A Temporal, Deontic, Conditional Logic with Typicality: a Preliminary Report

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Abstract
In this paper we introduce a temporal, deontic, conditional logic with typicality, $LTL_D^{\mathbf{T}}$. It combines a multi-preferential conditional logic, which can be used for defeasible reasoning, with a temporal and deontic logic. The combination provides a formalism which is able to capture the dynamics of a system, through its strict and defeasible temporal properties, and also to reason about obligations and permissions. Temporal ranked knowledge bases are introduced for strengthening preferential entailment.

Keywords
Preferential and Conditional reasoning, Temporal logic, Deontic logic, Typicality

1. Introduction

In this paper we aim at introducing a temporal, deontic, conditional logic with typicality, called $LTL_D^{\mathbf{T}}$, based on a preferential approach to commonsense reasoning [1, 2, 3, 4, 5, 6]. The logic $LTL_D^{\mathbf{T}}$ combines a *typicality operator*, which allows defining conditional implications, with *temporal operators* from the Linear Time Temporal Logic (LTL) [7], and with the *deontic operators* from Standard Deontic Logic [8], to represent obligations and permissions.

Preferential approaches to commonsense reasoning have their roots in conditional logics [9], and have been used to provide axiomatic foundations of non-monotonic or defeasible reasoning. They allow the representation of conditional statements of the form "normally if α holds, β holds", which allow to represent properties of the world that admit exceptions (e.g., that normally students have classes, but but there are worlds in which this is not the case). In preferential semantics such as in Kraus, Lehmann and Magidor (KLM) semantics [4], a preference relation on worlds allows identifying the less exceptional worlds ($w < w'$ means that world w is less exceptional than world w').

Extending the conditional logic with the temporal and deontic operators allows considering the temporal dimension when reasoning about the defeasible properties of a system, e.g., to represent statements such as "normally students will get a degree", and also to capture rules describing obligations admitting exceptions (for instance the rule that normally people have the obligation to pay taxes within a deadline, but, in case of violation, they have the obligation to pay with fine).

The formalism can be exploited for explanation, and for reasoning about fulfillment of obligations, in the verification of the compliance of a process (like a business process) to norms. It is well known from the literature that "many normative rules allow for exceptions. Without defeasibility, it would be impossible to distinguish exceptions from violations" [10], and defeasibility is needed for modelling temporal normative rules.

In this regard, in this work we aim at a conditional extension of a fragment of the Deontic Dynamic Linear Time Temporal Logic (Deontic DLTL) studied in [11], a fragment in which the regular expressions of DLTL [12] (enriching the temporal operators) are not allowed. For simplicity, here we only consider

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the LTL operators *next*, *until*, *eventually* and *always*, and we develop a conditional extension of a Deontic LTL. We call the presented formalism $LTL^{\mathbf{T}}_{D}$, as the formalism includes a typicality operator $\mathbf{T}$ for expressing conditionals, as well as obligations and permissions.

Preferential extensions of LTL with defeasible temporal operators have been recently studied [13, 14, 15] to enrich temporal formalisms with non-monotonic reasoning features, by considering defeasible versions of the LTL operators. Our approach, instead, adds the standard LTL operators to a conditional logic with typicality, an approach similar to the preferential extension considered for Description Logics (DLs), where the logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ [16] extends the temporal description logic $LTL_{\mathcal{ALC}}$ [17] with a *typicality operator* to allow for conditional reasoning.

As in the Propositional Typicality Logic (PTL) by Booth et al. [18] (and in the DLs with typicality [19]) the conditionals are formalized based on material implication (resp., concept inclusions) plus the *typicality operator* $\mathbf{T}$. A conditional statement "normally if α holds, β holds" is represented by a *conditional implication* $\mathbf{T}(\alpha) \rightarrow \beta$, meaning that "in the typical situations in which α holds, β also holds". For instance, in this temporal formalism, the conditional implication: $\mathbf{T}(Student) \rightarrow \Diamond get\_degree$ means that, normally, students will eventually get a degree (although not all students will).

When α and β are formulas of the propositional calculus, an implication $\mathbf{T}(\alpha) \rightarrow \beta$ is intended to correspond to the conditional $\alpha \mathrel{|\!\sim} \beta$ in KLM logics [4, 6]. As a major difference with KLM logics, in this paper, we consider a *multi-preferential semantics*, which exploits *multiple preference relations* $<_{\alpha}$ with respect to different formulas α, along the lines of previous *multi-preferential semantics* exploiting preferences with respect to different aspects [20], with respect to different modules [21] and, for DLs, with respect to different concepts, based on ranked or weighted knowledge bases (KBs) [22, 23]. For instance, a world w may represent a more typical situation describing a student, compared to w' ($w <_{stud} w'$) but, vice-versa, world w' may represent a more typical situation describing an employee, compared to w, ($w' <_{emp} w$). Under this respect, the semantics we consider is a generalization of the KLM preferential semantics, which exploits a single preference relation on worlds (see [24] for details).

After extending the conditional logic with typicality with deontic and temporal modalities, the paper provides a decidability result for the deontic temporal conditional logic $LTL^{\mathbf{T}}_{D}$. Then, for strengthening preferential entailment, it introduces *ranked knowledge bases* (i.e., knowledge bases in which conditional implications are associated with a rank) and develops an approach for reasoning from a ranked knowledge base in the temporal setting, based on a closure construction in the style of the lexicographic closure [25] for KLM logics, by exploiting the strategy $\#$ from Brewka's framework for qualitative preferences [26].

The schedule of the paper is the following. Section 2 introduces the two-valued preferential logic with typicality following [24]. Section 3 extends such a logic with the LTL modalities and with the deontic modalities to develop a temporal deontic conditional logic, and it proves the decidability of the logic. Section 4 focuses on ranked KBs and discusses a closure construction for the temporal case. Section 5 concludes the paper.

2. A Multi-Preferential Logic with Typicality

In this section, we recall the definition of a two-valued preferential logic with typicality from [24] and slightly generalize it. The preferential semantics of the logic generalizes Kraus Lehmann and Magidor's (KLM) preferential semantics [4, 6], allowing for multiple preference relations (i.e., preferences with respect to multiple aspects), rather than a single one.

We consider a propositional language L, whose formulae are built from a set $Prop$ of propositional variables using the boolean connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$ of propositional logic. We assume that $\bot$ (representing falsity) and $\top$ (representing truth) are formulae of L.

A typicality operator is introduced following the approach used in the description logic $\mathcal{ALC} + \mathbf{T}$ [27] as well as in the Propositional Typicality Logic (PTL), by Booth et al. [18]. We let $L^{\mathbf{T}}$ be the language with typicality. Intuitively, "a sentence of the form $\mathbf{T}(\alpha)$ is understood to refer to the *typical situations in which* α *holds*" [18]. As in PTL [18], the typicality operator cannot be nested. In an implication $\alpha \rightarrow \beta$,

α and β may contain occurrences of the typicality operator. When $\mathbf{T}$ does not occur in α nor β, the implication $\alpha \rightarrow \beta$ is called *strict*. When an implication has the form $\mathbf{T}(\alpha) \rightarrow \beta$, it is called a *defeasible implication*, whose meaning is that "normally, if α then β", and corresponds to KLM conditional $\alpha \mid\sim \beta$.

The KLM preferential semantics [4, 6, 3] exploits *a set of worlds* $\mathcal{W}$, with their valuation and a preference relation $<$ among worlds (where $w < w'$ means that world w is more normal than world w'). A conditional $A \mid\sim B$ is satisfied in a KLM preferential interpretation, if B holds in all the most normal worlds satisfying A, i.e., in all $<$-minimal worlds satisfying A.

Here, instead, we consider a *multi-preferential semantics*, where preference relations are associated with distinguished propositional formulas $A_1, \ldots, A_m$ (called *distinguished propositions* in the following). The idea is that how much a situation (a world) is normal (or less atypical) with respect to another one, depends on the aspects considered for comparison. The semantics introduced below exploits a set of preference relations $<_{A_i}$, each associated to a distinguished proposition A_i, where $w <_{A_i} w'$ means that world w is less atypical than world w' concerning aspect A_i. As mentioned above, for two worlds w and w' in $\mathcal{W}$, it may be the case that $w <_{student} w'$, but $w' <_{employee} w$.

In the following, we limit our consideration to *finite* KBs, and restrict our attention to a finite set of *distinguished propositions* $A_1, \ldots, A_m$. Preferential interpretations are equipped with a finite set of preference relations $<_{A_1}, \ldots, <_{A_m}$, one for each distinguished proposition A_i. For each A_i, we let $<_{A_i} \subseteq W \times W$ be a *strict partial order* on the set of worlds $\mathcal{W}$. So far we assume that, in any typicality formula $\mathbf{T}(A)$, A is a distinguished proposition, but we will lift this restriction in Section 4.2.

Definition 1. *A* (multi-)preferential interpretation *is a triple* $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ *where:*

- *$\mathcal{W}$ is a non-empty set of worlds;*
- *for each A_i, $<_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ is an irreflexive and transitive relation on $\mathcal{W}$;*
- *$v : \mathcal{W} \longrightarrow 2^{Prop}$ is a valuation function, assigning to each world $w \in \mathcal{W}$ a set of propositional variables in $Prop$ (the variables which are true in w).*

A *ranked interpretation* is a (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v \rangle$ for which all preference relations $<_{A_i}$ are *modular*, that is: for all x, y, z, if $x <_{A_i} y$ then $x <_{A_i} z$ or $z <_{A_i} y$. A relation $<_{A_i}$ is *well-founded* if it does not allow for infinitely descending chains of worlds $w_0, w_1, w_2, \ldots$ with $w_{i+1} <_{A_i} w_i$.

The valuation v is inductively extended to all formulae of $L^{\mathbf{T}}$:

$$\mathcal{M}, w \models \top \qquad \mathcal{M}, w \not\models \bot$$

$$\mathcal{M}, w \models p \text{ iff } p \in v(w), \quad \text{for all } p \in Prop$$

$$\mathcal{M}, w \models A \wedge B \text{ iff } \mathcal{M}, w \models A \text{ and } \mathcal{M}, w \models B$$

$$\mathcal{M}, w \models A \vee B \text{ iff } \mathcal{M}, w \models A \text{ or } \mathcal{M}, w \models B$$

$$\mathcal{M}, w \models \neg A \text{ iff } \mathcal{M}, w \not\models A$$

$$\mathcal{M}, w \models A \rightarrow B \text{ iff } \mathcal{M}, w \models A \text{ implies } \mathcal{M}, w \models B$$

$$\mathcal{M}, w \models \mathbf{T}(A_i) \text{ iff } \mathcal{M}, w \models A_i \text{ and } \nexists w' \in \mathcal{W} \text{ s.t. } w' <_{A_i} w \text{ and } \mathcal{M}, w' \models A_i.$$

Whether $\mathbf{T}(A_i)$ is satisfied at a world w also depends on the other worlds of the interpretation $\mathcal{M}$.

Let $[[A]]^{\mathcal{M}}$ be the set of all the worlds in $\mathcal{M}$ satisfying a formula A (i.e., $[[A]]^{\mathcal{M}} = \{w \in \mathcal{W} : \mathcal{M}, w \models A\}$) and let $Min_{<_A}(\mathcal{S})$ be the set of $<_A$-minimal worlds in $\mathcal{S}$, for any set of worlds $\mathcal{S} \subseteq \mathcal{W}$, and strict partial order $<_A$, that is: $Min_{<_A}(\mathcal{S}) = \{w \in \mathcal{S} \mid \text{ there is no } w' \in \mathcal{S}, \text{ such that } w' <_A w\}$. For a well-founded preference relation $<_A$, one can reformulate the semantic condition for the typicality operator as follows:

$$\mathcal{M}, w \models \mathbf{T}(A_i) \text{ iff } w \in Min_{<_{A_i}}([[A_i]]^{\mathcal{M}}).$$

In this work, we do not assume that all the preference relations $<_{A_i}$ are well-founded, so that the semantics above is more general than the one in [24], and than the usual KLM semantics [4, 6].

A formula A *is satisfiable* in the multi-preferential semantics if there exist a multi-preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v\rangle$ and a world $w \in \mathcal{W}$ such that $\mathcal{M}, w \models A$. A formula A *is valid in an interpretation* $\mathcal{M}$ (written $\mathcal{M} \models A$) if, for all worlds $w \in \mathcal{W}$, $\mathcal{M}, w \models A$. A formula A is *valid* in the multi-preferential semantics (simply, A *is valid*) if A is valid in any multi-preferential interpretation $\mathcal{M}$. Restricting our consideration to modular interpretations leads to the notions of satisfiability and validity of a formula in the *ranked (or rational) multi-preferential semantics*.

When an implication has the form $\mathbf{T}(A) \rightarrow B$, with B in L, it corresponds to a conditional $A \mid\sim B$ in KLM logics [4]. Note that, for well-founded preference relations, a defeasible implication $\mathbf{T}(A) \rightarrow B$ is valid in a preferential interpretation $\mathcal{M}$ (i.e., $\mathcal{M} \models \mathbf{T}(A) \rightarrow B$) *iff* for all worlds $w \in \mathcal{W}$, $w \in Min_{<_A}([[A]]^{\mathcal{M}})$ implies $w \in [[B]]^{\mathcal{M}}$, *iff* $Min_{<_A}([[A]]^{\mathcal{M}}) \subseteq [[B]]^{\mathcal{M}}$ holds. When all the preference relations $<_{A_i}$ coincide with a single well-founded preference relation $<$, a multi-preferential interpretation $\mathcal{M}$ corresponds to a KLM preferential interpretation [4], and a defeasible implication $\mathbf{T}(A) \rightarrow B$ (with B in L) has the same semantics as KLM conditional $A \mid\sim B$. The multi-preferential semantics is, therefore, a generalization of the KLM preferential semantics.

Let a *knowledge base* K be a set of (strict or defeasible) implications. A *preferential model of* K is a multi-preferential interpretation $\mathcal{M}$ such that $\mathcal{M} \models A \rightarrow B$, for all implications $A \rightarrow B$ in K. Given a knowledge base K, we say that an implication $A \rightarrow B$ *is preferentially entailed from* K if $\mathcal{M} \models A \rightarrow B$ holds, for all preferential models $\mathcal{M}$ of K. We say that $A \rightarrow B$ *is rationally entailed from* K if $\mathcal{M} \models A \rightarrow B$ holds, for all ranked models $\mathcal{M}$ of K.

It is well known that preferential entailment and rational entailment are weak. As with the rational closure [6] and the lexicographic closure [25] for KML conditionals, also in the multi-preferential case one can strengthen entailment by restricting to specific preferential models, based on some *closure constructions*, which allow to define the preference relations $<_{A_i}$ from a knowledge base K, e.g., by exploiting the ranks and weights of conditional implications, when available. Some examples of closure constructions for the multi-preferential case have been considered, e.g., for multi-preferential variants of the rational closure [20] and of the lexicographic closure [21], and for (multi-preferential) ranked or weighted defeasible DLs with typicality [22, 23]. We will come back to consider a construction for reasoning from ranked temporal deontic KBs later, in Section 4.

3. A Temporal Deontic Preferential Logic with Typicality

In this section we further extend the language $L^{\mathbf{T}}$ with the *temporal operators* X (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of Linear Time Temporal Logic (LTL) [7].

We also introduce in the language a *deontic operator* $\mathbf{O}$, which is read “it is obligatory that", and the possibility operator $\mathbf{P}$ (“it is permitted that") with a semantics as in Standard Deontic Logic [8]. We allow temporal operators, deontic operators and the typicality operator to occur in the formulas, with the only restriction that $\mathbf{T}$ should not be nested.

The syntax for *temporal deontic conditional formulas*, shortly, $LTL^{\mathbf{T}}_D$ formulas, is the following:

$$A ::= p \mid A \wedge B \mid A \vee B \mid \neg A \mid XA \mid A\,\mathcal{U}\,B \mid \Diamond A \mid \Box A \mid \mathbf{O}(A) \mid \mathbf{P}(A) \mid \mathbf{T}(A_i)$$

where A and B are well-formed formulas and A_i is a distinguished formula.

Combining the modalities allows formulating conditions on the evolution of a system, taking into consideration the obligations of agents, their fulfillment or violation. For instance, normally, when receiving an invoice, one has the obligation to pay within a deadline (dl_1)

$$\mathbf{T}(received\_invoice) \rightarrow \mathbf{O}(\neg dl_1\,\mathcal{U}\,pay)$$

and the violation to pay within deadline dl_1 generates an obligation to pay with a fine within a new deadline dl_2:

$$\mathbf{O}(\neg dl_1 \,\mathcal{U}\, pay)) \wedge dl_1 \wedge \neg pay \rightarrow \mathbf{O}(\neg dl_2 \,\mathcal{U}\, pay\_fine)$$

The implication above is strict, but it might as well be formulated as a defeasible implication. Other examples of $LTL^{\mathbf{T}}_{D}$ formulae are:

$$\Box(\mathbf{T}(professor) \rightarrow teaches \,\mathcal{U}\, retired)$$
$$lives\_in\_town \wedge young \rightarrow \mathbf{T}(\Diamond granted\_loan)$$

3.1. Semantics of the temporal deontic logic with typicality

Compared with the preferential semantics in the previous section, the semantics of a temporal deontic logic with typicality has also to consider the temporal dimension, through a set of time points in $\mathbb{N}$. The valuation function assigns, at each time point $n \in \mathbb{N}$, a truth value to each propositional variable in a world $w \in \mathcal{W}$; the preference relations $<^n_{A_i}$ (with respect to each A_i) are relative to time points. Evolution in time may change the valuation of propositions at the worlds, and it may also change the preference relations between worlds (w might represent a typical situation for a student at time point 0, but not at time point 50). At each time point n an accessibility relation R^n is used for evaluating obligations at time point n.

Definition 2. *A* temporal deontic (multi-)preferential interpretation *(or $LTL^{\mathbf{T}}_{D}$ interpretation) is a triple $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, \{R^n\}_{n\in\mathbb{N}}, v\rangle$ where:*

- *$\mathcal{W}$ is a non-empty set of worlds;*
- *for each A_i and $n \in \mathbb{N}$, $<^n_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ is an irreflexive and transitive relation on $\mathcal{W}$;*
- *$v : \mathbb{N} \times \mathcal{W} \longrightarrow 2^{Prop}$ is a valuation function assigning, at each time point n, a set of propositional variables in $Prop$ to each world $w \in \mathcal{W}$;*
- *for $n \in \mathbb{N}$, $R^n \subseteq \mathcal{W} \times \mathcal{W}$ is a serial accessibility relation.*

For $w \in \mathcal{W}$ and $n \in \mathbb{N}$, $v(n, w)$ is the set of the propositional variables which are true in world w at time point n. When there is no $w' \in \mathcal{W}$ s.t. $w' <^n_A w$, we say that w is a normal situation for A at time point n. R^n is the accessibility relation of the deontic modality $\mathbf{O}$ at time point n. We have assumed that, for all time points $n \in \mathbb{N}$, R^n is *serial*, that is: for all $w \in \mathcal{W}$, there is a $w' \in \mathcal{W}$ such that $(w, w') \in R^n$. This is the usual assumption in SDL. $\mathbf{O}(A)$ is true in a world w at time point n if A is true in all the worlds which are ideal with respect to w at time point n (i.e., in all the worlds w' such that $(w, w') \in R^n$).

Given an $LTL^{\mathbf{T}}_{D}$ interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, \{R^n\}_{n\in\mathbb{N}}, v\rangle$, we define inductively the *truth of a formula A in a world w at time point n* (written $\mathcal{I}, n, w \models A$), as follows:

$\mathcal{I}, n, w \models \top \qquad\qquad \mathcal{I}, n, w \not\models \bot$

$\mathcal{I}, n, w \models p$ iff $p \in v(n, w)$, for all $p \in Prop$

$\mathcal{I}, n, w \models A \wedge B$ iff $\mathcal{I}, n, w \models A$ and $\mathcal{I}, n, w \models B$

$\mathcal{I}, n, w \models A \vee B$ iff $\mathcal{I}, n, w \models A$ or $\mathcal{I}, n, w \models B$

$\mathcal{I}, n, w \models \neg A$ iff $\mathcal{I}, n, w \not\models A$

$\mathcal{I}, n, w \models A \rightarrow B$ iff $\mathcal{I}, n, w \models A$ implies $\mathcal{I}, n, w \models B$

$\mathcal{I}, n, w \models XA$ iff $\mathcal{I}, n+1, w \models A$

$\mathcal{I}, n, w \models \Diamond A$ iff there is an $m \geq n$ such that $\mathcal{I}, m, w \models A$

$\mathcal{I}, n, w \models \Box A$ iff for all $m \geq n$, $\mathcal{I}, m, w \models A$

$\mathcal{I}, n, w \models A\mathcal{U}B$ iff there is an $m \geq n$ such that $\mathcal{I}, m, w \models B$ and, for all k such that $n \leq k < m$, $\mathcal{I}, k, w \models A$

$\mathcal{I}, n, w \models \mathbf{O}(A)$ iff for all $w' \in \mathcal{W}$, such that $(w, w') \in R^n$, $\mathcal{I}, n, w' \models A$

$\mathcal{I}, n, w \models \mathbf{T}(A_i)$ iff $\mathcal{I}, n, w \models A_i$ and $\nexists w' \in \mathcal{W}$ s.t. $w' <^n_{A_i} w$ and $\mathcal{I}, n, w' \models A_i$.

Note that a temporal interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, \{R^n\}_{n \in \mathbb{N}}, v \rangle$ can be regarded as a sequence of (non-temporal) deontic preferential interpretations $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2, \ldots$ where each $\mathcal{M}_n$ is defined as follows: $\mathcal{M}_n = \langle \mathcal{W}, \{<^n_{A_i}\}, R^n, v^n \rangle$, where $w <^n_{A_i} w'$ holds in $\mathcal{M}_n$ iff $w <^n_{A_i} w'$ holds in $\mathcal{I}$, for all $w, w' \in \mathcal{W}$; $v^n(w) = v(n, w)$, for all $w \in \mathcal{W}$, and R^n in $\mathcal{M}^n$ is the accessibility relation at time point n in $\mathcal{I}$.

A *temporal deontic conditional KB* is a set of $LTL^{\mathbf{T}}_D$ formulas. We evaluate the satisfiability of a temporal formula in a temporal preferential interpretation $\mathcal{I}$, by verifying its truth at the initial time point 0 of the interpretation $\mathcal{I}$.

Definition 3 (Satisfiability and entailment). *A $LTL^{\mathbf{T}}_D$ formula α is* satisfied *in a temporal preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ if $\mathcal{I}, 0, w \models \alpha$, for some world $w \in \mathcal{W}$.*

A $LTL^{\mathbf{T}}_D$ formula α is valid *in the temporal preferential interpretation $\mathcal{I}$ (written $\mathcal{I} \models \alpha$) if $\mathcal{I}, 0, w \models \alpha$, for all worlds $w \in \mathcal{W}$.*

A preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ is a model *of a temporal deontic conditional knowledge base K, if $\mathcal{I} \models \alpha$ holds, for all the formulas α in K.*

A temporal deontic conditional knowledge base K entails *a formula α if $\mathcal{I} \models \alpha$ for all the models $\mathcal{I}$ of K.*

3.2. Decidability and complexity

The temporal deontic logic with typicality introduced in this section can be proven to be decidable. when the preference relations $<_{A_i}$ are well-founded. The problem of deciding the satisfiability of a $LTL^{\mathbf{T}}_D$ formula α can be polynomially reduced to deciding the satisfiability of a concept C_α in the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ introduced in [16], which extends the temporal description logic $LTL_{\mathcal{ALC}}$ [17] with the typicality operator. $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ has been shown to be decidable when a finite set of well-founded preference relations $<_{A_1}, \ldots, <_{A_m}$ is considered. This approach allows borrowing the decidability and complexity results from $LTL^{\mathbf{T}}_{\mathcal{ALC}}$.

Concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ can be polynomially reduced to concept satisfiability in $LTL_{\mathcal{ALC}}$, when a finite set of well-founded preference relations $<_{A_1}, \ldots, <_{A_m}$ is considered, and concept inclusions are regarded as global temporal constraints [16]. It has been proven that the decidability of concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ relies on the result that concept satisfiability in $LTL_{\mathcal{ALC}}$ w.r.t. TBoxes is in ExpTime (and, actually, it is ExpTime-complete), both with expanding domains [28] and with constant domains [17].

To provide a sketch of the decidability result for $LTL^{\mathbf{T}}_D$, let us introduce in the language of the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ a concept name P_j, for each proposition $p_j \in Prop$, and a role R_d, associated to the deontic operator $\mathbf{O}$. Given an $LTL^{\mathbf{T}}_D$ formula α, we define the concept C_α associated to α in the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, as follows (by induction on the structure of the formula):

$C_{p_j} = P_j$, if $p_j \in Prop$	$C_{\mathbf{T}(A_i)} = \mathbf{T}(C_{A_i})$
$C_{\neg A} = \neg C_A$.	$C_{XA} = X\ C_A$
$C_{A \wedge B} = C_A \sqcap C_B$	$C_{\Box A} = \Box\ C_A$
$C_{A \vee B} = C_A \sqcup C_B$	$C_{\Diamond A} = \Diamond\ C_A$
$C_{\mathbf{O}(A)} = \forall R_d.C_A$	$C_{\mathbf{P}(A)} = \exists R_d.C_A$

In order to enforce seriality for the deontic modality $\mathbf{O}$, we let the concept inclusion $\top \sqsubseteq \exists R_d.\top$ belong to the TBox T. As assumed before, such an inclusion is global and holds for all individuals at any time point. Note that the typicality operator can be used in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, so that any formula $\mathbf{T}(A)$ can be mapped to a concept $\mathbf{T}(C_A)$. The encoding above of a formula α into a concept C_α is clearly polynomial in the size of the formula α, and the TBox T only contains one axiom.

It can be proven that a $LTL^{\mathbf{T}}_D$ formula A is satisfiable if and only if the concept C_A is satisfiable in the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBox T. We omit the proof and refer to [16] for the semantics of the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ and for the proof that concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBoxes is ExpTime-complete. The following proposition provides an upper-bound on the complexity of satisfiability in $LTL^{\mathbf{T}}_{D}$.

Proposition 1. *The satisfiability of an $LTL^{\mathbf{T}}_{D}$ formula is in ExpTime, both with expanding domains and with constant domains.*

4. Ranked Conditional Knowledge Bases

As for KLM logics, the notion of preferential entailment considered in this section is rather weak. For KLM logics some different closure constructions have been proposed to strengthen entailment by restricting to a subset of the preferential models of a conditional knowledge base K. Let us just mention the rational closure [6], the lexicographic closure [25], and the MP-closure [20]. In the following we will consider a construction, based on *conditionals with ranks*, which can be used to associate a preference relation $<_{A_i}$ to the distinguished formulas $A_1, \ldots, A_m$.

In particular, here we will focus on *ranked knowledge bases*, that were first explored by Brewka in his framework for qualitative preferences [29, 26], where basic preference relations $\geq_K$ are associated with different ranked knowledge bases K, and new preference relations are defined by combining the basic preference relations. More precisely, we allow for *ranked temporal conditional KBs*, in which conditional implications have a *rank*, a natural number. Each distinguished formula A_i is associated with a set of conditionals, which are used for defining the *preorders* $\leq_{A_i}$, as well as the strict preference relations $<_{A_i}$ (the associated strict partial orders). An approach using ranks was adopted in [22] in a conditional extension of a lightweight description logic, and in [21] in a framework for modular, multi-concept preferential semantics based on the lexicographic closure (where ranks are determined by the rational closure construction). Here we extend the construction to the temporal case, and we exploit user defined ranks for the conditionals, based on the strategy $\#$ from Brewka's framework for qualitative preferences [26], extended to the temporal case.

4.1. Inducing preferences

Let us introduce the ranked conditional KBs for the temporal case, through an example. Let $K_{student}$ be a set of conditional implications, with their ranks, for the distinguished formula $student$. They describe the typical properties of students. The higher is the rank, the higher is the priority of the conditional formula:

d_1: $\mathbf{T}(student) \rightarrow \mathbf{O}(have\_Classes)$, 0

d_2: $\mathbf{T}(student) \rightarrow \Diamond get\_Degree$, 0

d_3: $\mathbf{T}(student) \rightarrow \neg has\_Boss$, 0

d_4: $\mathbf{T}(student \wedge employee) \rightarrow has\_Boss$, 1

Normally, a student has the obligation to have classes, she will eventually get a degree, and she does not have a boss; while a student who is also an employee has normally a boss. The rules above are intended to define a preference relation $<_{student}$. In particular, worlds violating conditionals with lower ranks are preferred with respect to worlds violating conditionals with higher ranks. We use the same relation, concerning defaults, as in Lehmann's lexicographic closure [25], but here we assume that the ranks of the conditionals are given. For instance, a world w describing a student having classes and not having a boss, and that will eventually get a degree, is more typical than a world w' describing a student and employee having classes and a boss, and that will eventually get the degree. In particular, w' violates the conditional d_3, while w does not violate any conditional (the formal definition is given below).

In a ranked conditional knowledge base, the sets of ranked conditionals $K_{A_1}, \ldots, K_{A_m}$ associated with the distinguished formulas $A_1, \ldots, A_m$, coexist with a *strict part* of the knowledge base K_S, i.e., a set of formulas which do not contain the typicality operator. For instance, by the strict implication

$$student \rightarrow \mathbf{O}(paid\_taxes\ \mathcal{U}\ (get\_degree \vee withdraw))$$

in K_S, all students have the obligation to pay the enrollment taxes until they either get the degree or withdraw, with no exceptions. A ranked conditional knowledge base is represented by a tuple $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$, with K_S the strict part, and $K_{A_1}, \ldots, K_{A_m}$ (the defeasible part of the KB) are sets of ranked conditionals. Each K_{A_i} is a set of pairs $(\mathbf{T}(A) \rightarrow B,\ l)$ associating defeasible implications with a rank. Note that A is not required to coincide with A_i. As in the example above we allow any formula A in a ranked conditional implication $(\mathbf{T}(A) \rightarrow B,\ l)$.

Given a ranked conditional knowledge base $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$, and a preferential temporal interpretation $\mathcal{I}$, one can define a preorder $\leq^n_{A_i}$ for each distinguished proposition A_i at time point n, by considering the rank l of each defeasible implication $(\mathbf{T}(A_i) \rightarrow B,\ l)$ in K_{A_i}.

We let $\mathcal{T}^l_{A_i}(w, l)$ be the set of typicality inclusions in K_{A_i} with rank l, being satisfied by world w at time point n in a temporal interpretation $\mathcal{I}$, that is:

$$\mathcal{T}^l_{A_i}(w, n) = \{\mathbf{T}(A) \rightarrow B \mid (\mathbf{T}(A) \rightarrow B,\ l) \in K_{A_i} \text{ and } \mathcal{I}, n, w \not\models A \text{ or } \mathcal{I}, n, w \models B\}.$$

We define the preference relations $\leq^n_{A_i}$ as follows:

Definition 4. *Given a ranked conditional knowledge base $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$, and an $LTL^{\mathbf{T}}_D$ interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, \{R^n\}_{n \in \mathbb{N}}, v \rangle$, for all worlds $w_1, w_2 \in \mathcal{W}$, we let*

$$w_1 \leq^n_{A_i} w_2 \quad \textit{iff} \quad \textit{either } |\mathcal{T}^l_{A_i}(w_1, n)| = |\mathcal{T}^l_{A_i}(w_2, n)|, \textit{for all } l,$$
$$\textit{or } \exists l \textit{ such that } |\mathcal{T}^l_{A_i}(w_1, n)| > |\mathcal{T}^l_{A_i}(w_2, n)| \textit{ and}, \forall h > l, |\mathcal{T}^h_{A_i}(w_1, n)| = |\mathcal{T}^h_{A_i}(w_2, n)|.$$

Informally, the preference relation $\leq^n_{A_i}$ gives higher preference to worlds violating a smaller number of conditional implications with higher rank for A_i at time point n. It corresponds to the strategy $\#$ in Brewka's framework for qualitative preferences [26], transferred to the temporal case. The preorder $\leq^n_{A_i}$ is total. The strict preference relation $<^n_{A_i}$ is defined as usual from the preorder relation as: $w <^n_{A_i} w'$ iff $w \leq^n_{A_i} w'$ and $w' \not\leq^n_{A_i} w$.

Definition 5. *An $LTL^{\mathbf{T}}_D$ interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, \{R^n\}_{n \in \mathbb{N}}, v \rangle$, is a* model *of the ranked knowledge base $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$, if $\mathcal{I}$ is a model of K_S according to Definition 3 and, for each distinguished proposition A_i, the preference relation $\leq^n_{A_i}$ is defined according to Definition 4.*

A formula α is entailed from a ranked $LTL^{\mathbf{T}}_D$ knowledge base *$K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$ if $\mathcal{I} \models \alpha$ for all the models $\mathcal{I}$ of the ranked knowledge base K.*

Let us continue with our example. Whether an obligation is fulfilled or not at a world, depends on the trajectory starting at that world. One can represent a situation in which an obligation, normally, has to be fulfilled, but it may have exceptions. For instance, the obligation $\mathbf{O}(paid\_taxes\ \mathcal{U}\ (got\_degree \vee withdrawn))$ should by normally fulfilled but, when it is not, there is a new obligation to pay with a fine within (e.g.) the next May (a contrary to duty obligation). Also, the obligation is normally canceled if there is a tax amnesty. This policy can be encoded by a set of defeasible implications (for sake of conciseness, we will use $\mathbf{O}(o_1)$ as a short name for the formula $\mathbf{O}(paid\_taxes\ \mathcal{U}\ (got\_degree \vee withdrawn))$):

d_4: $\mathbf{T}(\mathbf{O}(o_1)) \rightarrow paid\_taxes\ \mathcal{U}\ (got\_degree \vee withdrawn),\ 0$
d_5: $\mathbf{T}(\mathbf{O}(o_1) \wedge paid\_taxes \wedge \neg got\_degree \wedge \neg withdraw) \rightarrow X\mathbf{O}(o_1),\ 0$
d_6: $\mathbf{T}(\mathbf{O}(o_1) \wedge \neg paid\_taxes \wedge \neg got\_degree \wedge \neg withdrawn)$
$\rightarrow X\mathbf{O}(\neg may\ \mathcal{U}\ pay\_fine) \wedge X\neg\mathbf{O}(o_1),\ 1$
d_7: $\mathbf{T}(\mathbf{O}(o_1) \wedge tax\_amnesty) \rightarrow X\neg\mathbf{O}(o_1),\ 2$

By conditional d_5, the obligation $\mathbf{O}(o_1)$ normally persists to the next state if it has not been violated ($paid\_taxes$ holds) and it is not already fulfilled (by reaching a degree or withdrawing). Conditional d_6, with rank 1, states that, in a typical situation in which obligation $\mathbf{O}(o_1)$ is violated (i.e., $\mathbf{O}(o_1)$ holds, but taxes have not been paid, and the student neither has received a degree nor has withdrawn), in the next state a new obligation to pay with fine within May is added, and $\mathbf{O}(o_1)$ is canceled. By d_7 the obligation $\mathbf{O}(o_1)$ is normally canceled if there is a tax amnesty (conditional with rank 2).

In this example, we may assume that the conditionals concerning the payment of taxes for students belong as well to the set of ranked conditionals $K_{student}$. Based on the semantics above, this set of ranked conditionals induces a preference relation $<_{student}$, which enable us to prove, for instance the property that normally students will pay taxes until they get a degree or withdraw:

$$\mathbf{T}(Student) \rightarrow paid\_taxes\ \mathcal{U}\ (got\_degree \vee withdrawn)$$

(i.e., that the obligation for students to pay taxes is normally fulfilled).

4.2. Combining preferences

In the general case, we may want to verify conditional properties of the form $\mathbf{T}(A) \rightarrow B$, where A is not a distinguished proposition. For instance, we may want to check whether the conditional

$$\mathbf{T}(Student \wedge Employee) \rightarrow paid\_taxes\ \mathcal{U}\ (got\_degree \vee withdrawn)$$

is entailed from a ranked KB also in case $Student \wedge Employee$ is not a distinguished proposition. When general typicality formulas of the form $\mathbf{T}(A)$ are admitted (provided A does not contain the typicality operator), we also needs to generalize the semantic condition for evaluating the typicality operator $\mathbf{T}(A)$.

The semantic condition in Definition 2 can be extended to all typicality formulas, as follows:

$$\mathcal{I}, n, w \models \mathbf{T}(A) \ \text{ iff } \ \mathcal{I}, n, w \models A \text{ and } \nexists w' \in \mathcal{W} \ \text{ s.t. } w' <^n_A w \text{ and } \mathcal{I}, n, w' \models A.$$

This requires to provide a definition of the preference relation $<^n_A$ also for the cases when A is not a distinguished formula. New preference relations can be obtained by combining the preference relations $<^n_{A_1}, \ldots, <^n_{A_n}$, based on Brewka's framework for preference combination [26].

In Brewka's framework, qualitative preferences [29, 26] are defined, starting from basic preference descriptions to define preorders on models (propositional interpretations). More precisely, a logical preference description language, LPD, is introduced, to combine basic preference descriptions d_1 and d_2 into complex ones. If d_1 and d_2 are preference descriptions in LPD, also $d_1 \wedge d_2$, $d_1 \vee d_2$, $\neg d_1$ and $d_1 > d_2$ are preference descriptions in LPD, where: $d_1 \wedge d_2$ is defined as the set theoretic intersection of relations d_1 and d_2; $d_1 \vee d_2$ is defined as the transitive closure of the set theoretic union of d_1 and d_2; $\neg d_1$ is defined as the inverse of relation d_1, and $d_1 > d_2$ is intended to express preferences among expressions (in particular, preference d_1 has higher priority with respect to preference d_2; we refer to [26] for details).

In our context, the preference relations $\leq_{A_1}, \ldots, \leq_{A_n}$, associated with the distinguished propositions $A_1, \ldots, A_n$ play the role of basic preference descriptions to be combined, based on the framework above. Given the preferences $\leq_A$ and $\leq_B$, we let: $\leq_{A \wedge B} = \ (\leq_A \cap \leq_B)$, $\leq_{A \vee B} = \ (\leq_A \cup \leq_B)^+$, and $\leq_{\neg A} = \ (\leq_A)^{-1}$, where $(\leq_A)^{-1}$ is the inverse of preference relation $\leq_A$, $(\leq_A \cup \leq_B)^+$ is the transitive closure of the set theoretic union of $\leq_A$ and $\leq_B$, and $(\leq_A \cap \leq_B)$ is the set theoretic intersection of $\leq_A$ and $\leq_B$.

Note that, when $\leq_A$ and $\leq_B$ are *total preorders*, $\leq_{\neg A}$ and $\leq_{\neg B}$ are total preorders; and the union $\leq_A \cup \leq_B$ is transitive, and is as well a total preorder (in this case, applying the transitive closure is not needed). Note also that $\leq_{A \wedge B}$ is a preorder, but not necessarily total. When $\leq_A$ and $\leq_B$ are ranked preference relations, it holds that:

$$w \leq_{A \wedge B} w' \ \text{ iff } \ w \leq_A w' \text{ and } w' \leq_B w'$$
$$w \leq_{A \vee B} w' \ \text{ iff } \ w \leq_A w' \text{ or } w' \leq_B w'$$
$$w \leq_{\neg A} w' \ \text{ iff } \ w' \leq_A w$$

As observed by Brewka, the preference description $\neg(d_1 \vee d_2)$ is different from $\neg d_1 \wedge \neg d_2$ and, in our context, the preference relation $\leq_{\neg(A_1 \vee A_2)}$ differs from $\leq_{\neg A_1 \wedge \neg A_2}$. To avoid the problem that equivalent formulas may be associated to different preference relations, we let the relation $\leq_A$ associated to a boolean formula A be the preference relation associated to its *conjunctive normal form* (CNF). The preference relation associated to a formula in CNF can be computed from the basic ranked preferences, taking their inverse (still total preorders), then computing the preferences of the disjunctions as unions of total preorders, and finally, computing the intersections of the preferences for the disjuncts.

Once a preference relation $\leq_A$ has been associated to each formula A, the induced strict partial order $<_A$ can be used in the evaluation of the typicality formula $\mathbf{T}(A)$ while computing entailment.

This approach for dealing with ranked KBs provides an example of the constructions which can be adopted for reasoning in the temporal deontic conditional logic with ranked knowledge bases. Alternative constructions could be used for ranked knowledge bases, including, for instance, exploiting different lexicographic orders, while approaches, based on weighted knowledge bases can also be considered (as done for temporal defeasible Description Logics [16]).

5. Conclusions

The paper proposes a temporal, deontic, conditional logic with typicality, $LTL_D^{\mathbf{T}}$, based on a preferential semantics and exploiting the operators of LTL and the deontic operators of SDL. The interpretation of the typicality operator is based on a multi-preferential semantics, and exploits an extension of ranked conditional knowledge bases to the temporal deontic case.

Our starting point for defining $LTL_D^{\mathbf{T}}$ is a *multi-preferential logic with typicality*, a logic which allows for conditional reasoning based on a multi-preferential semantics. It is defined along the lines of preferential logics with typicality, such as the description logic $\mathcal{ALC} + \mathbf{T}$ [27] and the Propositional Typicality Logic (PTL) [18] and, more precisely, along the lines of *multi-preferential logics with typicality*, which have been used for conditional reasoning about multilayer networks [23] and about gradual argumentation semantics [30, 31], and have been recently exploited in a conditional extension of Answer Set Programming (Conditional ASP) [24]. The paper borrows and extends the propositional multi-preferential semantics in [24].

Future work includes studying different closure constructions, including constructions based on weighted KBs for temporal conditionals, as well as considering extensions of Answer Set Programming with temporal conditionals and providing reasoning tools.

On a different route, in the two-valued case, a preferential logics with defeasible LTL operators has been studied in [14, 32]. The decidability of different fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [13, 32]. Our approach does not consider defeasible temporal operators nor preferences over time points, but it combines standard LTL operators with the typicality operator in a temporal logic.

Related work also include the many-valued conditional logic for the verification of temporal properties of gradual argumentation graphs under a gradual argumentation semantics developed in [33, 34]. In this paper we have considered a two-valued temporal conditional logic, also including the deontic operators and we have proved a decidability result. We have as well developed a closure construction based on ranked knowledge bases, and their combination.

The Temporal Modal Defeasible Logic [10] and Temporal Defeasible Logic [35] are temporal extensions of Defeasible Logic [36], a formalism which extends logic programming (without negation) to deal with exceptions, exploiting defeasible rules, priorities between them, and defeaters. Whether priorities between conditionals can be accommodated in our preferential approach is a matter of investigation.

A Dynamic Deontic and Temporal Logic has been proposed by Dignum and Kuiper [37] to reason about obligations and deadlines. In particular, they provide a formalization of achievement obligations as obligations with an until formula as argument. This idea has been later exploited in a simple Deontic Dynamic Temporal Logic (DDLTL) [11] showing that several kinds of obligations which are relevant for business process verification can be formulated. In this paper, we have developed a conditional extension of the LTL fragment of DDLTL, which does not allow for program expressions. Extending the formalism with complex program expressions (as in DLTL) is a subject of future investigation.

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Declaration on Generative AI

The authors have not employed any Generative AI tool.

References

[1] J. Delgrande, A first-order conditional logic for prototypical properties, Artificial Intelligence 33 (1987) 105–130.
[2] D. Makinson, General theory of cumulative inference, in: Non-Monotonic Reasoning, 2nd International Workshop, Grassau, FRG, June 13-15, 1988, Proceedings, 1988, pp. 1–18.
[3] J. Pearl, Probabilistic Reasoning in Intelligent Systems Networks of Plausible Inference, Morgan Kaufmann, 1988.
[4] S. Kraus, D. Lehmann, M. Magidor, Nonmonotonic reasoning, preferential models and cumulative logics, Artificial Intelligence 44 (1990) 167–207.
[5] J. Pearl, System Z: A natural ordering of defaults with tractable applications to nonmonotonic reasoning, in: TARK'90, Pacific Grove, CA, USA, 1990, pp. 121–135.
[6] D. Lehmann, M. Magidor, What does a conditional knowledge base entail?, Artificial Intelligence 55 (1992) 1–60.
[7] E. M. Clarke, O. Grumberg, D. A. Peled, Model checking, MIT Press, 1999.
[8] G. von Wright, Deontic logic, Mind 60 (1951) 1–15.
[9] D. Nute, Topics in conditional logic, Reidel, Dordrecht (1980).
[10] G. Governatori, J. Hulstijn, R. Riveret, A. Rotolo, Characterising deadlines in temporal modal defeasible logic, in: Australian Conference on Artificial Intelligence, LNCS 4830, 2007, pp. 486–496.
[11] L. Giordano, A. Martelli, D. Theseider Dupré, Temporal deontic action logic for the verification of compliance to norms in ASP, in: International Conference on Artificial Intelligence and Law, ICAIL '13, Rome, Italy, June 10-14, 2013, ACM, 2013, pp. 53–62.
[12] J. Henriksen, P. Thiagarajan, Dynamic Linear Time Temporal Logic, Annals of Pure and Applied logic 96 (1999) 187–207.
[13] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, A one-pass tree-shaped tableau for defeasible LTL, in: TIME 2021, September 27-29, 2021, Klagenfurt, Austria, volume 206 of *LIPIcs*, 2021.
[14] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, On the decidability of a fragment of preferential LTL, in: TIME 2020, September 23-25, 2020, Bozen-Bolzano, Italy, volume 178 of *LIPIcs*, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2020.
[15] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, Defeasible linear temporal logic, J. Appl. Non Class. Logics 33 (2023) 1–51.
[16] M. Alviano, L. Giordano, D. Theseider Dupré, Preferential temporal description logics with

typicality and weighted knowledge bases, in: Proc. 38th Italian Conf. on Computational Logic (CILC 2023), volume 3428 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2023.
[17] C. Lutz, F. Wolter, M. Zakharyaschev, Temporal description logics: A survey, in: TIME, 2008, pp. 3–14.
[18] R. Booth, G. Casini, T. Meyer, I. Varzinczak, On rational entailment for propositional typicality logic, Artif. Intell. 277 (2019).
[19] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, ALC+T: a preferential extension of Description Logics, Fundamenta Informaticae 96 (2009) 1–32.
[20] L. Giordano, V. Gliozzi, A reconstruction of multipreference closure, Artif. Intell. 290 (2021).
[21] L. Giordano, D. Theseider Dupré, A framework for a modular multi-concept lexicographic closure semantics, in: Proc. 18th Int. Workshop on Non-Monotonic Reasoning, 2020.
[22] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning in a concept-aware multipreferential lightweight DL, TPLP 10(5) (2020) 751–766.
[23] M. Alviano, F. Bartoli, M. Botta, R. Esposito, L. Giordano, D. Theseider Dupré, A preferential interpretation of multilayer perceptrons in a conditional logic with typicality, Int. Journal of Approximate Reasoning 164 (2024). URL: https://doi.org/10.1016/j.ijar.2023.109065.
[24] M. Alviano, L. Giordano, D. Theseider Dupré, A framework for conditional reasoning in answer set programming, CoRR abs/2506.03997 (2025). URL: https://doi.org/10.48550/arXiv.2506.03997, accepted for publication at ICLP 2025 as Technical Communication.
[25] D. J. Lehmann, Another perspective on default reasoning, Ann. Math. Artif. Intell. 15 (1995) 61–82.
[26] G. Brewka, A rank based description language for qualitative preferences, in: 6th Europ. Conf. on Artificial Intelligence, ECAI'2004, Valencia, Spain, August 22-27, 2004, 2004, pp. 303–307.
[27] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Preferential Description Logics, in: LPAR 2007, volume 4790 of *LNAI*, Springer, Yerevan, Armenia, 2007, pp. 257–272.
[28] K. Schild, Combining terminological logics with tense logic, in: EPIA, 1993, pp. 105–120.
[29] G. Brewka, Preferred subtheories: An extended logical framework for default reasoning, in: Proceedings of the 11th IJCAI, 1989, pp. 1043–1048.
[30] M. Alviano, L. Giordano, D. Theseider Dupré, Typicality, conditionals and a probabilistic semantics for gradual argumentation, in: Proc. 21st International Workshop on Non-Monotonic Reasoning, volume 3464 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2023, pp. 4–13.
[31] M. Alviano, L. Giordano, D. Theseider Dupré, Weighted knowledge bases with typicality and defeasible reasoning in a gradual argumentation semantics, Intelligenza Artificiale 18 (2024) 153–174.
[32] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, Defeasible linear temporal logic, J. Appl. Non Class. Logics 33 (2023) 1–51.
[33] M. Alviano, L. Giordano, D. Theseider Dupré, Towards temporal many-valued conditional logics for gradual argumentation: a preliminary report, in: Proc. of the 8th Workshop on Advances in Argumentation in Artificial Intelligence 2024, volume 3871 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2024.
[34] M. Alviano, L. Giordano, D. Theseider Dupré, Temporal many-valued conditional logics: a preliminary report, CoRR abs/2409.09069 (2024). URL: https://doi.org/10.48550/arXiv.2409.09069.
[35] G. Governatori, A. Rotolo, Computing temporal defeasible logic, in: Theory, Practice, and Applications of Rules on the Web - 7th International Symposium, RuleML 2013, Proceedings, volume 8035 of *Lecture Notes in Computer Science*, Springer, 2013, pp. 114–128.
[36] G. Antoniou, D. Billington, G. Governatori, M. J. Maher, Representation results for defeasible logic, ACM Trans. Comput. Log. 2 (2001) 255–287.
[37] F. Dignum, R. Kuiper, Combining dynamic deontic logic and temporal logic for the specification of deadlines, in: HICSS (5), 1997, pp. 336–346.