LEO Satellite Orbit Study to Improve SOOP-based Positioning Precision

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Abstract

LEO satellite's position state given by TLE files has had its uncertainty analyzed based on the satellite's initial keplerian elements. In this research, the set of keplerian elements of a LEO satellite that minimizes its position state uncertainty has been calculated, which allows the improvement of a SOOP-based global positioning algorithm. In the first place, LEO satellite's dynamical evolution is studied to code a numerical propagator. Afterwards, a function Δd, whose output characterizes satellite's position state uncertainty , is implemented by making use of the numerical propagator. Finally, a genetic optimization algorithm has served to minimize Δd, and it converged to a single set of keplerian elements. The results show the existence of a Δd function minimum, which is related with the asymmetric gravitational potential of the Earth for being the strongest asymmetric force acting on the satellite.

Keywords

LEO satellite constellation, SOOP-based positioning, orbital propagation model, keplerian elements, TLE files, genetic optimization algorithm

1. Introduction

In the last few years, global positioning research has begun trying to use LEO satellite constellations instead of MEO constellations like GNSS. This change's objective is to take advantage of properties that LEO satellites can bring. These advantages come from differences in altitude between LEO and MEO satellites. LEO satellites are placed in an altitude region from 400 km to 1.500 km, while MEO satellites can be found near a 20.000 km altitude. This difference of an order of magnitude affects the satellite's signal power when it arrives at Earth's surface, for example, Iridium constellation signal at the surface has nearly 600 times more power than MEO's [1]. Lower altitude also requires higher speed to rest in orbit, which contributes to better avoid the multi-path effect and to reach a higher diversity in constellation satellites' position diversity, minimizing the Dilution Of Precision (DOP) parameter [2]. In conclusion, using LEO satellites for global positioning allows locating receivers in indoor or cities with big buildings where GNSS systems fail.

Many recent researches, about global positioning using LEO satellites, have worked with satellites' uncertainty in their position state [3, 4, 5, 6], since there is essential information to obtain the receiver's position. Unlike GNSS, LEO Signals Of OPportunity (SOOP) do not contain satellite's ephemeris, so satellite's position state becomes an unknown. This fact means that more LEO satellites are needed than MEO to do global positioning [7], and that precision in receiver's position will depend on how precise LEO satellites' position state can be calculated.

In this work, receiver's position precision has been studied taking into account uncertainty in measuring the LEO satellite's position state. The objective was to calculate a LEO satellite orbit that minimizes a function Δd, which gives qualitative values of precision in positioning, in order to make a

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selector algorithm, which would only take satellites whose orbit was similar to the calculated before, and use them in a global positioning algorithm to make it more precise.

Function Δd has been taken as the difference between the maximum and minimum distance from the receiver to the satellite, among all possible distances for all possible satellite's state positions inside an uncertainty region. To calculate the objective orbit, it will be necessary to minimize Δd and, to afford that, a genetic algorithm has been used.

2. Fundamentals

In order to obtain the uncertainty region where inside of it the position of a LEO satellite could be placed, it will be necessary to know the temporal evolution of the satellite position state. In this article, Newton's second law of dynamics has been used, since the forces acting on the satellite are known.

2.1. Force model

The force acting on the satellite can be taken as a sum of forces, where each of these represents a different interaction. However, some interactions are stronger than others, which allows us to focus on higher orders of magnitude terms. A more detailed discussion about the following contents might be found in [8, 9, 10].

LEO satellites are strongly affected by gravitational interactions due to their proximity to the Earth. A stationary model of the gravity force between the Earth and the satellite can be derived from:

$$\Phi_E(r,\phi,\lambda) = \frac{GM_\oplus}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{R_\oplus^n}{r^n}P_{nm}(\sin(\phi))(C_{nm}\cos(m\lambda)+S_{nm}\sin(m\lambda)) \tag{1}$$

which is a Legendre polynomial expansion potential expressed in geocentric coordinates, r for radial distance from the center of mass of the Earth, ϕ for latitude and λ for longitude. C_{nm} and S_{nm} are coefficients that can be obtained from experimental measurements. P_{nm} are associated Legendre polynomials of order m and degree n. G is the universal gravitational constant, while M and R are the mass and the radius respectively, and they are both referred to the Earth because of the symbol $\oplus$.

It is also important to take into account the gravitational force produced by Solar System bodies over the Earth and the satellite. The most relevant gravitational force is a consequence of the Moon and the Sun, which are taken as punctual objects and with their positions known to avoid a many-body problem. The acceleration over the satellite due to this interaction can be expressed as:

$$\ddot{\vec{r}}_{S+M} = G\left[m_{☾}\left(\frac{\vec{r}_{☾}-\vec{r}}{\|\vec{r}_{☾}-\vec{r}\|^3}-\frac{\vec{r}_{☾}}{\|\vec{r}_{☾}\|^3}\right)+m_\odot\left(\frac{\vec{r}_\odot-\vec{r}}{\|\vec{r}_\odot-\vec{r}\|^3}-\frac{\vec{r}_\odot}{\|\vec{r}_\odot\|^3}\right)\right] \tag{2}$$

where m and $\vec{r}$ are mass and position, when the symbol ☾ shows up, these magnitudes refer to the Moon, when the symbol $\odot$ shows up, they are referred to the Sun, and when there is no symbol they are referred to the satellite.

For the sake of considering temporal variations of Earth's gravitational potential, a study of Earth tides can be carried out. In its simplest form, the acceleration associated with this interaction is:

$$\ddot{\vec{r}}_t = \frac{\kappa_2 G M_c R_\oplus^5}{2r_c^3 r^4}(3-15\cos^2\theta)\frac{\vec{r}}{r}+6\cos\theta\frac{\vec{r}_c}{r_c} \tag{3}$$

where κ_2 is a coefficient related to Earth's elasticity, θ is the angle between $\vec{r}$ and $\vec{r}_c$, and c subindex means that the magnitude is referred to a certain body that produces the interaction, such as the Sun and the Moon.

LEO satellite constellations are the closest constellations to Earth's surface, that is why drag force becomes relevant. A simple aerodynamic model for the acceleration associated with atmospheric drag is:

$$\ddot{\vec{r}}_D = -\frac{1}{2} C_D \frac{A}{m_s} \rho_{at}(\vec{r}, t) v_r \vec{v}_r \tag{4}$$

where C_D is the drag coefficient, ρ_{at} is the density of the atmosphere near the satellite, and A, m_s and $\vec{v}_r$ are the cross-sectional area, the mass and the speed in the movement direction of the satellite, respectively.

Last interaction taken into account is the radiation pressure related to the energy of photons absorbed by the satellite. The acceleration due to photons emitted by the Sun can be expressed as:

$$\ddot{\vec{r}}_{rad} = \nu P_{\odot} C_r \frac{O}{m_s} (\mathrm{AU})^2 \frac{(\vec{r} - \vec{r}_{\odot})}{\|\vec{r} - \vec{r}_{\odot}\|^3} \tag{5}$$

where $\nu \in [0, 1]$ is a real value that takes values near 0 if few solar rays hit the satellite and near 1 if many solar rays hit it. C_r is a reflectivity coefficient, O is the cross-section area in the Sun's direction, and $P_{\odot} = L_{\odot}/c$ is related to solar flux ($L_{\odot}$). AU are astronomical units.

2.2. Uncertainty region

Positioning based on SOOP precision depends on how precise the satellite position state can be calculated over time, which depends on Newton's second law solver (usually known as propagator) accuracy and on initial conditions precision. In this work, focus has been set on the study of uncertainty due to initial conditions precision.

A common way to initialize a LEO satellite propagator is by making use of Two-Line Elements (TLE) files updated daily on the internet by the North American Aerospace Defense Command (NORAD). Those files contain the keplerian elements associated with a satellite at a specific epoch, apart from giving some information about atmospheric conditions. With that information, the initial position state of a LEO satellite can be obtained, but always with some uncertainty because the measurement of the keplerian elements can not be infinitely precise. These keplerian elements are: semi-major axis (a), eccentricity (e),inclination (i), right ascension of the ascending node (Ω), argument of perigee (ω) and the true anomaly (ν), which have been taken as in figure 1.

Initial uncertainty region of the position of a LEO satellite is not known. In this work, a spherical uncertainty region has been set at the initial instant corresponding to the newest TLE's epoch. Propagating some points inside this spherical region over time, the uncertainty region at any moment can be calculated, and this region will be the largest 24 hours after the initial instant. After 24 hours, TLE file will update the satellite's position and epoch, which can be set again as the initial instant.

3. Numeric orbital propagator implementation

In order to solve Δd function, whose output is the difference between the minimal and maximal distance from the receiver to the possible position state of the LEO satellite, the function has been implemented in MATLAB. The code calculates the difference between distances after obtaining a set of position states where the satellite could be 24 hours after the lecture of the TLE file. To calculate the set of states at a certain time, a numerical orbit propagator has also been implemented based on the force model explained in section 2.1.

In the first place, two functions (`r_moon` and `r_sun`) have been coded to give the position state of the Moon and Sun at a certain time, and to use that information in equations: (2), (3), and (5).

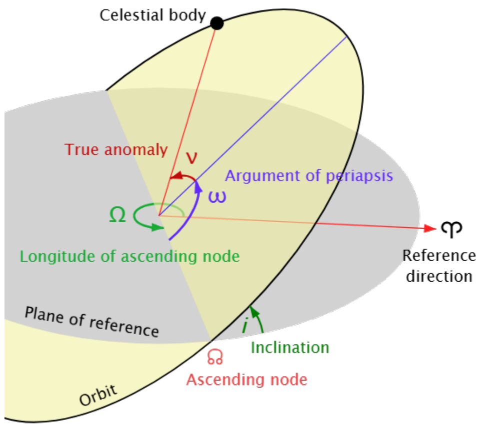

Figure 1: Keplerian elements.

Table 1
Atmosphere density for some altitudes values [9].

Height km	Density g/km^3	Height km	Density g/km^3
100	497.400	600	0.081 – 0.639
200	255 – 316	700	0.020 – 0.218
300	17 – 35	800	0.007 – 0.081
400	2.2 – 7.5	900	0.003 – 0.036
500	0.4 – 2.0	1000	0.001 – 0.018

For small displacements of the Moon and Sun in their orbit, a Chebyshev polynomial expansion can be used to approximate the position state of both bodies [8].

The terrestrial gravity term, equation (1), is calculated by using `gravitysphericalharmonic` function from the *Aerospace Toolbox* in MATLAB. This function's output is the acceleration of the satellite due to Earth's gravity in the Earth-Centered Earth-Fixed (ECEF) coordinate frame, and its inputs are the order and degree of the last Legendre polynomial to consider, and the gravity model (the EGM2008 model has been taken) that gives C_{nm} and S_{nm} coefficients. To transform ECEF acceleration to Earth-Centered Inertial (ECI) acceleration, `ecef2eci` function from the same MATLAB toolbox has been used.

To calculate accelerations in equations (3), (4) and (5), it has been considered a spherical satellite of mass $m_s = 260$ kg and cross-section area $A = O = 2.2$ m^2 both for along-track cross-section (A) and for Sun direction's cross-section (O). The fact of the satellite being spherical implies fixed values for some coefficients, these are: $\kappa_2 = 3$, $C_D = 2.2$ and $C_r = 1.3$. Moreover, low Earth orbit is inside a region of the atmosphere called the heterosphere where gases are separated in layers. Because of that, atmosphere's density near the satellite (ρ_{at}) can be taken as constant values for some altitude intervals, and these constants have been extracted from table 1.

Finally, to solve Newton's equations with the total force calculated, the Runge-Kutta method is implemented by using `ode45` function in MATLAB.

4. Results

To reach the objective of this work, and find the orbit that minimizes Δd it has been used a genetic algorithm (table 2), because it is a robust and simple method to calculate good solutions for optimization problems of non-linear functions [11]. It has been taken order and degree 4 for

Table 2
Genetic algorithm configuration.

Parameter	Value
Population's size	20
Convergence criteria	Generation's number = 200
Generational gap	0.9
Inputs value precision	[0, 4, 4, 4, 4, 4]
Interbreeding probability	0.7
Mutation probability	0.001
Interbreeding method	Random single point
Selection method	Universal stochastic sampling
Adjustement method	Linear ranking with selective pressure of 2

Table 3
Allowed value intervals for each keplerian element.

Parameter	Minimum value	Maximum value
a	6821 km	7821 km
e	0	0.01
i	0	π
Ω	0	2π
ω	0	2π
ν	0	2π

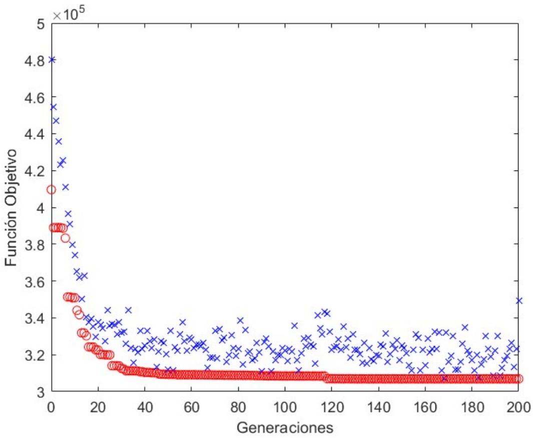

Figure 2: Genetic algorithm convergence through generations.

$$\begin{cases} \boxed{a = 1442863\ \mathrm{m}} & \boxed{e = 0{,}01} & \boxed{i = 0{,}6871\ \mathrm{rad}} \\ \boxed{\Omega = 4{,}3198\ \mathrm{rad}} & \boxed{\omega = 1{,}6690\ \mathrm{rad}} & \boxed{\nu = 3{,}1907\ \mathrm{rad}} \end{cases}$$

Figure 3: Set of keplerian elements which minimizes Δd.

`gravitysphericalharmonic` function since convergence time increases strongly with these parameters. Also, to characterize LEO satellite's allowed positions (they are confined into low Earth orbit region), some keplerian element's restrictions have been taken into account by considering these satellites safely (in terms of drag function's altitude interval defined) between a 450 and a 1.450 kilometers altitude (table 3).

With the selected configuration, genetic algorithm took 27 hours to converge to a set of keplerian elements that are Δd function's inputs and they define a LEO satellite's orbit. Convergence and orbit solution are shown in figures 2 and 3.

Objective function's uni-dimensional variations might be studied by calculating Δd function's value for each keplerian element variation. In figures from 4 to 9 it can be seen that Δd becomes minimum for only one or two keplerian element's values. The fact of this solution's existence is due

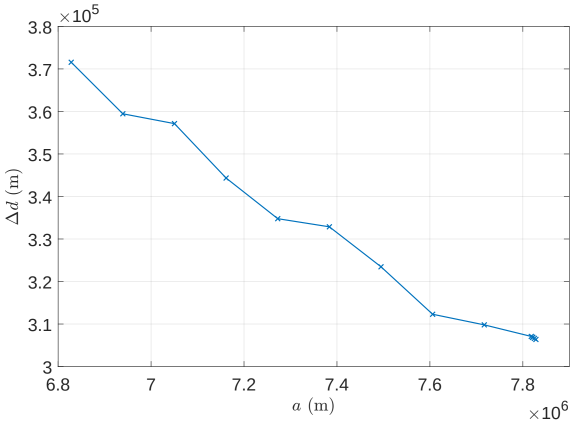

Figure 4: Δd function variation with a element.

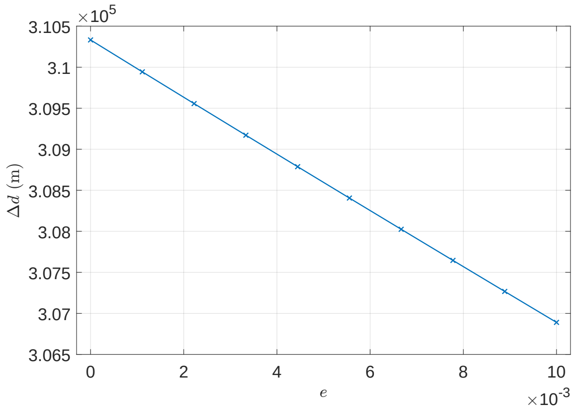

Figure 5: Δd function variation with e element.

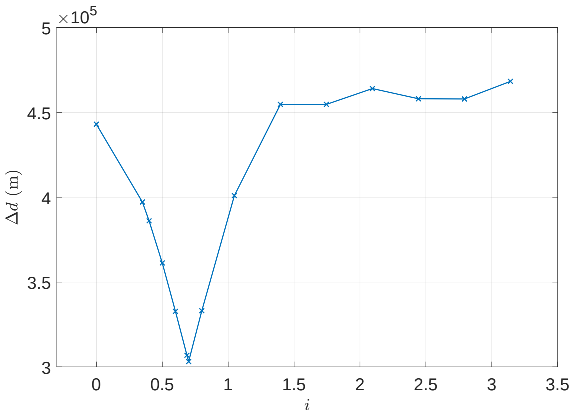

Figure 6: Δd function variation with i element.

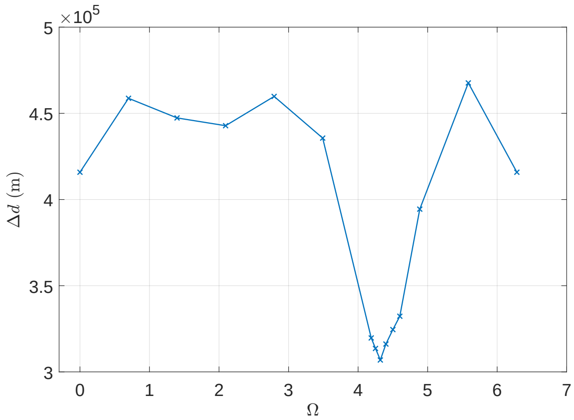

Figure 7: Δd function variation with Ω element.

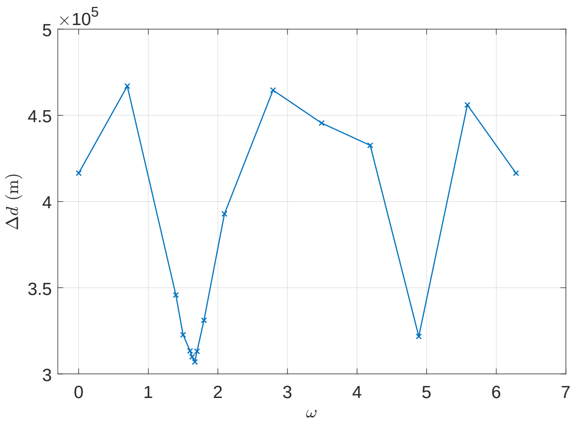

Figure 8: Δd function variation with ω element.

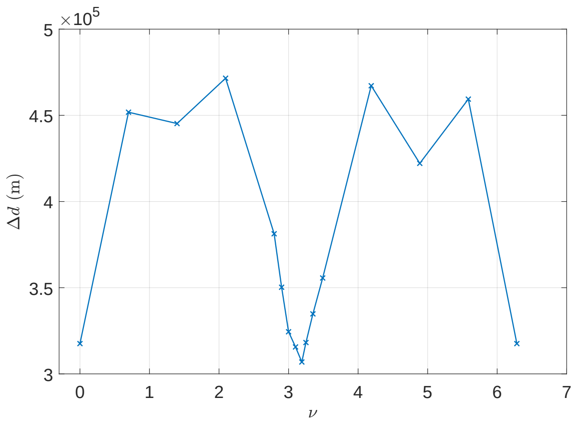

Figure 9: Δd function variation with ν element.

to the asymmetric Earth's gravitational potential, because this is the strongest interaction with the satellite that changes with its angular position.

The eccentricity behavior (figure 5) is the only one affected by the orbit's mathematical geometry. Since Δd calculates a distance in a straight line and not in a curved line, for higher values of eccentricity, the ellipse starts having sides where two points are separated by longer curved line distances than straight line distances (figure 10).

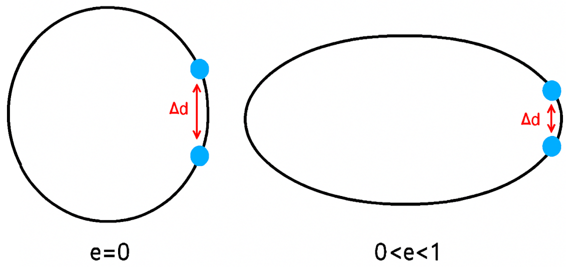

Figure 10: Δd function for different eccentricity values.

5. Conclusions

After this work, the existence of a LEO orbit that minimizes the lack of precision in the receiver's position due to initial conditions' uncertainty has been proven. Nevertheless, the precision taken for the terrestrial gravity term is not enough to study other perturbations' contributions to Δd variation.

The future objective of this research is to try to enhance terrestrial gravity precision without increasing convergence time for the genetic algorithm. To achieve that, other orbit propagators will be used, such as SGP4 model or optimized numerical propagators. Moreover, other algorithms will be executed to minimize Δd, for example, simulated annealing or tabu search.

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Declaration on Generative AI

During the preparation of this work, the authors used X-GPT-4 and Gramby in order to: Grammar and spelling check. Further, the authors used X-AI-IMG for figure 10 in order to: Generate images. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

References

[1] T. G. Reid, T. Walter, P. K. Enge, D. Lawrence, H. S. Cobb, G. Gutt, M. O'Connor, D. Whelan, Navigation from Low Earth Orbit, John Wiley Sons, Ltd, 2020, pp. 1359–1379. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/9781119458555.ch43a. doi:https://doi.org/10.1002/9781119458555.ch43a. arXiv:https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781119458555.ch43a.

[2] R. B. Langley, Dilution of precision, 1999. URL: https://api.semanticscholar.org/CorpusID:16850160.

[3] S. Hayek, J. Saroufim, Z. M. Kassas, Ephemeris error correction for tracking non-cooperative LEO satellites with pseudorange measurements, in: 2024 IEEE Aerospace Conference, 2024, pp. 1–9. doi:10.1109/AERO58975.2024.10521364.

[4] J. Saroufim, S. Hayek, S. Kozhaya, Z. M. Kassas, Improved leo pnt accuracy enabled by long baseline ephemeris corrections, in: Proceedings of the 37th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2024), 2024, pp. 1219–1229.
[5] N. Khairallah, Z. M. Kassas, Ephemeris tracking and error propagation analysis of leo satellites with application to opportunistic navigation, IEEE Transactions on Aerospace and Electronic Systems 60 (2023) 1242–1259.
[6] F. Caldas, C. Soares, Precise and efficient orbit prediction in leo with machine learning using exogenous variables, in: 2024 IEEE Congress on Evolutionary Computation (CEC), IEEE, 2024, pp. 1–8.
[7] F. Van Diggelen, A-GPS: Assisted GPS, GNSS, and SBAS, 2009.
[8] O. Montenbruck, E. Gill, Satellite Orbits, volume 1, 2000. doi:10.1007/978-3-642-58351-3.
[9] G. Seeber, Satellite Geodesy, De Gruyter, Berlin, New York, 2003. URL: https://doi.org/10.1515/9783110200089. doi:doi:10.1515/9783110200089.
[10] J. Mena, J. Berrios, Geodesia superior, Centro Nacional de Información Geográfica, 2008. URL: https://books.google.es/books?id=DfdlQwAACAAJ.
[11] D. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison Wesley series in artificial intelligence, Addison-Wesley, 1989. URL: https://books.google.es/books?id=2IIJAAAACAAJ.