:

$$u'_{\Sigma d}(t_i) = (U_1, U_{2,\dots}, U_{i,\dots}, U_n) \tag{6}$$

where U_i - instantaneous signal values $u'_{\Sigma}(t)$ at the *i*-th moment of time, obtained as a result of analog-to-digital conversion;

n - the number of samples obtained during analog-to-digital conversion of the $u'_{\Sigma}(t)$ in the interval from 0 to $\frac{T}{2}$.

The time diagram of the vector of instantaneous values $u'_{\Sigma d}(t_i)$ corresponding to certain moments of time t_i is shown in Fig. 5.

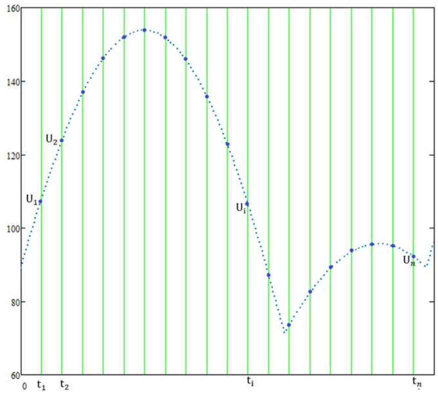

Figure 5: Time diagram of the signal of the vector of instantaneous values $u'_{\Sigma d}(t_i)$ corresponding to certain moments of time t_i.

A vector$c$ arrives at a computing device.

In the computing device, an operation is performed to determine the period T_{Σ} of the signal $u'_{\Sigma}(t)$. Based on the known values of the sampling frequency f_{Σ} and the number of samples (*N*), the signal period $u'_{\Sigma d}(t_i)$ is determined by the following expression:

$$T_{\Sigma} = \frac{T}{2} = \frac{n}{f_d} \tag{7}$$

To reduce computational operations and simplify the procedure for determining $\Delta\varphi$, it is advisable to implement an operation in the computing device to transition the vector

$u'_{\Sigma d}(t_i) = (U_1, U_{2,\dots}, U_{i,\dots}, U_n)$ from time samples to phase samples of the signal.

Then, based on the known value of the phase sampling step $\Delta\varphi_D$, it is determined as:

$$\Delta\varphi_D = \frac{\Delta t \cdot 2\pi}{T} \tag{8}$$

To reduce the methodical component of the measurement error, it is proposed to normalize the instantaneous values of the signal $u'_{\Sigma d}(t_i)$ to the average value $U'_{\Sigma mid}$, which can be determined using the following relation:

$$U'_{\Sigma mid} = \frac{1}{n}\sum_{i=1}^{n} U_i \tag{9}$$

Then, the normalized function $u'_{\Sigma dn}(\varphi_i)$ will have the form:

$$u'_{\Sigma dn}(\varphi_i) = \left(\frac{U_1}{U'_{\Sigma mid}}, \dots, \frac{U_i}{U'_{\Sigma mid}}, \dots, \frac{U_n}{U'_{\Sigma mid}}\right) = (U_{1.n}, U_{2.n}, \dots, U_{i.n}, \dots, U_{n.n} \tag{10}$$

As a multivalued measure of the phase shift, a set of reference functions synthesized in the computing device is proposed. Given that the amplitudes of the signals $u_1(t)$ and $u_2(t)$ are close, then as a reference function $u'_{\Sigma e}(\varphi_i)$, it is proposed to use the function of the summed signal obtained after performing half-wave rectification, assuming the equality of amplitudes U_{me}, according to the phase samples, which will have the following form:

$$u'_{\Sigma e}(\varphi_i) = \begin{cases} U_{e.min} + \left(U_{e1\max} - U_{e\,min}\right) \times \sin\left(\left(1 - \dfrac{\Delta\varphi}{\pi}\right)\varphi_i\right) & for\ 0 \le \varphi_i < \pi - \Delta\varphi \\ U_{e.min} + \left(U_{e2\max} - U_{e\,min}\right) \times \sin\left(\left(\dfrac{\Delta\varphi}{\pi}\right)\left(\varphi_i - (\pi - \Delta\varphi)\right)_i\right) & for\ \pi - \Delta\varphi \le \varphi_i < \pi \end{cases} \tag{11}$$

Where $U_{e.min} = U_{m\,e}\, sin\,\Delta\,\varphi$ are the discontinuity points of the reference function in the interval from 0 to π;

$U_{e1.max} = 2U_{m\,e}\cos(\,2\Delta\,\varphi)$ is the maximum value of the reference function in the interval $0 \le \varphi_i < \pi - \Delta\varphi$;

$U_{e2.max} = 2U_{m\,e}\sin(\,2\Delta\,\varphi)$ is the maximum value of the reference function in the interval $\pi - \Delta\varphi \le \varphi_i < \pi$

Performing the calculation according to the above expression for each value of φ_i, for a certain value of the phase shift $\Delta\varphi$, and performing normalization to the average value of the reference function, we obtain a vector of the normalized reference function:

$$u'_{\Sigma en}(\varphi_i) = (U_{1.e}, U_{2.e,\dots}, U_{i.e,\dots}, U_{n.e}) \tag{12}$$

Then, taking into account the above, the problem of determining the phase shift of harmonic signals $u_1(t)$ and $u_2(t)$ is formulated as follows: from the entire set of normalized reference functions $u'_{\Sigma en}(\varphi_i)$, select the function $u'_{\Sigma en.j}(\varphi_i) = (U_{1.e.j}, U_{2.e.j,\dots}, U_{i.e.j,\dots}, U_{n.e.j})$that most fully corresponds to the normalized signal $u'_{\Sigma Dn}(\varphi_i) = (U_{1.n}, U_{2.n,\dots}, U_{i.n,\dots}, U_{n.n})$.

The method of least squares (MLS), due to its wide range of applications, occupies an exceptional place among the methods of mathematical statistics. MLS plays a particularly important role in solving measurement problems. The task of MLS is to estimate the regularities observed against the background of random fluctuations and to use this estimation for further calculations, in particular, for the approximation of measured quantities.

Given that the amplitude-time conversion of the signal $u'_{\Sigma}(t)$ is performed by a single analog-to-digital converter under the same conditions, it can be argued that the root mean square deviation of the error in determining the instantaneous values U_i of the signal $u'_{\Sigma}(t)$ is constant ($\sigma_{U_i} = \mathrm{const\ for\ i} = 1 \dots \mathrm{n}$). Then, the observations of the instantaneous values U_i refer to measurements of equal precision.

As a parameter of the degree of coincidence between the vector of the normalized signal $u'_{\Sigma d}(\varphi_i)$, and the j-th reference normalized function $\mathrm{u'}_{\Sigma \mathrm{enj}}(\varphi_i)$, according to MLS, we will use the sum of the squares of the discrepancies for the corresponding element of the signal $u'_{\Sigma d}(\varphi_i)$ and the corresponding element of the function $u'_{\Sigma enj}(\varphi_i)$, that is:

$$S\left[u'_{\Sigma dn}(\varphi_i), u'_{\Sigma e\,nj}(\varphi_i)\right] = \sum_{i=1}^{n}\left(U_{i.n} - U_{i.e.j}\right)^2 \tag{13}$$

Alright, based on the preceding discussion, the measurement problem of determining the phase shift $\Delta\varphi$ of the signals $u_1(t)$ and $u_2(t)$ can be formulated as follows: from the entire set of reference phase shift functions, we will select the function $u'_{\Sigma enj}(\varphi_i)$, that provides the minimum value of the sum of squared deviations between the discrete normalized signal $u'_{\Sigma dn}(\varphi_i)$ and itself, that is:

$$S\left[u'_{\Sigma dn}(\varphi_i), u'_{\Sigma e\,nj}(\varphi_i)\right] = \sum_{i=1}^{n}\left(U_{i.n} - U_{i.e.j}\right)^2 \to min \tag{14}$$

From the formulation of the measurement problem, it is evident that this is a problem of finding the minimum value of the sum of squared deviations. This class of problems can be solved using analytical or numerical methods. An analysis of well-known software tools [17, 18] that are widely used at present shows that they employ numerical methods for finding the extremum.

In turn, numerical methods for finding the extremum of a function are divided into gradient methods, methods using second derivatives, and direct methods.

As a rule, when solving extremum search problems using gradient methods and methods using second derivatives, faster convergence is achieved than when using direct methods.

However, the application of methods using derivatives to solve this problem leads to difficulties due to the functional dependence of the investigated function.

Direct methods do not require the fulfillment of the conditions of regularity and continuity of the investigated function and the existence of a derivative.

An analysis of the change in the sum of squared deviations when determining the phase shift shows that this function is quasi-convex [19, 20].

The determination of which reference function provides the minimum value of the sum of squared deviations will be carried out using the golden section search method. The choice of this method compared to known methods, such as the dichotomic search method, is because it requires fewer iterations.

The following algorithm for determining $\Delta\varphi$ using the golden section search method is proposed:

Preliminary Stage.

Determination of the permissible final length of uncertainty l.

The selection of the minimum value of this parameter is intended to be carried out based on the requirements for the error of the phase shift measurement problem solution, taking into account the accuracy characteristics of the technical means involved in the process of analog-to-digital conversion of the signal and additional operations, as well as rounding errors during calculations.

As can be seen from the conditions for measuring the phase shift and the list of conversion operations of the input harmonic signals $u_1(t)$ and $u_2(t)$, the initial interval of uncertainty is $[0, \pi]$.

To explain the order of synthesis of the algorithm for determining the length of the new interval of uncertainty for the first iteration, Figure 6 is proposed.

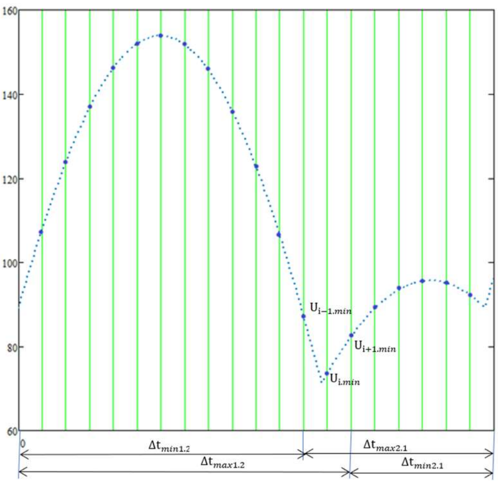

Figure 6: Graphical explanation of the sequence for determining the length of the uncertainty interval for the first iteration.

Determination of the length of the uncertainty interval l for the first iteration, its beginning $\Delta\varphi_{\lambda 1}$, and its end $\Delta\varphi_{\mu 1}$ will be carried out based on the following observations and assumptions:

The number of samples of the instantaneous values U_i of the signal $u'_{\Sigma}(t)$ obtained during the analog-to-digital conversion that belong to the time intervals $t_{1.2}$ and $t_{2.1}$ are different.

Based on the properties of the function obtained by summing the signals after performing full-wave rectification of two harmonic signals $u_1(t)$ and $u_2(t)$, as shown in [18], the values of the time intervals $t_{1.2}$ and $t_{2.1}$, depending on the value of the phase shift $\Delta\varphi$, are determined respectively using the following relations:

$$t_{1.2} = \frac{1}{2f} - \frac{\Delta\varphi}{2\pi f}; \tag{15}$$

$$t_{2.1} = \frac{\Delta\varphi}{2\pi f} \tag{16}$$

The discontinuity points $U_{1\min}$ and $U_{2\min}$ of the function $u'_{\Sigma}(t)$ in the interval from 0 to $\frac{T}{2}$ will, in the general case, be located within the quantization Δt_i interval and will not coincide with the values of $U_{1.i\min}$ and U_{n}.

The point $U_{i-1.min}$ will always be located in the time interval $t_{1.2}$, and accordingly, the point $U_{i+1.min}$ will always be located in the time interval $t_{2.1}$.

Then, based on the above observations, the following sequence of steps is proposed:

Using the vector $u'_{\Sigma d}(t_i)$ of instantaneous values U_i, the value of $U_{i.min}$, is determined, for example, by applying a sequential search method.

Determination of the end $\Delta\varphi_{\mu 1(1.2)}$ of the new uncertainty interval for the first iteration for the function in the time interval $t_1 \le t < t_2$. To do this, we perform the following additional operations: determine $U_{i-1.min}$, and based on the known sample number Δt , and the quantization interval, calculate $\Delta t_{min1.2}$ using the relation:

$$\Delta t_{min1.2} = (i_{min} - 1)\Delta t$$

Then, based on the known values of $\Delta t_{min1.2}$ and T_{Σ}, the end of the interval $\Delta\varphi_{\mu 1(1.2)}$ is calculated using expression (16) as:

$$\Delta\varphi_{\mu 1(1.2)} = \pi \frac{T_{\Sigma} - \Delta t_{min1.2}}{T_{\Sigma}} \tag{17}$$

Determination of the beginning $\Delta\varphi_{\lambda 1(1.2)}$ of the new uncertainty interval for the first iteration for the function in the time interval $t_1 \le t < t_2$. To do this, we perform the following additional operations: determine $U_{i+1.min}$, and based on the known sample number and the quantization interval Δt, calculate $\Delta t_{max1.2}$ using the relation:

$$\Delta t_{max1.2} = (i_{min} + 1)\Delta t$$

Then, based on the known values of $\Delta t_{max1.2}$ and T_Σ, the end of the interval $\Delta\varphi_{\lambda1(1.2)}$ is calculated using expression (16) as:

$$\Delta\varphi_{\lambda1(1.2)} = \pi\frac{T_\Sigma - \Delta t_{max1.2}}{T_\Sigma} \tag{18}$$

Determination of the beginning $\Delta\varphi_{\lambda1(2.1)}$ of the new uncertainty interval for the first iteration for the function in the time interval $t_2 \leq t < t_1$. To do this, we perform the following additional operations: determine $U_{i+1.min}$, and based on the known sample number and the quantization interval Δt, calculate $\Delta t_{min2.1}$ using the relation:

$$\Delta t_{min2.1} = \Delta t + (i_{min} - 1)\Delta t$$

Then, based on the known values of $\Delta\varphi_{\mu1(1.2)}$ the beginning of the interval $\Delta t_{min1.2}$ and T_Σ is calculated using expression (17) as:

$$\Delta\varphi_{\lambda1(2.1)} = \pi\frac{\Delta t_{min2.1}}{T_\Sigma} \tag{19}$$

Determination of the beginning $\Delta\varphi_{\lambda1(2.1)}$ of the new uncertainty interval for the first iteration for the function in the time interval $t_2 \leq t < t_1$. To do this, we perform the following additional operations: determine $U_{i+1.min}$, and based on the known sample number and the quantization interval Δt, calculate $\Delta t_{min2.1}$ using the relation:

$$\Delta t_{max1.2} = \Delta t + (i_{min} - 1)\Delta t$$

Then, based on the known values of $\Delta t_{max2.1}$ and T_Σ, the end of the interval $\Delta\varphi_{\lambda1(1.2)}$is calculated using expression (17) as:

$$\Delta\varphi_{\lambda1(1.2)} = \pi\frac{\Delta t_{max2.1}}{T_\Sigma} \tag{20}$$

Determination of the beginning $\Delta\varphi_{\lambda1}$ of the new uncertainty interval for the first iteration will be carried out based on the following conditions:

If $i_{min} \approx (n - i_{min})$, then $\Delta\varphi_{\lambda1} = \Delta\varphi_{\lambda1(1.2)}$, in case $\Delta\varphi_{\lambda1(1.2)} \geq \Delta\varphi_{\lambda1(2.1)}$then

$\Delta\varphi_{\lambda1} = \Delta\varphi_{\lambda1(2.1)}$,

if $i_{min}\ \ (n - i_{min})$ then $\Delta\varphi_{\lambda1} = \Delta\varphi_{\lambda1(2.1)}$,

if $i_{min}\ \ (n - i_{min})$ then $\Delta\varphi_{\lambda1} = \Delta\varphi_{\lambda1(1.2)}$.

Determination of the end $\Delta\varphi_{\mu1}$ of the new uncertainty interval for the first iteration will be carried out based on the following conditions:

If $i_{min} \approx (n - i_{min})$, then $\Delta\varphi_{\mu1} = \Delta\varphi_{\mu1(2.1)}$, in case $\Delta\varphi_{\mu1(1.2)} < \Delta\varphi_{\mu1(2.1)}$, in case $\Delta\varphi_{\mu1(1.2)} \geq \Delta\varphi_{\mu1(2.1)}$, then $\Delta\varphi_{\mu1} = \Delta\varphi_{\mu1(1.2)}$,

If $i_{min}\ \ (n - i_{min})$ then $\Delta\varphi_{\mu1} = \Delta\varphi_{\mu1(2.1)}$,

If $i_{min}\ \ (n - i_{min})$ then $\Delta\varphi_{\mu1} = \Delta\varphi_{\mu1(1.2)}$.

The next step is to calculate the value of the deviation of the sum of squares $S_{\lambda1}\left[u'_{\sum dn}(\varphi_i), u'_{\sum en\lambda1}(\varphi_i)\right]$ between the vector $u'_{\sum dn}(\varphi_i)$ and the reference function $u'_{\sum en\lambda1}(\varphi_i)$, provided that the phase shift is equal to $\Delta\varphi_{\lambda1}$, and value $S_{\mu1}\left[u'_{\sum dn}(\varphi_i), u'_{\sum en\mu1}(\varphi_i)\right]$ provided that the phase shift equals $\Delta\varphi_{\mu1}$ using the ratio (11) and (14).

We will now calculate $S_{\lambda1}\left[u'_{\sum dn}(\varphi_i), u'_{\sum en\lambda1}(\varphi_i)\right]$ and $S_{\mu1}\left[u'_{\sum dn}(\varphi_i), u'_{\sum en\mu1}(\varphi_i)\right]$, assume that $k = 1$ and move onto the main stage of calculation.

Main Stage.

Step 1. If $\Delta\varphi_{b.k} - \Delta\varphi_{a.k} \leq 1$, then stop and accept the value of the phase shift that is equal to

$$\Delta\varphi = \frac{\Delta\varphi_{b.k} - \Delta\varphi_{\mathrm{a.k}}}{2} \tag{21}$$

Where $\Delta\varphi_{\mathrm{a.k}}$ - beginning of the uncertainty interval for the k iteration;
$\Delta\varphi_{b.k}$ - end of the uncertainty interval for the k iteration.
In other case, if

$$S_{\lambda1}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\lambda1}(\varphi_{\mathrm{i}})\right] < S_{\mu1}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\mu1}(\varphi_{\mathrm{i}})\right] \tag{22}$$

then move onto the 3rd step, if

$$S_{\lambda1}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\lambda1}(\varphi_{\mathrm{i}})\right] \geq S_{\mu1}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\mu1}(\varphi_{\mathrm{i}})\right] \tag{23}$$

Then move onto the step 2.
Step 2. Determine:

$$\begin{gathered} \Delta\varphi_{\mathrm{a.(k+1)}} = \Delta\varphi_{\lambda.\mathrm{k}}, \Delta\varphi_{\mathrm{b.(k+1)}} = \Delta\varphi_{b.\mathrm{k}}, \Delta\varphi_{\lambda.\mathrm{(k+1)}} = \Delta\varphi_{\mu.\mathrm{k}}, \\ \Delta\varphi_{\mu.\mathrm{(k+1)}} = \Delta \mathrm{t}_{\mathrm{a.k}} + \alpha(\Delta\varphi_{\mathrm{b.(k+1)}} - \Delta\varphi_{\mathrm{a.(k+1)}} \end{gathered} \tag{24}$$

Calculate $S_{\mu1}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\mu(k+1)}(\varphi_{\mathrm{i}})\right]$ and go to step 4.
Step 3. Determine:

$$\begin{gathered} \Delta\varphi_{\mathrm{a.(k+1)}} = \Delta\varphi_{a.\mathrm{k}}, \Delta\varphi_{\mathrm{b.(k+1)}} = \Delta\varphi_{\mu.\mathrm{k}}, \Delta\varphi_{\mu.\mathrm{(k+1)}} = \Delta\varphi_{\lambda.\mathrm{k}} \\ \Delta\varphi_{\lambda.\mathrm{(k+1)}} = \Delta\varphi_{\mathrm{a.k}} + (1 - \alpha) \times (\Delta\varphi_{\mathrm{b.(k+1)}} - \Delta\varphi_{\mathrm{a.(k+1)}} \end{gathered} \tag{25}$$

Calculate $S_{\lambda(k+1)}\left[\mathrm{u}'_{\sum \mathrm{dn}}(\varphi_{\mathrm{i}}), \mathrm{u}'_{\sum \mathrm{en}\lambda(k+1)}(\varphi_{\mathrm{i}})\right]$ and go to step 4.
Step 4. Replace k with $\mathrm{k} + 1$ and move onto the step 1.

The main sources of errors in the proposed measurement method are: the error component due to amplitude-time conversion; the error component due to the formation of the start and end of the amplitude-time conversion; the influence of external noise and noise from the internal environment of the measuring device; the rounding error when searching for the minimum value of the sum of squares deviation; and the error due to the discreteness of the reference function generation.

4. Experiments, Results, and Discussion

Given that the use of full-wave rectification for solving the measurement problem of phase shift determination is a relatively new direction in the field of phasometry, the main objective of this material is to propose a methodological approach for determining phase shift using this transformation, which can be used in the construction of information-measuring systems.

Paper [16] considers the possibility of using full-wave rectification of harmonic signals to determine phase shift. This material identifies the ways and directions with which it is possible to implement an oscillographic measurement method using full-wave rectification of harmonic signals. However, a significant disadvantage of oscillographic measurement methods is the need for visual reading. This drawback does not allow the use of this method in information-measuring systems, which can be a component of a system for diagnosing and predicting the technical condition of complex technical systems.

Compared to known methods of implementing the compensation method for measuring phase shift, the proposed method allows reducing costs by up to 70 percent in the manufacturing of diagnostic and prognostic systems for technical condition, increasing accuracy indicators, and minimizing its size, by using a function synthesized by the computing means of the information-measuring system as a reference measure of the phase shift, and using MLS as a null indicator. Based on the characteristics of well-known mathematical software packages that have found wide application for modeling, such as Mathcad, MATLAB, and Electronics Workbench, it is known that it is possible to perform calculations with an accuracy of up to the 16th decimal place, which significantly exceeds the error indicators of known reference measures of phase shift and methods for implementing physical means of a null indicator.

The proposed method for determining the length of the uncertainty interval for the first iteration, compared to the method proposed in [21], will significantly reduce the number of iterations by up to 50% when searching for the minimum value of the discrepancy between the vectors of the discrete normalized signal and the normalized reference function using the golden section search method. To improve the sensitivity of the proposed phase shift determination method, it is proposed to use the vector of the signal normalized to the average value.

Additive signal processing, multiplicative signal processing, and Hilbert transform-based phase meters involve an auto-adjustment operation of signal levels for each of the measuring channels. The presence of this operation leads to a significant complication of the measuring channels, and accordingly, to a component of error caused by the phase asymmetry of the signal transmission channels, as well as the need for synchronization of the analog-to-digital conversion operation.

To assess the possible measurement error of the proposed phase shift determination method, a computer measurement experiment was carried out. The computer measurement experiment was performed using the Monte Carlo method, with the universal mathematical package MathCAD. The computer measurement experiment was conducted for the following initial data: noise did not exceed 0.2 of the signal level, 0.2, U_{m1} =10V, U_{m2} =12V, the analog-to-digital conversion error followed a uniform distribution law with an interval of 0.01 V, f=1 Hz, the error in forming the reference function vector followed a uniform distribution law with an interval of 0.0001 V, and 10 instantaneous values were determined per measurement period. Each measurement was performed 5 times. The maximum and minimum error sizes of the measurement results for different phase shift values are given in Table 1.

Table 1
Results of the computer measurement experiment.

MathResult, radians	0.1	0.5	0.7	0.9	1.1	1.4
Minimum Error, minutes	19	17	16	15	17	19
Maximum Error, minutes	25	23	21	23	22	28

A disadvantage of the proposed method for implementing the compensation measurement method is that the phase shift measurement range is within $[0, \pi]$ radians, and there is a limitation on the frequency range, which will be determined by the characteristics of the full-wave rectifier.

Further research, in our opinion, should be directed towards the synthesis of a mathematical model of the phase shift determination error. This model can subsequently be considered as a mathematical basis for synthesizing a methodology for justifying the requirements for filtering input signals $u_1(t)$ and $u_2(t)$, analog-to-digital conversion, computing hardware, and software.

After carrying out the indicated work, it is advisable to conduct computational experiments through simulation modeling, for example, in MATLAB and Electronics Workbench, in order to determine the correctness (adequacy) of the proposed models and the feasibility and type of use of digital filtering of the vector of instantaneous values $u'_{\sum d}(t_i)$, both under the influence of external electromagnetic interference on the measuring system and with the simulation of internal noise.

5. Conclusions

A structural diagram for implementing a method to determine the phase shift between two harmonic signals has been synthesized. The method is based on comparing the shape of a normalized signal—obtained by summing the harmonic signals after their double half-period transformation—with a set of normalized reference functions synthesized by computational means. The required list of technical tools and computational operations needed to solve the task has been identified.

As a criterion for matching the shapes of the analyzed signal and the reference function, the minimum deviation of the sum of squares between them is proposed. To solve the problem, an algorithm is suggested for determining the uncertainty interval in the first iteration, as well as an algorithm for finding the extremum of the deviation function between the reference function set and the analyzed signal using the golden section method.

Compared to conventional methods based on analog-to-digital amplitude-time transformation, the proposed method offers several advantages: it reduces the error component caused by phase asymmetry of the transmission channels due to their shorter length; it significantly lowers the requirements for automatic input signal level adjustment; it allows for the synthesis of a single analog-to-digital conversion channel for the signal under analysis instead of two, eliminating the need for synchronization between channels; and it significantly reduces the cost of creating a multivalued phase shift reference by using a set of synthesized reference functions and a system for matching the reference function with the test signal.

The main sources of error in this phase shift measurement method have been identified. The use of the proposed method for determining phase shift significantly reduces the complexity of control systems by simplifying circuit design, thereby saving up to 10% of material and technical resources for monitoring the parameters of military equipment and its components during the testing phases of development and production, without compromising control quality. Additionally, this method can serve as a methodological foundation for the development of diagnostic and measurement systems used in the operation of complex technical systems.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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