On Polymorphic Attacks in the $ASPIC^+$ and $ASPIC^R$ Formalisms

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Abstract

The recently proposed *ASPIC*^{R}, standing for *ASPIC*$^+$ *revisited* is an evolution of the prominent *ASPIC*$^+$ formalism for structured argumentation, which overcomes some limitations of *ASPIC*$^+$ by resorting in particular to an alternative notion of attack referring to sets of arguments. While this notion of attack is a key element for achieving the technical advantages offered by *ASPIC*^{R}, it also gives rise to some peculiar situations, where some attacks are, in a sense, polymorphic since there are multiple reasons by which a given set of arguments can attack an argument. After pointing out that polymorphic attacks are also possible in *ASPIC*$^+$, we provide some examples of polymorphic attacks in *ASPIC*$^+$ and *ASPIC*^{R}, discuss the underlying technical and conceptual issues and provide a preliminary discussion about how to revise the formalisms in order to encompass alternative options for their management.

Keywords

Structured argumentation, Attack relation, $ASPIC^+$ formalism

1. Introduction

ASPIC$^+$ [1] is a prominent rule-based formalism for structured argumentation, featuring remarkable expressiveness properties, like a generic contrariness relation, the ability to encompass the existence of a preference relation between arguments, and the classification of attacks in three types (undercutting, rebutting, and undermining) in turn distinguished (with the exception of undercutting) into preference-dependent and preference-independent. In virtue of its expressiveness and generality, *ASPIC*$^+$ is able to encompass other structured argumentation formalisms (e.g. Assumption-Based Argumentation [2] or logic-based argumentation [3]) as special cases and has been the subject of a large corpus of studies, including the investigation of several variants (see, e.g., [4, 5, 6]).

In recent years, however, it has also been pointed out that *ASPIC*$^+$ suffers from some specific technical problems concerning, in particular, the behavior in the presence of multiple contradictories [7, 8] and the occurrence of spurious preferences [9]. *ASPIC*$^+$ *revisited* (*ASPIC*^{R} for short), introduced in [8], overcomes these limitations by resorting to alternative technical solutions which, while not affecting (and indeed enhancing) the expressiveness of the formalism, involve a substantially different way of identifying and representing the conflicts among arguments and then of constructing the argumentation framework [10] which is the basis for the evaluation of their acceptability. In particular, *ASPIC*^{R} resorts to a notion of attack referring to sets of arguments. Leaving aside many technical aspects, to be recalled later, the core idea is that a set of arguments S attacks an argument α when there is a reason to exclude that all arguments in S and the argument α can be accepted at the same time.

While this idea is a key factor in enabling the advantageous technical properties of *ASPIC*^{R}, it also makes some peculiar situations possible. In particular, there may be multiple reasons for the same set S to attack an argument α, and each different reason corresponds to a different type of attack. In such a case, we say that the attack from S to α is polymorphic, since it potentially can be ascribed to multiple classes. We point out, however, that, though this has never been evidenced before in the literature, polymorphic attacks may also occur in the context of *ASPIC*$^+$.

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This peculiar feature has not been evidenced in previous works on *ASPIC*$^+$ and *ASPIC*^{R} and, while not affecting, as we will discuss, the technical correctness of the formalisms, poses both technical and conceptual questions and opens, in particular, the way to the study of alternative options in the definition of attacks in *ASPIC*^{R}. The present work aims at providing the basis for this investigation line by carrying out a first analysis of the phenomenon of polymorphic attacks and discussing their conceptual and technical status in perspective. The paper is organised as follows. Section 2 recalls the necessary background notions on argumentation frameworks and *ASPIC*$^+$, while Section 3 reviews *ASPIC*^{R}. Sections 4 and 5 discuss polymorphic attacks in *ASPIC*$^+$ and *ASPIC*^{R} respectively. Section 6 provides some perspectives on their treatment, and Section 7 concludes the paper.

2. Background: Dung's argumentation frameworks and ASPIC$^+$

We briefly review Dung's theory of argumentation frameworks.

Definition 1. *An argumentation framework (AF) is a pair $\mathcal{F} = \langle A, \twoheadrightarrow \rangle$, where A is a set of arguments and $\twoheadrightarrow \subseteq (A \times A)$ is a binary relation on A.*

When $(\alpha, \beta) \in \twoheadrightarrow$ (also denoted as $\alpha \twoheadrightarrow \beta$) we say that α attacks β. For a set $X \subseteq A$ and an argument $\alpha \in A$ we write $\alpha \twoheadrightarrow X$ if $\exists \beta \in X : \alpha \twoheadrightarrow \beta$ and $X \twoheadrightarrow \alpha$ if $\exists \beta \in X : \beta \twoheadrightarrow \alpha$, and we denote the arguments attacking X as $X^- \triangleq \{\alpha \in A \mid \alpha \twoheadrightarrow X\}$ and the arguments attacked by X as $X^+ \triangleq \{\alpha \in A \mid X \twoheadrightarrow \alpha\}$. An *extension-based* argumentation semantics σ specifies the criteria for identifying, for a generic AF, a set of *extensions*, where each extension is a set of arguments considered to be acceptable together. Given a generic argumentation semantics σ, the set of extensions prescribed by σ for a given AF $\mathcal{F}$ is denoted as $\mathcal{E}_\sigma(\mathcal{F})$. Several argumentation semantics are recalled in Definition 2, along with some basic underlying notions. For more details, the reader is referred to [11].

Definition 2. *Let $\mathcal{F} = \langle A, \twoheadrightarrow \rangle$ be an AF, $\alpha \in A$ and $X \subseteq A$. X is* conflict-free*, denoted as $X \in \mathcal{E}_{\mathrm{CF}}(\mathcal{F})$, iff $X \cap X^- = \emptyset$. α is* acceptable *with respect to X (or α is defended by X) iff $\{\alpha\}^- \subseteq X^+$. The function $F_\mathcal{F} : 2^A \to 2^A$ which, given a set $X \subseteq A$, returns the set of the acceptable arguments with respect to X, is called the* characteristic function *of $\mathcal{F}$. X is* admissible *(denoted as $X \in \mathcal{E}_{\mathrm{AD}}(\mathcal{F})$) iff $X \in \mathcal{E}_{\mathrm{CF}}(\mathcal{F})$ and $X \subseteq F_\mathcal{F}(X)$. X is a* complete *extension (denoted as $X \in \mathcal{E}_{\mathrm{CO}}(\mathcal{F})$) iff $X \in \mathcal{E}_{\mathrm{CF}}(\mathcal{F})$ and $X = F_\mathcal{F}(X)$. X is the* grounded *extension (denoted as $X = GR(\mathcal{F})$ or $X \in \mathcal{E}_{\mathrm{GR}}(\mathcal{F})$) iff X is the least fixed point of $F_\mathcal{F}$ (equivalently, the least complete extension). X is a* preferred *extension (denoted as $X \in \mathcal{E}_{\mathrm{PR}}(\mathcal{F})$) iff X is a maximal (with respect to set inclusion) admissible set. X is a* stable *extension (denoted as $X \in \mathcal{E}_{\mathrm{ST}}(\mathcal{F})$) iff $X^+ = A \setminus X$. X is a* semi-stable *extension (denoted as $X \in \mathcal{E}_{\mathrm{SST}}(\mathcal{F})$) iff it is a complete extension such that $X \cup X^+$ is maximal (wrt $\subseteq$) among all complete extensions.*

Argument justification status is defined on the basis of extension membership.

Definition 3. *Given a set S a justification labeling of S is a function $J : S \to \Sigma_J$, where $\Sigma_J = \{Sk, Cr, No\}$. Given an AF $\mathcal{F} = \langle A, \twoheadrightarrow \rangle$ and a semantics σ, the justification labeling of A according to σ is defined as follows[1]: $J(\alpha) = Sk$ iff $\alpha \in \bigcap_{\mathsf{E} \in \mathcal{E}_\sigma(\mathcal{F})} \mathsf{E}$; $J(\alpha) = Cr$ iff $\alpha \in \bigcup_{\mathsf{E} \in \mathcal{E}_\sigma(\mathcal{F})} \mathsf{E}$ and $\alpha \notin \bigcap_{\mathsf{E} \in \mathcal{E}_\sigma(\mathcal{F})} \mathsf{E}$; $J(\alpha) = No$ iff $\alpha \notin \bigcup_{\mathsf{E} \in \mathcal{E}_\sigma(\mathcal{F})} \mathsf{E}$.*

In words, we will say respectively that α is skeptically justified, credulously justified, and not justified. We now recall the essential notions of the *ASPIC*$^+$ formalism.

Definition 4. *An* argumentation system *is a tuple $AS = (\mathcal{L}, \bar{\ }, \mathcal{R}, n)$ where:*

1. *$\mathcal{L}$ is a logical language*

[1]We avoid reference to $\mathcal{F}$ and σ in J for ease of notation. Moreover, with respect to the traditional notion of justification we keep skeptical and credulous justification disjoint for reasons which will be clear later.

2. $^{-}$ *is a* contrariness *function from* $\mathcal{L}$ *to* $2^{\mathcal{L}}$ *such that: (i)* φ *is a* contrary *of* ψ *if* $\varphi \in \overline{\psi}$, $\psi \notin \overline{\varphi}$*; (ii)* φ *is a* contradictory *of* ψ *(denoted by* $\varphi = -\psi$*) if* $\varphi \in \overline{\psi}$, $\psi \in \overline{\varphi}$*; (iii) each* $\varphi \in \mathcal{L}$ *has at least one contradictory*
3. $\mathcal{R} = (\mathcal{R}_S, \mathcal{R}_D)$ *is a pair of sets of strict (*$\mathcal{R}_S$*) and defeasible (*$\mathcal{R}_D$*) inference rules of the form* $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ *and* $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ *respectively (where* φ_i, φ *are meta-variables ranging over wff in* $\mathcal{L}$*), and* $\mathcal{R}_S \cap \mathcal{R}_D = \emptyset$
4. $n : \mathcal{R}_D \rightarrow \mathcal{L}$ *is a naming convention for* $\mathcal{R}_D$*.*

In the following, given a set $S \subseteq \mathcal{L}$ with a little abuse of notation, we will denote the set of its contraries and contradictories as $\overline{S} = \bigcup_{\varphi \in S}\{\psi \mid \psi \in \overline{\varphi}\}$. Given a rule $r = \varphi_1, \ldots, \varphi_n \rightarrow (\Rightarrow)\varphi$, we will say that φ is the consequent of the rule, denoted as $cons(r)$ and that $\{\varphi_1, \ldots, \varphi_n\}$ is the set of the antecedents of the rule denoted as $ant(r)$.

Closure under transposition of strict rules is a desirable property as it ensures (together with other conditions) that an argumentation system satisfies some *rationality postulates* [12] (we do not recall all the relevant details as not necessary for this paper, see Definition 12 of [1]).

Definition 5. *Given* $AS = (\mathcal{L}, ^{-}, \mathcal{R}, n)$*, the set of strict rules* $\mathcal{R}_S$ *is* closed under transposition *iff if* $\varphi_1, \ldots, \varphi_n \rightarrow \psi \in \mathcal{R}_S$ *then, for* $i = 1 \ldots n$, $\varphi_1, \ldots, \varphi_{i-1}, -\psi, \varphi_{i+1}, \ldots, \varphi_n \rightarrow -\varphi_i \in \mathcal{R}_S$*. Given a set of strict rules* $\mathcal{R}_S$*, its closure under transposition is defined as* $Cl_{tr}(\mathcal{R}_S) \triangleq \mathcal{R}_S \cup \bigcup_{r \in \mathcal{R}_S} tr(r)$*, where for any* $r = \varphi_1, \ldots, \varphi_n \rightarrow \psi$, $tr(r) = \bigcup_{i=1\ldots n}\{\varphi_1, \ldots, \varphi_{i-1}, -\psi, \varphi_{i+1}, \ldots, \varphi_n \rightarrow -\varphi_i\}$*.*

A knowledge base is a subset of $\mathcal{L}$ including certain (called *axioms*) and defeasible (called *ordinary*) premises. It gives rise to the notion of argumentation theory. Arguments are built from a knowledge base using rules.

Definition 6. *A* knowledge base *in* $AS = (\mathcal{L}, ^{-}, \mathcal{R}, n)$ *is a set* $\mathcal{K} \subseteq \mathcal{L}$ *consisting of two disjoint subsets* $\mathcal{K}_n$ *(the* axioms*) and* $\mathcal{K}_p$ *(the* ordinary premises*). The tuple* $AT = (AS, \mathcal{K})$ *is called an* argumentation theory.

Definition 7. *An argument* α *on the basis of a knowledge base* $\mathcal{K}$ *in* $AS = (\mathcal{L}, ^{-}, \mathcal{R}, n)$ *is:*

1. φ *if* $\varphi \in \mathcal{K}$ *with:* $Prem(\alpha) = \{\varphi\}$*;* $Conc(\alpha) = \varphi$*;* $Sub(\alpha) = \{\varphi\}$*;* $Rules(\alpha) = \emptyset$*;* $Top(\alpha) =$ *undefined.*
2. $\alpha_1, \ldots, \alpha_n \rightarrow (\Rightarrow)\ \psi$ *if* $\alpha_1, \ldots, \alpha_n$ *are arguments such that there exists a strict (defeasible) rule* $Conc(\alpha_1), \ldots, Conc(\alpha_n) \rightarrow (\Rightarrow)\ \psi$ *in* $\mathcal{R}_S$ *(*$\mathcal{R}_D$*) with:* $Prem(\alpha) = Prem(\alpha_1) \cup \ldots \cup Prem(\alpha_n)$*;* $Conc(\alpha) = \psi$*;* $Sub(\alpha) = Sub(\alpha_1) \cup \ldots \cup Sub(\alpha_n) \cup \{\alpha\}$*;* $Rules(\alpha) = Rules(\alpha_1) \cup \ldots \cup Rules(\alpha_n) \cup \{Conc(\alpha_1), \ldots, Conc(\alpha_n) \rightarrow (\Rightarrow)\ \psi\}$*;* $Top(\alpha) = Conc(\alpha_1), \ldots, Conc(\alpha_n) \rightarrow (\Rightarrow)\ \psi$*;* $DefRules(\alpha) = \{r \mid r \in Rules(\alpha) \cap \mathcal{R}_D\}$*;* $StRules(\alpha) = \{r \mid r \in Rules(\alpha) \cap \mathcal{R}_S\}$*.*

For any argument α, $Prem_n(\alpha) = Prem(\alpha) \cap \mathcal{K}_n$*;* $Prem_p(\alpha) = Prem(\alpha) \cap \mathcal{K}_p$*.* α *is:* strict *if* $DefRules(\alpha) = \emptyset$, defeasible *if* $DefRules(\alpha) \neq \emptyset$*;* firm *if* $Prem(\alpha) \subseteq \mathcal{K}_n$*;* plausible *if* $Prem(\alpha) \nsubseteq \mathcal{K}_n$*;* finite *if* $Rules(\alpha)$ *is finite.*

Notation 1. *Some further notations are useful. Given* $S \subseteq \mathcal{L}$, $S \vdash \varphi$ *denotes that there exists a strict argument* α *such that* $Conc(\alpha) = \varphi$*, with* $Prem(\alpha) \subseteq S$*.* $S \vdash_{min} \varphi$ *denotes that* $S \vdash \varphi$ *and* $\nexists T \subsetneq S : T \vdash \varphi$*. Given a set of arguments* X, $Prem(X) \triangleq \bigcup_{\alpha \in X} Prem(\alpha)$*, and similarly for* $Conc(X)$, $Sub(X)$, $Rules(X)$, $Top(X)$, $DefRules(X)$, $StRules(X)$*.*

Three kinds of attack between arguments are considered.

Definition 8. *An argument* α attacks *an argument* β *iff* α undercuts, rebuts*, or* undermines β *where:* α undercuts β *(on* β'*) iff* $Conc(\alpha) \in \overline{n(r)}$ *for some* $\beta' \in Sub(\beta)$ *such that* $r = Top(\beta')$ *is defeasible.* α rebuts β *(on* β'*) iff* $Conc(\alpha) \in \overline{\varphi}$ *for some* $\beta' \in Sub(\beta)$ *of the form* $\beta''_1, \ldots, \beta''_n \Rightarrow \varphi$*. In such a case* α contrary-rebuts β *iff* $Conc(\alpha)$ *is a contrary of* φ*.* α undermines β *(on* β'*) iff* $Conc(\alpha) \in \overline{\varphi}$ *for some* $\beta' = \varphi$, $\varphi \in Prem_p(\beta)$*. In such a case* α contrary-undermines β *iff* $Conc(\alpha)$ *is a contrary of* φ*.*

In some cases, attack effectiveness depends on a preference ordering $\preceq$ over arguments (assumed to be a preorder as in [13]). As usual $\alpha \prec \beta$ iff $\alpha \preceq \beta$ and $\beta \npreceq \alpha$; $\alpha \simeq \beta$ iff $\alpha \preceq \beta$ and $\beta \preceq \alpha$. Effective attacks give rise to defeat.

Definition 9. *Let α attack β on β'. If α undercuts, contrary-rebuts, or contrary-undermines β on β', then α* preference-independent *attacks β on β', otherwise α* preference-dependent *attacks β on β'. Then, α* defeats *β iff for some β' either α* preference-independent *attacks β on β' or α* preference-dependent *attacks β on β' and $\alpha \nprec \beta'$.*

Then a *structured argumentation framework* (SAF) can be defined from an argumentation theory,[2] using the attack relation. Using the defeat relation, an AF is then derived from a SAF.

Definition 10. *Let $AT = (AS, \mathcal{K})$ be an argumentation theory. A* structured argumentation framework *(SAF), defined by AT is a triple $(\mathsf{S}, \mathsf{C}, \preceq)$ where S is the set of all finite arguments constructed from $\mathcal{K}$ in AS (called the set of arguments on the basis of AT), $\mathsf{C} \subseteq \mathsf{S} \times \mathsf{S}$ is such that $(\alpha, \beta) \in \mathsf{C}$ iff α attacks β, and $\preceq$ is an ordering on S.*

Definition 11. *Let $\Delta = (\mathsf{S}, \mathsf{C}, \preceq)$ be a SAF, and $\mathsf{D} \subseteq \mathsf{S} \times \mathsf{S}$ be the defeat relation according to Definition 9. The AF corresponding to Δ is defined as $\mathcal{F}_\Delta = (\mathsf{S}, \mathsf{D})$.*

Given an argumentation semantics σ, the justification status of arguments in S according to σ is determined by the set of extensions $\mathcal{E}_\sigma(\mathcal{F}_\Delta)$ according to Definition 3.

3. ASPIC$^+$ revisited

In this section, we present *ASPIC*$^+$ revisited (in the following *ASPIC*^{R}), introduced in [8].

The main differences of *ASPIC*^{R} with respect to *ASPIC*$^+$ can be summarized in two points:

- an original form of closure, not involving any structural constraint on the set of rules, but rather resorting to a sort of extension of the contrariness function based on the strict rules;
- an alternative notion of attack referring to sets of arguments, which leads to the construction of a substantially different argumentation framework.

In addition, *ASPIC*^{R} relaxes some requirements and adjusts some hypotheses, as illustrated step by step in the following.

As to Definition 4, *ASPIC*^{R} adopts the same basic elements of *ASPIC*$^+$ but does not require that every element of the language has a contradictory, while adding the constraint that the name of a rule cannot be the contrary of anything, i.e. for every rule $r \nexists \varphi \in \mathcal{L}$ such that $n(r) \in \overline{\varphi}$. Similarly, the names of the rules cannot be included in the antecedent of a rule or in the knowledge base[3].

Thus, in Definition 4 items 1 and 4 are unmodified, while item 2 is reduced to: $^-$ is a function from $\mathcal{L}$ to $2^{\mathcal{L} \setminus \mathcal{RN}}$ where $\mathcal{RN} = \{n(r) \mid r \in \mathcal{R}_D\}$, and in item 3 φ_i are meta-variables ranging over wff in $\mathcal{L} \setminus \mathcal{RN}$ while φ is a meta-variable ranging over wff in $\mathcal{L}$.

In Definition 6 it is prescribed $\mathcal{K} \subseteq \mathcal{L} \setminus \mathcal{RN}$ while Definition 7 and Notation 1 are left unmodified.

Among the properties considered in Definition 12 of [1], *ASPIC*^{R} keeps only the property that axioms are consistent with respect to strict rules and a relaxed notion of well-formedness, which involves axioms and their strict consequences rather than all the consequents of strict rules. More precisely, the property of *weak well-formedness* is defined as follows.

Definition 12. *Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$. AT is weakly well-formed if whenever $\varphi \in \overline{\psi}$ and $\psi \in \mathcal{K}_n$ or ψ can be obtained from $\mathcal{K}_n$ by applying strict rules, it also holds that $\psi \in \overline{\varphi}$.*

[2]We do not consider here the alternative notion of c-structured argumentation framework in [1].

[3]The constraints on the rule names prevent that some peculiar formal situations, void of practical meaning, may occur, as evidenced in [8]. Similar requirements, though in a slightly different form, have been adopted in [14].

One of the main standpoints of *ASPIC*^{R} is avoiding the requirement that strict rules are closed under transposition. Rather, the satisfaction of the rationality postulates is ensured (as shown in [8]) by exploiting an extended contrariness relation at the level of sets of language elements whose definition is based on a general notion of strict derivability from language elements, independently of the presence of these elements in the knowledge base.

Definition 13. *Given an argumentation system* $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$ *the strict knowledge base* $\mathcal{K}^*_{AS}$ *for* AS *is given by* $\mathcal{K}_n = \mathcal{L}$*,* $\mathcal{K}_p = \emptyset$ *and the corresponding argumentation theory is defined as* $AT^*_{AS} = (AS, \mathcal{K}^*_{AS})$*.* $S \vdash^* \varphi$ *and* $S \vdash^*_{min} \varphi$ *denote respectively that* $S \vdash \varphi$ *and* $S \vdash_{min} \varphi$ *in* AT^*_{AS}*.*

The extended contrariness relation $\mathsf{EC}(AS)$ refers to sets of language elements and is obtained by applying a sort of closure with respect to strict derivability, together with a requirement of minimality.

Definition 14. *Given an argumentation theory* $AT = (AS, \mathcal{K})$ *with* $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$*, let* $\mathsf{EC}^*(AS) \subseteq 2^{\mathcal{L}} \times 2^{\mathcal{L}}$ *be defined as* $\mathsf{EC}^*(AS) = \{(S,T) \mid S \vdash^* \varphi, T \vdash^* \psi \text{ and } \varphi \in \overline{\psi}\}$*. Letting for* $S \subseteq \mathcal{L}$*,* $\widehat{S} = S \setminus \mathcal{K}_n$*, the extended contrariness relation is defined as* $\mathsf{EC}(AS) = \{(\widehat{S}, \widehat{T}) \mid (S,T) \in \mathsf{EC}^*(AS) \text{ and } \forall (S', T') \in \mathsf{EC}^*(AS) \text{ s.t. } \widehat{S'} \subseteq \widehat{S} \text{ and } \widehat{T'} \subseteq \widehat{T}, \widehat{S'} = \widehat{S} \text{ and } \widehat{T'} = \widehat{T}\} \subseteq 2^{\mathcal{L}} \times 2^{\mathcal{L}}$*.* U *is a contrary of* V *if* $(U,V) \in \mathsf{EC}(AS)$ *and* $(V,U) \notin \mathsf{EC}(AS)$*;* U *is a contradictory of* V *if* $(U,V) \in \mathsf{EC}(AS)$ *and* $(V,U) \in \mathsf{EC}(AS)$*.*

In words, $\mathsf{EC}^*(AS)$ corresponds to the completion of the contrariness relation $^-$ on the basis of the set of strict rules, i.e. a set S is regarded as 'being against' a set T if a strict consequence of S 'is against' a strict consequence of T. Then for each pair $(S,T) \in \mathsf{EC}^*(AS)$ the corresponding pair $(\widehat{S}, \widehat{T})$ where axioms are excluded is considered. Among these pairs, only those which respect a minimality condition, and hence are not redundant, belong to $\mathsf{EC}(AS)$.

Definition 14 can be equivalently expressed in the form encompassed by the following more constructive definition.

Definition 15. *Given an argumentation theory* $AT = (AS, \mathcal{K})$ *with* $AS = (\mathcal{L}, ^-, \mathcal{R}, n)$*, let* $\mathsf{EC}^1(AS)$*,* $\mathsf{EC}^2(AS)$*,* $\mathsf{EC}^3(AS)$ *be the following subsets of* $2^{\mathcal{L}} \times 2^{\mathcal{L}}$

- $\mathsf{EC}^1(AS) = \{(\{\varphi\}, \{\psi\}) \mid \varphi \in \overline{\psi}\}$*;*
- $\mathsf{EC}^2(AS) = \{(S, \{\psi\}) \mid S \vdash^*_{min} \varphi \text{ and } \varphi \in \overline{\psi}\}$*;*
- $\mathsf{EC}^3(AS) = \{(S,T) \mid T \vdash^*_{min} \psi \text{ and } (S, \{\psi\}) \in \mathsf{EC}^1(AS) \cup \mathsf{EC}^2(AS)\}$*.*

Letting $\mathsf{EC}^{*m}(AS) = \mathsf{EC}^1(AS) \cup \mathsf{EC}^2(AS) \cup \mathsf{EC}^3(AS)$*, and, for* $S \subseteq \mathcal{L}$*,* $\widehat{S} = S \setminus \mathcal{K}_n$*, the extended contrariness relation is defined as* $\mathsf{EC}^m(AS) = \{(\widehat{S}, \widehat{T}) \mid (S,T) \in \mathsf{EC}^{*m}(AS) \text{ and } \forall (S', T') \in \mathsf{EC}^{*m}(AS) \text{ s.t. } \widehat{S'} \subseteq \widehat{S} \text{ and } \widehat{T'} \subseteq \widehat{T}, \widehat{S'} = \widehat{S} \text{ and } \widehat{T'} = \widehat{T}\} \subseteq 2^{\mathcal{L}} \times 2^{\mathcal{L}}$*.* U *is a contrary of* V *if* $(U,V) \in \mathsf{EC}^m(AS)$ *and* $(V,U) \notin \mathsf{EC}^m(AS)$*;* U *is a contradictory of* V *if* $(U,V) \in \mathsf{EC}^m(AS)$ *and* $(V,U) \in \mathsf{EC}^m(AS)$*.*

Let us comment on the above definition. First, $\mathsf{EC}^{*m}(AS)$ is defined in a constructive incremental way starting from $\mathsf{EC}^1(AS)$ which mirrors, at level of singletons, the contrariness relation $^-$. Then $\mathsf{EC}^2(AS)$ is defined taking into account $\mathsf{EC}^1(AS)$, i.e. the contrariness relation $^-$, and the strict rules: basically a set S is put in relation to $\{\psi\}$, i.e. is regarded as being against $\{\psi\}$, if it is possible to strictly derive a contradiction to ψ from S. Finally $\mathsf{EC}^3(AS)$ is defined taking into account $\mathsf{EC}^1(AS)$ and $\mathsf{EC}^2(AS)$: a set S is put in relation to T, i.e. is regarded as being against T, if S is against some $\{\psi\}$ according to $\mathsf{EC}^1(AS)$ or $\mathsf{EC}^2(AS)$ and ψ strictly follows from T. Then, as in Definition 14, for each pair $(S,T) \in \mathsf{EC}^{*m}(AS)$ the corresponding pair $(\widehat{S}, \widehat{T})$ where axioms are excluded is considered. Among these pairs, only those that are not redundant belong to $\mathsf{EC}^m(AS)$.

The equivalence between the two definitions is proved in [8].

Proposition 1 (Prop. 24 of [8]). *Definitions 14 and 15 are equivalent, i.e.* $(S,T) \in \mathsf{EC}(AS)$ *iff* $(S,T) \in \mathsf{EC}^m(AS)$*.*

The various forms of attack are then revised, replacing Definition 8 with the following definition concerning attacks at the level of sets of arguments.

Definition 16. *Given an argumentation theory $AT = (AS, \mathcal{K})$, a set of arguments X* attacks *an argument β iff X* undercuts*,* rebuts*, or* undermines *β where:*

- X undercuts β *(on β') iff for some $\beta' \in Sub(\beta)$ such that $r = Top(\beta') \in \mathcal{R}_D$, the following condition holds: $\exists T, U$ such that $T \cup U = Conc(X) \cup \{n(r)\}$, $(T, U) \in \mathsf{EC}(AS)$ and $n(r) \in U$.*
- X rebuts β *(on β') iff for some $\beta' \in Sub(\beta)$ of the form $\beta''_1, \ldots, \beta''_n \Rightarrow \varphi$ the following condition holds: $\exists T, U$ such that $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ and $\varphi \in U$. In this case X* contrary-rebuts *β if $(U, T) \notin \mathsf{EC}(AS)$.*
- X undermines β *(on β') iff for some $\beta' = \varphi$, $\varphi \in Prem_p(\beta)$ the following condition holds: $\exists T, U$ such that $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ and $\varphi \in U$. In this case X* contrary-undermines *β if $(U, T) \notin \mathsf{EC}(AS)$.*

As the effectiveness of some attacks depends on the preference relation, the notion of preference ordering needs to be generalized to sets of arguments. Note, however, that in *ASPIC*^{R} there are no requirements on the preference relation (in [1] it must satisfy several conditions to be considered *reasonable*) and it is only assumed that a preorder between arguments is given.

Definition 17. *Given a preorder $\preceq$ on a set of arguments X, we extend $\preceq$ to $2^X \times X$ as follows. An argument α is at least as preferred as a set of arguments Y, denoted $Y \preceq \alpha$, iff $\exists \beta \in Y$ such that $\beta \preceq \alpha$. α is strictly preferred to Y, denoted $Y \prec \alpha$, iff $\exists \beta \in Y$ such that $\beta \prec \alpha$, not strictly preferred to Y, denoted $Y \not\prec \alpha$ iff $\nexists \beta \in Y$ such that $\beta \prec \alpha$.*

On this basis, an extended notion of defeat is introduced, replacing Definition 9.

Definition 18. *Let the set of arguments Y attack an argument β on β' according to Definition 16. If Y undercuts, contrary-rebuts, or contrary-undermines β on β', then Y* preference-independent *attacks β on β', otherwise Y* preference-dependent *attacks β on β'. Then, Y* defeats *β iff either Y* preference-independent *attacks β on β' or Y* preference-dependent *attacks β on β' and $Y \not\prec \beta'$. Y* minimally defeats *β, denoted as $Y \rightsquigarrow \beta$, if Y defeats β and $\nexists Y' \subsetneq Y$ such that Y' defeats β.*

An argumentation framework based on the notion of defeat provided in Definition 18 is then defined to evaluate the justification status of arguments. The idea is that the framework nodes represent relevant sets of arguments. In particular, we need a node for each singleton corresponding to a produced argument, and a node for each set of ultimately fallible arguments (as per Definition 19) that minimally defeats some produced argument.

Definition 19. *Given an argumentation theory $AT = (AS, \mathcal{K})$, let S be the set of the arguments produced in AS on the basis of $\mathcal{K}$. The set of ultimately fallible arguments of AT is defined as $\mathsf{UF}(\mathsf{S}) \triangleq \mathcal{K}_p \cup \{\alpha \in \mathsf{S} \mid Top(\alpha) \in \mathcal{R}_D\}$.*

Definition 20. *Given an argumentation theory $AT = (AS, \mathcal{K})$ with ordering $\preceq$, let S be the set of the arguments produced in AS on the basis of $\mathcal{K}$. The set of relevant sets of arguments of AT, denoted as $\mathsf{RS}(AT)$, is defined as $\mathsf{RS}(AT) = \{\{\alpha\} \mid \alpha \in \mathsf{S}\} \cup \{X \mid X \subseteq \mathsf{UF}(\mathsf{S})$ and $\exists \beta \in \mathsf{S} : X \rightsquigarrow \beta\}$.*

Then, a relevant set of ultimately fallible arguments attacks another one simply if it minimally defeats one of its members.

Definition 21. *Let $X, Y \in \mathsf{RS}(AT)$ for an argumentation theory $AT = (AS, \mathcal{K})$ and S be the set of the arguments produced in AS on the basis of $\mathcal{K}$. X D-attacks Y, denoted as $\|X\| \twoheadrightarrow \|Y\|$, iff $X \subseteq \mathsf{UF}(\mathsf{S})$ and $\exists \alpha \in Y : X \rightsquigarrow \alpha$.*

The relevant set based argumentation framework is defined accordingly.

Definition 22. *Given an argumentation theory $AT = (AS, \mathcal{K})$, the* RS*-based argumentation framework induced by AT is defined as* $\mathsf{RS}{-}\mathsf{F}(AT) = (\{\|X\| \mid X \in \mathsf{RS}(AT)\}, \twoheadrightarrow)$.

Finally, given an argumentation semantics σ, the justification status of an argument α corresponds to the one of $\|\{\alpha\}\|$ in $\mathsf{RS}{-}\mathsf{F}(AT)$ according to Definition 3.

It is proved in [8] that complete extensions of $\mathsf{RS}{-}\mathsf{F}(AT)$ ensure the satisfaction of a generalized form of the rationality postulates[4] proved in [1] for *ASPIC*$^+$.

4. Polymorphic attacks in ASPIC$^+$

In *ASPIC*$^+$ the potential occurrence of an attack betwen two arguments (Definition 8) depends essentially on a relation of contrariness between the conclusion of the attacking argument and an attackable element (either the conclusion or the name of a defeasible rule or a defeasible premise) of (a subargument of) the attacked argument.

Three types of attack (undercut, rebut, and undermining) are identified, depending on the attackable element involved. Moreover, rebutting and undermining attacks are partitioned into two subclasses, depending on whether the relevant contrariness relation is symmetric or asymmetric (in the latter case the attack is called a *contrary*-attack).

It is worth noting[5], that given two arguments α and β, Definition 8 does not per se guarantee that the three types of attack are mutually exclusive. In fact, it is formally possible, in specific situations, that more than one of the conditions establishing the occurrence of an attack hold at the same time for the same pair of arguments α and β.

In more detail, in order to have that α undercuts and rebuts β at the same time, the conclusion of α should be a contrary both of a conclusion of a subargument of β and of the name of a defeasible rule used in a subargument of β, as shown in Example 1.

Example 1. *Consider an argumentation system $AS_1 = (\mathcal{L}, ^-, \mathcal{R}, n)$ such that: $\mathcal{L} = \{\varphi, \psi, \chi, \omega\}$; $\omega \in \overline{\psi}$; $\omega \in \overline{\chi}$; $\mathcal{R} = \mathcal{R}_D = \{r_1\}$ where $r_1 = \varphi \Rightarrow \psi$; $n(r_1) = \chi$. Consider then the following knowledge base in AS_1: $\mathcal{K} = \mathcal{K}_p = \{\varphi, \omega\}$. We have then the following arguments: $\alpha_1 = \varphi$, $\alpha_2 = \omega$, $\alpha_3 = \alpha_1 \Rightarrow \psi$. It turns out that α_2 both rebuts and undercuts α_3*

Similarly, in order to have that α undercuts and undermines β at the same time, the conclusion of α should be a contrary both of an ordinary premise of β and of the name of a defeasible rule used in a subargument of β, as shown in Example 2

Example 2. *Consider an argumentation system $AS_2 = (\mathcal{L}, ^-, \mathcal{R}, n)$ which is a variation of AS_1 with the only difference that $\omega \in \overline{\varphi}$ instead of $\omega \in \overline{\psi}$. Consider then the same knowledge base in AS_2: $\mathcal{K} = \mathcal{K}_p = \{\varphi, \omega\}$. We have, similarly to above, the following arguments: $\alpha_1 = \varphi, \alpha_2 = \omega, \alpha_3 = \alpha_1 \Rightarrow \psi$. It turns out that α_2 both undermines and undercuts α_3.*

Turning to a further case, in order to have that α rebuts and undermines β at the same time, the conclusion of α should be a contrary both of an ordinary premise of β and of the conclusion of a subargument of β, as shown in Example 3

Example 3. *Consider an argumentation system $AS_3 = (\mathcal{L}, ^-, \mathcal{R}, n)$ which is a variation of AS_1 with the only difference that $\omega \in \overline{\varphi}$ instead of $\omega \in \overline{\chi}$. Consider then the same knowledge base in AS_3: $\mathcal{K} = \mathcal{K}_p = \{\varphi, \omega\}$. We have, similarly to above, the following arguments: $\alpha_1 = \varphi, \alpha_2 = \omega, \alpha_3 = \alpha_1 \Rightarrow \psi$. It turns out that α_2 both undermines and rebuts α_3.*

[4] More precisely, in [8] it is shown that any LAF-ensemble satisfying a given set of conditions obeys the rationality postulates and that *ASPIC*^{R} is an instance of LAF-ensemble complying with the required conditions.

[5] As to our knowledge this aspect has never been pointed out explicitly in the literature. We were stimulated to analyse this aspect by the occurrence of polymorphic attacks in *ASPIC*^{R}, discussed later.

Finally, note that, as is probably already evident, it is also possible to have a situation where an argument attacks another argument in all three possible ways.

Example 4. *Consider an argumentation system* $AS_3 = (\mathcal{L}, ^-, \mathcal{R}, n)$ *which is a variation of* AS_1 *with the only difference that* $\omega \in \overline{\varphi}$ *in addition to* $\omega \in \overline{\psi}$ *and* $\omega \in \overline{\chi}$*. Consider then the same knowledge base in* AS_4*:* $\mathcal{K} = \mathcal{K}_p = \{\varphi, \omega\}$*. We have, similarly to above, the following arguments:* $\alpha_1 = \varphi$*,* $\alpha_2 = \omega$*,* $\alpha_3 = \alpha_1 \Rightarrow \psi$*. It turns out that* α_2 *undermines, rebuts, and undercuts* α_3*.*

In all the examples above, there is a language element that belongs to the contrariness function of at least two other elements. One may wonder whether this is a necessary condition for $polymorphic$ attacks to occur in *ASPIC*$^+$, i.e., whether $polymorphic$ attacks may occur even under the constraint that each language element belongs to at most one contrariness function. The answer is positive, though rather peculiar situations must be considered, as shown by the following examples.

Example 5. *Consider an argumentation system* $AS_5 = (\mathcal{L}, ^-, \mathcal{R}, n)$ *such that:* $\mathcal{L} = \{\varphi, \psi, \chi, \omega\}$*;* $\omega \in \overline{\chi}$*;* $\mathcal{R} = \mathcal{R}_D = \{r_1\}$ *where* $r_1 = \varphi, \chi \Rightarrow \psi$*;* $n(r_1) = \chi$*. Consider then the following knowledge base in* AS_1*:* $\mathcal{K} = \mathcal{K}_p = \{\varphi, \chi, \omega\}$*. We have then the following arguments:* $\alpha_1 = \varphi$*,* $\alpha_2 = \omega$*,* $\alpha_3 = \chi$*,* $\alpha_4 = \alpha_1, \alpha_3 \Rightarrow \psi$*. It turns out that* α_2 *both undermines and undercuts* α_4*.*

Example 6. *Consider an argumentation system* $AS_6 = (\mathcal{L}, ^-, \mathcal{R}, n)$ *such that:* $\mathcal{L} = \{\varphi, \psi, \chi, \omega, \eta\}$*;* $\omega \in \overline{\chi}$*;* $\mathcal{R} = \mathcal{R}_D = \{r_1, r_2\}$ *where* $r_1 = \varphi, \chi \Rightarrow \psi$ *and* $r_2 = \eta \Rightarrow \chi$*;* $n(r_1) = \chi$*. Consider then the following knowledge base in* AS_1*:* $\mathcal{K} = \mathcal{K}_p = \{\varphi, \omega, \eta\}$*. We have then the following arguments:* $\alpha_1 = \varphi$*,* $\alpha_2 = \omega$*,* $\alpha_3 = \eta$*,*$\alpha_4 = \alpha_3 \Rightarrow \chi$*,* $\alpha_5 = \alpha_1, \alpha_4 \Rightarrow \psi$*. It turns out that* α_2 *both rebuts and undercuts* α_5*.*

For the sake of brevity and simplicity, the examples presented above are rather compact. More articulated examples, in particular involving longer rule chains, can easily be generated. Thus, it emerges that a variety of polymorphic attacks are actually possible in the context of *ASPIC*$^+$.

Two main questions then arise. On the application side, one may wonder whether polymorphic attacks may occur in practice, i.e., if there are realistic reasoning contexts giving rise to them. On the technical side, one has, in any case, to consider whether they are problematic in any sense and how they are managed by the current definition of the formalism.

As to the first question, consider the following situation. Suppose you have a defeasible rule r_1 stating that people wearing overalls are usually workers, and another defeasible rule r_2 stating that workers are typically adults. Moreover you know that r_1 is not applicable to children (since they may wear overalls in masquerade or for other fun reasons) and that children are not adults. Now you have a picture including a person who seems to be wearing overalls and to be a child: being a child is both a rebut and an undercut for the argument that the person is an adult, as shown in the example below.

Example 7. *Consider an argumentation system* $AS_w = (\mathcal{L}, ^-, \mathcal{R}, n)$ *defined as follows.* $\mathcal{L} = \{Wo, Wk, Ch, Ad, N_1, N_2\}$ *where* Wo *means* wears overalls*,* Wk *means* is a worker*,* Ch *means* is a child*,* Ad *means* is an adult*, and* N_1 *and* N_2 *are symbols used for rule names. Then we assume* $Ch \in \overline{Ad}$*,* $Ad \in \overline{Ch}$ *and* $Ch \in \overline{N_1}$*. We consider a set of two defeasible rules:* $\mathcal{R} = \mathcal{R}_D = \{r_1, r_2\}$ *where* $r_1 = Wo \Rightarrow Wk$ *with* $n(r_1) = N_1$ *and* $r_2 = Wk \Rightarrow Ad$ *with* $n(r_2) = N_2$*. We assume the following knowledge base in* AS_w*:* $\mathcal{K} = \mathcal{K}_p = \{Wo, Ch\}$*. We have then the following arguments:* $\alpha_1 = Wo$*,* $\alpha_2 = Ch$*,* $\alpha_3 = \alpha_1 \Rightarrow Wk$*,* $\alpha_4 = \alpha_3 \Rightarrow Ad$*. It emerges that* α_2 *undercuts* α_3 *and then* α_2 *both rebuts* α_4 *(on* α_4*) and undercuts* α_4 *(on* α_3*). Moreover* α_4 *undermines* α_2*.*

Example 7 provides an instance of polymorphic attack (rebut and undercut) arising from a realistic reasoning situation. While a systematic analysis of realistic examples (if any) giving rise to other combinations of types of attack is left to future work, we suggest that Example 7 is sufficient to show that polymorphic attacks are more than a formal curiosity and should not be overlooked.

We turn, therefore, to consider the questions raised by polymorphic attacks from a technical and conceptual perspective.

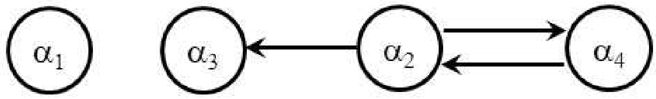

Figure 1: The argumentation framework $\mathcal{F}_w$ corresponding to Example 7.

Starting from the technical side, we remark first that Definition 8 is agnostic with respect to polymorphic attacks: it says that an argument α *attacks* an argument β iff α *undercuts*, *rebuts*, *or undermines* β, and does not explicitly assume nor implicitly relies on the fact that the disjunction is exclusive.

Definition 9 then distinguishes preference-dependent and preference-independent attacks from an argument α to an argument β with reference to the subargument β' to which the attack is directed. Thus, it turns out that an argument α can attack another argument β in a preference-dependent and in a preference-independent way at the same time (on different subarguments). For instance, in Example 7, α_2 preference-dependent attacks α_4 on α_4 and preference-independent attack α_4 on α_3.

While this is a bit peculiar, it is, *per se*, not problematic for the notion of defeat introduced in Definition 9. It states that an argument α defeats an argument β if there is some $\beta' \in Sub(\beta)$ on which the attack is successful. As previously discussed, it may happen that: (i) there can be several such subarguments β' and (ii) preferences may or may not play a role for the success of attacks concerning different subarguments. However the facts (i) and (ii) do not represent a problem: there will in any case a single defeat from α to β (possibly corresponding to more than one attack).

The defeat relation is then the basis for the construction of the argumentation framework in Definition 11, on which the assessment of the acceptability of arguments is based. Thus, technically speaking, polymorphic attacks do not pose problems nor require a revision of the relevant definitions in *ASPIC*$^+$.

We argue, however, that from the conceptual point of view, some issues arise. As remarked above, Definition 9 admits that an argument α can both preference-independent and preference-dependent attack another argument since the quality of being preference-(in)dependent depends on the pair (α, β'), where β' is the subargument of β which is attacked, rather than on the pair (α, β).

Let us refer to Example 7. The undercutting attacks from α_2 to α_3 and from α_2 to α_4 are, of course, preference-independent, while the rebutting attacks from α_2 to α_4 and vice versa are preference-dependent. According to Definition 9, α_2 then defeats both α_3 and α_4 independently of any preference while α_4 defeats α_2 only if $\alpha_4 \not\prec \alpha_2$. Assuming $\alpha_4 \not\prec \alpha_2$, we get that in the argumentation framework $\mathcal{F}_w$ prescribed by Definition 11 for Example 7 (see Figure 1) there is a formally symmetric defeat between α_4 and α_2 whose branches are however conceptually asymmetric: one branch has a polymorphic origin, while the other arises from a rebutting attack.

Let us examine the outcomes of the evaluation of the arguments in $\mathcal{F}_w$ according to standard semantics. The grounded extension consists of the unique unattacked argument α_1 ($GR(\mathcal{F}_w) = \{\alpha_1\}$), while in the case of preferred, stable, and semi-stable semantics, we have two extensions: $\mathcal{E}_{\mathrm{PR}}(\mathcal{F}_w) = \mathcal{E}_{\mathrm{ST}}(\mathcal{F}_w) = \mathcal{E}_{\mathrm{SST}}(\mathcal{F}_w) = \{\{\alpha_1, \alpha_2\}, \{\alpha_1, \alpha_3, \alpha_4\}\}$.

In words, α_1 is the only argument skeptically accepted with any semantics, while α_2 (namely the evidence that the person is a child) is regarded as dubious, due to the mutual attack with α_4. It is interesting to note that α_4 defends its own subargument α_3 from the undercutting attack coming from α_2. This is a bit peculiar: a consequence that the undercutting attack is meant to prevent to draw is used to rebut the source of the undercutting attack itself.

Whether this is conceptually legitimate or problematic can be regarded as a matter of discussion, which we leave to future work. In this paper, we limit ourselves to suggesting that further investigation on polymorphic attacks in *ASPIC*$^+$ is worth pursuing and that it may stimulate the study of alternative notions of defeat in *ASPIC*$^+$. In particular, one might consider a revision of Definition 9 where the fact that an attack whose source is an argument α gives rise to a successful defeat takes into account the possible existence of polymorphic attacks involving proper subarguments of α, which, in a sense, might override the self-defence capability of α against them.

5. Polymorphic attacks in ASPIC^R

In the context of *ASPIC*^{R}, the notions of undercutting, rebutting, and undermining attacks refer to the relation between a set of arguments, in the role of attacker, and an individual argument, in the role of attackee, as specified in Definition 16. Essentially, the starting point for the identification of attacks is the extended contrariness relation $\mathsf{EC}(AS)$, which consists of pairs of sets of language elements, say pairs (T, U) with $T, U \subseteq \mathcal{L}$. Intuitively, each such pair indicates that the elements in $T \cup U$ cannot stay together and, more precisely, indicates that if all the elements in T hold, then an least one element of U cannot hold. It follows that if there are arguments such that all their conclusion or names of top rule coincide with the set $T \cup U$, one of the arguments whose conclusion or name of top rule is included in U cannot be accepted if all the others are. The three kinds of attacks realize this idea in different ways depending on the role of the language element involved and on the structure of the attacked argument. Thus:

- a set X[6] undercuts an argument β on a subargument β' when β' has a defeasible top rule whose name is included in U;
- a set X rebuts an argument β on a subargument β' when β' has a defeasible top rule and its conclusion is included in U;
- a set X undermines an argument β on a subargument β' when β' is an ordinary premise which is included in U.

Concerning the occurrence of polymorphic attacks in *ASPIC*^{R}, it is first of all interesting to examine what happens in the examples introduced in Section 4 for *ASPIC*$^+$.

As a general observation, with reference to Definition 15 we can remark that since the examples do not contain any strict rule in all cases $\mathsf{EC}^2(AS)$ and $\mathsf{EC}^3(AS)$ do not contain any additional element with respect to $\mathsf{EC}^1(AS)$. i.e. $\mathsf{EC}^{*m}(AS) = \mathsf{EC}^1(AS)$. Moreover, since the examples do not contain any axiom, for any set S of arguments it follows that $\widehat{S} = S$. We get then that $\mathsf{EC}^m(AS) = \mathsf{EC}^{*m}(AS) = \mathsf{EC}^1(AS)$ and by Proposition 1 $\mathsf{EC}(AS) = \mathsf{EC}^1(AS)$. In words this amounts to the fact that, in the examples, the extended contrariness function $\mathsf{EC}(AS)$ of *ASPIC*^{R} essentially coincides with the contrariness function of *ASPIC*$^+$, the only difference being that the pair of language elements are replaced by the pairs of the corresponding singletons. For instance, in Example 1 $\omega \in \overline{\psi}$ becomes $(\{\omega\}, \{\psi\}) \in \mathsf{EC}(AS)$ (and similarly for $\omega \in \overline{\chi}$ and for all the pairs in the other examples).

Since every pair in $\mathsf{EC}(AS)$ consists of two singletons, in turn the attacks prescribed by Definition 16 have necessarily a singleton as a source, and skipping some relatively straightforward steps, we reach the conclusion that all the examples introduced in Section 4 are treated by *ASPIC*^{R} essentially in the same way as in *ASPIC*$^+$, since (modulo the replacement of individual arguments with the corresponding singletons) the attacks are the same and hence the argumentation frameworks prescribed by Definitions 11 and 22 are, in these examples, isomorphic.

In summary, all the considerations on polymorphic attacks drawn in Section 4 are still valid for *ASPIC*^{R}.

In particular, it is worth remarking that Definition 16, like Definition 8 is agnostic with respect to polymorphic attacks and does not explicitly assume nor implicitly relies on the fact that the disjunction of the kinds of attacks is exclusive.

Definition 18, similarly to Definition 9, is compatible with the fact that a set of arguments attacks an argument both in a preference-independent and in a preferemce-dependent way and the notion of defeat simply requires that at least one successful attack exists, not excluding that there are several. Subsequent definitions from 19 to 22 are not affected by polymorphic attacks.

While the discussion above suggests a substantial similarity between *ASPIC*$^+$ and *ASPIC*^{R} concerning polymorphic attacks, it is worth noting that extending the consideration to examples where strict rules are present, *ASPIC*^{R} raises further questions on polymorphic attacks. To see this, consider the following example.

[6]For brevity, we do not repeat here the condition of coverage of $T \cup U$ that the set X, together with the attacked argument, must satisfy.

Example 8. *Consider an argumentation system* $AS_r = (\mathcal{L}, ^-, \mathcal{R}, n)$ *such that:* $\mathcal{L} = \{\varphi, \neg\varphi, \psi, \chi, \omega, \eta, \theta\}$; $\varphi \in \overline{\neg\varphi}$; $\neg\varphi \in \overline{\varphi}$; $\psi \in \overline{\chi}$; $\mathcal{R} = \mathcal{R}_S = \{r_1, r_2, r_3, r_4\}$ *where* $r_1 = \omega \to \varphi$; $r_2 = \eta, \theta \to \neg\varphi$; $r_3 = \eta \to \psi$; $r_4 = \omega, \theta \to \chi$. *Consider then the following knowledge base in* AS_r: $\mathcal{K} = \mathcal{K}_p = \{\omega, \eta, \theta\}$. *We have then the following arguments:* $\alpha_1 = \omega$; $\alpha_2 = \eta$; $\alpha_3 = \theta$; $\alpha_4 = \alpha_1 \to \varphi$; $\alpha_5 = \alpha_2, \alpha_3 \to \neg\varphi$; $\alpha_6 = \alpha_2 \to \psi$; $\alpha_7 = \alpha_1, \alpha_3 \to \chi$. *According to Definition 15 we get:*

- $\mathsf{EC}^1(AS) = \{(\{\varphi\}, \{\neg\varphi\}), (\{\neg\varphi\}, \{\varphi\}), (\{\psi\}, \{\chi\})\}$;
- $\mathsf{EC}^2(AS) \setminus \mathsf{EC}^1(AS) = \{(\{\omega\}, \{\neg\varphi\}), (\{\eta, \theta\}, \{\varphi\}), (\{\eta\}, \{\chi\})\}$;
- $\mathsf{EC}^3(AS) \setminus (\mathsf{EC}^1(AS) \cup \mathsf{EC}^2(AS)) = \{(\{\neg\varphi\}, \{\omega\}), (\{\eta, \theta\}, \{\omega\}), (\{\varphi\}, \{\eta, \theta\}), (\{\omega\}, \{\eta, \theta\}), (\{\psi\}, \{\omega, \theta\}), (\{\eta\}, \{\omega, \theta\})\}$.

Given that $\mathcal{K}_n = \emptyset$ *and that all pairs listed above respect the minimality condition specified in Definition 15 we get that the set* $\mathsf{EC}(AS)$ *consists exactly of these pairs.*

Let us now focus on the attacks involving ultimately fallible arguments (namely $\alpha_1, \alpha_2, \alpha_3$*) which, in virtue of Definitions 16, 20, 21, are the basis for the construction of the argumentation framework* $\mathsf{RS{-}F}(AT)$ *(Definition 22).*

In particular, we note that there are two distinct conditions entailing that the set $\{\alpha_2, \alpha_3\}$ *undermines argument* α_1 *(on* α_1*) according to Definition 16. The first condition is the presence in* $\mathsf{RS{-}F}(AT)$ *of the pair* $(\{\eta, \theta\}, \{\omega\})$*, the second condition is the presence of the pair* $(\{\eta\}, \{\omega, \theta\})$.

The two conditions differ concerning the kind of undermining: in the first case we have that also $(\{\omega\}, \{\eta, \theta\}) \in \mathsf{RS{-}F}(AT)$*, while in the second case* $(\{\eta\}, \{\omega, \theta\}) \notin \mathsf{RS{-}F}(AT)$*. It follows that the first condition corresponds to a preference-dependent undermining attack, while the second condition corresponds to a preference-independent contrary-undermining attack.*

Definition 16 does not consider explicitly the possible occurrence of situations like the one presented in Example 8, however it implicitly gives the priority to contrary-undermining (and contrary-rebutting) attacks: if there is a pair $(T, U) \in \mathsf{RS{-}F}(AT)$ satisfying the specified conditions and $(U, T) \notin \mathsf{RS{-}F}(AT)$ then the attack is of the contrary kind, and hence is preference-independent. This is not affected by the possible existence of other pairs $(T', U') \in \mathsf{RS{-}F}(AT)$ still satisfying the required conditions and such that also $(U', T') \in \mathsf{RS{-}F}(AT)$.

At a general level, this appears a sort of conservative choice: it prevents the possible preference-dependent removal of the attacks whose nature is not univocal. In turn, given that attacks are motivated by the contrariness relation, this choice can be regarded as favoring the respect of consistency properties.

There are however also potential downsides of conservativeness: on one hand, it seems to demote the role of preferences, on the other hand, it tends to increase undecidedness, since it can preserve some mutual attacks (which intuitively correspond to an undetermined conflict) in situations where they could become a unidirectional attack (corresponding to a more determined conflict situation, where one element dominates the other).

In general, it emerges that polymorphic attacks pose more questions, but possibly also more opportunities, in *ASPIC*^{R} than in *ASPIC*$^+$, for the reasons discussed in the following.

We have already remarked that all the examples of polymorphic attacks presented in Section 4 for *ASPIC*$^+$, not involving strict rules nor axioms, have a direct correspondence in *ASPIC*^{R}. In the presence of strict rules, the significant differences between *ASPIC*$^+$ and *ASPIC*^{R} emerge.

First of all, it must be recalled that, in order to ensure the satisfaction of the rationality postulates, *ASPIC*$^+$ imposes several constraints limiting its expressiveness. In particular, Example 8, which is fully legitimate and manageable in *ASPIC*^{R}, cannot be correctly handled and is simply forbidden in *ASPIC*$^+$. The reason is that the language element χ has a contrary (namely ψ) and is the consequent of a strict rule (namely r_4). This is not allowed in *ASPIC*$^+$ since it violates the condition of *well formedness* (fifth item of Definition 12 in [1]): elements which have a contrary cannot be axioms nor consequents of strict rules. This restriction is related to the fact that, otherwise, it would be easy to build examples violating the postulates of direct or indirect consistency[7]. In *ASPIC*^{R}, a relaxed condition of *weak well-formedness*

[7]Discussing in detail this matter is beyond the scope of this paper. In a nutshell, let ψ be a contrary of χ: then, if, for instance,

is adopted (see Definition 12) which allows an element that has a contrary to be consequent of strict rules, provided that there is no strict derivation of this element from axioms. Example 8 satisfies weak well-formedness and is therefore legitimate in *ASPIC*^{R}, as already mentioned.

Thus, it turns out that the larger variety of polymorphic attacks in *ASPIC*^{R} is also related to its greater expressiveness.

Another point to be remarked, concerns the different nature of attacks in *ASPIC*^{R}, with respect to *ASPIC*$^+$, when strict rules are involved. Both in *ASPIC*$^+$ and in *ASPIC*^{R} the so-called *restricted rebut* is adopted, namely attacks can be directed only towards ordinary premises or arguments whose last rule is defeasible and then on their superarguments (see Definitions 8 and 16).

The use of restricted rebut is crucial to ensure some desirable properties (see [12]), but in turn entails that some attacks which are not 'directly' encompassed by the definitions are recovered in some other way.

The, so to speak, 'recovery strategy' in *ASPIC*$^+$ is based on 'inverting' strict rules so as to obtain additional arguments which can attack other arguments as needed. For this reason, in *ASPIC*$^+$ the requirement of closure under transposition of the set of strict rules, recalled in Definition 5, has been introduced[8]. This approach is based on the traditional notion of binary attack between arguments.

ASPIC^R adopts a radically different 'recovery strategy', which, in a nutshell, aims at identifying the sets of ultimately fallible arguments which are the roots of the occurring inconsistencies. The fact that these arguments are incompatible is captured through an innovative notion of attack at the level of sets.

A side effect of the strategy illustrated above is that the same ultimately fallible arguments can be identified as the roots of distinct inconsistencies. This is exactly what happens in Example 8: the arguments α_1, α_2, and α_3 are at the basis both of the inconsistency between α_4 and α_5 and of the one between α_6 and α_7. The different nature of these inconsistencies (roughly speaking, one is symmetric while the other is not) leads to the existence of a polymorphic attack.

It is then evident that, at least in principle, the cases of polymorphic attacks one can conceive are virtually unlimited, given that nothing prevents having very different derivations based on the same premises. It is also rather evident that most of these cases will hardly have a meaningful counterpart in practice: while, as already discussed, the notion of polymorphic attacks is not just a technical curiosity, it has also to be acknowledged that the practical interest of its instances is likely to decrease with the increase of their complexity.

6. Perspectives on the technical treatment of polymorphic attacks

After having evidenced and discussed the phenomenon of polymorphic attacks, in this section we carry out a preliminary discussion on possible variations of the *ASPIC*$^+$ and *ASPIC*^{R} formalisms oriented to the treatment of polymorphic attacks.

Starting from *ASPIC*$^+$, the first point of possible revision would be Definition 8. Here the key point is that the notion of attack is equated to the disjunction of three conditions (undercut, rebut or undermining). This implicitly prevents to distinguish the occurrence of one condition from the occurrence of more of them, i.e. somehow 'hides' or 'flattens' the possible existence of polymorphic attacks. To avoid this flattening, one could simply avoid, at the beginning, the reference to the common notion of attack and simply introduce all the types of attack relation, with a more explicit emphasis on the role of the subargument β', i.e., as ternary relations. In the following (as in [1]) we assume an argumentation theory $AT = (AS, \mathcal{K})$ and we refer to the set of all finite arguments constructed from $\mathcal{K}$ in AS, denoted as S and called the set of arguments on the basis of AT.

Definition 23. *Let* S *be the set of arguments on the basis of an argumentation theory* AT*. We define the following relations on* $\mathsf{S} \times \mathsf{S} \times \mathsf{S}$*. Given arguments* $\alpha, \beta, \beta' \in \mathsf{S}$ *:*

χ is an axiom while ψ is an ordinary premise, according to *ASPIC*$^+$ there is no attack between them and both would be accepted, leading to an inconsistency.

[8]In alternative, the requirement of closure under contraposition has been considered. It has not been recalled here as it is not constructive and not necessary for the present paper. The interested reader may refer to Definition 12 of [1].

- α undercuts β *on* β' *iff* $Conc(\alpha) \in \overline{n(r)}$, $\beta' \in Sub(\beta)$ *and* $r = Top(\beta')$ *is defeasible. This is denoted as* $(\alpha, \beta, \beta') \in \mathcal{C}$.
- α rebuts β *on* β' *iff* $Conc(\alpha) \in \overline{\varphi}$, $\beta' \in Sub(\beta)$ *and* β' *has the form* $\beta''_1, \ldots, \beta''_n \Rightarrow \varphi$. *In such a case* α contrary-rebuts β *on* β' *iff* $Conc(\alpha)$ *is a contrary of* φ*, denoted as* $(\alpha, \beta, \beta') \in \mathcal{R}_c$*; otherwise* α plain-rebuts β *on* β' *denoted as* $(\alpha, \beta, \beta') \in \mathcal{R}$.
- α undermines β *on* β' *iff* $Conc(\alpha) \in \overline{\varphi}$ *and* $\beta' = \varphi$ *for some* $\varphi \in Prem_p(\beta)$. *In such a case* α contrary-undermines β *iff* $Conc(\alpha)$ *is a contrary of* φ *denoted as* $(\alpha, \beta, \beta') \in \mathcal{M}_c$*; otherwise* α plain-undermines β *on* β' *denoted as* $(\alpha, \beta, \beta') \in \mathcal{M}$.

Then, to explicitly take into account the possible occurrence of polymorphic attacks, one can introduce the notion of attack set for a pair of arguments.

Definition 24. *The set* $\mathcal{AR}$ *of attack relations for* ASPIC$^+$ *is defined as* $\mathcal{AR} = \{\mathcal{C}, \mathcal{R}, \mathcal{M}, \mathcal{R}_c, \mathcal{M}_c\}$. *Given a pair of arguments* (α, β) *the set of attacks from* α *to* β *denoted as* $\mathcal{SA}(\alpha, \beta)$ *is defined as* $\mathcal{SA}(\alpha, \beta) = \{(\alpha, \beta, \beta', \mathcal{X}) \mid \mathcal{X} \in \mathcal{AR} \text{ and } (\alpha, \beta, \beta') \in \mathcal{X}\}$. *Given a set of arguments* A*, the set of all the attacks relevant to* A *is defined as* $\Omega(A) = \bigcup_{\alpha, \beta \in A} \mathcal{SA}(\alpha, \beta)$.

For instance, in Example 7, $\mathcal{SA}(\alpha_2, \alpha_4) = \{(\alpha_2, \alpha_4, \alpha_4, \mathcal{R}), (\alpha_2, \alpha_4, \alpha_3, \mathcal{C})\}$, while $\mathcal{SA}(\alpha_4, \alpha_2) = \{(\alpha_4, \alpha_2, \alpha_2, \mathcal{M})\}$. Moreover, $\mathcal{SA}(\alpha_2, \alpha_3) = \{(\alpha_2, \alpha_3, \alpha_3, \mathcal{C})\}$ and, as an example of the absence of any attack, $\mathcal{SA}(\alpha_1, \alpha_3) = \emptyset$.

Then, in place of Definition 9, one can introduce the notion of a function for deriving the defeat relation from the set of attack tuples.

Definition 25. *A defeat generation function for* ASPIC$^+$ $\mathcal{DF}$ *is a function which, given a set of arguments* A*, its relevant set of attacks* $\Omega(A)$ *and a preorder on* A*, returns a binary relation on* A.

Note that we do not make any assumption on the defeat generation function, leaving open, at this stage, all possible alternatives for its design.

Definitions 10 and 11 can then be modified as follows.

Definition 26. *Let* $AT = (AS, \mathcal{K})$ *be an argumentation theory. A* structured argumentation framework *(*SAF*), defined by* AT *is a triple* $(\mathsf{S}, \Omega(\mathsf{S}), \preceq)$ *where* S *is the set of all finite arguments constructed from* $\mathcal{K}$ *in* AS *(called the set of arguments on the basis of* AT*),* $\Omega(A)$ *is the set of all relevant attacks according to Definition 24, and* $\preceq$ *is an ordering on* S.

Definition 27. *Let* $\Delta = (\mathsf{S}, \Omega(A), \preceq)$ *be a* SAF *and* $\mathcal{DF}$ *be a defeat generation function. The* AF *corresponding to* Δ *is defined as* $\mathcal{F}_\Delta = (\mathsf{S}, \mathcal{DF}(\mathsf{S}, \Omega(\mathsf{S}), \preceq))$.

In particular, the defeat generation function corresponding to the traditional version of *ASPIC*$^+$, denoted as $\mathcal{DF}^{\mathrm{A}^+}$, can be characterized as follows.

Definition 28. *Given a set of arguments* A*, a relevant set of attacks* $\Omega(A)$ *and a preorder* $\preceq$ *on* A*, the defeat generation function* $\mathcal{DF}^{\mathrm{A}^+}$ *is defined as follows:* $(\alpha, \beta) \in \mathcal{DF}^{\mathrm{A}^+}(A, \Omega(A), \preceq)$ *iff* $\exists(\alpha, \beta, \beta', \mathcal{X}) \in \Omega(A)$ *such that* $\mathcal{X} \in \{\mathcal{C}, \mathcal{R}_c, \mathcal{M}_c\}$ *or* $\exists(\alpha, \beta, \beta', \mathcal{X}) \in \Omega(A)$ *such that* $\mathcal{X} \in \{\mathcal{R}, \mathcal{M}\}$ *and* $\alpha \not\prec \beta'$.

The parametric scheme introduced above easily accommodates alternative treatments of polymorphic attacks, by varying the function $\mathcal{DF}$. We leave this investigation to future work, remarking that, in particular, it would be interesting to identify properties of the defeat generation function which provide necessary or sufficient conditions for the satisfaction of the rationality postulates. Ideally, one could achieve the characterization of a family of defeat generation functions, such that rationality postulates are satisfied for every choice of a function $\mathcal{DF}$ in that family.

Turning to *ASPIC*^{R}, a modification along the same lines can be devised.

First, we revise Definition 16 analogously to Definition 23.

Definition 29. *Given an argumentation theory $AT = (AS, \mathcal{K})$, let* S *be the set of arguments on the basis of* AT*. We define the following relations on* $2^{\mathsf{S}} \times \mathsf{S} \times \mathsf{S}$*. Given a set of arguments* $X \subseteq \mathsf{S}$ *and arguments* $\beta, \beta' \in \mathsf{S}$*:*

- X undercuts β *on* β' *iff* $\beta' \in Sub(\beta)$ *is such that* $r = Top(\beta') \in \mathcal{R}_D$ *and the following condition holds:* $\exists T, U$ *such that* $T \cup U = Conc(X) \cup \{n(r)\}$, $(T, U) \in \mathsf{EC}(AS)$ *and* $n(r) \in U$*. This is denoted as* $(X, \beta, \beta') \in \mathcal{C}^R$.
- X *(plain-)*rebuts β *on* β' *iff* $\beta' \in Sub(\beta)$ *has the form* $\beta''_1, \ldots, \beta''_n \Rightarrow \varphi$ *and the following condition holds:* $\exists T, U$ *such that* $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ *and* $\varphi \in U$ *and* $(U, T) \in \mathsf{EC}(AS)$*. This is denoted as* $(X, \beta, \beta') \in \mathcal{R}^R$.
- X contrary-rebuts β *on* β' *iff* $\beta' \in Sub(\beta)$ *has the form* $\beta''_1, \ldots, \beta''_n \Rightarrow \varphi$ *and the following condition holds:* $\exists T, U$ *such that* $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ *and* $\varphi \in U$ *and* $(U, T) \notin \mathsf{EC}(AS)$*. This is denoted as* $(X, \beta, \beta') \in \mathcal{R}^R_c$.
- X *(plain-)*undermines β *on* β' *iff* $\beta' = \varphi$ *for some* $\varphi \in Prem_p(\beta)$ *and the following condition holds:* $\exists T, U$ *such that* $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ *and* $\varphi \in U$ *and* $(U, T) \in \mathsf{EC}(AS)$*. This is denoted as* $(X, \beta, \beta') \in \mathcal{M}^R$.
- X contrary-undermines β *on* β' *iff* $\beta' = \varphi$ *for some* $\varphi \in Prem_p(\beta)$ *and the following condition holds:* $\exists T, U$ *such that* $T \cup U = Conc(X) \cup \{\varphi\}$, $(T, U) \in \mathsf{EC}(AS)$ *and* $\varphi \in U$ *and* $(U, T) \notin \mathsf{EC}(AS)$*. This is denoted as* $(X, \beta, \beta') \in \mathcal{M}^R_c$.

On this basis, the notion of attack set for *ASPIC*^{R} can be introduced.

Definition 30. *The set* $\mathcal{AR}^R$ *of attack relations for* ASPIC^R *is defined as* $\mathcal{AR} = \{\mathcal{C}^R, \mathcal{R}^R, \mathcal{M}^R, \mathcal{R}^R_c, \mathcal{M}^R_c\}$*. Given a pair* (X, β) *where* $X \subseteq \mathsf{S}$ *and* $\beta \in \mathsf{S}$ *the set of attacks from* X *to* β *denoted as* $\mathcal{SA}^R(X, \beta)$ *is defined as* $\mathcal{SA}^R(X, \beta) = \{(X, \beta, \beta', \mathcal{X}) \mid \mathcal{X} \in \mathcal{AR}^R$ *and* $(X, \beta, \beta') \in \mathcal{X}\}$*. Given a set of arguments* A*, the set of all the attacks relevant to* A *is defined as* $\Omega^R(A) = \bigcup_{X \subseteq A, \beta \in A} \mathcal{SA}^R(X, \beta)$*.*

Then, in place of Definition 18, one can introduce the notion of a function for deriving the defeat relation from the set of attack tuples.

Definition 31. *A defeat generation function for* ASPIC^R $\mathcal{DF}^R$ *is a function which, given a set of arguments* A*, its relevant set of attacks* $\Omega^R(A)$ *and a preorder on* A*, returns a binary relation on* $2^A \times A$*, denoted as* $\rightsquigarrow$*.*

Definitions 20, 21, and 22 are then unchanged, under the assumption that a defeat generation function is used to generate the defeat relation $\rightsquigarrow$.

Similarly to the case of *ASPIC*$^+$, this generalization paves the way to several possible developments which are left to future work. In particular, also for *ASPIC*^{R}, it will be very interesting to investigate general properties of the defeat generation function ensuring the satisfaction of rationality postulates.

7. Conclusions

In this paper we have carried out a preliminary analysis concerning polymorphic attacks, namely the occurrence of multiple attacks of different kinds between the same pair of entities in the context of the *ASPIC*$^+$ and *ASPIC*^{R} formalisms for structured argumentation. As to our knowledge, this peculiar phenomenon has not been considered in previous literature. We presented and discussed several examples of its occurrence, showing in particular that there are realistic reasoning cases where it may occur and that the increased expressiveness of *ASPIC*^{R} with respect to *ASPIC*$^+$ also gives rise to a richer variety of polymorphic attacks.

From a technical viewpoint, we have commented how polymorphic attacks are (in a sense, implicitly) treated in the current versions of *ASPIC*$^+$ and *ASPIC*^{R} and suggested that dealing with them explicitly may lead to considering alternative options for handling them. Towards this investigation direction, we presented a preliminary proposal of suitable revisions of some definitions of $ASPIC^{+}$ and $ASPIC^{R}$ in order to make the notion of defeat relation parametric with respect to a defeat generation function.

In addition to those already discussed in previous sections, among the directions of future work we mention a broader analysis of the potential importance of polymorphic attacks in practical reasoning contexts and the study of the possible relationships with formalisms where attacks are associated with a numerical weight, like for instance in weighted argument systems [15].

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Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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