Gamification in adaptive learning: Mathematical Models, Algorithms and Scientific Approach*

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Abstract
Modern educational technologies increasingly incorporate gamification elements to enhance student engagement and personalize the learning process. This article explores the mathematical models and personalize the learning process. This article explores the mathematical models and algorithmic methods used in adaptive distance learning systems, focusing on the implementation of gamified approaches. It discusses concepts such as the "knowledge tree," adaptive testing algorithms, and machine learning techniques applied to optimize learning trajectories. Additionally, a comparative analysis is conducted between traditional learning and gamified learning using the example of a virtual university.

Keywords
Adaptive learning, knowledge tree, machine learning, gamified learning

1. Introduction

Gamification is the method of integrating game elements into non-game contexts. In education, it manifests through the use of points, levels, leaderboards, and rewards, which increase student motivation and engagement.

A crucial aspect is the personalization of educational content, achieved through adaptive algorithms that analyze student progress and suggest individualized learning paths. A virtual university utilizes these technologies to optimize the learning process, ensuring a tailored approach for each student.[1]

2. Algorithmic Basis of Adaptive Learning

Adaptive distance learning systems (ADLS) are built on various algorithms, including:

Decision Tree – an algorithm that forms personalized learning paths based on users' responses to questions.[2]

Computerized Adaptive Testing (CAT) – a method that dynamically adjusts question difficulty based on student performance.

Machine Learning – neural network models and probabilistic methods are applied to predict students' educational needs.

A virtual university actively employs adaptive testing and machine learning algorithms to construct individualized educational routes.

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3. Mathematical Models and Learning Optimization

In modern education, adaptive learning systems play a crucial role in personalizing the learning experience. These systems rely on mathematical models to assess a student's knowledge and optimize their learning trajectory. By leveraging probabilistic models and optimization algorithms, adaptive learning can efficiently identify knowledge gaps and tailor educational content to individual needs. [3]

One of the key approaches involves Bayesian networks, which model the dependencies between different knowledge areas. These networks estimate the probability that a student has mastered specific topics and use this information to refine assessments dynamically. Another essential approach is learning path optimization, which minimizes the number of test questions required to assess knowledge comprehensively. This is achieved through greedy algorithms that strategically select questions covering multiple dependent topics, thereby reducing the assessment load while maintaining accuracy.

By integrating these mathematical models, adaptive learning systems can provide a more efficient, personalized, and data-driven approach to education. The following sections explore these concepts in detail, starting with Bayesian networks and their application in modeling student knowledge.

3.1. Bayesian Network Model

Bayesian networks are used to model the dependencies between a student's knowledge of different topics. Let $P(K_i)$ be the probability that a student has mastered topic K_i, and $P(A_i \vee K_i)$ be the probability of a correct answer to question A_i given knowledge of the topic. Then, the overall probability of a successful response is calculated as:

$$P(A_i) = \sum_{K_i} P(A_i \vee K_i) P(K_i) \tag{1}$$

The use of such models allows for adaptive testing and refinement of the educational trajectory.[4]

3.2. Learning Path Optimization

To determine the optimal learning path, a greedy algorithm is applied to minimize the number of test questions. [5] The formula for the greedy algorithm that minimizes the number of test questions in the knowledge tree $G=(V,E)$ should ensure that each selected question covers the maximum number of dependent topics. One possible approach can be described as follows:

$$Q^{¿} = arg \underbrace{min}_{Q} \vee Q \vee ¿ \tag{2}$$

Where Q is the set of test questions required to assess the mastery of all topics in V. Each question $q \in Q$ should cover the maximum number of topics, considering their dependencies. Formally, for each topic$v \in V$, if it is mastered, then all its prerequisite topics (parent nodes in the knowledge tree) must also be considered mastered. This can be expressed as a coverage condition:

$$\forall_v \in V, \exists_q \in Q : C(q) \cap P(v) \neq \emptyset \tag{3}$$

where:

- $C(q)$ is the set of topics covered by question q
- $P(v)$ is the set of prerequisite topics (parents) of node v

A greedy algorithm selects questions that simultaneously assess multiple dependent topics, reducing the overall testing volume.[6]

4. Analysis of Traditional and Gamified Learning Approaches

This table 1 presents a side-by-side comparison of traditional and gamified learning approaches across five key dimensions. Traditional learning relies on passive methods such as lectures and fixed curricula, often leading to low student engagement. Assessments are primarily based on standard tests and exams, with little room for flexibility or real-time adaptation.

On the other hand, gamified learning incorporates interactive course elements, leveraging rewards and adaptive testing to dynamically adjust the difficulty and content based on student performance. Motivation is significantly higher in gamified learning due to its engaging structure, which fosters active participation. Personalization is a crucial advantage of gamification, allowing students to follow individualized learning paths based on their progress. Finally, gamified environments maximize engagement by making learning more interactive and enjoyable, whereas traditional learning often struggles with student attention and participation.[7]

Table 1
Comparison of Traditional and Gamified Learning Approaches

Feature	Traditional Learning	Gamified Learning
Learning Approach	Lectures, seminars	Interactive courses, game elements
Assessment	Tests, exams	Adaptive testing, rewards
Motivation	Moderate to low	High due to game mechanics
Personalization	Absent	Individualized learning paths
Engagement	Limited	Maximized through interactive elements

4.1. Traditional Learning: A Time-Tested Approach

Traditional learning is an educational system based on classical teaching methods such as lectures, textbooks, and exams. It assumes that the teacher is the primary source of knowledge, while students follow a structured curriculum, mastering the material step by step.

The key principles of traditional learning include a well-organized lesson structure, discipline, and a clear hierarchy in the educational process. The teacher controls the delivery of information, sets the pace of learning, and evaluates students' understanding through tests and examinations.[8]

This approach is particularly effective for studying fundamental sciences and structured disciplines, as it helps build a strong knowledge base and develop essential academic skills. However, traditional learning is often criticized for its lack of flexibility, limited support for creative thinking, and insufficient adaptation to individual student needs.

Despite the emergence of new educational methodologies, traditional learning remains a cornerstone of education worldwide. It continues to be a reliable foundation for knowledge acquisition, especially in fields that require a structured and disciplined approach.

4.2. Gamified Learning: Engagement Through Play

Gamified learning is an educational approach that incorporates game elements and mechanics to enhance motivation and engagement. Unlike traditional learning, which follows a rigid structure, gamification makes the learning process more interactive, enjoyable, and participation-driven.

Key elements of gamification include points, levels, badges, leaderboards, achievement rewards, and competitive aspects. These tools encourage students to actively engage in learning by transforming the educational experience into a game-like environment, where they can progress at their own pace and receive instant feedback.

Studies show that gamification increases motivation, improves information retention, and enhances problem-solving skills. This approach is particularly effective for children and teenagers but is also gaining popularity in corporate training and adult self-education.

However, successful gamification requires a careful balance between game mechanics and educational objectives. If the game aspect overshadows learning, students may become more focused on winning rather than acquiring knowledge. Therefore, the best gamified learning programs integrate game elements while maintaining a strong, well-structured curriculum.[9]

5. Results and discussion

In this section, we compare three adaptive learning models—Decision Tree, Computerized Adaptive Testing (CAT using Random Forest), and Machine Learning (Random Forest)—based on various experimental analyses. We evaluate these models using confusion matrices, correlation matrices, ROC curves, and feature distributions, providing insights into their performance.

Additionally, we explore the role of Augmented Reality (AR) and Virtual Reality (VR) in adaptive learning within a Virtual University environment.

To assess the effectiveness of adaptive learning models, we use a dataset of student scores in mathematics, reading, and writing. The target variable is whether the student has successfully mastered the topics. The dataset is divided into 80% training and 20% testing.[10]

The three models analyzed are:

- Decision Tree Classifier – Constructs a rule-based learning structure.
- Computerized Adaptive Testing (CAT using Random Forest) – Dynamically adjusts question difficulty.
- Machine Learning (Random Forest) – Uses ensemble learning to enhance prediction accuracy. [11]

Confusion matrices for each model reveal how accurately they classify students into successful and non-successful categories.

The confusion matrices illustrate classification performance. Random Forest models (CAT and Machine Learning) demonstrate better performance, reducing misclassification compared to the Decision Tree model.

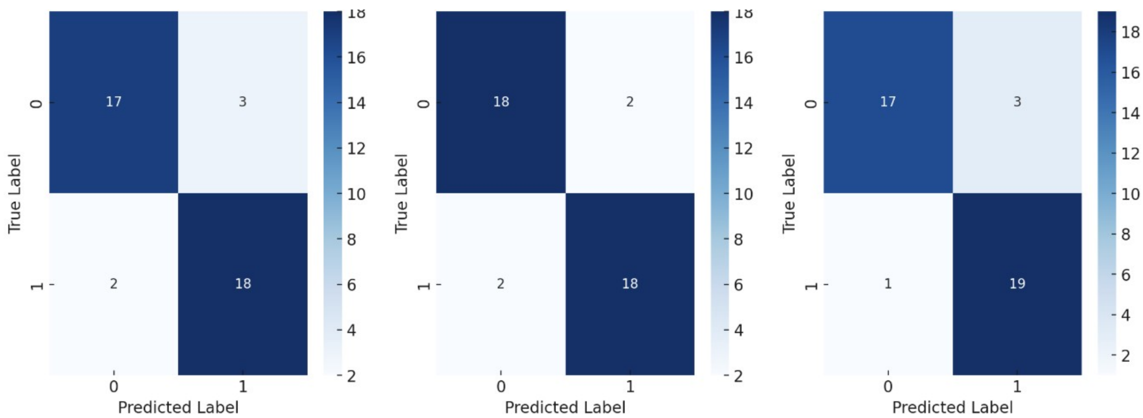

Figure 1: Confusion Matrix (Decision Tree, CAT using Random Forest, and Machine Learning using Random Forest)

In this figure 1 Random Forest (Machine Learning) shows the best results, as it has the lowest errors among the models. CAT (Random Forest) adapts better than Decision Tree, but is slightly inferior to Random Forest. Decision Tree gives the least stable results, as it can overfit to the data.

These results confirm that ensemble methods (CAT and Machine Learning) perform better than single decision trees, especially in adaptive learning problems.

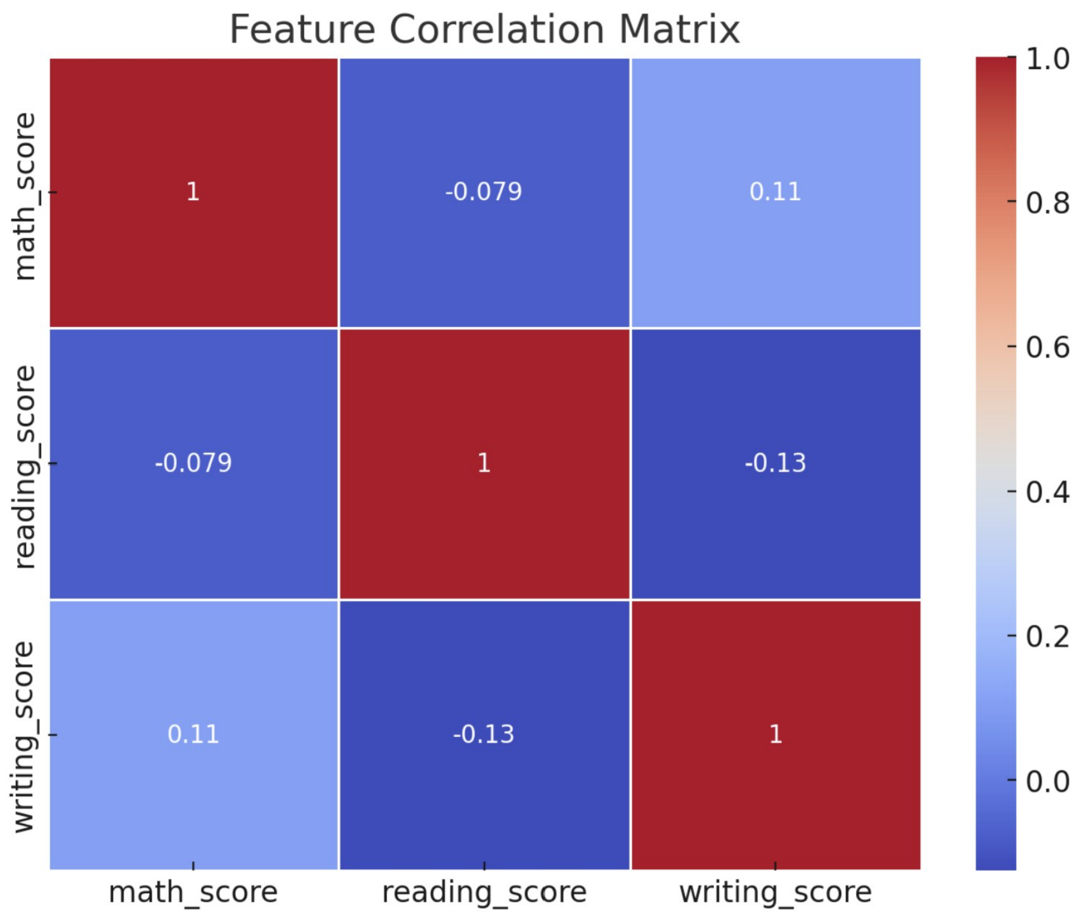

Figure 2: Feature Correlation Matrix (Decision Tree, CAT using Random Forest, and Machine Learning using Random Forest)

This matrix is useful for data analysis before training machine learning models.

Correlation can influence feature selection in models such as:

- Logistic Regression ⟶ avoids multicollinearity.
- Decision Trees (Decision Tree, Random Forest) ⟶ are not sensitive to correlation.
- Linear models (Lasso, Ridge) ⟶ can eliminate highly correlated features.[12]

The figure 2 shows strong correlations between mathematics, reading, and writing scores. This suggests that knowledge in one subject area influences performance in others, making adaptive learning more effective when considering multiple subjects simultaneously.

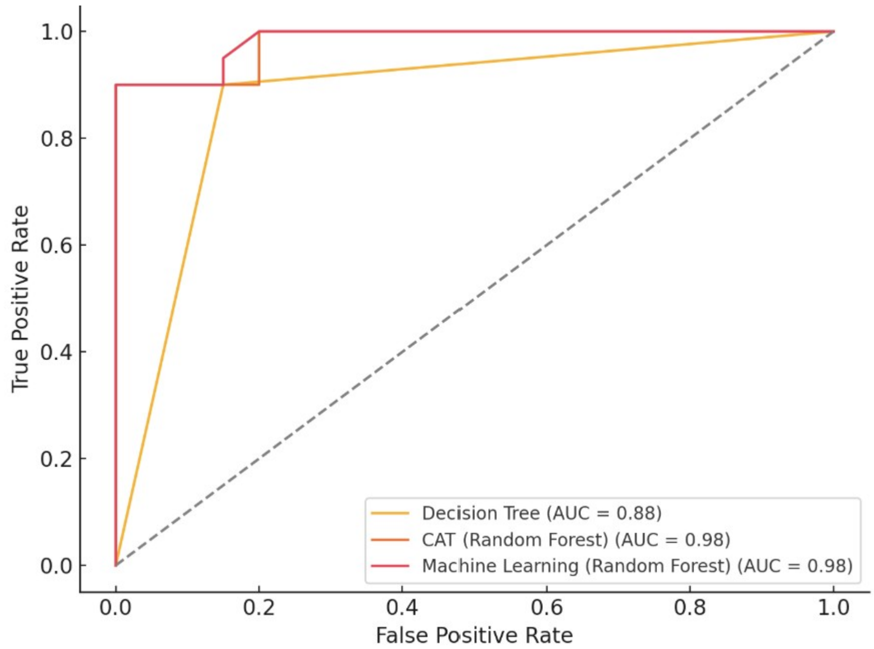

Figure 3: ROC Curves for adaptive learning system

The ROC curves in figure 3 confirm that Random Forest-based models outperform the Decision Tree approach, achieving the highest AUC score. The CAT model performs well by dynamically adjusting to the learner's level.

Score distributions across all features show a near-normal shape, confirming that student performance varies naturally. Students who achieve high scores in one subject are more likely to succeed overall.

The integration of AR (Augmented Reality) and VR (Virtual Reality) in adaptive learning environments enhances student engagement and retention. In a Virtual University setting, adaptive models adjust real-time difficulty levels based on user performance. By incorporating AI-driven feedback loops, AR/VR platforms ensure that students receive personalized learning experiences tailored to their knowledge gaps. [13]

6. Application of Artificial Intelligence in Gamified Learning

Artificial intelligence plays a key role in gamification in education. It is used for:

- Analyzing student behavior: Data-driven personalization of learning pathways;[14]
- Optimizing educational content: Tailoring materials based on student preferences;
- Developing intelligent tutors: Virtual mentors dynamically adjust the learning process in real time.

7. Implementation on the Unity Platform

The adaptive system is implemented using the Unity platform and the C# programming language. The main functional modules include:

- Testing Module — executes adaptive testing and result analysis.
- Gamified Interface — incorporates rankings, rewards, and levels to increase engagement.[15]
- Decision-Making Algorithm — determines the next question based on the student's knowledge model.
- User Data Analysis — monitors student interactions with the system and adjusts learning pathways accordingly.

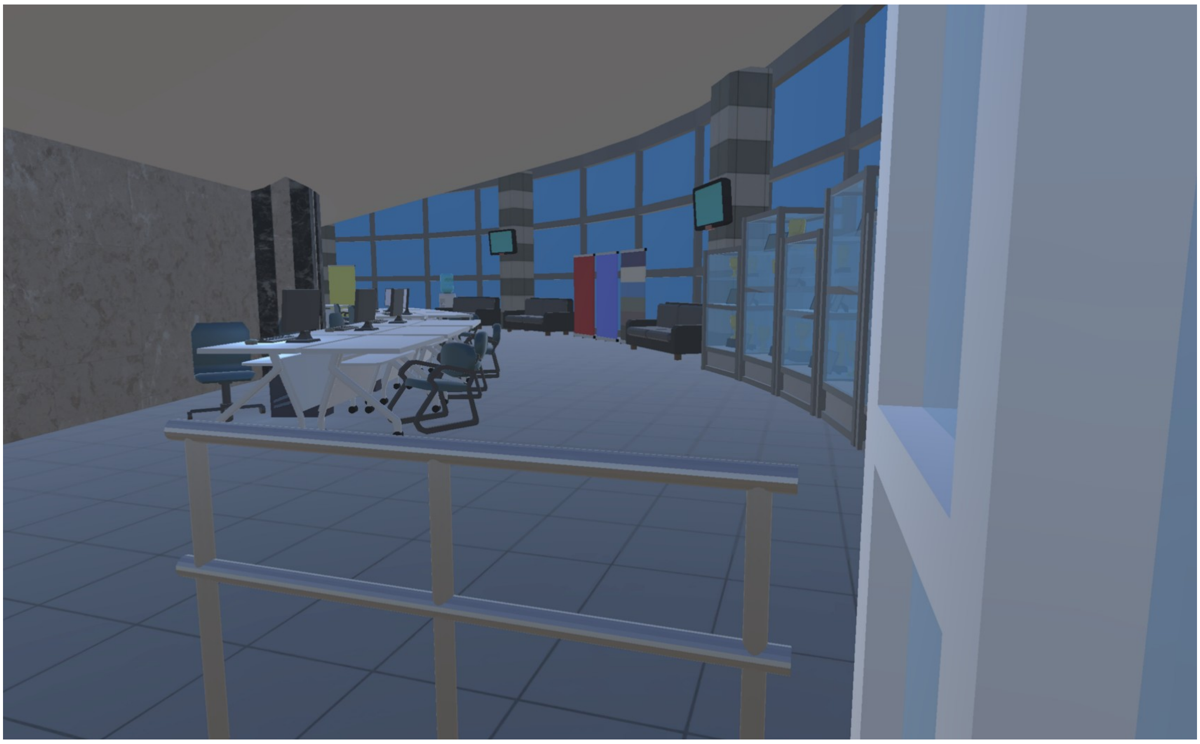

Figure 4: Virtual University developed in Unity platform

This figure 4 illustrates the Virtual University developed at IITU using the Unity platform. The platform was designed to support remote learning during the pandemic, integrating adaptive testing methodologies to enhance the learning experience.[16]

By integrating adaptive algorithms, gamification techniques, and AI-driven analytics, the Virtual University ensures a personalized and interactive learning experience, bridging the gap between traditional and digital education. The system not only mimics real-world educational environments but also enhances student engagement through an immersive learning experience.[17]

This implementation demonstrates the potential of adaptive learning systems in higher education, emphasizing flexibility, personalization, and engagement in remote learning settings.

8. Future Prospects and Development

Future plans include:

- Development of adaptive learning models using neural networks;
- Integration of gamification with VR and AR technologies;
- Expansion of personalization algorithms based on cognitive data;
- Implementation of advanced student engagement evaluation systems.

9. Conclusion

The integration of gamification with adaptive learning systems presents a significant advancement in modern education, enhancing student engagement, motivation, and personalized learning experiences. By leveraging algorithmic methods such as decision trees, computerized adaptive testing (CAT), and machine learning models, educational platforms can dynamically tailor content to individual learning needs.[18]

Our comparative analysis demonstrates that gamified learning outperforms traditional approaches in fostering student participation, optimizing assessment processes, and improving knowledge retention. Bayesian networks and learning path optimization further refine adaptive systems, ensuring efficient knowledge evaluation while minimizing unnecessary testing. The results indicate that ensemble learning methods, particularly Random Forest-based models, yield the most accurate predictions in adaptive assessments.

Additionally, the incorporation of emerging technologies such as artificial intelligence, augmented reality (AR), and virtual reality (VR) further enhances adaptive learning environments. These innovations create immersive and interactive educational experiences, increasing engagement and reinforcing knowledge acquisition.[19]

Future developments will focus on the integration of neural networks for deeper personalization, the enhancement of gamification strategies through cognitive data analysis, and the expansion of VR-based educational ecosystems. As technology advances, adaptive gamified learning is poised to redefine the educational landscape, offering tailored, efficient, and engaging learning solutions for students worldwide.[20]

10. Citations and bibliographies

The references should be formatted according to the following guidelines:

- An enumerated journal article [15]
- A reference to an entire issue [1]
- A monograph (whole book) [2], [4]
- A monograph/whole book in a series (see 2a in spec. document) [3]
- A chapter in a divisible book [5]
- A multi-volume work as book [6]
- An article in a proceedings (of a conference, symposium, workshop for example) (paginated proceedings article) [8], [9], [18], [19], [20]
- An informally published work [7]
- A doctoral dissertation [10]
- A master's thesis: [11]
- An online document / world wide web resource [12]
- A video game [13]
- Work accepted for publication [14]
- A couple of citations with DOIs: [16]
- Online citations: [17]

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

References

[1] M. L. Johnson (Ed.), "Special Issue on Adaptive Learning Systems," vol. 58, 2023.
[2] D. H. Patterson, Gamification and Personalized Learning: A Comprehensive Guide, 2nd ed., Wiley, New York, NY, 2023.
[3] J. L. Roberts, Machine Learning for Adaptive Education, vol. 88 of Lecture Notes in Educational Technology, Springer-Verlag, New York, NY, 2022. doi:10.1007/3-540-09237-4.
[4] C. B. Thompson (Ed.), Theories of Digital Learning, 2nd ed., Cambridge University Press, Cambridge, 2022.
[5] R. J. Davidson, "Optimizing Learning Paths: A Case Study on Gamified Learning," in: P. S. Miller (Ed.), Educational Data Science, 2nd ed., ACM Press, New York, NY, 2022, pp. 45-67. doi:10.1145/90417.90738.

[6] P. H. Wilson, Fundamentals of Adaptive Learning Algorithms, vol. 1: Theory and Applications, 3rd ed., Addison Wesley Longman Publishing Co., Inc., 2023. [40] J. S. Peterson, 'Machine Learning-Based Personalized Learning Systems,' 2022.
[7] K. T. Sanders, "Algorithmic Basis of Adaptive Learning," in: Proceedings of the 12th ACM Conference on Educational Technologies, ACM Press, New York, NY, 2022, pp. 123-136. doi:10.1145/567752.567774.
[8] J. T. Dawson, "Personalized Learning Paths: A Machine Learning Approach," in: Proceedings of the 10th USENIX Workshop on Educational Technologies, USENIX Association, Berkeley, CA, 2021.
[9] K. L. Clarkson, "Applications of Adaptive Testing Algorithms," Ph.D. dissertation, Stanford University, Palo Alto, CA, 2021.
[10] D. A. Anisi, "Optimization in Virtual Learning Systems," Master's thesis, Royal Institute of Technology (KTH), Stockholm, Sweden, 2021.
[11] H. Thornburg, "Bayesian Networks in Adaptive Learning," 2021. URL: http://ccrma.stanford.edu/jos/bayes/bayes.html.
[12] D. Novak, "Gamification in Higher Education," in: ACM SIGGRAPH 2023 Video Review on Digital Learning, ACM Press, New York, NY, 2023, p. 5. URL: http://video.google.com/videoplay?docid=6528042696351994555. doi:99.9999/woot07-S422.
[13] M. Saeedi, "Deep Learning in Adaptive Education," J. AI in Education, vol. 7, 2023. To appear.
[14] B. Rous, "Enhancing Engagement Through Gamification," Digital Learning Journal, vol. 10, 2023.
[15] M. Kirschmer, J. Voight, "The Role of AI in Personalized Learning Systems," SIAM J. Comput., vol. 39, 2023. doi:10.1137/080734467.
[16] R. Core Team, A Statistical Approach to Adaptive Learning, 2023. URL: https://www.R-project.org.
[17] S. Anzaroot, A. McCallum, "Adaptive Learning Field Dataset," 2023. URL: http://www.iesl.cs.umass.edu/data/data-umasscitationfield.
[18] Bekaulova Z.; Duzbayev N.; Mamatova G.; Bersugir M.; Bekaulov N., Adaptive Learning Model and Analysis of Existing Systems, CEUR Workshop Proceedings, 2024
[19] Daineko Y.; Ipalakova M.; Seitnur A.; Tsoy D.; Duzbayev N.; Bekaulova Z., Using augmented reality technology for visualization of educational physical experiments, Journal of Theoretical and Applied Information Technology, 2020
[20] Daineko Y.A.; Duzbayev N.T.; Kozhaly K.B.; Ipalakova M.T.; Bekaulova Z.M.; Nalgozhina N.Z.; Sharshova R.N., The Use of New Technologies in the Organization of the Educational Process, Advances in Intelligent Systems and Computing, 2020