Automatic Adjustment of the Dynamic Positioning System Redundant Structure by Determinant

Serhii Zinchenko[1,*,†], Oleh Tovstokoryi[1,†], Pavlo Nosov[1,†] and Ihor Popovych[2,†]

[1] Kherson State Maritime Academy, Kanatna 99, Odesa, Ukraine
[2] Kherson State University, Universytetska 27, Kherson, Ukraine

Abstract

The purpose of the study is to reduce dynamic positioning deviations of the offshore vessel with two stern azipods and a bow thruster. The following methods were used in the study: linear algebra, to finding the determinant of the actuators redundant structure minors, singular states of the structure and states maximally distant from singularities; conditional optimization, to determine the optimal states of the structure with the maximum determinant; mathematical modeling, to verify the operability and effectiveness of the developed model. A process mathematical model for adjusting the redundant structure has been developed, which allows, during dynamic positioning operations, to reduce dynamic deviations and increase accuracy. The obtained result is explained by: using the on-board controller in the control system, finding, at each step of the on-board controller, the optimal state of the structure, determined by the maximum determinant, by solving the conditional optimization problem, taking into account the constraints of the equalities type (for simultaneous formation of the necessary controls), and the inequalities type (for taking into account the physical constraints of the structure on the thrust force and the propellers rotation angles); reconfiguring the structure to a certain optimal position.

Keywords

intelligent transportation systems; redundant structures; optimal settings, dynamic positioning, structure determinant

1. Introduction

Automation of movement control processes allows to significantly increase the efficiency of control systems, due to the use of modern methods of information processing, including optimization. The components of the control systems efficiency are accuracy, maneuverability, economy, reliability, navigational safety. There are various ways to solve these issues, among which the main ones can be distinguished: power plants improvement; design solutions; hydrodynamic solutions; use of wind energy [1]; movement trajectory optimization; use of automated and automatic modules to monitor the state of the ship's navigator, increasing navigation safety [2,3], including in stormy conditions [4–6], human factor influence reduction [7,8]. New opportunities for increasing the efficiency of control systems have appeared with the use of redundant control [9].

Traditionally, redundant control structures have been used to increase reliability [10,11]. Later, they began to be used to optimize control processes in space, aviation [12] and other sectors of the national economy [13,14].

Redundant control structures are also widely used in modern ships. Redundancy of control RC means that the number of independent controls NIC exceeds the number of freedom degrees NFD to be controlled [15]. For most modern ships, the number of freedom degrees to be controlled is

ICST-2025: Information Control Systems & Technologies, September 24-26, 2025, Odesa, Ukraine

* Corresponding author.
† These authors contributed equally.
srz56@ukr.net (S. Zinchenko); otovstokory@gmail.com (O. Tovstokoryi); pason@ukr.net (P. Nosov); Ihorpopovych999@gmail.com (I. Popovych)
0000-0001-5012-5029 (S. Zinchenko); 0000-0003-3048-0028 (O. Tovstokoryi); 0000-0002-5067-9766 (P. Nosov); 0000-0002-1663-111X (I. Popovych)

$NFD = 3$ (longitudinal motion, lateral motion, and angular motion in the yaw channel). The number of independent controls for different types of ships is different. Controls $\mathbf{U}_j = (P_{xj}, P_{yj}, M_{zj})$, $j = 1..n$ are independent if they are not collinear, where P_{xj}, P_{yj}, M_{zj} are the longitudinal force, lateral force, and yaw moment of the j - th actuator.

For conventional single-screw vessels, the number of independent controls is $NIC = 2$ (power plant telegraph deviation and rudder deviation), and the control redundancy is $RC = 2 - 3 = -1$. Such structures have insufficient control, and the controllability of the vessel is ensured by the organization of dual-circuit control (lateral displacement is worked out by angular deviation, with subsequent return to the previous course).

For vessels with a bow or stern thruster, the number of independent controls is $NIC = 3$ (power plant telegraph deviation, rudder deviation, bow or stern thruster telegraph deviation), and the control redundancy is $RC = 3 - 3 = 0$.

For vessels with bow and stern thrusters, the number of independent controls is $NIC = 4$ (power plant telegraph deviation, rudder deviation, bow thruster telegraph deviation, stern thruster telegraph deviation), and the control redundancy is $RC = 4 - 3 = 1$.

For vessels with two stern azipods and a bow thruster, the number of independent controls is $NIC = 5$ (the thrust force and the rotation angle of the first azipod propeller, the thrust force and the rotation angle of the second azipod propeller, the thrust force of the bow thruster), and the control redundancy is $RC = 5 - 3 = 2$. Redundant control structures in the maritime industry are used in dynamic positioning systems on passenger ships, military ships, platform support vessels, anchor handling tags, pipelayers, cablelayers, etc., which have special requirements for maneuverability and reliability [16,17].

Fig. 1 shows exclusive photos of the anchor handling tug AHT Jascon 11 (IMO 9386847) and its stern azipods, provided by the co-author of the article, deep sea captain Oleh Tovstokoryi.

Figure 1: Anchor handling tug AHT Jascon 11 (IMO 9386847) and its stern azipods.

2. Related Works

Control processes optimization has been considered previously by many authors. In work [18] the issues of optimal control of jib cranes used on ships are considered. The model of a dynamic jib crane is linearized by expansion in a Taylor series around the operating point, which is recalculated at each control step. To calculate the gain in the feedback channel of the controller, the algebraic Riccati equation is solved. The proposed approach to optimal control provides fast and accurate tracking of the variable states of jib cranes with moderate fluctuations of the input control signals.

New opportunities to improve the efficiency of control systems have emerged with the use of redundant control. Redundant structures of actuators have traditionally been used to provide redundancy and increase the reliability of control systems by eliminating failed devices. In [19], the issues of increasing the reliability of magnetically suspended bearings in redundant designs are considered, due to the reconfiguration of the design in the event of failure of individual components. The displacement current coefficient is one of the key factors in fault-tolerant control. The authors developed a method for optimizing the displacement current coefficient in the redundant structure. Using mathematical analysis, the existence of an optimal solution is proven. Search algorithm for the optimal solution are developed.

In article [20], the issues of creating a fault-tolerant steering system for unmanned underwater vehicles are considered. The analysis conducted by the authors showed that the reliability of the control system using the strategy and algorithms of redundancy control is significantly better than the traditional configuration.

Methods for controlling the redundand structure of electro-hydraulic drives based on fuzzy aggregation, Mamdani fuzzy logic rules, and fuzzy neural network theory were investigated in [21].

In [22], a fault-tolerant controller for an adaptive high-rise structure is developed, which is able to adapt to multiple actuator failures and is an important step towards the automation of adaptive structures. The proposed control law consists of two parts: static displacement compensation, which counteracts the constant force applied by the failed actuators to the mechanical structure, and a reconfigurable linear quadratic controller, which optimally minimizes the vibrations of the structure using the remaining functioning actuators. The proposed approach is verified by simulation under wind disturbance conditions. The simulation results showed that the fault-tolerant control scheme provides more efficient system operation (up to 33% compared to the nominal controller).

In article [23], the problem of aircraft control system synthesis is related to the possibility of structural reconfiguration of control laws while preserving the dynamic properties of the closed loop "aircraft – control system" in the event of failures and damage.

Modular Aerial Robotic Systems (MARS) consist of many drone modules assembled into a single integrated flying platform. Thanks to the built-in redundancy, MARS can independently change its configuration to mitigate faults and maintain stable flight. Existing works on self-reconfiguration of MARS often do not take into account the practical controllability of intermediate structures, which limits their applicability. In [24], a dynamic model of MARS is considered taking into account the constraints of intermediate structures and a robust and efficient self-healing algorithm is proposed that maximizes the controllability margin at each intermediate stage.

Control surface redundancy allows the control system to reconfigure the control law when faults occur during flight. In [25], it is proposed to divide possible faults into two categories: predictable and unpredictable. Predictable faults are handled by an adaptive model switching scheme. Unpredictable faults are handled by a simple adaptive control scheme to force the faulty station to track a given reference model. Simulation results showed that the developed reconfiguration strategy is able to quickly detect the fault and stabilize the aircraft.

Modular self-reconfigurable spacecraft (MSRS) consist of homogeneous or heterogeneous modules that can autonomously transform their configuration without external intervention. The paper [26] reviews the key technologies of MSRS. The concepts of modular reconfigurable spacecraft (MRS) and MSRS are described and the advantages of the latter are analyzed. Typical MSRS are introduced. The key technologies of MSRS are analyzed: assembly structure design technology, mission configuration optimization technology, self-reconfiguration planning technology, and cooperative position control technology. The development trends of MSRS are presented.

Subsequently, redundant structures began to be used also for optimization of control processes. Thus, in [27], optimization of manipulability using a metaheuristic-based recurrent neural network MRNN was proposed, which can directly process a nonlinear and non-convex problem with constraints and ensure achievement of a global optimum. Existing kinematic schemes do not take into account optimization of manipulability, or require transformation of a non-convex problem into a convex one, which can affect optimal manipulability. Also, existing kinematic schemes rarely take into account obstacles. The results of computer modeling and physical experiments are presented, which demonstrate the advantages of the proposed scheme.

Optimization of functional redundancy is one of the most effective ways to improve robot performance. The trade-off between the smoothness of robot motion and other important aspects makes the optimization problem too nonlinear, which cannot be solved in milling operations. In [28], it is shown that the robot motion will be approximately smooth when the functional redundancies of the cutter are variables of the smoothing function. On this basis, a surrogate Legendre model was constructed for planning the optimal functional redundancy, which allows to significantly reduce the calculations. The results of a real experiment on milling three-dimensional S-shaped grooves by the proposed method showed that, compared with known methods, higher milling performance and computational efficiency are achieved.

3. Materials and Methods

The object of the research is the processes of automatic adjustment of the vessel's actuators redundant structure with two stern azipods and a bow thruster by determinant. The subject of the research is process mathematical model of automatic adjustment of the vessel's actuators redundant structure with two stern azipods and a bow thruster by determinant. The purpose of the study is to reduce dynamic positioning deviations of a marine vessel with two stern azipods and a bow thruster. The following methods were used in the study: analysis, synthesis, abstraction, formalization, generalization, observation, comparison, experiment; methods of linear algebra, for finding the minors determinant of the redundant structure, singular states of the structure and states maximally distant from singularities; methods of conditional optimization, to determine the optimal states of the structure with the maximum determinant; methods of automatic control theory, mathematical modeling, to verify the operability and efficiency of the developed algorithms; requirements for dynamic positioning systems, theoretical works and practical experience of other authors.

4. Results

The vessel's control scheme with two stern azipods and a bow thruster is shown in Fig. 2.

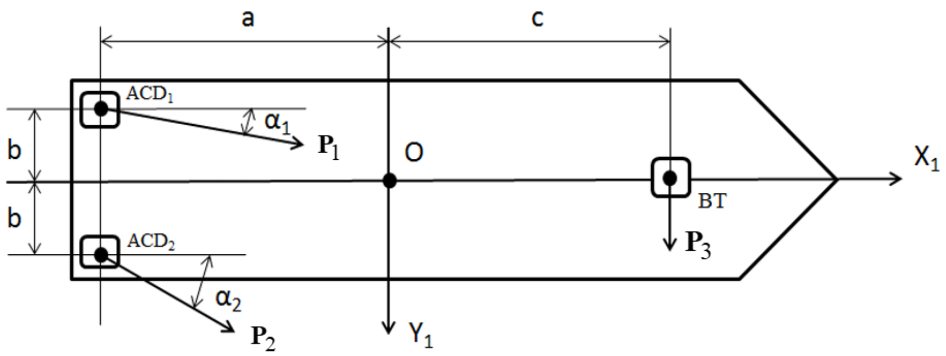


Figure 2: Control scheme for a vessel with two stern azipods and a bow thruster

The figure shows a coupled coordinate system (CCS) $OX_1Y_1Z_1$, the origin of which is located at the rotation center of the vessel. The axis OX_1 lies in the diametrical plane of the vessel, parallel to the deck of the vessel and directed to the bow of the vessel. The axis OY_1 is perpendicular to the diametrical plane of the vessel and directed towards the starboard side. The axis OZ_1 complements the system to the "right". In the projection, Fig. 2, the OZ_1 axis is perpendicular to the image plane and directed away from the observer.The figure also shows two stern azipods ACD_1, ACD_2 and bow thruster BT. The position of the first azipod ACD_1 in the CCS is determined by the coordinates $(-a,-b,0)$; the position of the second azipod ACD_2 in the CCS is determined by the coordinates $(-a,b,0)$; the position of the bow thruster BT in the CCS is determined by the coordinates $(c,0,0)$.

The first azipod ACD_1 creates a propeller thrust force vector $\mathbf{P}_1 = (P_1 \cos\alpha_1, P_1 \sin\alpha_1)$, where P_1 is the modulus and α_1 is the rotation angle of the propeller thrust force. Control constraints are $|P_1| \le P_1^{\max}, |\alpha_1| \le \pi$. The second azipod ACD_2 creates a propeller thrust force vector $\mathbf{P}_2 = (P_2 \cos\alpha_2, P_2 \sin\alpha_2)$, where P_2 is the modulus and α_2 is the rotation angle of the propeller thrust force. Control constraints are $|P_2| \le P_2^{\max}, |\alpha_2| \le \pi$. The bow thruster creates a lateral force $\mathbf{P}_3 = (0, P_3)$, which is limited by the value $|P_3| \le P_3^{\max}$. Let's write the state matrix of the structure from the vectors $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$

$$\mathbf{S} = \begin{bmatrix} P_1 \cos\alpha_1 & P_1 \sin\alpha_1 \\ P_2 \cos\alpha_2 & P_2 \sin\alpha_2 \\ 0 & P_3 \end{bmatrix}. \tag{1}$$

The matrix $\mathbf{S}[3,2]$ is rectangular and its rank is determined by the highest order of the non-zero minor. Let's write the second-order minors of the matrix $\mathbf{S}[3,2]$.

First minor is

$$M_1 = \begin{bmatrix} P_1 \cos\alpha_1 & P_1 \sin\alpha_1 \\ P_2 \cos\alpha_2 & P_2 \sin\alpha_2 \end{bmatrix}, \tag{2}$$

$$\det M_1 = P_1 P_2 \sin\alpha_2 \cos\alpha_1 - P_1 P_2 \sin\alpha_1 \cos\alpha_2 = P_1 P_2 \sin(\alpha_2 - \alpha_1), \tag{3}$$

$$\det M_1 = 0 \rightarrow (\alpha_2 - \alpha_1) = \pm \pi n, n = 0,1,2,.. \tag{4}$$

Second minor is

$$M_2 = \begin{bmatrix} P_1 \cos\alpha_1 & P_1 \sin\alpha_1 \\ 0 & P_3 \end{bmatrix}, \tag{5}$$

$$\det M_2 = P_1 P_3 \cos\alpha_1, \tag{6}$$

$$\det M_2 = 0 \rightarrow \alpha_1 = \frac{\pi}{2} \pm \pi n, n = 0,1,2,.. \tag{7}$$

Third minor is

$$M_3 = \begin{bmatrix} P_2 \cos\alpha_2 & P_2 \sin\alpha_2 \\ 0 & P_3 \end{bmatrix}, \tag{8}$$

$$\det M_3 = P_2 P_3 \cos\alpha_2, \tag{9}$$

$$\det M_3 = 0 \rightarrow \alpha_2 = \frac{\pi}{2} \pm \pi n, n = 0,1,2,.. \tag{10}$$

In the general case, all three determinants of the minors are not equal to zero and the state matrix of the structure has rank $rang(\mathbf{S}) = 2$, i.e. the structure in all cases, except for singular (degenerate) states, can create control in the plane OX_1Y_1. The singular states of the structure are determined by the equation

$$\alpha_1 = \alpha_2 = \frac{\pi}{2} \pm \pi n, n = 0,1,2,.., \tag{11}$$

obtained from equations (4), (7), (10), when all three second-order minors are equal to zero. In singular states, the vectors $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$ are collinear, the rank of the state matrix is $rang(\mathbf{S}) = 1$ and the structure loses the ability to create a control force along the axis OX_1. In practical applications, it is desirable to keep the state of the structure away from the singular, which is achieved by maximizing the determinants of the minors

$$\left|\det M_1\right| = P_1P_2\left|\sin(\alpha_2-\alpha_1)\right| \to \max, (\alpha_2-\alpha_1) = \frac{\pi}{2} \pm \pi n, n = 0,1,2,.., \quad (12)$$

$$\left|\det M_2\right| = P_1P_3\left|\cos\alpha_1\right| \to \max, \alpha_1 = \pm\pi n, n = 0,1,2,..., \quad (13)$$

$$\left|\det M_3\right| = P_2P_3\left|\cos\alpha_2\right| \to \max, \alpha_2 = \pm\pi n, n = 0,1,2,... \quad (14)$$

Conditions (12) – (14) do not have a compatible solution, however, the use of any of them guarantees the maximum distance of the structure from the singular state and the creation of the best controllability of the vessel. This allows to reduce the dynamic delay of the control system and the errors caused by this delay. Also, conditions (12) – (14) can be used to reconfigure the structure in the event of failure of one of the actuators, namely, condition (12) – in the event of failure of the BT, condition (13) – in the event of failure of ACD_2, condition (14) – in the event of failure of ACD_1.

Conditions (12) – (14) are objective functions that maximize the determinant of the structure and ensure the best controllability of the vessel. Also, it is necessary to additionally take into account the constraints of the equalities type that ensure the creation of the necessary control forces and moments by the structure, as well as the constraints of the inequalities type take into account the permissible ranges of changes in the parameters of the structure

$$\begin{cases} Q = \left|\sin(\alpha_2-\alpha_1)\right| \to \max \\ P_1\cos\alpha_1 + P_2\cos\alpha_2 - P_x^* = 0 \\ P_1\sin\alpha_1 + P_2\sin\alpha_2 + P_3 - P_y^* = 0 \\ P_1b\cos\alpha_1 - P_2b\cos\alpha_2 - P_1a\sin\alpha_1 - P_2a\sin\alpha_2 + P_3c - M_z^* = 0 \\ \left|P_1\right| \le P_1^{\max}, \left|\alpha_1\right| \le \pi \\ \left|P_2\right| \le P_2^{\max}, \left|\alpha_2\right| \le \pi \\ \left|P_3\right| \le P_3^{\max} \end{cases}, \quad (15)$$

where - Q is the objective function to be optimized, P_x^*, P_y^*, M_z^* are the longitudinal force, lateral force and yaw moment required to work out deviations in the longitudinal, lateral and angular motion channels, are determined by PID controllers.

The solution of the optimization problem (15) is carried out in the on-board computer using a nonlinear optimization procedure with constraints such as equalities and inequalities.

4.1. Mathematical Modeling

To confirm the operability and effectiveness of the process mathematical model, mathematical modeling was carried out in the MATLAB environment. The main dimensions (Fig. 2) and the characteristics of the mathematical model correspond to the dimensions and characteristics of the offshore vessel ESNAAD-224 and are given in Table 1.

Table 1
The main characteristics of the mathematical model of the vessel

Parameter name	Parameter value
Vessel weight m, kg	$4.02e^6$
Vessel length L, m	70.4
Vessel width B, m	15.77

Vessel draft d, m	4.85
Maximum speed $V_{\max}$, m/s	5.5
The range of change in thrust force of the first P_1 and second P_2 azipod screw, N	$\pm 2.27e^5$
The range of change in the BT thrust force, N	$\pm 0.57e^5$
The range of change in the first α_1 and second α_2 azipod rotation angle, rad	$\pm \pi$
Dimension a, m	26.5
Dimension b, m	5.5
Dimension c, m	26.5

The following were used for modeling: task manager (organization of procedure calls, formation of arrays for plotting graphs); control object model (system of 17 differential equations, including 6 dynamic equations of linear and angular motion of the vessel, 6 kinematic equations of linear and angular motion of the vessel, 5 differential equations that take into account the inertia of the change in the propeller thrust force and rotation angle of the first and second azipods, as well as the bow thruster force); control system model (calculation of control forces and yaw moment necessary to maintain a given position or movement, solution of the conditional optimization problem (15); external influences model; 4th-order Runge-Kut integration method; etc. The MATLAB procedure was used as the conditional optimization procedure

$$f\min con\left(@\,func, \mathbf{x}0, \mathbf{A}, \mathbf{b}, \mathbf{A}_{eq}, \mathbf{b}_{eq}, \mathbf{l}b, \mathbf{u}b, @\,nonlcon\right), \quad (16)$$

where $@\,func$ is a reference to the objective function, $\mathbf{x}0 = \left(P_1(0), \alpha_1(0), P_2(0), \alpha_2(0), P_3(0)\right)$ is the initial state vector of the structure, $\mathbf{A}$ is the matrix of the inequality type linear constraints, in our case absent, $\mathbf{b}$ is the right-hand side vector of the inequality type linear constraints, in our case absent, $\mathbf{A}_{eq}$ is the matrix of the equality type linear constraints, in our case absent, $\mathbf{b}_{eq}$ is the right-hand side vector of the equality type linear constraints, in our case absent, $\mathbf{l}b = \left(-P_1^{\max}, -\pi, -P_2^{\max}, -\pi, -P_3^{\max}\right)$ is the lower values of the control parameters, $\mathbf{u}b = \left(P_1^{\max}, \pi, P_2^{\max}, \pi, P_3^{\max}\right)$ is the upper values of the control parameters, $@\,nonlcon$ is a reference to the file of the equality type nonlinear constraints.

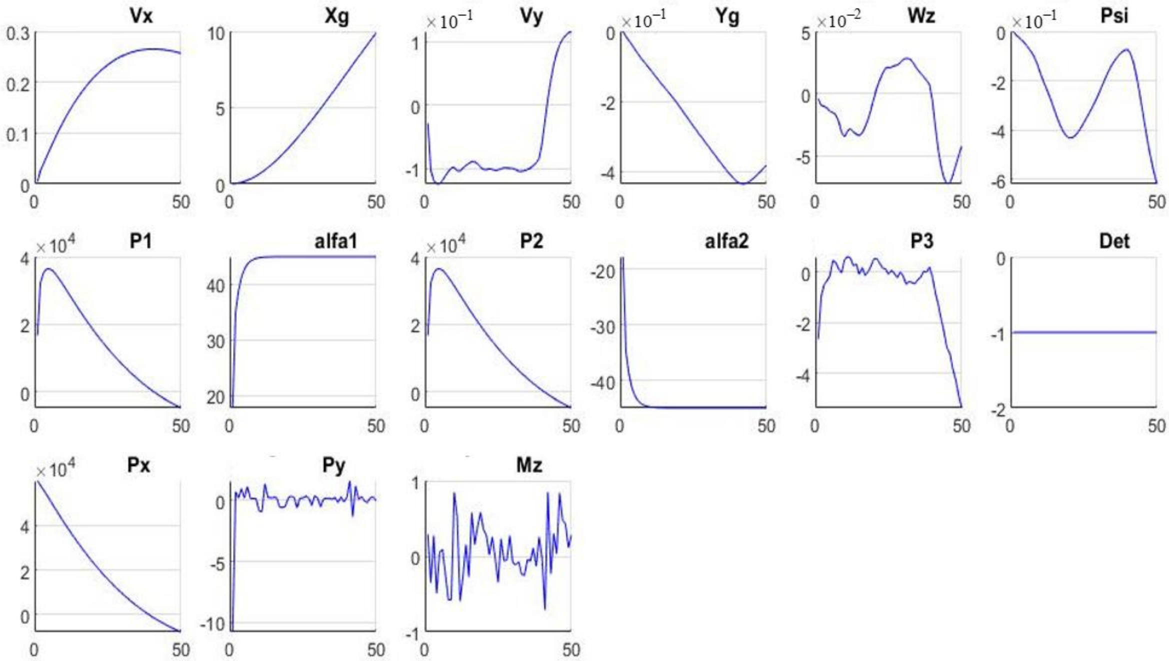
The objective function $Q = \left|\sin(\alpha_2 - \alpha_1)\right|$ has a single maximum, since $0 \le \alpha_2 - \alpha_1 \le \pi$ and optimization process always converges to the global extremum for any initial value $\mathbf{x}0$, for example, $\mathbf{x}0 = \mathbf{0}$. At the next steps of the on-board controller $\mathbf{x}0(n+1) = \mathbf{x}^*(n)$, where $\mathbf{x}^*(n)$ is the optimal state vector from the previous calculation step.

4.1.1. Experiment 1

The purpose of the experiment is to check the possibility of reconfiguring the structure in non-stationary modes of ship motion. Experimental conditions: during the longitudinal movement of the vessel, reconfiguring the structure to a state with the maximum determinant.

The simulation results are shown in Fig. 3 in the form of graphs of changes in time: longitudinal speed V_x [m/s] and longitudinal displacement X_g [m] of the vessel, lateral speed V_y [m/s] and lateral displacement Y_g [m] of the vessel, angular rate ω_z [dg/s] and yaw angle Psi [dg] of the vessel, thrust force modulus P_1 [N] and the rotation angle α_1 [dg] of the first azipod, thrust force modulus P_2 [N] and the rotation angle α_2 [dg] of the second azipod, thrust force modulus P_3 [N] of the bow thruster, determinant of the structure Det, total longitudinal force of the structure P_x [N], total lateral force of the structure P_y [N] and total yaw moment M_z [N·m].

Figure 3: Structural reconfiguration during unsteady vessel motion



As can be seen from the graphs $V_x(t), X_g(t)$, the vessel increases speed and moves in the longitudinal direction. At the same time, the structure is readjusted - the thrust force vector of the first azipod propeller returns from the initial position $\alpha_1(0) = 0^\circ$ to the position $\alpha_1(t) = 45^\circ$, and the thrust force vector of the second azipod propeller returns from the initial position $\alpha_2(0) = 0^\circ$ to the position $\alpha_2(t) = -45^\circ$. At the time of the reconfiguration completion, the angle between the thrust vector of the first azipod and the thrust vector of the second azipod is $\alpha_1 - \alpha_2 = \frac{\pi}{2}$ (the vectors are perpendicular), and the determinant of the structure is $Det(t) = -1$ (the structure is maximally distant from the singular state). During the reconfiguration, the deviations in the lateral motion and angular motion channels are insignificant. Also, the graphs show insignificant changes in the lateral force and the control moment of the structure in the yaw channel . Significant changes in the longitudinal force of the structure are caused by the need to accelerate and brake the vessel during movement.

Conclusion on the experiment 1. The developed process mathematical model allows reconfiguring the redundant structure in non-stationary modes of vessel motion.

4.1.2. Experiment 2

The purpose of the experiment is to determine the dynamic deviations of the vessel's positioning when using a redundant structure tuned to the maximum determinant. Experimental conditions: modeling of dynamic positioning processes in the presence of wind gusts. The results of mathematical modeling are presented in the form of graphs in time, Fig. 4.

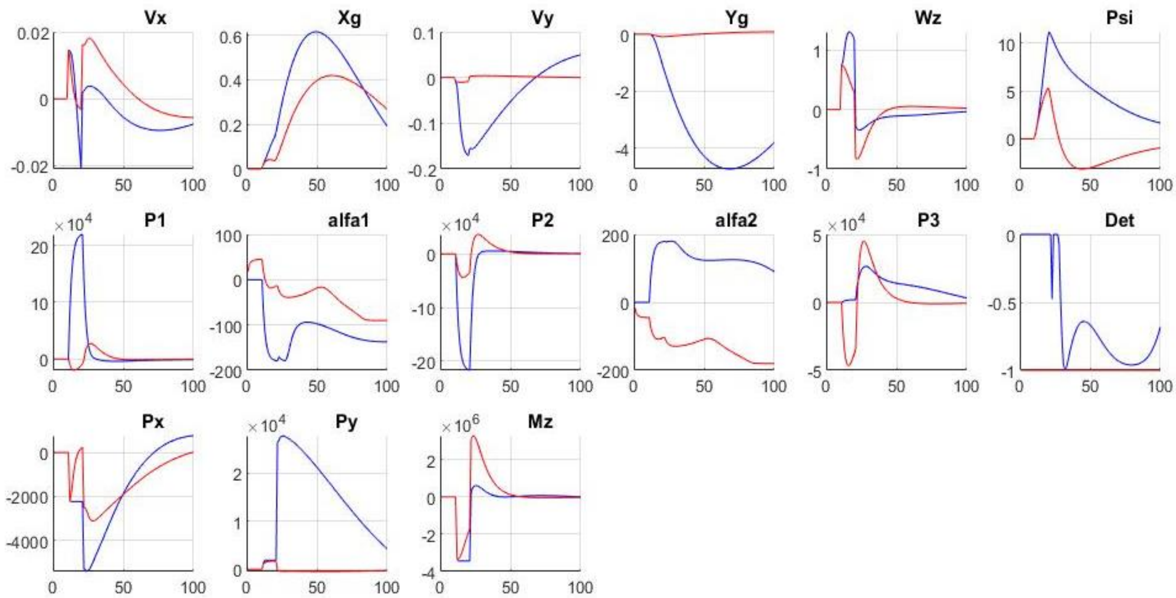


Figure 4: Dynamic positioning faults when using a redundant structure optimized for determinant and minimum energy consumption

The graphs of the processes of optimizing the structure by the determinant are shown in red, and the graphs of the processes of optimizing energy consumption [9] are shown in blue. The initial values of all parameters are zero. The first wind gust with speed $W = 15$ m/s and direction $K_W = 45$ dg is simulated in the time interval $t \in [10, 12]$ s. The second wind gust with speed $W = 15$ m/s and direction $K_W = -45$ dg is simulated in the time interval $t \in [20, 22]$ s.

From the results presented, it is clear that in the time interval of the action of wind gusts $t \in [10, 12]$ s and $t \in [20, 22]$ s dynamic positioning deviations appear in the channels of longitudinal $V_x(t), X_g(t)$, lateral $V_y(t), Y_g(t)$ and angular $\omega_z(t), Psi(t)$ motion, as well as the control forces $P_x(t), P_y(t)$ and moment $M_z(t)$ of the structure to work out these deviations. From the graph $Det(t)$, fig. 4, it is clear that the control of the structure by the determinant constantly maintains the maximum (modulo) value of the determinant, while the objective function of minimum energy consumption allows the value of the determinant to change within $0 \leq |Det(t)| \leq 1$, including the value $Det(t) = 0$.

From the graphs $\alpha_1(t)$, $\alpha_2(t)$, it is also clear that $|\alpha_1(t) - \alpha_2(t)| = \frac{\pi}{2}$, i.e. the control vectors $\mathbf{P}_1, \mathbf{P}_2$ are always orthogonal.

Fig. 5 shows the results of simulation on the phase plane.

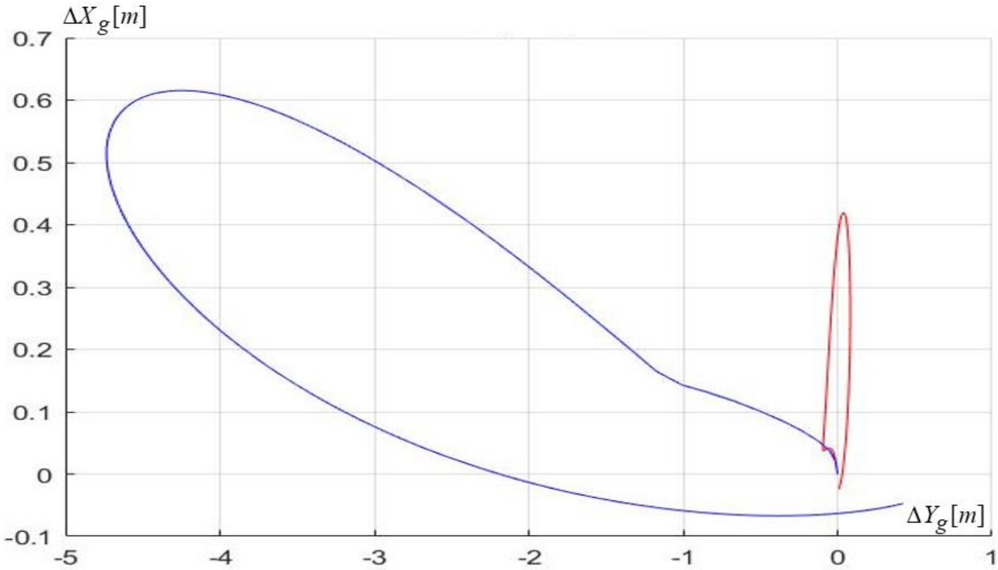


Figure 5: Dynamic positioning deviations on the phase plane

As can be seen from the graphs above, the dynamic faults when setting the structure to the maximum determinant are significantly smaller than the dynamic faults when setting the structure to the minimum energy consumption. The maximum deviations of the parameters under the action of external influences for the considered control laws are summarized in Table 2.

Conclusion on experiment 2. Tuning the redundant structure according to the maximum determinant allows reducing dynamic positioning deviations, compared to the known minimum energy consumption method.

Table 2
Dynamic positioning deviations

Control Law	ΔV_x m/s	ΔX_g m	ΔV_y m/s	ΔY_g m	$\Delta\omega_z$ dg/s	ΔPsi dg
$Q_1 = P_1^2 + P_2^2 + P_3^2 \rightarrow \min$	0.02	0.6	0.15	5.0	1.3	11.0
$Q_2 = \lvert\det M_1\rvert \rightarrow \max$	0.018	0.4	0.01	0.1	0.8	2.5

5. Discussion

A process mathematical model for adjusting the redundant structure of the vessel's actuators has been developed, which allows, during dynamic positioning operations, to reduce dynamic deviations and increase accuracy. The obtained result is explained by: using the on-board controller in the control system, finding, at each step of the on-board controller, the optimal state of the structure, determined by the maximum determinant, by solving the conditional optimization problem, taking into account the constraints of the equalities type (for simultaneous formation of the necessary controls), and the inequalities type (for taking into account the physical constraints of the structure on the thrust force and the azipod's rotation angles); re-adjusting the structure to a certain optimal position. The obtained results differ from known solutions in that they allow adjusting and maintaining the redundant structure in the position with the maximum determinant, which allows reducing the dynamic deviations of the control system. A process mathematical model is designed for use in the on-board controller of an automatic/automated vessel motion control system with redundant actuator structures and cannot be used for manual control, or on vessels without redundant control. The results obtained are reproducible and can be used in the development of automatic/automated vessel motion control systems with two stern azipods and a bow thruster. Further research may be related to the development of structure reconfiguration methods for other actuator structures.

6. Conclusion

An analysis of literary sources was conducted, which considered the issues of optimal control and the use of redundant structures to increase the reliability of control and optimization systems. It was established that the closest technical solutions are implemented in dynamic positioning systems, which allow the use of redundant structures both to increase reliability and to optimize control processes. At the same time, among the known solutions, the authors did not find any that allow the use of redundant structures to reduce dynamic deviations by adjusting and maintaining the structure in a position with the maximum determinant. A process mathematical model for controlling the redundant structure of an offshore vessel with two stern azipods and a bow thruster has been developed, which allows reducing dynamic positioning deviations. The operability and effectiveness have been verified by mathematical modeling in the MATLAB environment. The theoretical value of the results obtained is in the development of a process mathematical model for controlling a redundant structure that optimizes the determinant of the structure and ensures a reduction in dynamic positioning deviations. The practical value of the results obtained is in checking the operability and effectiveness of the process model by mathematical modeling, the possibility of using the process model in the development of automatic / automated control systems for vessels with redundant structures of actuators, to increase the accuracy of dynamic positioning.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

References

[1] Y. Ma, H. Bi, M. Hu, Y. Zheng, L. Gan, Hard sail optimization and energy efficiency enhancement for sail-assisted vessel, Ocean Engineering, 2019, 687–699. doi: 10.1016/j.oceaneng.2019.01.026.

[2] S. Zinchenko, V. Kobets, O. Tovstokoryi, K. Kyrychenko, P. Nosov, I. Popovych, Control of the Pivot Point Position of a Conventional Single-Screw Vessel, CEUR Workshop Proceedings 3513 (2023) 130–140. URL: https://ceur-ws.org/Vol-3513/paper11.pdf.

[3] S. Zinchenko, K. Kyrychenko, O. Grosheva, P. Nosov, I. Popovych, P. Mamenko, Automatic reset of kinetic energy in case of inevitable collision of ships, IEEE Xplore, 2023, 496–500. doi:: 10.1109/ACIT58437.2023.10275545.

[4] S. Zinchenko, O. Tovstokoryi, V. Mateichuk, P. Nosov, I. Popovych, V. Perederyi, Automatic Prevention of the Vessel's Parametric Rolling on the Wave, CEUR Workshop Proceedings 3668 (2024) 235–246. URL: https://ceur-ws.org/Vol-3668/paper16.pdf.

[5] S. Zinchenko, O. Tovstokoryi, K. Kyrychenko, P. Nosov, I. Popovych, Analytical Model of a Ship's Stability on a Regular Wave, in: Babichev, S., Lytvynenko, V. (eds) Lecture Notes in Data Engineering, Computational Intelligence, and Decision-Making, Vol.1, ISDMCI 2024. Lecture Notes on Data Engineering and Communications Technologies 2024, Vol. 219. Springer, Cham. doi: 10.1007/978-3-031-70959-3_3.

[6] S. Zinchenko, O. Tovstokoryi, V. Mateichuk, P. Nosov, I. Popovych, K. Kyrychenko, Application of the conditional optimization method in the problem of vessel stormy sailing, CEUR Workshop Proceedings 3790 (2024) 388–398. URL: https://ceur-ws.org/Vol-3790/paper34.pdf.

[7] M. Luo, S. Shin, Half-century research developments in maritime accidents: Future directions, Accident Analysis & Prevention, 2019, 448–460. doi: 10.1016/j.aap.2016.04.010.

O. Melnyk, Y. Bychkovsky, O. Onishchenko, S. Onyshchenko, Y. Volianska, Development of the method of shipboard operations risk assessment quality evaluation based on experts review, in Studies in Systems, Decision and Control, Springer, 2023, Vol. 481, pp. 695–710. doi: 10.1007/978-3-031-35088-7_40.
[8] S. Zinchenko, V. Kobets, O. Tovstokoryi, P. Nosov, I. Popovych, Intelligent System Control of the Vessel Executive Devices Redundant Structure, CEUR Workshop Proceedings 3403 (2023) 582–594. URL: https://ceur-ws.org/Vol-3403/paper44.pdf.
[9] W. Li, G. Shi, Redundancy management strategy for electro-hydraulic actuators based on intelligent algorithms, Advances in Mechanical Engineering, 2020. doi: 10.1177/1687814020930455.
[10] Ke. Wang, B. Qu, M. Gao, Redundancy Control Strategy for a Dual-Redundancy Steer-by-Wire System, Actuaturs 13 9 (2024) 378. doi: 10.3390/act13090378.
[11] G. Jacobellis, F. Gandhi, M. Floros, Using Control Redundancy for Power and Vibration Reduction on a Coaxial Rotor Helicopter at High Speeds, Journal of the american helicopter society, 2019. doi: 10.4050/JAHS.64.032008.
[12] W. Gao, Q. Tang, J. Yao, Y. Yang, Automatic motion planning for complex welding problems by considering angular redundancy, Robotics and Computer-Integrated Manufacturing, 2020. doi: 10.1016/j.rcim.2019.101862.
[13] X. Cheng, S. Deng, B. Cheng, L. Meiqian, R. Zhou, Optimization of bias current coefficient in the fault-tolerance of active magnetic bearings based on the redundant structure parameters, Automatika, 2020, 602–613. doi: 10.1080/00051144.2020.180.
[14] I. Gritsuk, P. Nosov, A. Bondarchuk, O. Bondarchuk, Using redundant control to optimize control torque, Technology audit and production reserves 3 2 71 (2023) 20–24. doi: 10.1558/2706-5448.2023.282042.
[15] F. Hubert, Dynamic Positioning Systems: Principles, Design and Applications, Editions OPHRYS, 1990.
[16] T. Perez, Dynamic Positioning Marine Manoeuvering, 2017. doi: 10.1002/9781118476406.emoe110.
[17] G. A. Rigatos, Nonlinear optimal control approach for underactuated offshore cranes, Ships and Offshore Structures, 2022. https://doi.org/10.1080/17445302.2022.2150420.
[18] X. Cheng, S. Deng, B. Cheng, L. Meiqian, R. Zhou, Optimization of bias current coefficient in the fault-tolerance of active magnetic bearings based on the redundant structure parameters, Automatika (2020) 602–613. doi: 10.1080/00051144.2020.1806012.
[19] W. Huang, H. Xu, J. Wang, C. Miao, Y. Ren, L. Wang, Redundancy Management for Fault-tolerant Control System of an Unmanned Underwater Vehicle, in Proc. 5th International Conference on Automation, Control and Robotics Engineering (CACRE), China, 19–20 Sept. 2020. doi: 10.1109/CACRE50138.2020.9230038.
[20] W. Li, G. Shi, Redundancy management strategy for electro-hydraulic actuators based on intelligent algorithms. Advances in Mechanical Engineering, 2020. doi: 10.1177/ 1687814020930455.
[21] S. Dakova, J. Wagner, A. Gienger, C. Tarín, M. Böhm, O. Sawodny, Reconfiguration Strategy for Fault-Tolerant Control of High-Rise Adaptive Structures, IEEE Robotics and Automation Letters 6 4 (2021). doi: 10.1109/LRA.2021.3093861.
[22] T. Gulay, V. Zakharov, R. Kron, D. Litvin, S. Popova, Mathematical model of reconfiguration of the aircraft integrated control system on the basis of embedding systems technology, AIP Conf. Proc. 2402 1 (2021) 070052. doi: 10.1063/5.0074028.
[23] R. Huang, S. Tang, Zh. Cai, L. Zhao, Robust Self-Reconfiguration for Fault-Tolerant Control of Modular Aerial Robot Systems, 2025. doi: 10.48550/arXiv.2503.09376.
[24] J. Wen, Ya. Song, H. Wang, D. Han, Ch. Yang, Hybrid Adaptive Control for Tiltrotor Aircraft Flight Control Law Reconfiguration, Aerospace 10 12 (2023) 1001. doi: 10.3390/aerospace10121001.

[25] G. Hu, Ya. Li, X. Li, G. Zhang, Zh. Zhang, X. Wang, W. Man, Modular self-reconfigurable spacecraft: Development status, key technologies, and application rospect, Acta Astronautica 207 (2023) 240−256. doi: 10.1016/j.actaastro.2023.03.012.

[26] J. Tan, ; M. Shang, L. Jin, Metaheuristic-Based RNN for Manipulability Optimization of Redundant Manipulators, IEEE Transactions on Industrial Informatics 20 4 (2024) 6489−6498. doi: 10.1109/TII.2023.3348830.

[27] Y. Liu, X. Chen, L. Wu, G. Huang, J. Luo, Yu. Huang, Global optimization of functional redundancy in a 6R robot for smoothing five-axis milling operations, Engineering Optimization 56 1 (2024) 138−154. doi: 10.1080/0305215X.2022.2147519.