Optimizing form of a piezoelectric transformer⋆

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Abstract

A method for optimizing the shape of a piezoelectric transducer based on its one-dimensional linear model is proposed. The meaning of optimization is to select the shape of the piezoelectric plate side surface in such a way as to provide maximum stresses in the maximum possible volume. Analytical results of calculating the shape of the piezoelectric transducer and its numerical model are presented. The results obtained for the developed one-dimensional model are compared with the stresses obtained as a result of numerical modeling in the FreeFem++ program. The stress distribution values obtained as a result of numerical modeling are very close to the stress values in the piezoelectric element obtained in the one-dimensional model, which indicates the adequacy of a simple one-dimensional model and the shape optimization process. The proposed approach to optimizing the shape of a piezoelectric transducer allows obtaining its design with higher technical and economic indicators compared to the classical shape and reducing the cost.

Keywords

Power supply unit, transformer, piezotransformer, optimization

1. Introduction

In modern information management systems, there is often a need to obtain very compact, cheap and at the same time relatively high-voltage distributed power sources, in particular for Internet of Things systems. There are other areas of small energy, which use small capacities of consumed energy: small-sized household appliances, telephone cellular communication, systems for monitoring and diagnosing the technical condition of remote objects, lighting devices [1], smart biosensors [2], smart home control equipment [3]. All of these devices do not require large power sources, so they can use autonomous sources of electrical energy [2], such as small batteries, power over communication lines, such as in PoE (Power over Ethernet) systems energy harvester device [4,5], ets.

Recently, more efficient self-regenerating autonomous sources of electrical energy, which are based on new principles, have also been used. One of these directions is the development of energy sources that use piezoceramic elements (PE) [6, 7] and piezotransformers (PT) [8, 9] - converters of environmental energy into electrical energy, with subsequent storage and transmission to a receiving device. The requirements for such energy sources (accumulators and converters of ambient energy) depend significantly on the types of devices for which they are intended.

Most of all, such energy sources can be used in such systems and devices as:

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- compact wireless electronic devices with an extended service life,
- low-power built-in and wireless communication devices (for example, for cellular phones and smartphones),
- household electromechanics devices and electronics,
- piezoelectric generators for local lighting and alarm systems at remote infrastructure facilities, etc.

An integral part of such systems is a small-sized transducer - a piezoelectric transformer, which is used when high reliability, high (more than 90%) efficiency, complete galvanic separation, high wear resistance, and non-flammability are required. Piezo transformers do not create electromagnetic barriers and are insensitive to electromagnetic fields [10].

But the use of piezo elements and piezotransformers is limited by the lack of adequate models that could allow to calculate their parameters with high reliability and determine the optimal parameters of their design.

The purpose of mathematical modelling of electromechanical oscillatory processes in piezoceramic elements is to present the quantitative and qualitative parameters of electric and elastic fields in the piezo plate for calculating their characteristics, determining optimal dimensions and predicting efficiency [11]. Most of the existing research methods include methods for determining the characteristics and parameters of both piezoelectric elements and piezoelectric materials, and can be used to solve only local problems [12]. This is explained by the fact that the relationship between the PT electromagnetic field and mechanical fluctuations in it complicates the calculation of both its electrical and structural parameters, primarily its shape and geometric dimensions.

Modelling the main processes and characteristics of the PT will allow the creation of specialized software that would make it possible to calculate the optimal geometric dimensions and shape of the PT based on predetermined initial parameters to obtain highly efficient electromechanical converters built on its basis[13,14].

2. Approved methodology

Consider the problem of optimizing the shape of a planar piezo current transformer (PT) with polarization according to the thickness of the plate. Let the plate have a thickness h, and its middle plane coincides with the x0y plane. The material has a density ρ.

Assume that the side surfaces of the piezotransformer are free of electrodes, and the upper and lower surfaces are covered with a system of electrodes, the gap between which approaches zero. To reduce energy losses, the piezotransformer is usually fixed in such a way that its surfaces do not transmit power to the fastening. This condition leads to the boundary condition: $\sigma_{ij} n_j = 0$, where n_j - is the external normal vector. In the case of one-dimensional oscillations with a circular frequency ω along the length (coordinate x) with variable width $b(x)$ of the piezotransformer and symmetry of the PT relative to the 0x axis, the equations describing it can be given in the form [10, 14]:

$$\frac{d}{dx}(b(x)\,\sigma_{11}) + \rho\omega^2 b(x) u_1 = 0 \quad (1)$$

$$\frac{d}{dx}u_1 = s_{11}\sigma_{11} + \frac{d_{31}}{h \quad b(x)} \int_{b/2}^{b/2} \varphi(x,y)dy \quad (2)$$

where $\varphi(x,y)$ - is the potential difference between the upper and lower electrodes of the PT, which depends, in general, on two coordinates. Since the piezotransformer most often works in a mode close to resonance, it can be assumed that the stress distribution in the transformer will be the same as with its own form of oscillations.

Then the system of equations will be simplified and can be written in the form of one equation of the second order:

$$\frac{d}{dx}\left(\frac{1}{b(x)}\frac{d}{dx}(b(x)\sigma_{11})\right)+\rho\omega^2 s_{11}\sigma_{11}=0 \tag{3}$$

Solving equations relative $b(x)$ to the given in advance $\sigma_{11}=\sigma_0[\sigma]$ we have:

$$b(x)=\frac{B}{\sigma_0}exp\left(-\int\left(\frac{\rho\omega^2 s_{11}\int\sigma_0 dx}{\sigma_0}\right)dx+\int\frac{A}{\sigma_0}dx\right) \tag{4}$$

In the case of the optimal shape of the PT, the stress σ_{11} in the PT material are equal to $[\sigma]$ and $\sigma_0=1$, then at interior points:

$$b(x)=B exp\left(-\frac{\rho\omega^2 s_{11}x^2}{2}+A\cdot x\right) \tag{5}$$

The unknown constant A only leads to the movement of the piezotransformer along the x axis, and when A=0, the unknown constant B is determined by the required PT power.

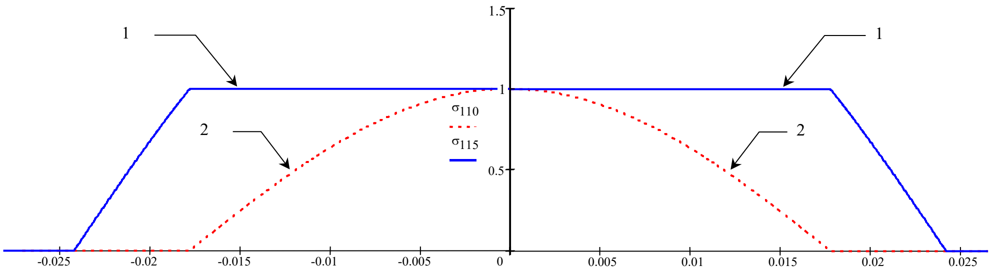

Figure 1: Distribution of width-averaged stress in the piezotransformer material in fractions of $[\sigma]$ (all dimensions in m) 1 - proposed piezotransformer, 2 - standard piezotransformer of constant width.

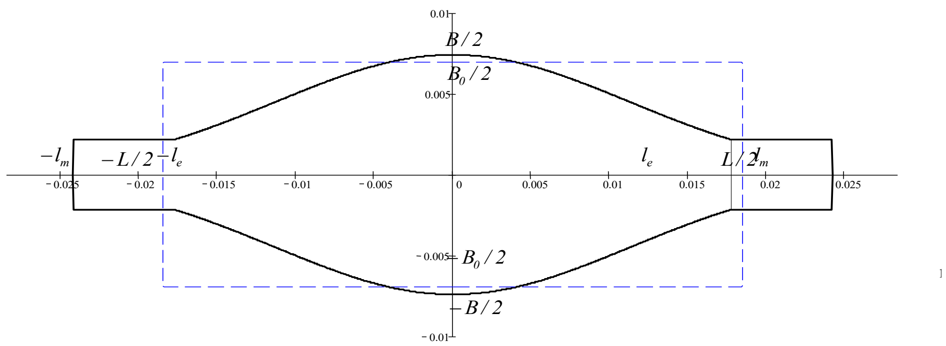

Figure 2: The shape of the piezotransformer proposed (1) compared to the classical one (2) (dimensions in meters).

It is easy to understand that the defect of such an optimal shape is that the optimal shape of the piezo plate must have an endless length, since otherwise the stress will never reach zero. But since the width in the final sections is small, it does not have a significant effect on the quality of the shape. Therefore, if the width is forcibly set at $|x|>l_e$ and the length $b(x)=b(l_e)$, of such a section is chosen such that the stress in the material decrease to zero, the resulting PT shape will be close to the optimal one, and the closer to the optimal one, the greater the l_e.

In the main part of such a piezo transformer, the stress σ_{11} are equal $[\sigma\sigma$, which follows from the method of its construction. However, to determine the size of the final ones, it is necessary to know the distribution of displacements in the main part of the piezo transformer. It follows from equation (1) that:

$$u(x) = -\frac{1}{\rho\omega^{2}\, b(x)}\frac{d}{dx}(b(x)\,\sigma_{11}) \tag{6}$$

Since in the main part $\sigma_{11} = [\sigma]$ and are constant, the displacement while maintaining the same boundary conditions at $x = 0$ in the main part will depend on the shape of the side surface and be defined as:

$$u(x) = -\frac{[\sigma]}{\rho\omega^{2}\, b(x)}\frac{d}{dx}b(x) = s_{11}[\sigma]x \tag{7}$$

Therefore, the displacement in the main part of the piezotransformer increases linearly with increasing distance from the center and proportionally to the product $s_{11}[\sigma\sigma$.

Let's write the boundary conditions on the contact side of the boundary section with the main part of the piezotransformer. At the boundary of the main part, the stress are equal $\sigma_{11} = [\sigma]$, and the displacements are equal to:

$$u(l_e) = s_{11}[\sigma]\, l_e \tag{8}$$

Usually, a piezotransformer is fixed so that no forces are applied to its outer surfaces, so the question arises about the size of the above-mentioned final regions where the stresses should drop to zero. We will consider the width of the final section to be constant and equal $b(l_e)$. In this section, the width is constant, and therefore the equations describing the free form of oscillations can be written in the form:

$$\frac{d}{dx}(\sigma_{11}) + \rho\omega^{2}u_1 = 0 \tag{9}$$

$$\frac{d}{dx}u_1 = s_{11}\sigma_{11} \tag{10}$$

The solution of the equations satisfying the boundary conditions for displacement and stress at the boundary of the main section can be written as:

$$\sigma = [\sigma]\cos\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right) - [\sigma]\sqrt{\rho\, s_{11}}\,\omega l_\varepsilon\, \sin\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right) \tag{11}$$

$$u = [\sigma]\left(\sqrt{\frac{s_{11}}{\rho\,\omega^{2}}}\sin\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right) + s_{11}\, l_\varepsilon \cos\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right)\right) \tag{12}$$

The stress will reach zero the first time the coordinate reaches a value:

$$l_m = l_e + \frac{1}{\sqrt{\rho\, s_{11}}\,\omega}\left(arctg\left(\frac{1}{\sqrt{\rho\, s_{11}}\,\omega\, l_e}\right)\right) \tag{13}$$

Thus, the length of the obtained piezotransformer becomes equal:

$$L(l_e) = 2l_m = \frac{\pi}{\sqrt{\rho\, s_{11}}\,\omega} + 2\, l_e - \frac{2}{\sqrt{\rho\, s_{11}}\,\omega}arctg\left(\sqrt{\rho\, s_{11}}\,\omega\, l_e\right) \tag{14}$$

that on:

$$\Delta L(l_e) = 2\, l_e - \frac{2}{\sqrt{\rho\, s_{11}}\,\omega} arctg\left(\sqrt{\rho\, s_{11}}\,\omega l_e\right) \qquad (15)$$

more than the length of a classical PT with the same resonance frequency, which is known to be

$$L_0 = \frac{\pi}{\sqrt{\rho\, s_{11}}\,\omega} \qquad (16)$$

It is easy to understand that, if $l_e = 0$, the piezotransformer is filled with a classical shape of a rectangle with a width B_0 and a length L_0. The amount of material used to create a piezotransformer of the proposed form is equal to:

$$V = 2\int_0^{l_e} b(x) h dx + 2\,(l_m - l_e)\, h\, b(l_e) \qquad (17)$$

In the unfolded form, the volume of the material is equal to:

$$V = \sqrt{\frac{2}{\pi}} L_0 B\, h\, erf\left(\frac{\pi\, l_e}{\sqrt{2} L_0}\right) + \frac{2B\, L_0 h}{\pi} exp\left(-\frac{\pi^2 l_e^{\,2}}{2\, L_0^{\,2}}\right)\left(arctg\left(\frac{L_0}{\pi\, l_e}\right)\right) \qquad (18)$$

where h is the thickness of the piezo plate.

Assume that the volume of the material of the piezo transformer is given and equal to V_0, then its maximum width is equal to:

$$B = \frac{V_0}{L_0\, h\left(\sqrt{\frac{2}{\pi}}\, erf\left(\frac{\pi\, l_e}{\sqrt{2} L_0}\right) + \frac{2}{\pi} exp\left(-\frac{\pi^2 l_e^{\,2}}{2\, L_0^{\,2}}\right)\left(arctg\left(\frac{L_0}{\pi\, l_e}\right)\right)\right)} \qquad (19)$$

3. Evaluation of the adequacy of the obtained design

Let's evaluate the quality of the resulting design. The criterion of quality will be the equality of the currents caused by mechanical oscillations through the upper surface of a conventional piezotransformer and the piezotransformer, the shape of which is proposed, with the same volume of material.

As is known, the current in the one-dimensional approximation will be defined as [10]:

$$I = -j\omega \int_S \left(d_{31}\, \sigma_{11} - \epsilon_{33}\frac{\Delta\varphi}{a}\right) ds \qquad (20)$$

Where S is the area of the entire piezo plate. The absolute value of the component caused by mechanical vibrations is determined by the formula:

$$I = \omega\, d_{31} \int_S \sigma_{11} ds \qquad (21)$$

Taking into account the stress distribution obtained above, the absolute value of the component caused by mechanical vibrations is determined by the formula:

$$\begin{aligned} I &= \omega\, d_{31} \int_{-l_e}^{l_e} B[\sigma]\, exp\left(-\frac{\rho\omega^2 s_{11} x^2}{2}\right) dx + \\ &+ 2B[\sigma] \int_{l_e}^{lm} exp\left(-\frac{\rho\omega^2 s_{11} l_e^{\,2}}{2}\right)\left(cos\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right) - \sqrt{\rho\, s_{11}}\,\omega l_\varepsilon\, sin\left(\sqrt{\rho\, s_{11}}\,\omega(x - l_e)\right)\right) dx \end{aligned} \qquad (22)$$

Whence, after appropriate transformations, we have:

$$I = \omega d_{31} B L_0 [\sigma] \sqrt{\frac{2}{\pi}} erf\left(\frac{\pi l_e}{\sqrt{2} L_0}\right) + 2\omega d_{31} B [\sigma] exp\left(-\frac{\pi^2 l_e^2}{L_0^2}\right)\left(\sqrt{l_e^2 + \frac{L_0^2}{\pi^2}} - l_e\right). \quad (23)$$

Considering the expression (19), we have:

$$I = \frac{\omega d_{31} V_0 [\sigma] \left(\sqrt{\frac{2}{\pi}} L_0 \, erf\left(\frac{\pi l_e}{\sqrt{2} L_0}\right) + 2 exp\left(-\frac{\pi^2 l_e^2}{L_0^2}\right)\left(\sqrt{l_e^2 + \frac{L_0^2}{\pi^2}} - l_e\right)\right)}{L_0 h \left(\sqrt{\frac{2}{\pi}} erf\left(\frac{\pi l_e}{\sqrt{2} L_0}\right) + \frac{2}{\pi} exp\left(-\frac{\pi^2 l_e^2}{2 L_0^2}\right)\left(arctg\left(\frac{L_0}{\pi l_e}\right)\right)\right)}. \quad (24)$$

Marking $l_e = x L_0$, the last expression can be written as follows:

$$I = \frac{\omega d_{31} V_0 [\sigma]}{h} \cdot \frac{\sqrt{\frac{2}{\pi}} erf\left(\frac{\pi x}{\sqrt{2}}\right) + 2 exp\left(\frac{-\pi^2 x^2}{2}\right)\left(\sqrt{x^2 + \frac{1}{\pi^2}} - x\right)}{\sqrt{\frac{2}{\pi}} erf\left(\frac{\pi x}{\sqrt{2}}\right) + \frac{2}{\pi} exp\left(-\frac{\pi^2 x^2}{2}\right)\left(arctg\left(\frac{1}{\pi x}\right)\right)}. \quad (25)$$

The graph of the dependence of the current in relative units is shown in Fig. 3. It is easy to understand that the value x = 0 corresponds to an ordinary rectangular piezotransformer.

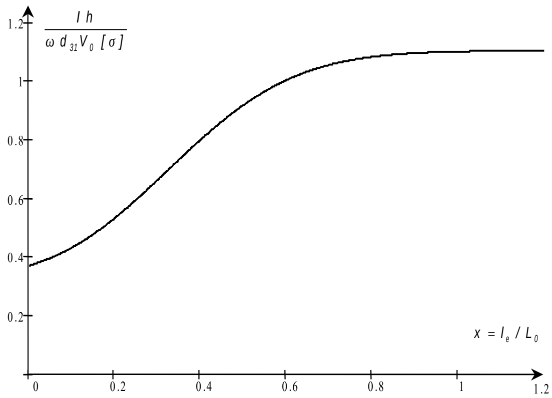

Figure 3: Dependence of the total current through the electrodes on the ratio of the geometric parameters of the piezotransformer.

With a significant increase in the parameter l_e the shape of the piezotransformer becomes non-technological, so it is worth limiting yourself to the value l_e = 0.5, at which, on the one hand, the current is already much larger than the current of an ordinary rectangular piezotransformer, and on the other hand, the shape is not yet so difficult to manufacture. The stress distribution in the material for a piezo transformer made of PZT-4 material with an operating frequency of 48 kHz is shown in Fig. 1. The sketch of the corresponding shape of the piezotransformer made of PZT-4 material is presented in Fig. 2 ($l_e = \frac{1}{4f\sqrt{\rho \cdot s_{11}}} = 0.018$). Dotted lines depict a "classical" piezotransformer of transverse-transverse type with the same value of electrical parameters.

The distribution of width-averaged mechanical stress σ_{11} (in fractions $[\sigma]$) in the piezotransformer is shown in Fig. 1. It can be seen from the graph that the stress in the material of the proposed piezotransformer are close to the maximum allowable practically throughout the PT material, except for the end regions, the contribution of which to the overall performance of the material is not significant.

4. Modeling in the FreeFem++ software environment

To confirm the developed provisions, a 2-d simulation of the piezotransformer was carried out using the FreeFem++ program. FreeFem++ is a program package designed to solve the problems of modelling physical fields in a multidimensional setting. The program performs numerical modelling of user-specified differential equations based on the finite element method. Unlike many other commercial programs, FreeFem++ allows you to specify an arbitrary domain of simulation, which can be specified, for example, analytically, and conduct simulations of arbitrary differential equations.

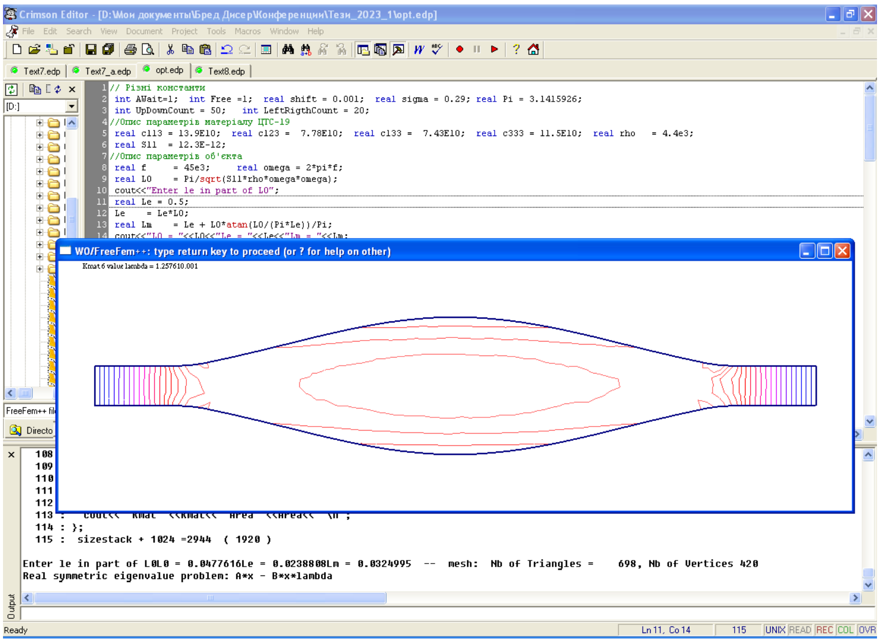

Figure 4: The main window of the FreeFem++ program during simulation.

This software package can run on all types of OS starting from UNIX (with G++ 2.95.2 or later and X11R6), Linux, Windows and Mac OS. Main features of FreeFem++:

- Problem description in real or complex numbers, problem assignment in variational formulations, with access to internal vectors and matrices if required.
- The possibility of solving two-dimensional (and with some limitations also 3-dimensional) problems of statics and dynamics, including solving problems on eigenvalues.
- Simple geometric input with analytical description of area boundaries.
- Automatic mesh generator, based on the De Longe algorithm. Moreover, the internal density of points can be specified in the process of partitioning or by means of the problem of the density of points on the border.
- A high-level easy-to-use input C-like language with an algebra of analytic functions and functions defined on finite elements.
- The possibility of using a set of finite-element grids within one problem with automatic interpolation of data on different grids and possible saving of interpolation matrices.
- Various triangular finite elements: linear and quadratic Lagrangian, elements with P1 discontinuities and Raviart-Thomas elements, elements of non-scalar type, mini-elements.
- A wide variety of linear direct and iterative solvers (LU, Cholesky, Crout, CG, GMRES, UMFPACK), as well as solvers for determining the eigenvalues and eigenvectors of the problem.

- Availability of operational graphics, formation of .txt, .eps, gnu, and mesh files for further manipulation of initial and received data. Although the system does not have a defined IDE, it configures itself to run and can work with any text editor.

The program package window during simulation is shown in Figure 4. For modelling, a functional was used that describes the oscillations of the piezo transformer in the following form [16, 17]:

$$I(u,v)=\int_S \left(\frac{\partial}{\partial x}v_1\right)\cdot\left(c_{11}\cdot\frac{\partial}{\partial x}u_1+c_{12}\cdot\frac{\partial}{\partial y}u_2\right)+\left(\frac{\partial}{\partial y}v_2\right)\cdot\left(c_{12}\cdot\frac{\partial}{\partial x}u_1+c_{11}\cdot\frac{\partial}{\partial y}u_2\right)+ \\ +\left(\frac{\partial}{\partial x}v_2+\frac{\partial}{\partial y}v_1\right)\cdot(c_{66})\cdot\left(\frac{\partial}{\partial x}u_2+\frac{\partial}{\partial y}u_1\right)+\rho\omega^2\lambda(u_1\cdot v_1+u_2\cdot v_2)ds\,, \tag{26}$$

where v - test movements.

The assignment of the area was carried out by standard FreeFem++ means - that is, the boundary restriction function was first set

```
Func real Border (real x1)
{ real C;
   if(abs(x1)<(Le)) {C = B*exp(-rho*omega*omega*S11*x1*x1/2)/2;}
    else {C = B*exp(-rho*omega*omega*S11*Le*Le/2)/2;}
  return C;
};
```

Then, on the basis of the boundary function, the boundaries themselves were constructed in a parametric form:

```
border BLeft (t=1,-1){x=-Lm;y=Border(-Lm)*t;label = Free;};//Left side
border BRigth (t=-1,1){x=Lm;y= Border( Lm)*t;label = Free;};// Right side
border BBottom(t=-1,1){x= Lm*t; y=-Border(t*Lm) ; label = Free; };// Bottom
border BTop (t=1,-1){x= Lm*t; y= Border(t*Lm) ; label = Free; };// Top
```

And then, based on the description of the boundary, a finite-element domain is constructed:

```
mesh TAll = buildmesh(BLeft(LeftRigthCount) + BBottom(UpDownCount)
                 +BRigth(LeftRigthCount) + BTop(UpDownCount));
```

On the basis of the obtained area, finite-element spaces for movements were constructed:

```
fespace Vh(TAll,[P2,P2]);
```

and stress in the material

```
fespace Sh(TAll,P1);
```

These spaces were used to define the displacement field

```
Vh [u1,u2], [v1, v2];
```

and stress in the material

```
Sh Sigma11, Sigma12, Sigma22, SigmaMax;
```

The problem was formulated as a variational one over the obtained areas, given by the operators:

```
varf VA([u1,u2],[v1,v2])= int2d(TAll)( dx(v1)*(c11*dx(u1) + c12*dy(u2))
             + dy(v2)*(c12*dx(u1) + c11*dy(u2))
             + (dx(v2)+dy(v1))*c66*(dx(u2)+dy(u1))
            + rho*omega*omega*shift*(u1*v1 + u2*v2));
varf VB([u1,u2],[v1,v2])= int2d(TAll)(rho*omega*omega*(u1*v1 + u2*v2));
```

Then, from the obtained variational operators, the corresponding matrices of coefficients were extracted using:

```
matrix MA= VA(Vh,Vh,solver=Crout,factorize=1,eps=1e-20);
matrix MB= VB(Vh,Vh,solver=CG,eps=1e-20);
```

Next, the problem of the eigenvectors of the obtained operators VA and VB was solved, and the eigenform of oscillations, which is necessary for our problem, was determined.

Then, based on the calculated forms of own movements, using ratios:

$$\begin{aligned}
\sigma_{11} &= c_{11} \cdot \frac{\partial}{\partial x} u_1 + c_{12} \cdot \frac{\partial}{\partial y} u_2 , \\
\sigma_{22} &= c_{12} \cdot \frac{\partial}{\partial x} u_1 + c_{11} \cdot \frac{\partial}{\partial y} u_2 , \\
\sigma_{12} &= c_{66} \cdot \frac{\partial}{\partial x} u_2 + c_{66} \cdot \frac{\partial}{\partial y} u_1 ,
\end{aligned} \tag{27}$$

values of stress in the material were obtained, which were immediately normalized so that the maximum stresses were not greater than $[\sigma]$.

For comparison, a piezo transformer of the usual shape was modeled in parallel. The distribution of the highest stresses in the material (σ_{11}) is presented in Fig. 5.

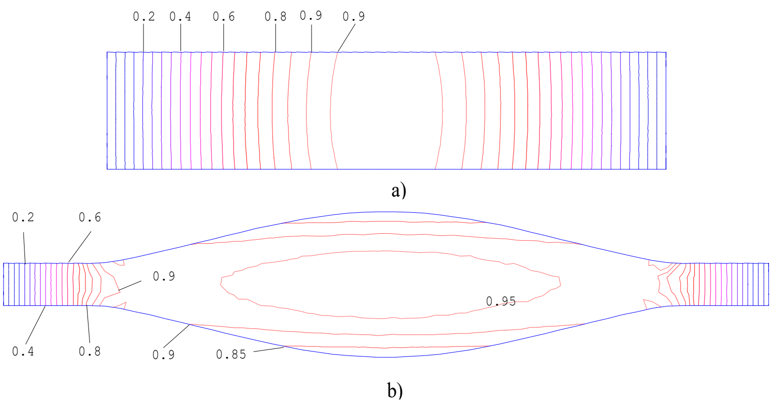

Figure 5: Distribution of σ11 along the width in the piezotransformer of the "normal" shape (a) and in the piezotransformer of the proposed shape (b).

From the comparison of the figures, it can be seen that the stress in the proposed piezotransformer are distributed much more evenly. This makes it possible to obtain a higher material utilization ratio, improve the technical and economic performance of piezotransformers, and reduce their cost.

5. Conclusions

As a result of the simulation, it was established that the stress in the material of the proposed design of the piezotransformer are close to the maximum allowable for almost the entire PT material, except for the end regions, the contribution of which to the overall performance of the material is not significant. To confirm the developed provisions, a 2-d simulation of the piezotransformer was carried out using the Free Fem++ program. Discretization of the piezotransformer region was carried out using the finite element method using second-order basis functions, corresponding global matrices were constructed, and a standard problem was solved regarding the eigenform of the PT mechanical oscillations. The proposed approach to the optimization of the shape of the piezoelectric transformer allows to obtain its design with higher, compared to the classical shape, technical and economic indicators and to reduce the cost of PT.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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