Adaptive Model of Economic Development with an Open Market*

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Abstract

The article develops an adaptive Mathematical Model (MM) of economic growth, which, being multidimensional, distances itself as much as possible from the traditional one-dimensional models of this type. The dimensionality of the dynamic nonlinear MM obtained in the article is explained by taking into account not only physical capital, but also public capital, savings dynamics and human capital. Particular attention is paid to the following factors: analysis of the impact of international trade, capital and labor flows between countries, interaction of economic sectors, government influence on the economy through taxation, administration of public capital and investment in it. It also takes into account the dynamics of foreign investment and the impact of economic cycles. The author emphasizes the use of computer modelling for the analysis (qualitative and quantitative) of nonlinear complex economic processes in the interaction of various factors based on large data sets.

Keywords

models of economic growth, classical (one-dimensional) and multidimensional mathematical models (MM), production function, open economy, investment, savings, IT in the economy

1. Introduction

At the current stage of studying the evolution of any type of economy, computer modelling plays an extraordinary role, as due to profound transformational changes in the world economy and ever-growing instability, it is crucial to have effective (efficient, accurate) tools for in-depth analysis and prediction of economic process trajectories. Growing globalization, rapid technological change and other factors make it difficult to manage economic development. Existing models of economic growth based on the concept of a closed economy are becoming less and less suitable for promptly making balanced and appropriate decisions.

Among the shortcomings of most models are the following: the conditions of a closed economy do not correspond to the realities of growing globalization and international division of labor; multidimensional models that better explain economic processes are rarely used, while unidimensional models are mostly used; the inclusion of savings in a certain period does not reflect their relationship to investment, even in a closed economy; the absence of foreign investment and government transfers in the models; insufficient assessment of the role of the "shadow economy"; lack of.

Purpose of the article. The aim is to develop and analyze an adaptive model of economic growth aimed at minimizing unrealistic assumptions inherent in existing models of economic growth. The main goal is to create a multidimensional dynamic model, taking into account not only physical capital, as is traditionally done (in the case of one-dimensional models), but also public capital,

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savings dynamics, and human capital. It is crucial to take into account the impact of international trade, capital and labor movements between countries. In addition, attention is paid to the interaction of economic sectors; the impact of the state on the economy through taxation, administration of public capital and investment in it; and the consideration of foreign investment, its dynamics and the impact of economic cycles.

Therefore, the development of new and modification of existing models becomes a necessary and sufficient condition for a better virtual reflection of current trends in economic development, providing a more accurate forecast of its behavior.

Outline of the main material. Basic assumptions of the model. According to a study [1], the vast majority of known models of economic growth have a large error. This is due to the fact that although the indicators correlate, they do not fully do so, and in practice, the conditions of a closed economy $Y = C + S$ and $S = I$ are not fulfilled precisely because of the active movement of capital, goods, services and labor on the international market.

With regard to the first premise, an open economy is characterized by imports (the process of buying goods or services from another country for use in the home or production sector) and exports (the process of selling goods or services from one's own country to other countries for profit or other benefits). First of all, the amount of goods consumed in the domestic market will consist of goods/services produced/provided in the economy C_d and goods/services that are imported $\Im$, that it:

$$C = C_d + \Im, \tag{1}$$

Similarly, for savings, in addition to the profit received from the sale of own products on the domestic market S_d export is added Ex:

$$S = S_d + Ex, \tag{2}$$

According to the methodology of calculating GDP by expenditures and taking into account expressions (1) and (2), we get:

$$\begin{gathered} Y = C + S_d + Xn + G = C_d + \Im + S_d + Ex - \Im + G = \\ = C_d + S_d + Ex + G = Y_p + G, \end{gathered} \tag{3}$$

Where $Xn = Ex - \Im$ are net exports (difference between exports and imports), G is the public spending, $Y_p = C_d + S_d + Ex$ is the production.

With regard to the second premise, equality between savings and investment is not correct, as there may often be a gap between the two. This may be due to several reasons: first, the economy is a complex nonlinear system and it is not feasible in practice to convert all savings into investments before the end of the period; second, external conditions, including economic cycles, have a significant impact on investors' willingness to invest; third, financial markets may be inefficient or underdeveloped, which leads to restrictions on investment.

The model introduced an indicator of total savings (M), which is responsible for the total savings available in the economy and includes not only the savings of the current period, but also of previous periods. This indicator is increased by the amount of current period savings and decreased by the amount of current period investments (external and internal). In other words, the general scheme of investment flows can be represented as follows $S \longrightarrow M \longrightarrow I$. The closest economic indicator to measure total savings is money supply.

Investments can be classified into domestic I_{in}, external I_{out} and foreign I_f according to their direction and source, with $I = I_{\text{in}} + I_{\text{out}}$. Domestic investment occurs when money from the savings of domestic economic actors, such as businesses or individuals, is invested in the same economy. External investment occurs when this money is spent on investments in other countries. Foreign investment reflects the situation when other countries invest in the domestic economy [2].

The other preconditions are similar to the Solow-Swann model [3–5] and similar models. To summarize the above, the model considers the following prerequisites:

- the country's economy produces only one product Y, which is used both for consumption C_d and for savings S, and in the international market.
- the economy is open to the movement of capital, labor, goods and services with other economies.
- enterprises operate in a competitive environment and maximize their profits.
- the economy is divided into productive and innovative sectors, with the productive sector being divided into primary (mining and agriculture), secondary (industry and construction) and tertiary (services), while the innovative sector is engaged in knowledge creation.
- consumption C consists of domestic consumption C_d (expenditure on consumption of goods and services produced and sold in the domestic market) and imports $\Im$.
- savings S consist of domestic savings S_d (income from the sale of goods and services in the domestic market) and exports Ex.
- investment is the main driver of economic growth I, which are divided into internal I_{in}, outer I_{out} and foreign I_f.
- savings are not automatically transformed into investments, and their transition depends on economic conditions and may be delayed in time (investment lag).
- time t is changing continuously.

Production function and factors. Factors of production are important components of the economic process that influence the production of goods and services. In the model, the main factors of production are private capital K_{pr}, public capital K_{gov}, human capital (knowledge) H, labor$L$ and the variable factor R.

Capital is a set of material resources used to produce goods and services. It is divided into private capital and public capital. Private capital K_{pr} Capital is a set of material resources used to produce goods and services. It is divided into private capital and public capital. Private capital K_{gov} includes material resources of the state invested in infrastructure development, education, healthcare and other sectors that also contribute to economic growth.

Human capital H is the totality of knowledge, skills, information, experience and intellectual potential available to the population. It is a key driver of economic growth, as it provides opportunities for innovation, increased productivity and a better quality of life. In the model, human capital is represented as a set of knowledge and information in the economy, this indicator grows as the R&D sector innovates and does not depreciate. The amount of innovation in the reporting period depends on the capital and labor invested in the sector.

Labor L is one of the main factors of production and describes the physical contribution to the production process.

Variable factor R in a single-sector production model is responsible for the land factor N. In this case, the land factor reflects the aggregate of agricultural land and natural resources, which is expressed in monetary terms [6].

The model uses an analogue of the Cobb-Douglas function, which was chosen due to its simplicity, versatility of application and high statistical quality. As a result, in our case, the production function is written as

$$Y_p = A K_{\text{pr}}^{\alpha} K_{\text{gov}}^{\beta} H^{\gamma} N^{\varphi} L^{1-\alpha-\beta-\gamma-\varphi}, \tag{4}$$

where α is the coefficient of elasticity of private capital, β is the public capital elasticity ratio, γ is the human capital elasticity coefficient, φ is the elasticity of the variable factor, in this case, land. Expression (4) generalizes the orthodox Cobb-Douglas production function.

In the multi-sector model, the factor R depends on the sector. For the primary sector Y_{agr} (covering industries that use natural resources, such as agriculture, forestry, fisheries, and mining), land is a factor, similar to the single-sector model. For the secondary sector Y_{ind} (covers industries engaged in the processing of raw materials and production of industrial products, such as metallurgy, chemical industry, light and food industry, machine building and construction) the factor is primary sector output Y_{agr}. For the tertiary sector Y_{serv} (covers industries that provide services to households and businesses, such as trade, transport, finance, etc.) secondary sector output is a factor Y_{ind}.

For a multi-sector model, the production function takes the form:

$$\begin{aligned} Y_p = {} & A_1 K_{agr}^{\alpha_1} K_{gov}^{\beta_1} H_{agr}^{\gamma_1} N^{\varphi_1} L_{agr}^{1-\alpha_1-\beta_1-\gamma_1-\varphi_1} + \\ & + A_2 K_{ind}^{\alpha_2} K_{gov}^{\beta_2} H_{ind}^{\gamma_2} Y_{agr}^{\varphi_2} L_{ind}^{1-\alpha_2-\beta_2-\gamma_2-\varphi_2} + \\ & + A_3 K_{serv}^{\alpha_3} K_{gov}^{\beta_3} H_{serv}^{\gamma_3} Y_{ind}^{\varphi_3} L_{serv}^{1-\alpha_3-\beta_3-\gamma_3-\varphi_3} \end{aligned} \tag{5}$$

where by $Y_p = Y_{agr} + Y_{ind} + Y_{serv}$, similarly $K_{pr} = K_{agr} + K_{ind} + K_{serv}$ and $H = H_{agr} + H_{ind} + H_{serv}$, $L = L_{agr} + L_{ind} + L_{serv}$.

The innovation sector in an economy plays a critical role in stimulating economic growth, as it is a source of new ideas, technologies and approaches to solving problems. This sector is the basis for creating new markets, jobs and stimulating production. Innovations can include the development of new products, processes or services, as well as the introduction of new management and organizational methods. Through innovation, companies can improve their competitiveness, increase productivity and create added value for society as a whole.

In the model, the innovation sector generates new knowledge according to the production function:

$$\Delta H = B K_{rd}^{\upsilon} L_{rd}^{1-\upsilon}, \tag{6}$$

where K_{rd} is the capital raised in the innovation sector, L_{rd} is Labor employed in the innovation sector, υ is elasticity of capital in the innovation sector. Total capital in the economy K_{full} can be calculated by the following formula: $K_{full} = K_{rd} + K_{pr} + K_{gov}$, similar to labor: $L_{full} = L_{rd} + L$.

The basic system of equations. The basic equation of the Solow model and similar models is a key tool in economic theory used to analyze economic growth in the long run. This differential equation describes changes in capital and other factors over time. Basically, it expresses how the total amount of capital in an economy changes as a function of capital inputs (investments), natural change in existing capital, and capital outflows due to depreciation. This equation is usually expressed in terms of specific indicators, such as capital intensity. For example, the equation in the Solow model [3–5] takes the form:

$$k^{\bullet} = sA k^{\alpha} - (d+n)k, k_0 = k(t_0),, \tag{7}$$

where k=$k(t)$ corresponds to the capital intensity, $k^{\bullet} = \frac{dk}{dt}$ is its first derivative; coefficient s is capital accumulation rate; constants A and α belong to the Cobb-Douglas function, respectively, A reflects indirect costs, and α is the elasticity; coefficient s is the rate of capital accumulation, d is the degree of capital retirement, n is the average growth rate of the employed population. And what is more $d + n = \lambda$. The equation is converted to this form using the transformation:

$$k^{\bullet} = \left(\frac{K}{L}\right)^{\bullet} = \frac{K^{\bullet} L - K L^{\bullet}}{L^2} = \frac{K^{\bullet} L}{\text{LL}} - \frac{K L^{\bullet}}{L^2} = \frac{K^{\bullet}}{L} - \frac{K}{L}\left(\frac{L^{\bullet}}{L}\right) = \frac{K^{\bullet}}{L} - nk. \tag{8}$$

For multidimensional models, the equations are similar. In the Mankiw-Romer-Weil model [7], the basic equation is:

$$\begin{aligned} k^{\bullet} &= s_k A k^{\alpha} h^{\beta} - (d+n)k \\ h^{\bullet} &= s_h A k^{\alpha} h^{\beta} - (d+n)h \end{aligned} \tag{9}$$

where s_k is the rate of accumulation of physical capital, and s_h is rate of human capital accumulation, h is human capital per unit of labor.

Since the model divides capital into private and public, and investment is made through aggregate savings, capital dynamics can be expressed through three indicators: private sector capital intensity, public sector capital intensity, and aggregate savings per unit of labor.

The capital intensity of the private sector, defined as the ratio of capital to labor, indicates the efficiency of capital use in production processes. It reflects how much capital is used to generate a unit of output compared to the amount of labor involved in production. A high capital intensity indicates an efficient use of capital and may indicate a high level of development of the country's economy. On the contrary, a low capital intensity may indicate underinvestment, inefficient use of capital, or an underdeveloped economy. The capital stock of the private sector grows through investment (domestic and foreign) and decreases through depreciation of fixed capital. In equation form, this can be written as:

$$\dot{k}_{pr} = i_{in} + i_f - (d_{pr} + n) k_{pr}, \tag{10}$$

where $k_{pr} = \frac{K_{pr}}{L}$ is the capital intensity of the private sector, d_{pr} is the depreciation ratio of private capital, n is the average growth rate of the employed workforce, $i_{in} = \frac{I_{in}}{L}$ is the domestic investment per unit of labor, $i_f = \frac{I_f}{L}$ is the foreign investment per unit of labor.

Public sector capital intensity measures the proportion of capital relative to labor used to provide public services and infrastructure provided by government agencies or institutions. It reflects the level of investment and efficiency in the management of capital resources in the public sector, such as buildings, roads, schools, healthcare facilities, etc. A high capital intensity of the public sector may indicate a high level of investment in the development of public services and infrastructure, which contributes to improving the quality of life of the population and stimulating economic development. The growth of this indicator is driven by taxes paid to the state budget, while the decrease, similar to the private sector, is driven by capital disposals (depreciation). Public international transfers in the form of international aid or international lending can be included in the brackets, which can either increase or decrease capital intensity [8]. The equation for the capital intensity of the public sector is as follows:

$$\dot{k}_{gov} = g - (d_{gov} + n) k_{gov} + tx, \tag{11}$$

where $k_{gov} = \frac{K_{gov}}{L}$ is the capital intensity of the public sector, $g = \frac{G}{L}$ are taxes per unit of labor, d_{gov} is the depreciation ratio for public equity, tx is the net government international transfers. The figures include $d_{pr} + d_{gov} = d$.

Gross savings per unit of labor is the amount of savings accumulated in relation to each employee. This measure takes into account the total amount of money saved by employees from their income after taxes and other expenses. It may include personal savings, investments in pension funds, shares or other financial instruments. Total savings per unit of labor can serve as an indicator of the financial stability of the economy as a whole and the ability of its agents to invest. Aggregate savings are increased by productive activity, taking into account their distribution between consumption and savings, and decreased by investment in fixed assets within the economy or in other economies, as well as by taxes. In mathematical terms:

$$\dot{m} = sA\, k_{pr}^{\alpha}\, k_{gov}^{\beta}\, h^{\gamma}\, n_N^{\varphi} - (g + nm + i_{in} + i_{out}), \tag{12}$$

where $m = \frac{M}{L}$ is the total savings per unit of labor, s is the accumulation rate, $n_N = \frac{N}{L}$ is the land factor per unit of labor, $i_{out} = \frac{I_{out}}{L}$ is the external investment per unit of labor.

Taking into account expressions (6) and (9), a equation for the model will take the form:

$$\dot{h} = B\, k_{\mathrm{rd}}^{\upsilon}\, l - nh, \tag{13}$$

where $k_{\mathrm{rd}} = \frac{K_{\mathrm{rd}}}{L_{\mathrm{rd}}}$ is the capital intensity in the innovation sector, $l = \frac{L_{\mathrm{rd}}}{L}$ is the labor force participation rate in the innovation sector.

Combining expressions (10)–(13), we obtain the general form of the model:

$$\begin{cases} \dot{k}_{\mathrm{pr}} = i_{\mathrm{in}} + i_f - (d_{\mathrm{pr}} + n) k_{\mathrm{pr}},\ \dot{k}_{\mathrm{gov}} = g - (d_{\mathrm{gov}} + n) k_{\mathrm{gov}} + tx, \\ \dot{m} = sf(k) - (g + nm + i_{\mathrm{in}} + i_{\mathrm{out}}),\ \dot{h} = B\, k_{\mathrm{rd}}^{\upsilon}\, l - nh. \end{cases} \tag{14}$$

For a one-sector model, the system of equations will be as follows:

$$\begin{cases} \dot{k}_{\mathrm{pr}} = i_{\mathrm{in}} + i_f - (d_{\mathrm{pr}} + n) k_{\mathrm{pr}},\ \dot{k}_{\mathrm{gov}} = g - (d_{\mathrm{gov}} + n) k_{\mathrm{gov}} + tx, \\ \dot{m} = sA\, k_{\mathrm{pr}}^{\alpha}\, k_{\mathrm{gov}}^{\beta}\, h^{\gamma}\, n_N^{\varphi} - (g + nm + i_{\mathrm{in}} + i_{\mathrm{out}}),\ \dot{h} = B\, k_{\mathrm{rd}}^{\upsilon}\, l - nh. \end{cases} \tag{15}$$

For multi-sector modification

$$f(k) = A_1 \frac{L_{\mathrm{agr}}}{L} k_{\mathrm{agr}}^{\alpha_1} k_{\mathrm{gov}}^{\beta_1} h_{\mathrm{agr}}^{\gamma_1} n_N^{\varphi_1} + A_2 \frac{L_{\mathrm{ind}}}{L} k_{\mathrm{ind}}^{\alpha_2} k_{\mathrm{gov}}^{\beta_2} h_{\mathrm{ind}}^{\gamma_2} y_{\mathrm{arg}}^{\varphi_2} + A_3 \frac{L_{\mathrm{serv}}}{L} k_{\mathrm{serv}}^{\alpha_3} k_{\mathrm{gov}}^{\beta_3} h_{\mathrm{serv}}^{\gamma_3} y_{\mathrm{ind}}^{\varphi_3}.$$

Forecasting and economic cycles. It is important for the model to take into account economic cycles, as they have a significant impact on the rate of economic growth. Economic cycles are regular fluctuations in economic activity that are reflected in changes in output, employment, prices, investment, and other indicators. These cyclical fluctuations arise from the interaction of various economic factors, such as changes in consumer demand, investment sentiment, money supply and other factors.

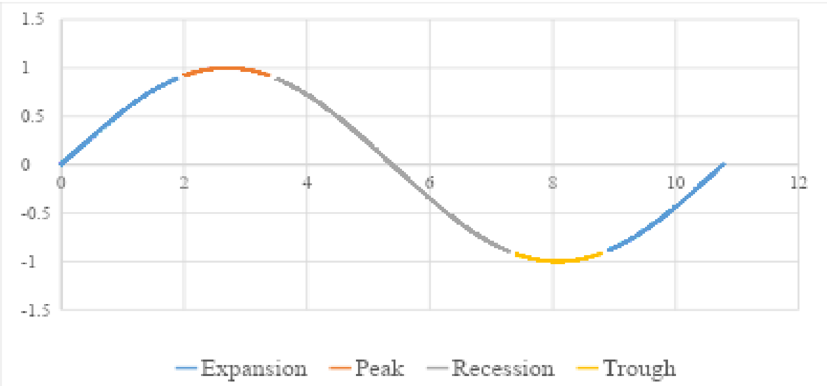

Figure 1: Phases of the economic cycle

The main phases of the economic cycle are the boom (expansion), peak, downturn (recession) and crisis (Fig. 1). During the expansion phase, the economy grows faster than usual, and there is an increase in production, employment and consumer demand. The peak marks the maximum level of economic activity, followed by a recession, when the economy typically slows down and production and employment decline. Finally, there is a recession, when the economy reaches its lowest level of activity before the next cycle begins. Growth phases motivate investment, while recession phases do the opposite, and the dynamics of economic growth depends on this.

According to the duration, the cycles can be divided into (Fig. 2):

- *Kitchin* [9] (or entrepreneurial) cycle: reflects periodic fluctuations in the mood of entrepreneurs and investors. It lasts from 3 to 7 years. The cycle is caused by changes in consumer preferences, technological innovation, and investment activity.
- *Juglar* [10] (or reverse) cycle: reflects changes in industrial output and employment. It usually lasts from 7 to 11 years and is caused by fluctuations in market supply and demand, as well as changes in credit conditions and pricing policies.
- *Kuznets* [11] (or business) cycle: reflects changes in the level of economic activity and business sentiment. The cycle lasts approximately 15–25 years. It is triggered by fundamental changes in production and economic structure, such as new technologies, industry development, and changes in the competitive environment.
- *Kondratiev* [10] (or long-term) cycle: reflects changes in the level of economic development and socio-economic transformations. The duration of the cycle is estimated at 40-60 years. This cycle is triggered by profound economic transformations, such as innovations, demographic changes, geopolitical shifts, and other major factors.

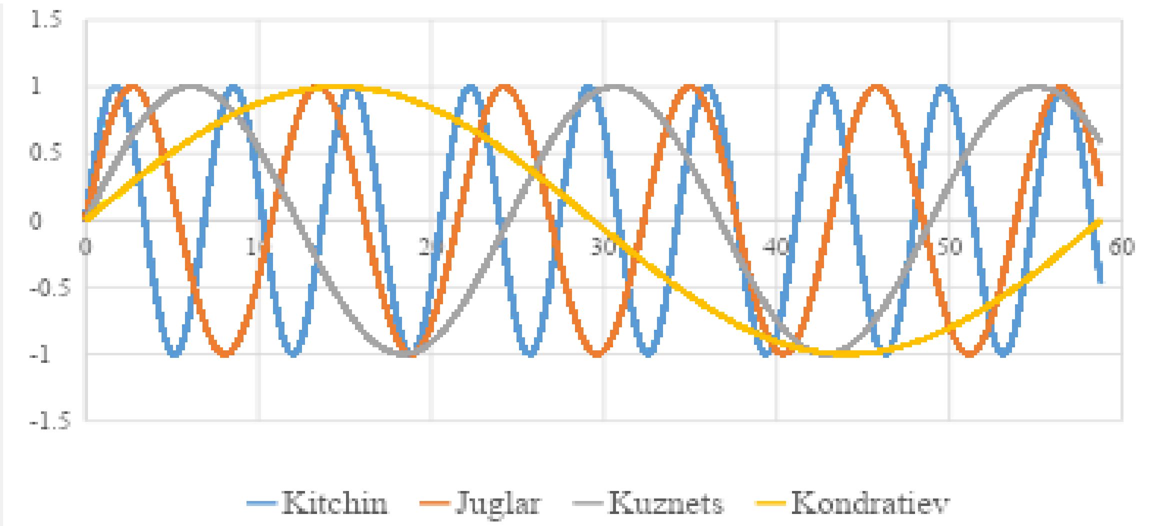

Figure 2: Economic cycles by duration

Forecasting indicators with several cyclicality whose periods do not coincide is a rather complicated task. In this case, the main task is to decompose the time series into a modelled (part of the value that depends on other indicators) and a cyclical component. This task is partially solved by time series forecasting models such as Holt-Winters [12, 13] or Theil-Wage [14]. For systems where economies of scale play an important role, the additive cyclic component is not suitable, so we modify the model with the multiplicative component as follows:

$$
\begin{gathered}
\hat{y}_{t+h \vee t} = (l_t + h b_t)\, s_{t+h+(k+1)} \\
l_t = \alpha (y_t / s_t) + (1-\alpha)(l_{t-1} + b_{t-1}) \\
b_t = \beta (l_t - l_{t-1}) + (1-\beta) b_{t-1} \\
s_t = s_{t_1 - m_1} * s_{t_2 - m_2} * \ldots * s_{t_n - m_n}, \\
s_{t_1} = \gamma_1 (y_t / (l_{t-1} + b_{t-1})) + (1-\gamma_1) s_{t_1 - m_1} \\
s_{t_2} = \gamma_2 (y_t / (l_{t-1} + b_{t-1}) s_{t_1}) + (1-\gamma_2) s_{t_2 - m_2} \\
\ldots \\
s_{t_n} = \gamma_n (y_t / (l_{t-1} + b_{t-1}) s_{t_1} * \ldots * s_{t_n}) + (1-\gamma_n) s_{t_n - m_n},
\end{gathered} \tag{16}
$$

where l_t is the time series level equation, b_t is the trend equation, $s_{t_1}, s_{t_2}, \ldots, s_{t_n}$ are seasonality/cyclicality equations, coefficients α, β, and $\gamma_1, \gamma_2, \ldots, \gamma_n$ are the degrees of smoothing of the time series, trend and seasonality/cyclicality, respectively, which can take values from 0 to 1, $m_1, m_2, \ldots, m_n$ are duration of seasonality periods.

At the present stage, during the recession, the production of most countries is not decreasing, but it is not growing either, so we can conclude that the problem is investment, which cannot ensure economic growth but is at a level that allows compensating for the cost of capital disposal. Therefore, it will be enough to take into account the cyclicality of investments. In addition to cyclicality, investment is influenced by the interest rate and the willingness of the population to invest. Taking these factors into account, the investment function for domestic investment will look like this:

$$i_{\text{in}} = \frac{s_t \omega_{\text{in}} m}{(1+r)}, \tag{17}$$

where ω_{in} is the willingness of economic agents to make domestic investments, r is the interest rate.

Taking into account the expression (17) in equation (15), we obtain:

$$\begin{cases} \dot{k}_{\text{pr}} = \frac{s_t \omega_{\text{in}} m}{(1+r)} + i_f - (d_{\text{pr}} + n) k_{\text{pr}}, \\ \dot{k}_{\text{gov}} = g - (d_{\text{gov}} + n) k_{\text{gov}} + tx, \ \dot{m} = sA k_{\text{pr}}^{\alpha} k_{\text{gov}}^{\beta} h^{\gamma} n_N^{\varphi} - \left(g + nm + \frac{s_t \omega_{\text{in}} m}{(1+r)} + i_{\text{out}} \right), \\ \dot{h} = B k_{\text{rd}}^{\upsilon} l - nh. \end{cases} \tag{18}$$

Using information technology in modelling. The use of information technology, programming and computational automation are becoming an integral part of modelling complex multidimensional economic systems, as the complexity of models and their data requirements increase, making it difficult and time-consuming to build a model analytically. These tools allow analysts and economists to effectively model a variety of economic scenarios, taking into account the multifactorial nature and interaction of various aspects of the economy. Programming is used to create computer models of economic processes, which allows analyzing and forecasting various scenarios of economic development. The use of programming makes it possible to create complex computing algorithms that ensure the accuracy and speed of analyzing large amounts of data. Automation of calculations in modelling economic systems allows for efficient use of computing resources and reduces the likelihood of errors with large amounts of data. This helps to improve the accuracy of forecasts and make informed economic decisions based on the analysis of model data [15

Modelling technology plays an important role in the analysis, including automatic data download from online repositories, processing of these data, and replacement of missing values using a group of regression models, which is described in more detail in [1]. After that, complex economic models are built to analyze various development scenarios. The technology also includes forecasting future trends based on calculations and creating dashboards based on the results obtained, which facilitates a convenient visual representation of data for making informed economic decisions.

The World Bank, as one of the leading international financial institutions, is used as an important online data repository for modelling economic processes. Its databases contain extensive information on the economies of various countries, including data on GDP, inflation, labor force, trade and other key indicators with sufficient accuracy of the data presented.

Using the R programming language as a modelling framework, it is easy to create complex economic models thanks to a rich set of functions and packages for data processing and statistical analysis. With R Shiny, you can create interactive dashboards and web applications to visualize modelling results, making the analysis accessible and understandable to a wide range of users. The advantages of this approach include rapid development and implementation of models, the ability to collaborate and share data between researchers, and the ability to respond to changes in real time using interactive visualizations.

Results: model testing on real data. First of all, the model was built on data for the Ukrainian economy (Fig. 3). Ukraine's economy for the period 1991–2021 is difficult to model because of the transition from a planned to a market economy, and every 4–5 years there are complex socio-political phenomena that significantly affect the country's economy.

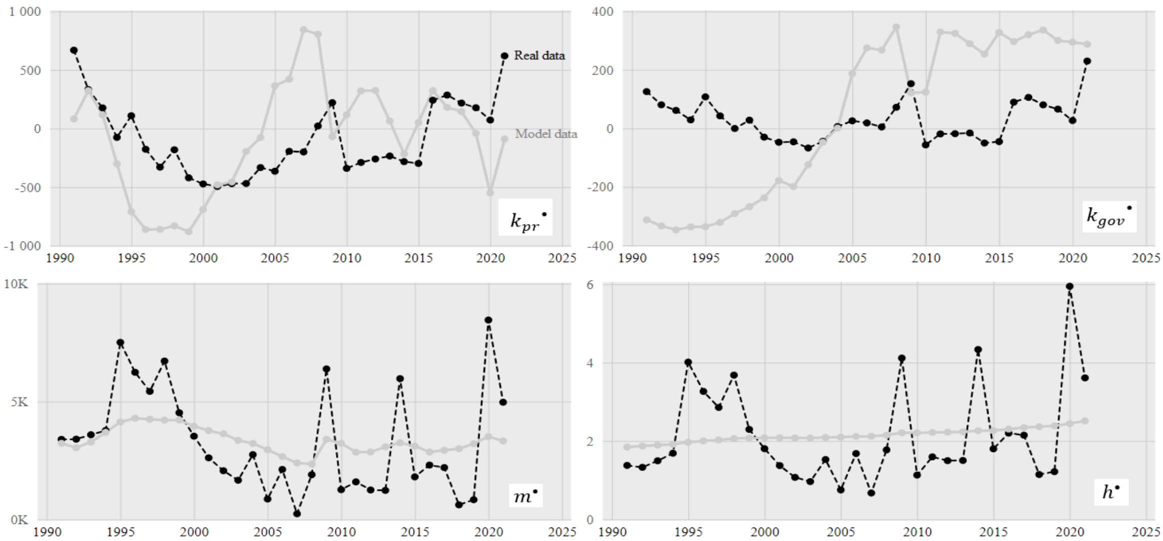

Figure 3: Model based on data for Ukraine

The model for Ukraine has a coefficient of determination of -0.93 for the growth of private sector capital intensity, while other economic growth models had a similar indicator of -40 on average. For the growth of the public sector capital intensity, the coefficient of determination is -15, for the growth of savings per unit of labor 0.2, and for the growth of human capital per unit of labour -0.3. Such indicators can be explained by the large share of the "shadow" and "grey" economy, the complexity of the transition to a market economy, inconsistent economic policy, etc. Also, Fig. 3 shows the pronounced crisis phenomena in 2008-2009 (global economic crisis), 2014-2015 (Revolution of Dignity and the outbreak of war in the eastern regions), 2020–2021 (global COVID-19 epidemic).

Models based on data from the economies of Poland, Hungary*, the Czech Republic, and Romania (Fig. 4) show generally similar dynamics between the model and real data.

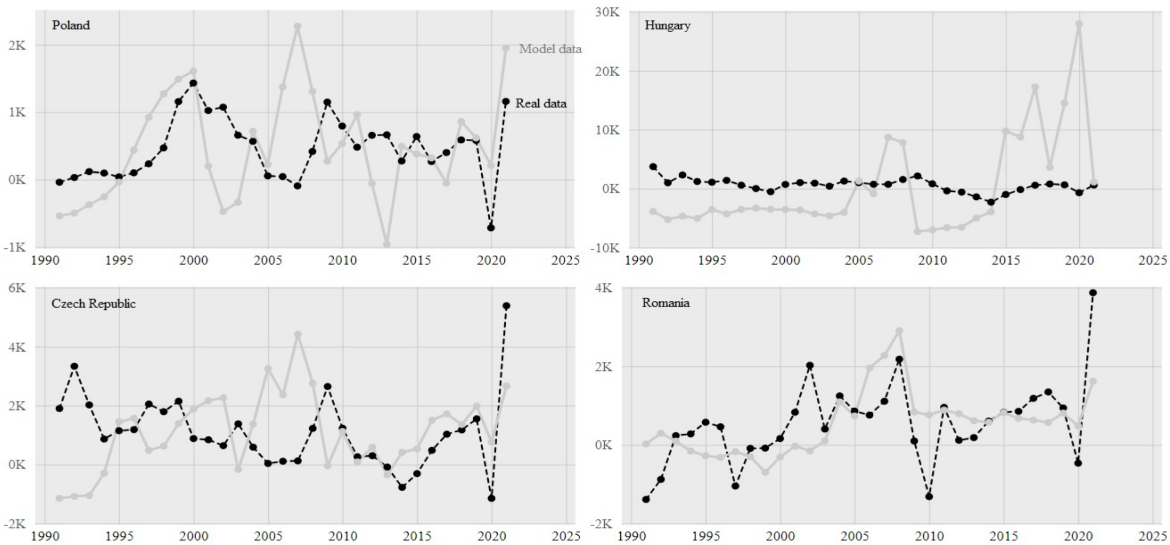

Figure 4: The model ($k_{pr}^{\bullet}$) on data for selected Eastern European countries

*Note** For the Hungarian economy, the modelled data differ significantly (Fig. 4) from the actual data. Although the main trend is similar, the deviations are too large. This can be explained by internal factors, such as the economy's dependence on EU funding, weak economic policy due to the authoritarian regime. These factors are not taken into account in the model, as they are difficult to account for mathematically, firstly, due to lack of data, and secondly, due to the difficulty of unambiguous mathematical interpretation of certain political phenomena.

As shown in Figs. 5 and 6, the open market growth model performs very well for highly developed countries with stable economies, with the exception of China**, compared to other growth models. This is due to several factors: the knowledge-based model of technological progress is quite realistic for highly developed countries, involvement in international trade, international capital and labor movements have a significant impact on the trajectory of economic development, and the inclusion of investment responses to changes in the phase of the economic cycle has a positive impact on the quality of the final model.

*Note*** Given the peculiarities of the Chinese government and a study [16] comparing economic growth rates with illumination, there are reasons to consider the official economic growth rates to be overstated. And since the official data does not correspond to the real data, a model based on market economy assumptions will not work correctly.

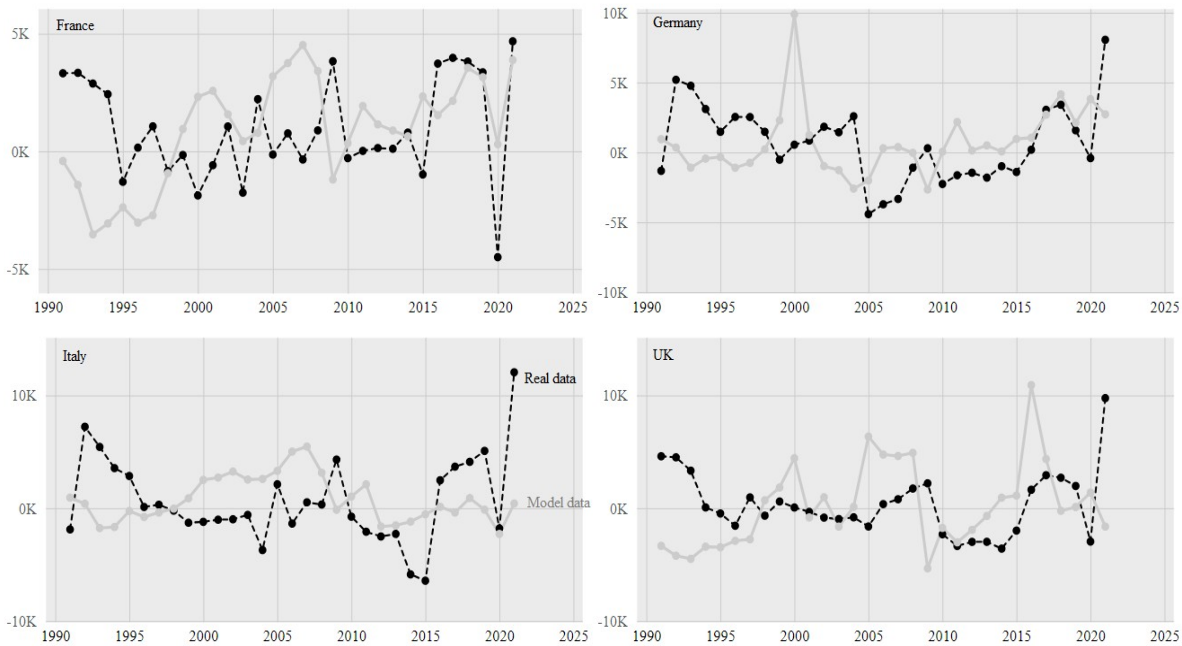

Figure 5: The model (k_{pr}^{*}) based on data for selected Western European countries

Figs. 5 and 6 show a similar situation with crises (not cyclical, but unexpected), which significantly increase the error. Accounting for crises in models of economic dynamics presents significant challenges due to their complexity and unexpectedness. Crises can arise for various reasons, such as financial distress, political instability, or natural disasters, and are often characterized by non-linear and unpredictable effects. Modelling such phenomena requires taking into account a wide range of factors, including the behavioral aspects of market participants, the interrelationships between different sectors of the economy and the impact of external factors. Moreover, the inclusion of crises in models requires constant updating and adaptation, as the economic environment is constantly changing and crises may take on new forms and characteristics. Similar results were reached by the authors in [17].

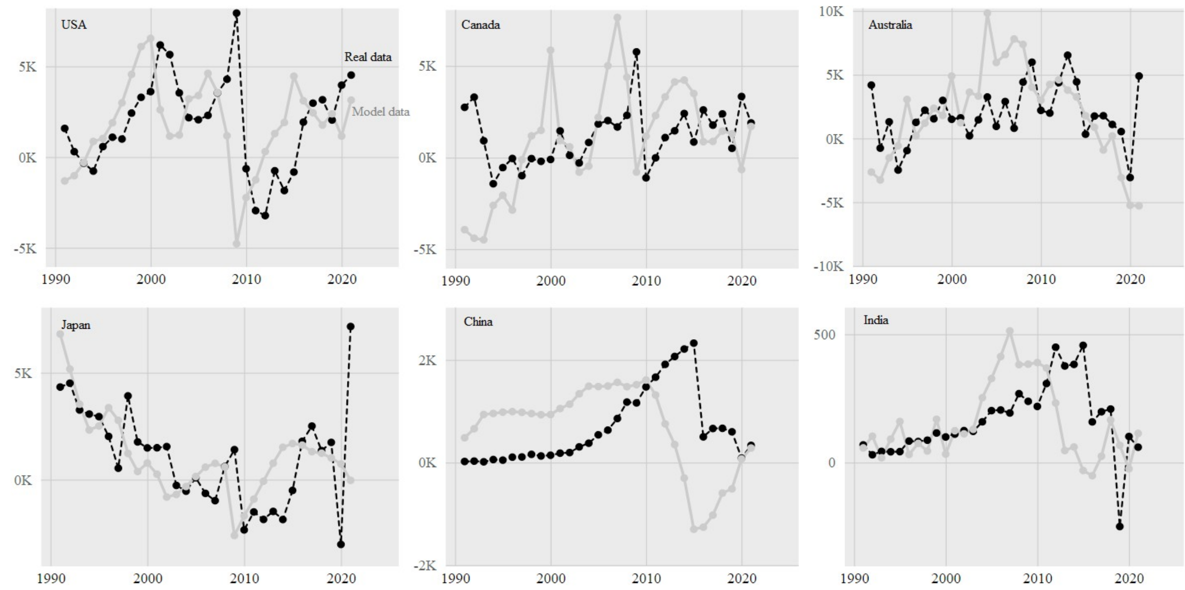

Figure 6: The model (k_{pr}^{*}) based on data for selected countries in the rest of the world

Modelling crisis shocks is a separate forecasting task, often unrelated to economic dynamics models, and a multiple increase in model complexity, in addition to data problems, can negatively affect periods of "stability". In the medium or long term, most crises will be levelled out over time, so taking them into account will create more risks than benefits for the final model.

Since the quality indicators of the models described above are much better than those of the other economic growth models (Fig. 7), we can conclude that the hypothesis that the assumptions of a closed economy, equality of investment and savings in each period, and the absence of an investment lag are unrealistic is true.

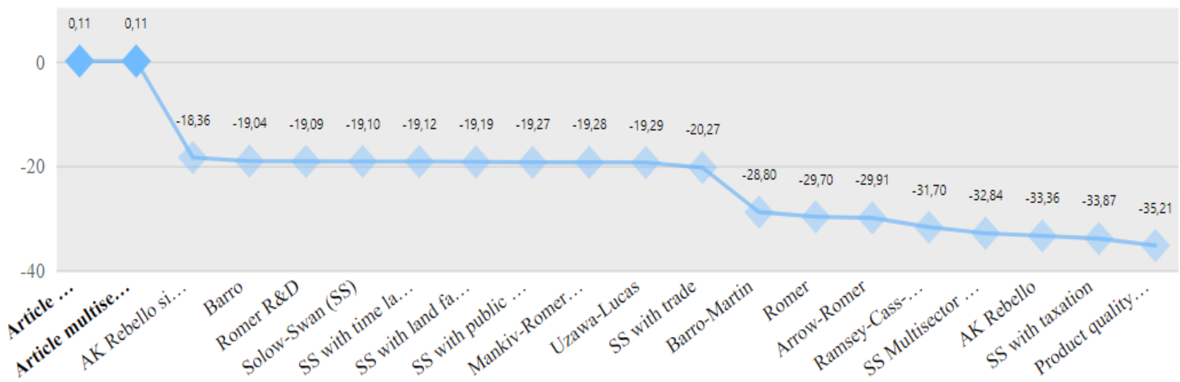

Figure 7: Average coefficient of determination by economic growth models

From Fig. 8, we can draw similar conclusions. In addition, the model has a significantly lower error for RMSLE (Fig. 8), but this is not the case for other indicators such as RMSE or MSE. For some economies, increasing the complexity of the model significantly increased the difference between model and real values, the vast majority of these countries are some underdeveloped countries in Africa and Asia, where there are socio-economic or political nuances that affect the indicators, or shortcomings in data collection that make the primary assumptions of the models unrealistic.

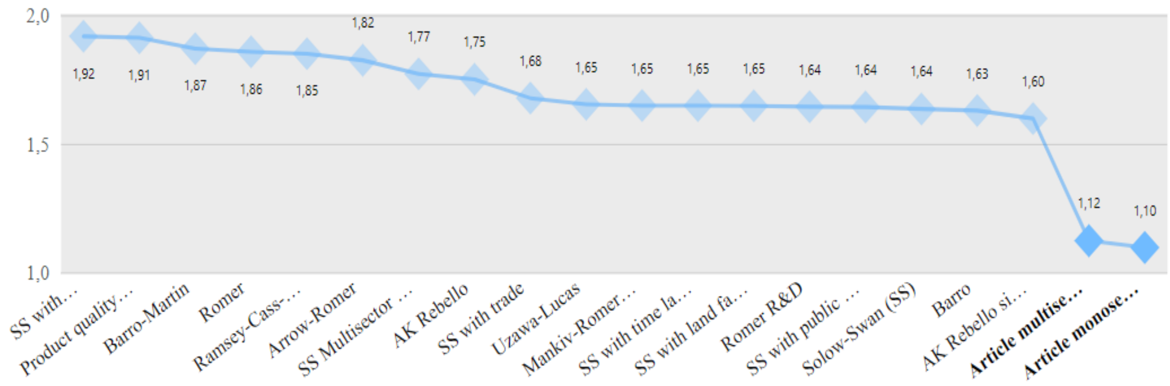

Figure 8: Average RMSLE by economic growth models

The quality of a short-run economic growth model is often higher than a medium- or long-term simulation (Fig. 9, n is the length of the simulation period in years, from 10 to 60), as it can be more accurate due to the limited time horizon and relatively stable conditions. In the short-term analysis of economic models, some factors, such as production capacity, consumer spending, or exchange rates, for example, may remain relatively constant or change little, allowing for more accurate modelling of economic dynamics. At the same time, a wider range of factors, such as technological changes, demographic trends and global events, affect the medium- and long-term analysis of models, which can complicate modelling and reduce the accuracy of the models in general.

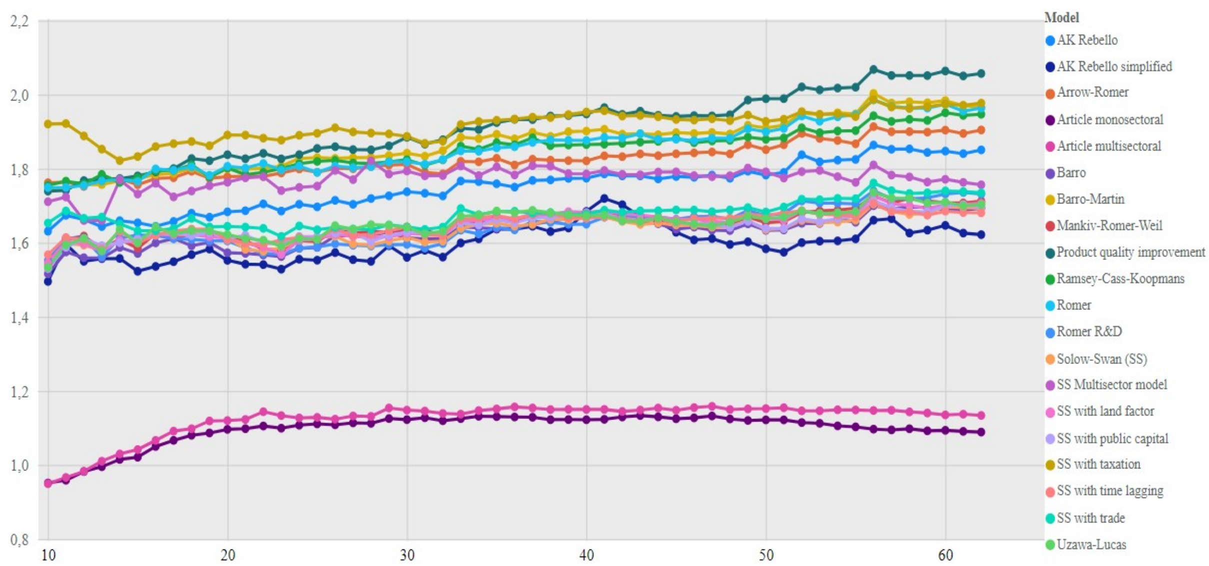

Figure 9: Average RMSLE by the length of the modelling period and economic growth models

A multi-sectoral growth model often works better in cases where the economy has undergone significant structural changes between sectoral shares during the period under study. This is because such models allow for a better account of the interrelationships and interactions between different sectors of the economy, taking into account their impact on overall economic dynamics. This is especially important in the long run, when structural changes can have a significant impact on economic growth trends. Such models allow for better analysis and forecasting of the impact of these changes on productivity, investment flows, and other key economic indicators on a long-term basis.

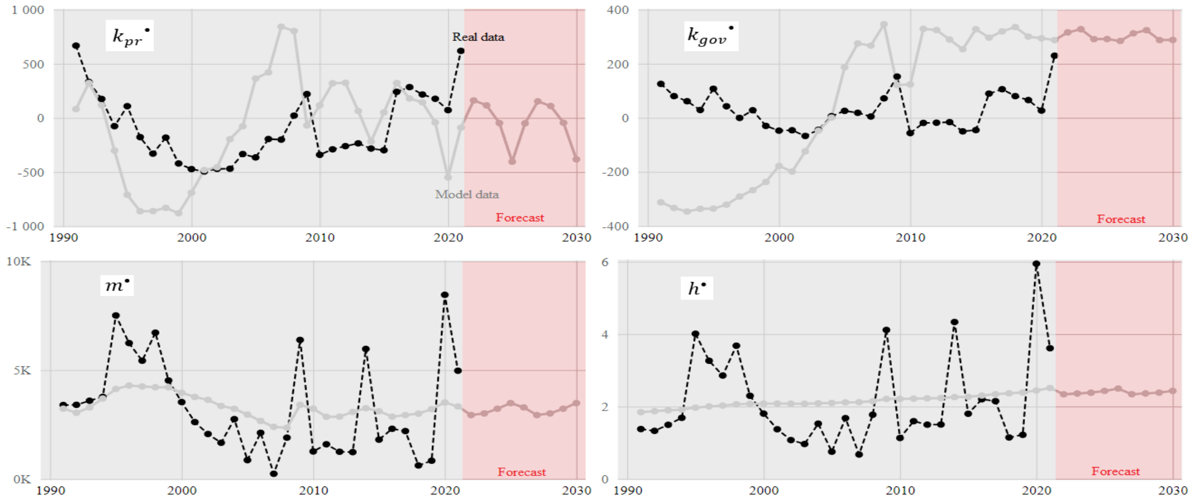

Figure 10: Forecast for the Ukrainian economy

Results: behavior dynamics of scalar parameters. Next, we will consider the parameters of the economic growth model separately. The analysis of the parameters was conducted in two areas: depending on the length of the period, i.e., models were built for the last 10 years to 62 years (full period) and depending on time (models were built for 10 years starting from 1960 in one-year increments).

Consider the parameter A, responsible for technological progress (Fig. 11).

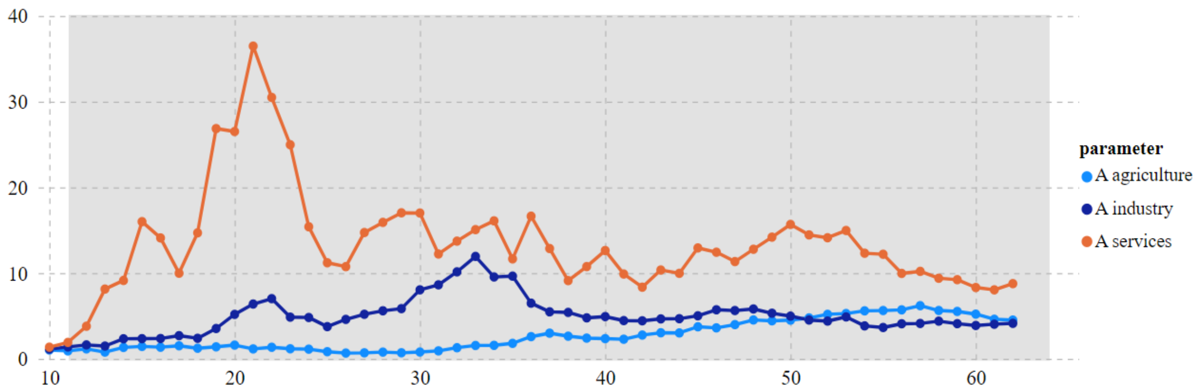

Figure 11: Average parameter A by economic sector by period length

The value of the parameter A decreases and fluctuates less with the length of the period (Fig. 11) due to the effect of gradual adaptation of innovations and saturation of technological processes. In short time intervals, technological breakthroughs can have a significant impact on the economy, causing sharp jumps in productivity and capital intensity. However, over time, the effect of innovation is smoothed out as the economic system adapts to the new technology and the pace of technological progress stabilizes. Longer periods cover the phases of market saturation with new technologies, when growth becomes more uniform and the impact of individual innovation breakthroughs is less.

Parameter A is highest for the services sector, lower for industry, and even lower for agriculture (Fig. 11), due to the different nature and speed of technological innovation in these sectors. The services sector is often characterized by a high rate of change and the integration of new technologies, such as information and communication technologies, which contribute to significant shifts in productivity and efficiency. In industry, technological progress, while also important, requires more time to implement new technologies in production processes due to significant capital investment and equipment modernization. In agriculture, despite the gradual impact of new technologies, the pace of progress is usually slower due to the more conservative nature of production practices and the lower intensity of technological innovation.

Next, we consider the parameters of the elasticity of production factors (Figs. 12–19).

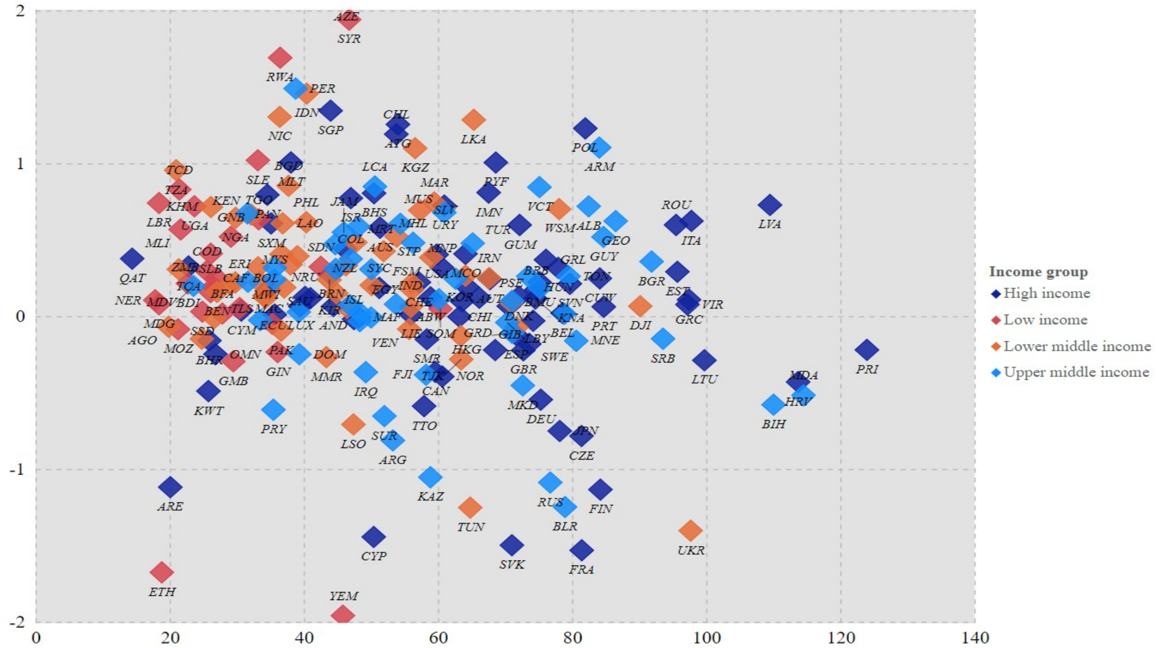

Figure 12: Dependence of the parameter α on the growth rate of human capital per unit of labor

Countries with lower knowledge growth rates tend to have higher capital elasticities, as their economic development depends more on capital investment than on innovation or technological progress (Fig. 12). In such countries, increased investment in infrastructure, equipment, or production capacity has a much greater impact on productivity growth and economic output, as new knowledge and technologies are adopted more slowly. The lack of rapid accumulation of knowledge means that capital becomes the main driver of development, and each additional capital investment has a stronger effect on the economy compared to countries where growth is based on technology and knowledge.

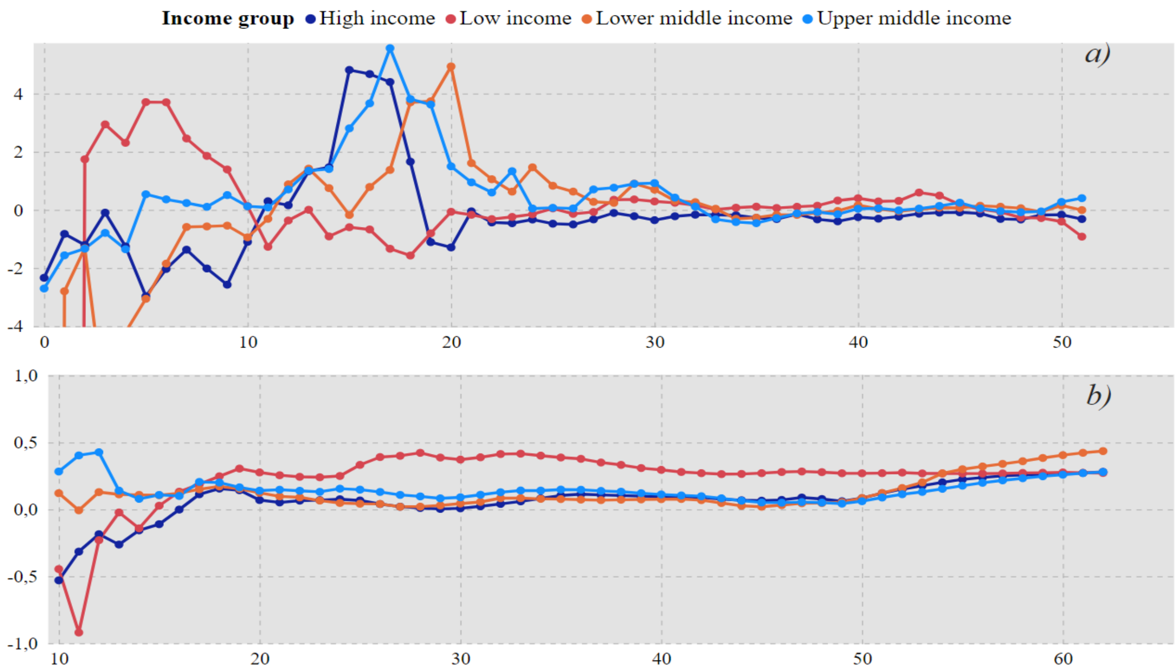

Figure 13: Average parameter α by time (a) and by period length (b)

Capital elasticities fluctuate much more in the period 1960–1990 (see Fig. 13-a) than in later periods, due to the economic and structural changes that characterized this era. After the Second World War, the world economy experienced a rapid recovery and industrialization, accompanied by rapid changes in investment and productivity. Additionally, this period included the oil crises of the 1970s, the ups and downs of industrial investment, as well as active processes of globalization and technological change. All these factors contributed to the high volatility of capital elasticity. In later periods, after the stabilization of global markets and the transition to a post-industrial economy, the rate of fluctuation of capital elasticity decreased due to the growing stability in capital investment and technological development.

The elasticity of capital is higher for underdeveloped countries in the medium and long run (see Fig. 13-b), as these economies have a large potential for productivity gains through capital investment. In these countries, even relatively small capital investments can lead to substantial productivity gains, as the baseline level of capital and technology is often low. Thus, the effect of new investments in production capacity, infrastructure, or technology is more pronounced than in developed economies, where the market is already saturated with capital. In the long run, growing economies may continue to exhibit higher capital elasticities due to rapid capital accumulation and the transition to more efficient technologies and practices.

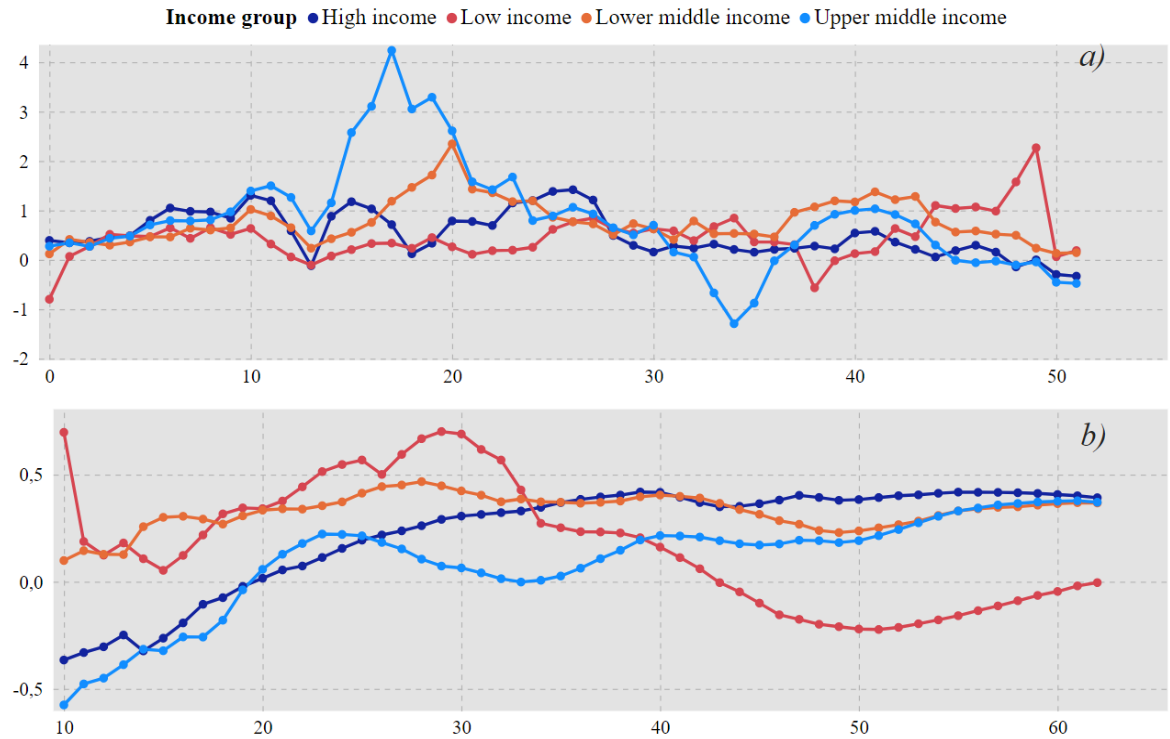

Figure 14: Average parameter β by time (a) and by period length (b)

The elasticity of public capital increased for middle-income countries in the 1980s and 1990s due to the large investments in infrastructure, energy, and education that underpinned their economic development during this period (see Fig. 14-a). These countries actively used public investment to modernize their economies and increase productivity, which led to an increase in the elasticity of public capital. For underdeveloped countries, a similar increase in elasticity occurred only in the 2000s and 2010s (see Fig. 14), when global initiatives such as international aid and investment programs aimed at improving infrastructure, health care, and education began to be actively implemented.

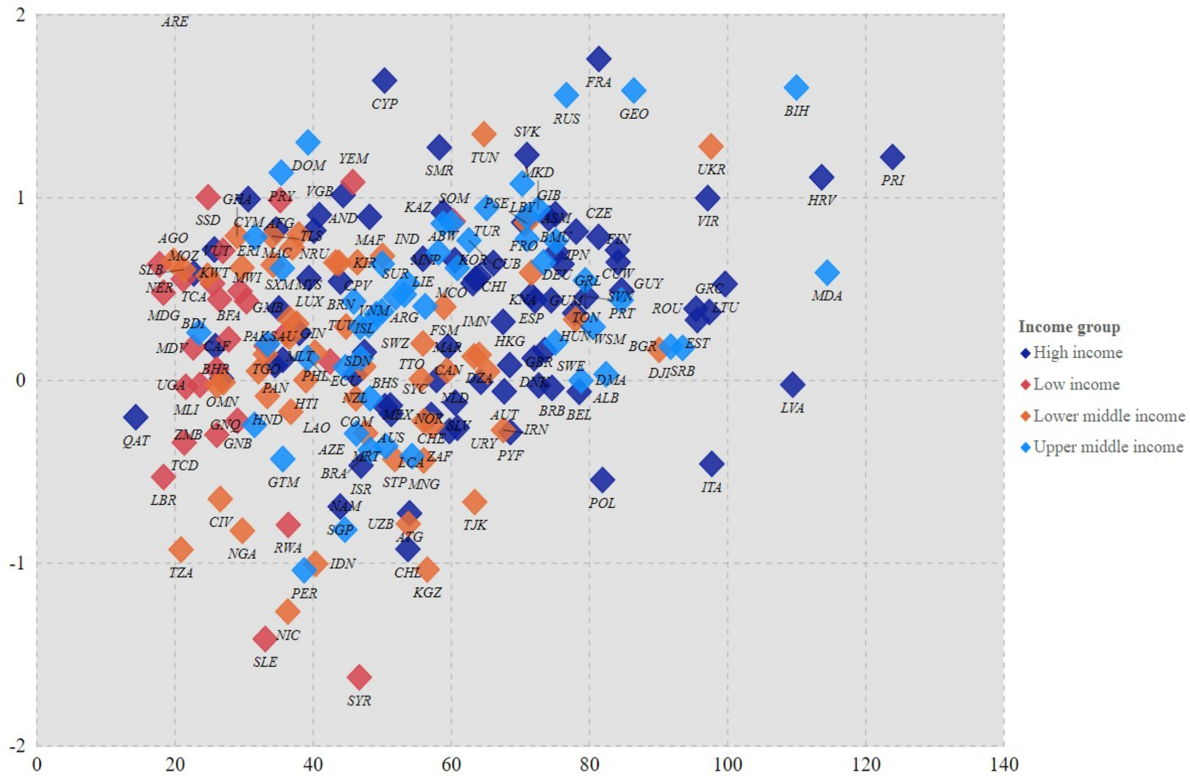

Figure 15: Dependence of the parameter γ on the growth rate of human capital per unit of labor

The higher the growth rate of human capital, the higher its elasticity (Fig. 15), as the rapid accumulation of knowledge, skills, and qualifications increases the economy's ability to adapt to new challenges and opportunities. When human capital grows rapidly, each additional investment in education, training, or innovation brings greater economic benefits by increasing labor productivity and facilitating the adoption of new technologies. In this case, the economy gains more from human capital, as highly educated and skilled workers are able to use resources more efficiently, create innovations and ensure sustainable growth.

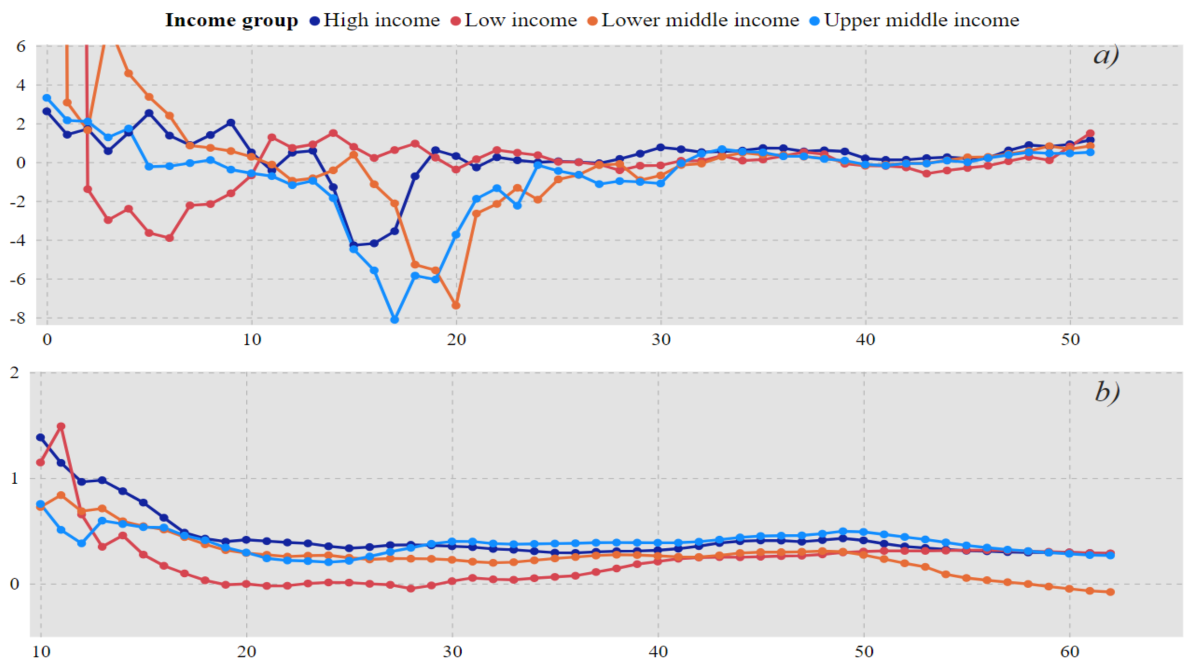

Figure 16: Average parameter γ by time (a) and by period length (b)

The elasticity of human capital is higher for highly developed countries regardless of the length of the period (see Fig. 16-b), as these countries have well-developed education, health care, and training systems that allow for efficient use of human resources. In such economies, investments in human capital, such as education, training, and innovation projects, are quickly translated into productivity growth and the adoption of new technologies. This impact is stable and long-lasting, as the high level of knowledge and skills of the workforce constantly supports the economy's competitiveness in the global market, regardless of the short or long term.

The decline in the elasticity of human capital in the 1980s and 1990s (see Fig. 16-a) was driven by several factors, including economic crises and structural changes in the global economy. During this period, significant economic restructuring took place, including the transition to a post-industrial economy and the automation of production, which reduced the demand for traditional jobs, particularly in the industrial sector. In addition, financial instability and budget cuts in many countries have led to limited investment in education and training systems. This reduced the rate of accumulation of new knowledge and skills, which directly affected the elasticity of human capital, as economies did not reap as many benefits from investments in human resources.

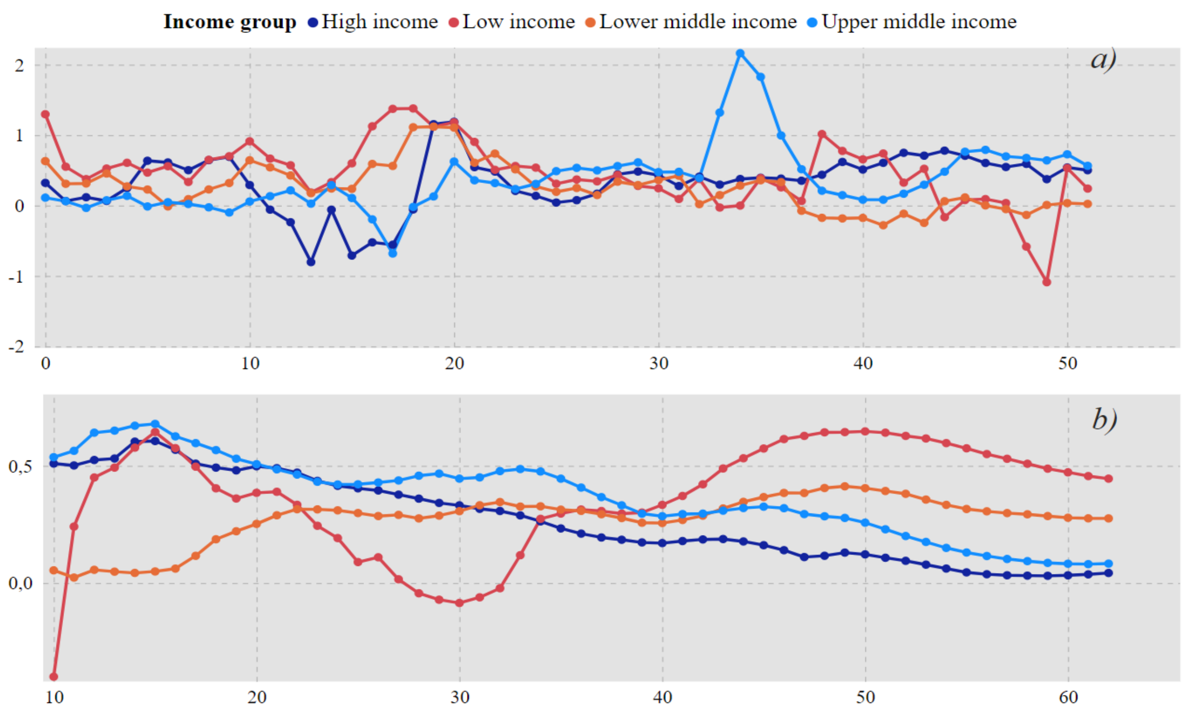

Figure 17: The average parameter φ over time (a) and over the duration of the period (b)

The elasticity of the land or natural resource factor is much higher for underdeveloped countries in the long run (see Fig. 17-b), as these economies have large untapped resources that can generate significant economic growth if they are used efficiently. However, this elasticity has declined in recent decades as many underdeveloped countries have reached a certain level of resource exploitation and are also facing constraints from environmental problems and natural resource depletion. At the same time, highly developed countries that use innovative technologies and efficient resource management methods have achieved a more stable and less volatile use of natural resources, which reduces their elasticity. As a result, the elasticity of the land or natural resource factor in underdeveloped countries is currently lower than in highly developed countries (see Fig. 17-a).

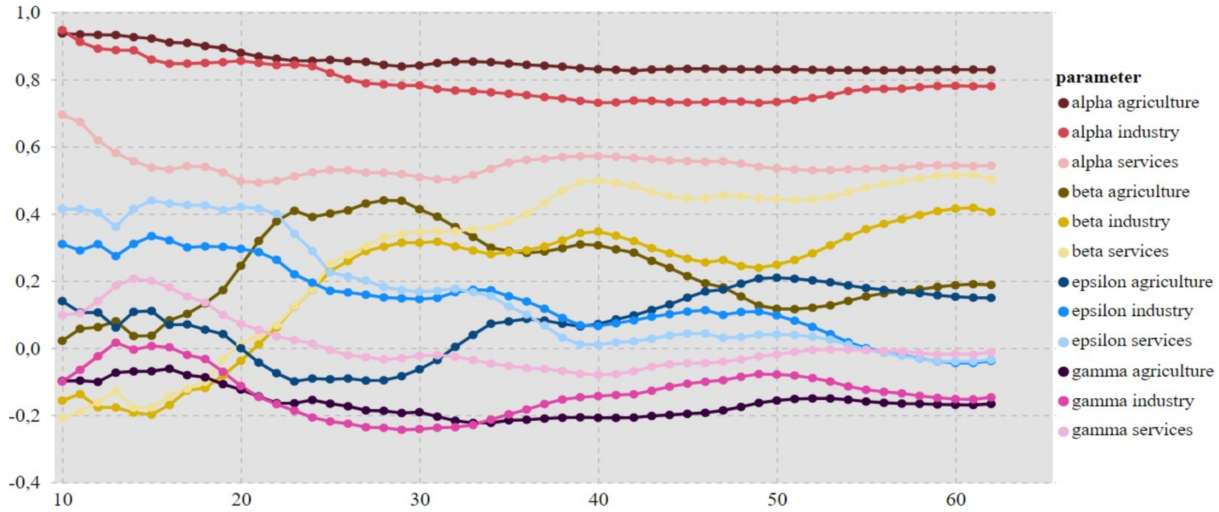

Figure 18: Average factor elasticity parameters by economic sector by period length

The elasticity of private capital is much higher for agriculture and industry than for services (Fig. 18), as these sectors are often characterized by a high need for capital investment to modernize equipment, infrastructure, and processes. In agriculture and industry, significant investments can significantly increase productivity through the introduction of new technologies and modernization of production facilities. In contrast, the service sector, where capital investment has a smaller direct impact on productivity due to the lower need for physical inputs and equipment, shows a lower elasticity of private capital.

As for public capital, its elasticity is higher for services in the long run (Fig. 18), as investments in infrastructure, education, and health care can generate significant benefits and efficiency gains in services over time. In the short run, however, the elasticity of public capital in this sector may be lower, as rapid changes in public spending may have less of an impact on short-term outcomes.

The elasticity of human capital is highest in the services sector (Fig. 18), as high levels of education and training in this sector significantly increase labor productivity and innovation potential, making human capital particularly important for the development of services.

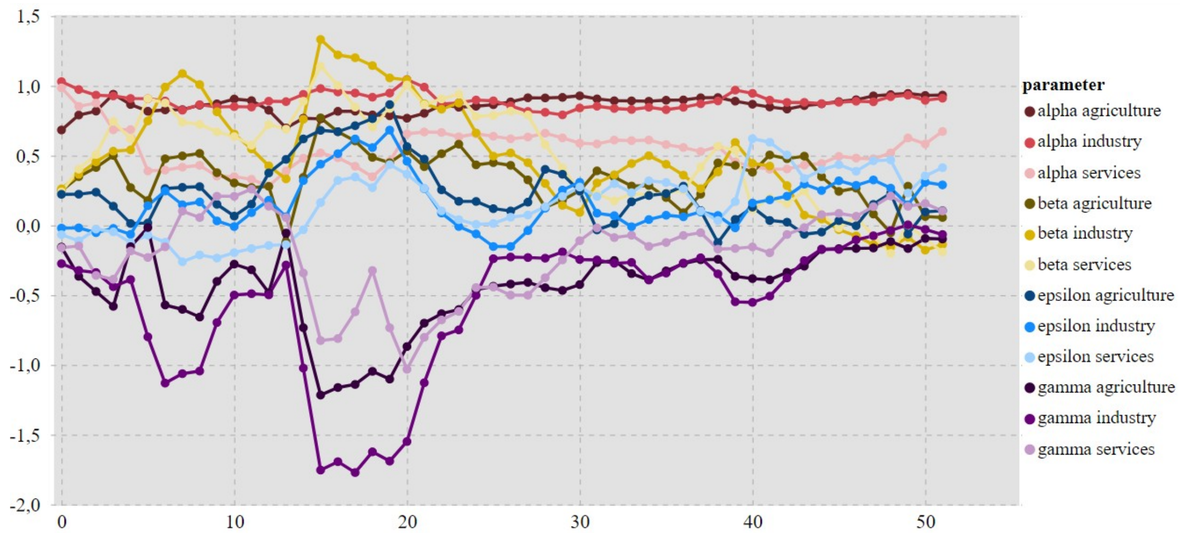

Figure 19: Average parameters of elasticity of production factors by economic sector over time

Over time, the elasticity of private capital and human capital increases (Fig. 19) due to the accumulation of experience, improved technology, and increased resource efficiency. Private capital becomes more adaptive as investments in new technologies and modernization of production facilities yield significant productivity gains. Similarly, human capital becomes more elastic over time due to advances in education and training, allowing workers to use new knowledge and skills more effectively. However, the elasticity of public capital decreases (Fig. 19) as initial investment in infrastructure and public services reaches saturation and its impact on economic growth diminishes. Once a baseline level of infrastructure and social services is reached, additional spending does not generate as much productivity gains or economic impact as in the first stages of investment.

Next, we consider the parameters of labor force growth, depreciation, and the rate of accumulation (Figs. 20–24).

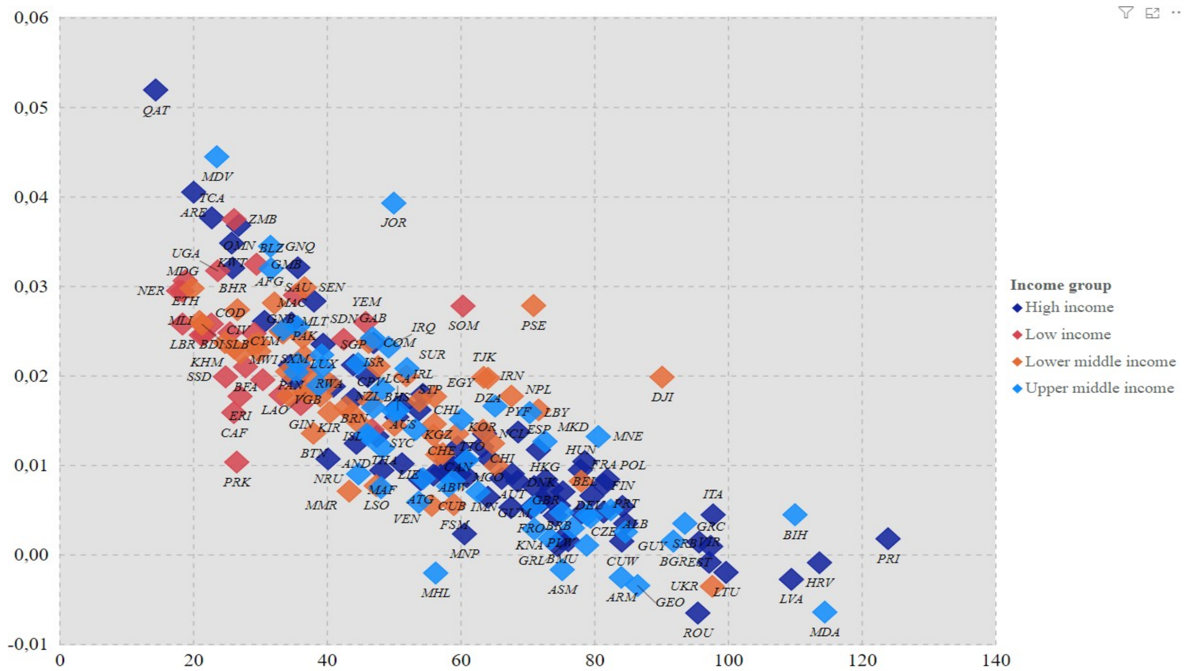

Figure 20: Dependence of the parameter n on the growth rate of human capital per unit of labor

Population is growing faster in underdeveloped countries, reinforcing the negative correlation between population growth and human capital per capita (Fig. 20). In such countries, where education and training systems already face limited resources, rapid population growth overwhelms these systems, which negatively affects the quality of education and training. As a result, the growth rate of human capital per capita is slowing down as more people are spread over fewer resources that cannot keep up with the high level of learning and skill development. This creates a vicious circle where rapid population growth in underdeveloped countries hinders the effective development of human capital, which in turn can hinder economic development and reduce the potential for further improvements in quality of life.

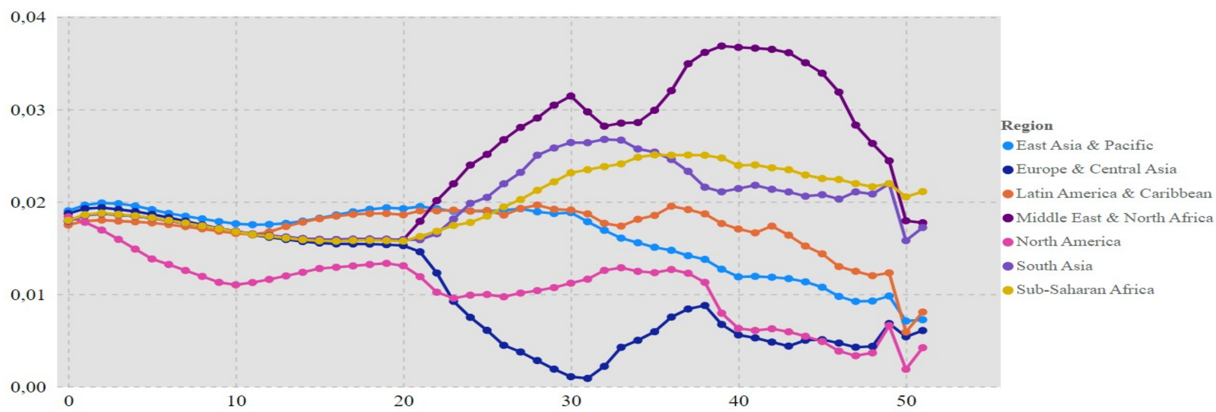

Figure 21: Average parameter n by region by time

Demographics depend on a number of key factors, such as the level of urbanization and quality of life (driven by economic development), health care and access to it, education, social and pension provision, and cultural characteristics. For example, underdeveloped countries, mainly in Africa and parts of Asia, are facing high population growth rates, often exceeding 2% per year, due to high birth rates and declining mortality. In comparison, developed countries in Europe and North America are experiencing much lower population growth rates or even population stagnation, due in part to low fertility and aging populations. Central and South America, as well as parts of Asia, are experiencing average growth, which is declining as living conditions improve and fertility rates decline (see Fig. 21).

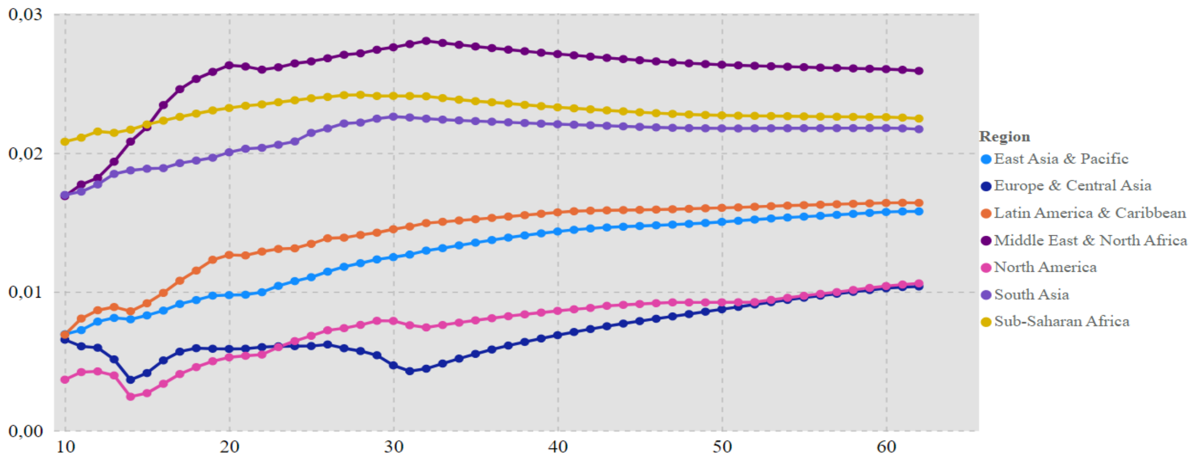

Figure 22: The average parameter n by region by the duration of the period

Population growth rates tend to decline (decrease) in all regions of the world, due to global demographic changes. In developed countries, this process is already clearly visible, as low fertility and aging of the population lead to stabilization or even decline in population size. At the same time, in underdeveloped countries, population growth rates are also gradually declining, thanks to improved living standards, access to health care and education, which affects the decline in fertility (Fig. 22).

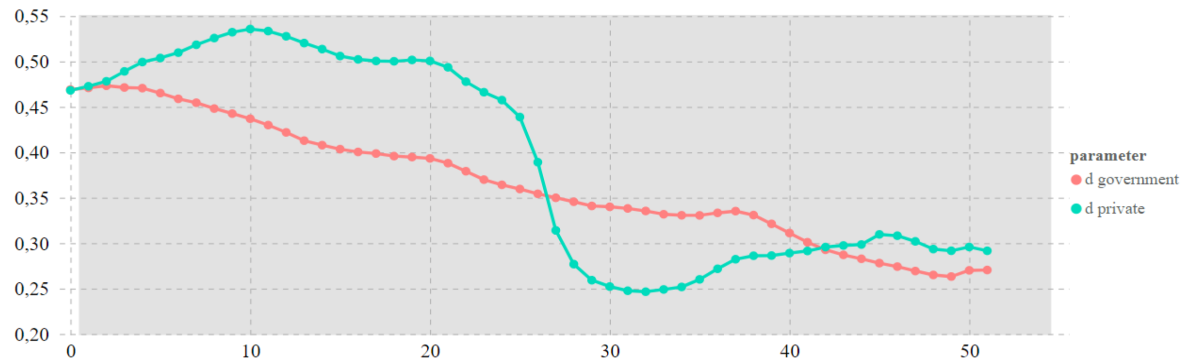

Figure 23: Average parameter d by region by time

As for capital depreciation, this indicator is gradually decreasing (Fig. 23) due to several important factors related to technological progress and changes in the structure of the economy. Modern technologies make it possible to create more durable and efficient capital assets, which reduces their physical wear and tear and the need for replacement. In addition, the growing share of intangible assets, such as information technology, patents, and innovative solutions, which are subject to less physical depreciation than traditional production assets, contributes to the overall decline in depreciation. The reorientation of the economy to more technology-intensive sectors, where capital depreciates more slowly, also plays a significant role in this process.

There are cyclical fluctuations when the depreciation rate of the private sector exceeds that of the public sector and vice versa (Fig. 23). During periods of economic recovery and increased private investment, enterprises actively modernize and replace equipment, which leads to higher depreciation costs in the private sector. In contrast, during economic downturns, the private sector may reduce investment and delay asset renewal, which reduces its depreciation costs. To summarize, these fluctuations are related to different economic conditions, government policies, and cyclical changes in investment activity.

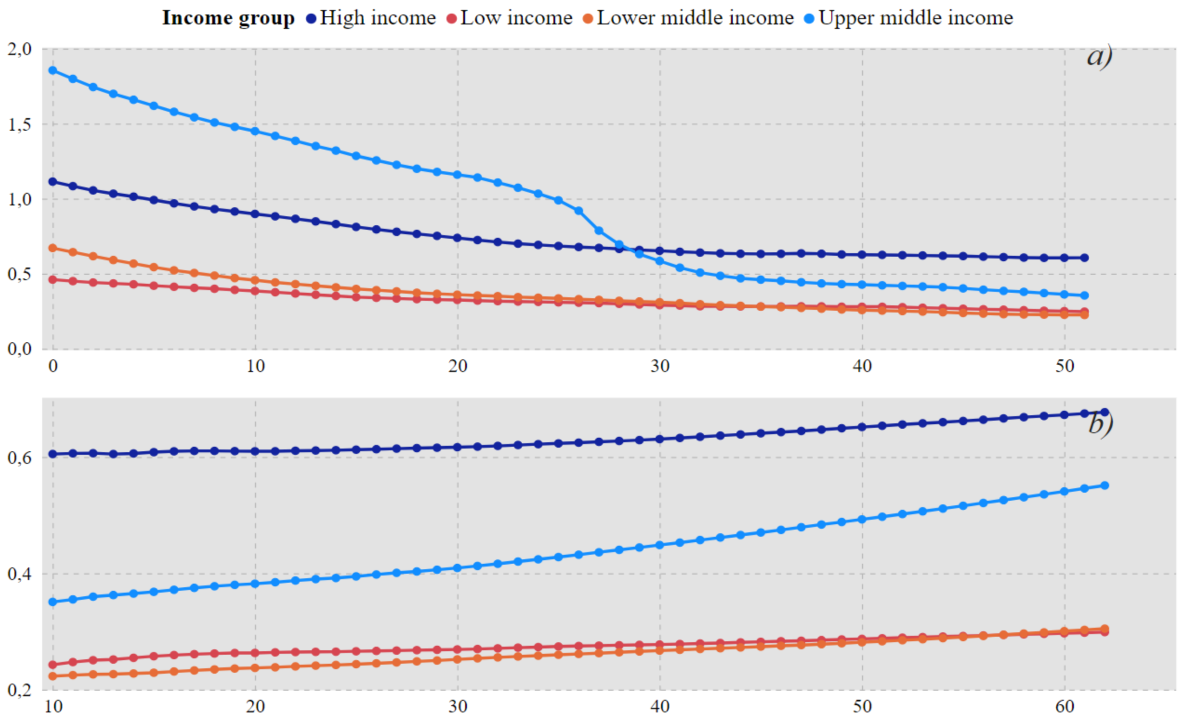

Figure 24: Average parameter s by time (a) and by period length (b)

The savings rate for highly developed countries is significantly higher than that of underdeveloped countries (Fig. 24) due to higher income levels, more stable economic conditions, and a developed financial infrastructure. Highly developed countries have greater opportunities for capital accumulation due to high levels of economic stability, access to financial instruments and savings programs, and cultural and social norms that encourage savings. In contrast, in underdeveloped countries, limited incomes, economic instability, and insufficient access to financial services prevent significant savings from being generated. These resource constraints and high levels of consumer spending in such countries reduce their ability to accumulate capital, which affects their economic development and investment activities.

Conclusions

The article presents an adaptive dynamic model of economic growth, which aims to increase the practical quality by taking into account the shortcomings of similar models by complicating its analytical form.

First of all, the model was transformed from a one-dimensional to a multidimensional model. The new assumptions of the model are that the economy is open to the movement of capital, labour, goods and services with other countries, the economy is divided into production and innovation sectors, consumption and savings are made up of different sources, including imports and exports, the main factor of economic growth is investment, which can be domestic, foreign and external, and its transition depends on conditions and may be delayed in time. The production function includes as inputs private capital, public capital, human capital, land (or the output of a sector of the economy) and labour.

The main equation of the model is a system of 4 components: growth of private capital intensity, growth of public sector capital intensity, dynamics of aggregate savings and dynamics of human capital (knowledge). The private sector increases its capital through domestic and foreign investment and decreases it through depreciation. The public sector increases its capital through taxes to the state budget, and decreases it through retirements. Public international transfers can both increase and decrease the total capital intensity. Aggregate savings increase through productive activity, but decrease through investment in fixed assets or in other economies, as well as through taxes.

We also drew the following conclusions regarding the dynamics of the parameters of the economic growth model:

- The technological progress parameter (A) is largest in the services sector, smaller in industry, and smallest in agriculture due to different speeds of technological innovation, complexity of innovative solutions, and size of the effect of implementation.
- The elasticity of private capital (α) is higher in agriculture and industry than in services, as these sectors require more capital investment to upgrade equipment and technology.
- The elasticity of public capital (β) is higher in services, especially in the long run, as investments in infrastructure, education, and health care provide significant benefits to the workforce, which is a key productive factor in this sector.
- The elasticity of human capital (ɣ) is higher for highly developed countries regardless of the period, as their advanced education and health care systems allow for the efficient use of human resources for sustainable productivity growth. Faster growth of human capital increases its elasticity, as the rapid accumulation of knowledge and skills increases the ability of the economy to adapt to new challenges and opportunities. The highest level of human capital elasticity is observed in the service sector.
- Over time, the elasticity of private and human capital increases due to improvements in technology and efficiency, while the elasticity of public capital decreases due to investment saturation.
- Amortization of capital (d) is gradually decreasing due to technological progress and changes in economic structure, as modern technology provides longer-lasting assets and intangible assets are less subject to physical wear and tear. The cyclical fluctuations in depreciation rates that are typical of private capital arise because during periods of economic expansion the private sector is more active in modernizing equipment and increasing depreciation expense, while during economic downturns, reduced investment leads to a reduction in depreciation expense.
- The savings rate (s) in highly developed countries is significantly higher than in underdeveloped countries due to higher incomes, more stable economic conditions, and a developed financial infrastructure.

Declaration on Generative AI

While preparing this work, the authors used the AI programs Grammarly Pro to correct text grammar and Strike Plagiarism to search for possible plagiarism. After using this tool, the authors reviewed and edited the content as needed and took full responsibility for the publication's content.

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